1. The Figure below gives the velocity component $v$ as a function of time for a particle moving in rectilinear motion along an axis. It is part of a sine curve. Point 1 has the maximum value on the curve; point 4 has the minimum value; and points 2 and 6 have the same value.

![Diagram of velocity-time graph]

a) What is the direction of travel at time $t = 0$?

Forward (or positive)

b) What is the direction of travel at point 4?

Backward (or negative)

c) At which of the numbered points does the particle reverse its direction of travel?

3, 5

d) Rank the six points according to the magnitude of the acceleration, greatest first.

$2, 6 > 3, 5 > 1, 4$
3. Person $A$ is moving forward with velocity vector $v$ as shown in the figure (overhead picture). The total acceleration vector of the person may be either of four vectors $a_1, a_2, a_3, a_4$.

![Figure showing acceleration vectors](image)

a) Check the box that correctly explains what is happening to the direction of motion at this moment:

<table>
<thead>
<tr>
<th></th>
<th>Direction remains unchanged</th>
<th>Turning to the person’s left</th>
<th>Turning to the person’s right</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>$\checkmark$</td>
<td></td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$a_3$</td>
<td></td>
<td>$\checkmark$</td>
<td></td>
</tr>
<tr>
<td>$a_4$</td>
<td>$\checkmark$</td>
<td></td>
<td>$\checkmark$</td>
</tr>
</tbody>
</table>

b) Check the box that correctly explains the change of the speed of the person at this moment:

<table>
<thead>
<tr>
<th></th>
<th>constant</th>
<th>Increasing</th>
<th>Decreasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$\checkmark$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td></td>
<td>$\checkmark$</td>
<td></td>
</tr>
<tr>
<td>$a_3$</td>
<td>$\checkmark$</td>
<td></td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$a_4$</td>
<td></td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
</tbody>
</table>

c) Rank the radius of curvature in increasing order (mention any ties and justify your answer)

$2, 1=4, 3$

Since the velocity is the same for all four cases, the normal component of acceleration determines the radius of curvature:

$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n}$$

therefore, larger normal component results in smaller radius of curvature.

d) Rank the rate of change of speed in increasing order (mention any ties and justify your answer)

$3=4, 2, 1$

Speed change is determined by the tangential component of acceleration. In 3 and 4, tangential acceleration is 2 units in the negative direction which make the speed decrease. In 2, there is no tangential acceleration therefore, the speed remains unchanged and in 1, the speed is increasing because of the positive tangential acceleration.
2. The small block is sliding on the inclined wedge as shown. For each of the following scenarios, draw the free-body and kinetic diagrams (FBD/KD) of
i) the block;
ii) the wedge.
Use \( F_k \) for kinetic friction and \( F_s \) for static friction wherever necessary and clearly indicate the directions of forces and accelerations whenever possible. The force of gravity is acting straight down in the Figure.

a) The incline is smooth, the horizontal surface is rough enough to prevent the wedge from moving.

<table>
<thead>
<tr>
<th>FBD Block</th>
<th>KD Block</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="FBD Block" /></td>
<td><img src="image2" alt="KD Block" /></td>
</tr>
</tbody>
</table>

b) The incline is rough, the horizontal surface is smooth.

<table>
<thead>
<tr>
<th>FBD Wedge</th>
<th>KD Wedge</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="FBD Wedge" /></td>
<td><img src="image4" alt="KD Wedge" /></td>
</tr>
</tbody>
</table>

Remarks: We only know directions of \( a_B \) and \( a_{R/B} \); \( a_R \) is determined by these two accelerations (Relative accln. of \( R \) with respect to \( B \))
c) All surfaces are rough, with the friction in the horizontal surface sufficient to prevent the wedge from moving.

\[
\begin{array}{cc}
\text{FBD Wedge} & \text{KD Wedge} \\
\begin{array}{c}
\text{W}_B \\
\text{N}_B \\
\text{F}_{hk}
\end{array} & \begin{array}{c}
\text{m}_B \\
\text{a}_B
\end{array}
\end{array}
\]

\[
\begin{array}{cc}
\text{FBD Block} & \text{KD Block} \\
\begin{array}{c}
\text{W}_R \\
\text{N}_R \\
\text{F}_{hk}
\end{array} & \begin{array}{c}
\text{m}_R \\
\text{a}_R
\end{array}
\end{array}
\]

\[
\begin{array}{cc}
\text{FBD Wedge} & \text{KD Wedge} \\
\begin{array}{c}
\text{W}_B \\
\text{N}_B \\
\text{F}_{hk}
\end{array} & \begin{array}{c}
\text{m}_B \\
\text{a}_B = 0
\end{array}
\end{array}
\]

d) All surfaces are smooth

\[
\begin{array}{cc}
\text{FBD Block} & \text{KD Block} \\
\begin{array}{c}
\text{W}_R \\
\text{N}_R
\end{array} & \begin{array}{c}
\text{m}_R \\
\text{a}_R
\end{array}
\end{array}
\]

\[
\begin{array}{cc}
\text{FBD Wedge} & \text{KD Wedge} \\
\begin{array}{c}
\text{W}_B \\
\text{N}_B \\
\text{F}_{hk}
\end{array} & \begin{array}{c}
\text{m}_B \\
\text{a}_B
\end{array}
\end{array}
\]

Remark: \( \alpha_R \) is not parallel to the incline; it needs to be indicated that \( \alpha_R \) has a vertical component.
a) Determine the speed $v$ of the plane when it has traveled 200 ft.

\[
A = -0.15s + 75
\]

\[
\int_0^s a \, ds = \int_0^v v \, dv
\]

\[
\frac{V^2}{2} - \frac{V_0^2}{2} = \int_0^2 (-0.15s + 75) \, ds
\]

\[
V^2 = V_0^2 + (-0.15s^2 + 150s)
\]

\[
\Rightarrow V = 162.78 \approx 163 \text{ ft/s}
\]

Note: those who calculated the area under curve directly (not integrating the a(s)) and then from that found the correct value for velocity at $s=200$ ft, are given 12 marks for part (a) at once.

b) How much time $t$ is required for it to travel 200 ft?

\[
V = \frac{ds}{dt} \Rightarrow dt = \frac{ds}{V}
\]

\[
V^2 = V_0^2 + 2 \int_0^s (-0.15s + 75) \, ds
\]

\[
V = \sqrt{-0.15s^2 + 150s + 2500}
\]

\[
\int dt = \int_0^{200} \frac{ds}{\sqrt{-0.15s^2 + 150s + 2500}}
\]

\[
t = \frac{1}{\sqrt{0.15}} \left[ \text{erf}^{-1} \left( \frac{0.3(200) - 150}{\sqrt{150^2 + 4(2500)(0.15)}} \right) - \text{erf}^{-1} \left( \frac{-150}{\sqrt{150^2 + 4(2500)(0.15)}} \right) \right]
\]

\[
t \approx 1.802 \approx 1.80 \text{ sec}
\]
5. During action movies, cars often leave the ground for a variety of dramatic reasons. In the scene under examination, the stunt driver has been asked to get the car up to a sufficient speed so that it can go up a 20° incline, launch into the air, and land on a platform located ahead of the car. The relevant dimensions are shown in the sketch. Treat the car as a mass particle,
   a. solve for the minimum speed $v$ that will allow the stunt to be successfully undertaken and
   b. determine the magnitude and direction of the velocity immediately before landing. The direction should be reported as an angle with respect to the horizontal

Solution:

Assume the car will launch at the edge of the platform, then $V_o$ is the minimum initial speed, and $V_f$ is the velocity before landing.

a) In horizontal direction: $V_0 \cos 20° = 79 \text{ ft}$  

b) In vertical direction: $V_0 \sin 20° - \frac{1}{2}gt^2 = 21 - 10 = 11 \text{ ft}$

Substitute $t = \frac{79 \text{ ft}}{V_0 \cos 20°}$ into (2):

$V_0 \sin 20° \frac{79}{V_0 \cos 20°} - \frac{1}{2}g\left(\frac{79}{V_0 \cos 20°}\right)^2 = 11 \text{ ft}$

$\Rightarrow V_0 = \sqrt{\frac{79^2 g}{2(\cos 20°)^2 (79 \tan 20° - 11)}} = 80.1 \text{ ft/s}$
b)

\[ V_{tx} = V_{0x} = V_0 \cos 20^\circ = 80.1 \times \cos 20^\circ = 75.3 \text{ ft/s} \]

\[ t = \frac{79 \text{ ft}}{V_0 \cos 20^\circ} = \frac{79 \text{ ft}}{75.3 \text{ ft/s}} = 1.05 \text{ s} \]

\[ V_{ty} = V_{0y} - gt = V_0 \sin 20^\circ - gt \]

\[ = 80.1 \times \sin 20^\circ - 32.2 \times 1.05 \]

\[ = -6.41 \text{ ft/s} = 6.41 \text{ ft/s} \ (\downarrow) \]

\[ V_t = \sqrt{V_{tx}^2 + V_{ty}^2} = 75.6 \text{ ft/s} \]

\[ \theta = \tan^{-1} \left( \frac{-6.41 \text{ ft/s}}{75.3 \text{ ft/s}} \right) = -4.87^\circ \ (\text{clockwise from horizontal direction}) \]
6. In the figure shown below, all blocks are moving. Block $B$ is moving to the left with a speed of 1 m/s and its speed is decreasing at a rate of 0.5 m/$s^2$. Block $C$ is moving to the right with a speed of 2 m/s and the speed of $C$ is decreasing at a rate of 0.2 m/$s^2$.

Determine the following:

a) The constraint equation relating the velocities and accelerations of the three blocks.

\[
(x_c - x_A) + (x_B - x_A) = \text{constant}
\]

\[
-2u_A + u_B + u_C = 0
\]

\[
-2a_A + a_B + a_C = 0
\]

b) The velocity and acceleration of block $A$, expressed as a magnitude and direction.

\[
u_A = \frac{1}{2} (u_B + u_C) = \frac{1}{2} (-1 + 2) = +0.5 \, \frac{m}{s} = 0.5 \, \frac{m}{s}
\]

\[
a_A = \frac{1}{2} (a_B + a_C) = \frac{1}{2} (+0.5 - 0.2) = +0.15 \, \frac{m}{s^2} = 0.15 \, \frac{m}{s^2}
\]

c) The relative velocity and acceleration of block $B$ with respect to block $C$, expressed as a magnitude and direction.

\[
\begin{align*}
u_{B.C} &= u_C + u_{B.C} ; \\
u_{B.C} &= u_B - u_C = -1 - 2 = -3 \\
\end{align*}
\]

\[
\begin{align*}
a_{B.C} &= a_C + a_{B.C} ; \\
a_{B.C} &= a_B - a_C = 0.5 - (-0.2) = +0.7 \\
a_{B.C} &= +0.7 \, \frac{m}{s^2}
\end{align*}
\]