Faculty of Engineering and Department of Physics

Engineering Physics 131

Final Examination

Saturday April 21, 2018; 14:00 pm – 16:30 pm

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1. Closed book exam. No notes or textbooks allowed.
2. Formula sheets are included (may be removed).
3. The exam has 9 problems and is out of **61.5 marks**. Attempt all parts of all problems.
4. *Questions 1 to 4 do not require detailed calculations and only the final answers to these questions will be marked.*
5. *For Questions 5 to 9, details and procedures to solve these problems will be marked.* Show all work in a neat and logical manner.
6. Write your solution directly on the pages with the questions. Indicate clearly if you use the backs of pages for material to be marked.
7. Only non-programmable calculator approved by the Faculty of Engineering permitted. Turn off all cell-phones, laptops, etc.

DO NOT separate the pages of the exam containing the problems.

LAST NAME: ______________________________

FIRST NAME: ______________________________

ID#: _______________

Please circle the name of your instructor:

EB01: Wheelock	**EB02: Jung**
EB03: Wang	**EB04: Kim**
EB05: Gingrich	**EB06: Tang**

Address all inquiries to a supervisor. Do not communicate with other candidates. If you become ill during the exam, contact a supervisor immediately. (You may not claim extenuating circumstances and request your paper to be cancelled after writing and handing in your examination.) You may not leave the exam until at least 30 minutes have elapsed

End of Exam: When the signal is given to end the exam, students must promptly cease writing. If a student does not stop at the signal, the instructor has the discretion either not to grade the exam paper or to lower the grade on the examination.

Exam Collection Procedure
•If you finish early, stay in your seat and raise your hand
•Someone will come to collect your exam (after which you may quietly leave)
•**In the final 10 minutes: please remain seated until ALL exams have been collected.**

Please do not write in the table below.

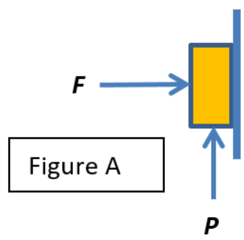
Question	**Value (marks)**	**Mark**
1	4.5	
2	4	
3	4	
4	4	
5	10	
6	10	
7	9	
8	8	
9	8	
Total	61.5	

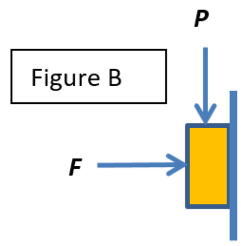
Q1. [4.5 marks]

A block **weighing** 22 N is held at rest against a vertical wall by a horizontal force ***F*** of magnitude 60 N. The coefficient of static friction between the wall and the block is 0.55, and the coefficient of kinetic friction between them is 0.38. A second force ***P*** is applied to the block and directed parallel to the wall with the magnitudes and directions shown in the figures and table below.

For each value of P in the table below, determine: the magnitude of the frictional force; whether the block moves up, down or remains stationary; and whether the frictional force (between block and wall) is directed downward. **Write your answers in the table.**

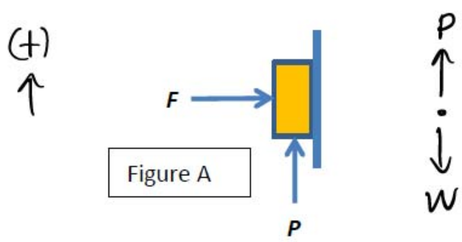
[0.5 marks for each answer, no partial marks]

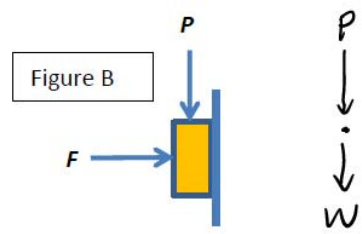




Force ***P*** (N)	Magnitude of frictional force (N)	Block moves (no/up/down)	Frictional force down the wall (yes/no)
12, up (figure A)			
62, up (figure A)			
10, down (figure B)			

Solution





$N = F = 60\,N$

$w = mg = 22\,N$

Force ***P*** (N)	Magnitude of frictional force (N)	Block moves (no/up/down)	Frictional force down the wall (yes/no)
12, up (figure A)	10	NO	NO
62, up (figure A)	22.8	UP	YES
10, down (figure B)	32	NO	NO

$f_s \le \mu_s N = (0.55)(60) = 33\,N$

$f_K \le \mu_K N = (0.38)(60) = 22.8\,N$

$P + W$: $12 - 22 = -10$; $f_s = +10$ UP ; NO MOVEMENT

$62 - 22 = 40 > f_s \Rightarrow$ MOVES UP

$> f_K = -22.8$ DOWN

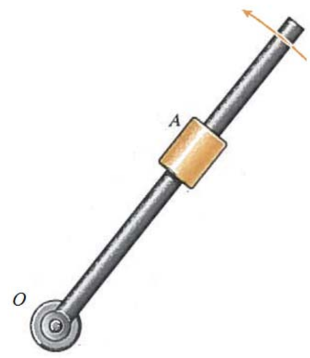
$-10 - 22 = -32$; $f_s = +32$ UP ; NO MOVEMENT

Q2. [4 marks]

For each of the scenarios below, circle ALL the forces that could contribute to the normal acceleration a_n of the specified particle. ***Please Note**: "Friction" may be static or kinetic, "Normal force" is the support force on the specified particle from the surface it is in contact with.*

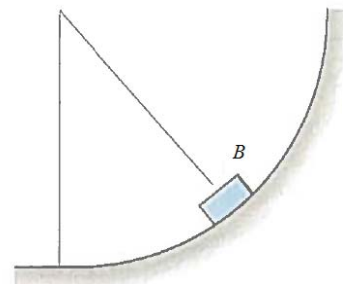
[*1 mark for each scenario, no partial marks, all correct answers must be selected in order to receive the 1 mark.*]

(a) A **rough** bar rotates **in the vertical plane** about point *O*. Collar *A* moves together with the bar without relative motion. Consider collar *A* as the particle and it is at the position shown.



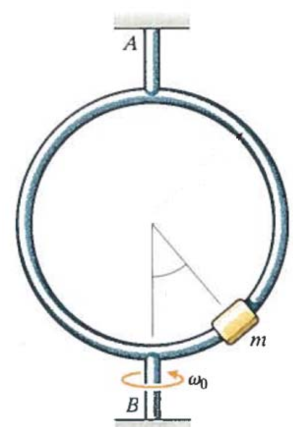
Normal force *Friction* *Gravity*

(b) Block *B* is sliding down the **rough** circular track, which is **in the vertical plane**. Consider block *B* as the particle and it is at the position shown.



Normal force *Friction* *Gravity*

(c) The **rough** circular bar rotates **about the vertical axis** *AB*. The mass *m* remains **stationary relative to the circular bar**. Consider mass *m* as the particle and it is at the position shown.



Normal force *Friction* *Gravity*

(d) A motorcycle (the particle considered here) moves along a circular track on a **flat rough** surface.

Normal force *Friction* *Gravity*

Solution:

1) Normal force **Friction** **Gravity**

2) **Normal force** Friction **Gravity**

3) **Normal force** **Friction** Gravity

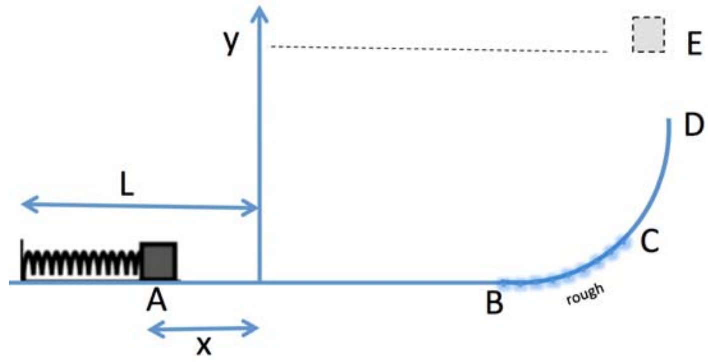
4) Normal force **Friction** Gravity

Q3. [4 marks]

A block is held at rest at location A against a massless spring that is initially compressed a distance x relative to its resting length L. When the spring is released, the block slides to the right, up a circular ramp, then exits the ramp vertically at D, and finally reaches its maximum elevation at location E. Assume that all surfaces are smooth except the rough portion of the ramp between B and C as shown in the diagram. Neglect air resistance.

As the block moves along each segment of its path, consider whether the <u>work</u> done by each force is <u>positive, negative or zero</u>. In the table below, circle the correct answer in each box.

No partial credit; for each row, all answers must be correct to receive credit.

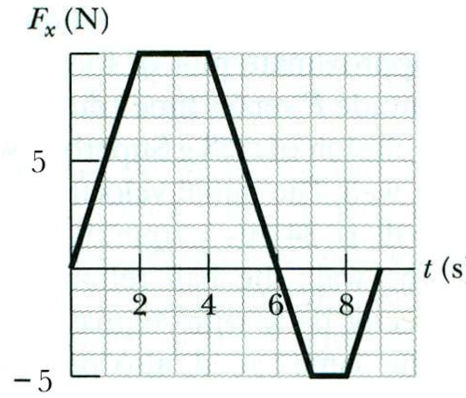


Segment	Gravitational force	Friction	Spring force	Normal force
(a) [1 mark] From A to B	+ **0** –	+ **0** –	**+** 0 –	+ **0** –
(b) [1 mark] From B to D	+ 0 **–**	+ 0 **–**	+ **0** –	+ **0** –
(c) [1 mark] From D to E	+ 0 **–**	+ **0** –	+ **0** –	+ **0** –

(d) [1 mark] Now consider the total <u>work done by all forces</u> acting on the block, as it moves from its initial position at A to location E. Is the total work positive, negative or zero? Circle the correct entry below.

Segment	Total Work
From A to E	+ **0** –

Q4. [4 marks]



At time $t = 0$, a 5-kg toy car moves in the x-y plane at a velocity given by $\mathbf{v} = -5\ \mathbf{i} + 2\ \mathbf{j}$ m/s. The graph gives the component F_x of the total force acting on the toy car. The component F_y is equal to zero.

Please include appropriate units. No partial marks. Only the final answers will be marked; detailed calculations are not required.

(a) [1 mark] What are the x and y components of the impulse on this toy car between $t = 0$ and 6 s?

I_x = ________________ I_y = ________________

(b) [1 mark] What are the x and y components of the impulse on this toy car between $t = 0$ and 9 s?

I_x = ________________ I_y = ________________

(c) [1 mark] What is the toy car's velocity at $t = 6$ s? (Give the velocity's magnitude and angle measured from the positive **i** direction counterclockwise.)

magnitude: ___________________ angle: ___________________

(d) [1 mark] What is the toy car's velocity at $t = 9$ s? (Give the velocity's magnitude and angle measured from the positive **i** direction counterclockwise.)

magnitude: ___________________ angle: ___________________

Solution

A. The x-component of the impulse is $I_{x,6} = \int_0^6 F_x(t)\,dt = \frac{1}{2}(2)(10) + 2(10) + \frac{1}{2}(2)(10) = \boxed{40\ \mathrm{N\cdot s}}$ and $\boxed{I_{y,6} = 0\ \mathrm{N\cdot s}}$

B. The x-component of the impulse is $I_{x,9} = \int_0^9 F(t)\,dt = I_6 + \frac{1}{2}(-5) + (-5) + \frac{1}{2}(-5) = \boxed{30\ \mathrm{N\cdot s}}$ and $\boxed{I_{y,9} = 0\ \mathrm{N\cdot s}}$

C. $I_y = 0$ implies $v_y = 2.0$ m/s at all times. For v_x, we use the principle of linear momentum and impulse to obtain

$$v_{2x} = v_{1x} + \frac{1}{m}\int \sum F_x(t)\,dt = \begin{cases} -5 + \frac{40}{5} = 3\ \text{m/s at 6 s,} \\ -5 + \frac{30}{5} = 1\ \text{m/s at 9 s.} \end{cases}$$

At $t = 6$ s, $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$ m/s so $\boxed{v = 3.6\ \text{m/s at } 34^\circ \text{ above } -x \text{ axis}}$

At $t = 9$, $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$ m/s so $\boxed{v = 2.2\ \text{m/s at } 63^\circ \text{ above } -x \text{ axis}}$

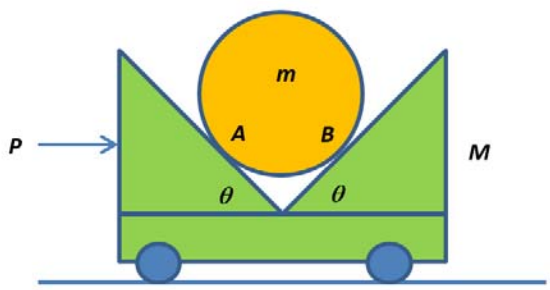
Q5. [10 marks]

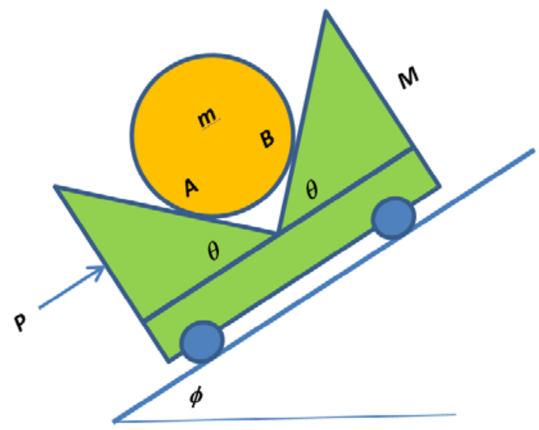
Consider the motion of a cart as shown. A uniform cylinder rests on the cart and remains stationary relative to the cart during the motion.

(a) [5 marks] Consider the figure on the left below. For a given horizontal force ***P***, determine the normal reaction forces at *A* and *B*. The angle $\theta = 60^o$. The mass of the cylinder is *m* and that of the cart is *M*. Neglect all friction.

(b) [5 marks] The system is now placed on the $\phi = 15^o$ incline as shown in the figure on the right below. What force ***P*** will cause the normal reaction force at *B* to be zero?

Express your answers in terms of P, M, m and g (gravitational acceleration). **Box your answer.**





Solution

$\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = \frac{1}{2}$

(A) FOR SYSTEM: $P = (M+m)a \Rightarrow a = \frac{P}{M+m}$

FOR CYLINDER $\sum_i \vec{F}_m = m\vec{a}$

x: $N_A \sin\theta - N_B \sin\theta = ma = \frac{m}{M+m}P \Rightarrow N_A - N_B = \frac{2}{\sqrt{3}}\frac{m}{M+m}P$ (1)

y: $N_A \cos\theta + N_B \cos\theta - mg = 0 \Rightarrow N_A + N_B = 2mg$ (2)

(1)+(2): $N_A = m\left[g + \frac{P}{\sqrt{3}(M+m)}\right]$

(2)−(1): $N_B = m\left[g - \frac{P}{\sqrt{3}(M+m)}\right]$

(B) FOR SYSTEM: $P - \sin\phi(M+m)g = (M+m)a \Rightarrow a = \frac{P}{M+m} - \sin\phi\, g$

FOR CYLINDER $\sum_i \vec{F}_m = m\vec{a}$

x: $N_A \sin\theta - N_B \sin\theta - mg\sin\phi = ma = \frac{m}{M+m}P - mg\sin\phi$

$N_A - N_B = \frac{2}{\sqrt{3}}\frac{m}{M+m}P$ (1)

y: $N_A \cos\theta + N_B \cos\theta - mg\cos\phi = 0 \Rightarrow N_A + N_B = 2mg\cos\phi$

(2)−(1): $N_B = mg\cos\phi - \frac{m}{\sqrt{3}(M+m)}P$

FOR $N_B = 0$, $P = \sqrt{3}\cos\phi(M+m)g$

$\cos(15^o) = \cos(60^o)\cos(45^o) + \sin(60^o)\sin(45^o)$

$= \frac{1}{2}\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$

$\therefore P = \frac{1}{2}\sqrt{\frac{3}{2}}(1+\sqrt{3})(M+m)g$

$P = 1.673(M+m)g$

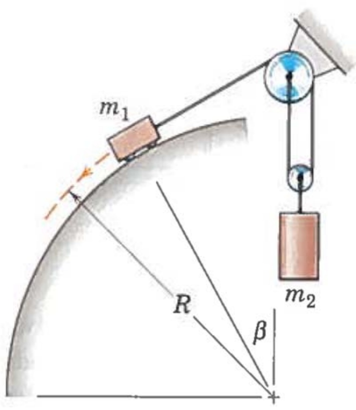
Q6. [10 marks]

At the instant shown, the cable attached to the cart of mass $m_1 = 0.4\,\text{kg}$ is tangent to the **smooth** circular path of the cart. The cylinder has mass $m_2 = 0.6\,\text{kg}$ and the mass of the pulley is negligible. $R = 1.75\,\text{m}$ and $\beta = 30^o$.

(a) [5 marks] Determine the acceleration of m_2 and the tension T in the cable.

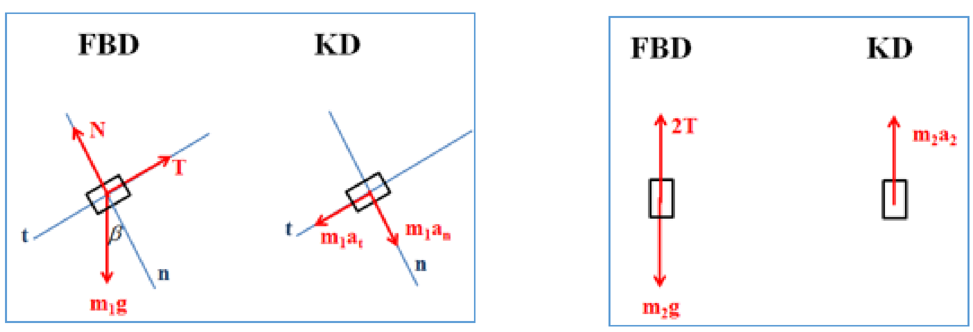
(b) [5 marks] At this instance what would be the maximum speed v_2 of m_2 for which m_1 remains in contact with the surface?

Box your answer.



Solution:

1) The FBD/KD are shown below for the cart (left) and the cylinder (right).



For the cart: $+]$ $\quad m_1 g\cos\beta - N = m_1 a_n = m_1 \dfrac{v_1^2}{R}$ (1)

$+[$ $\quad m_1 g\sin\beta - T = m_1 a_t$ (2)

For the cylinder: $+\uparrow$ $\quad 2T - m_2 g = m_2 a_2$ (3)

From the constraint of the pulley: $v_1 = 2v_2$, $\quad a_t = 2a_2$ (4)

Solving (1)-(4) gives:

$$N = m_1\left(g\cos\beta - \frac{4v_2^2}{R}\right), \qquad a_2 = \left(\frac{2m_1\sin\beta - m_2}{4m_1 + m_2}\right)g, \qquad T = \frac{m_1 m_2 g(\sin\beta + 2)}{4m_1 + m_2}$$

Substituting in the values of m_1, m_2, β, we get:

$T = 2.68$ N and $a_2 = -0.892$ m/s^2, i.e., 0.892 m/s^2 downward.

2) When the cart loses contact with the surface, $N = m_1\left(g\cos\beta - \dfrac{4v_2^2}{R}\right) = 0$, i.e.,

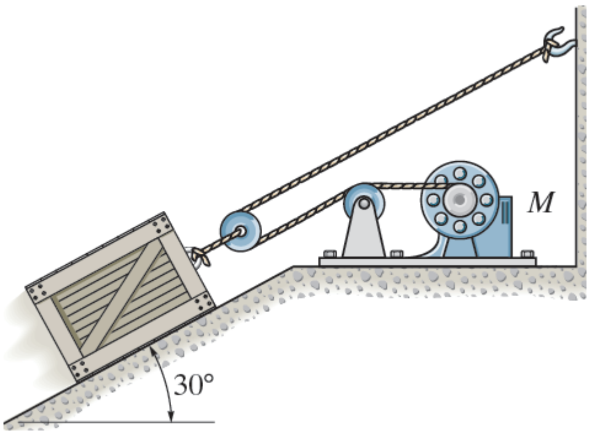
$$v_2 = \frac{\sqrt{gR\cos\beta}}{2} = 1.93 \text{ m/s}$$ is the maximum speed of m_2.

Q7. [9 marks]

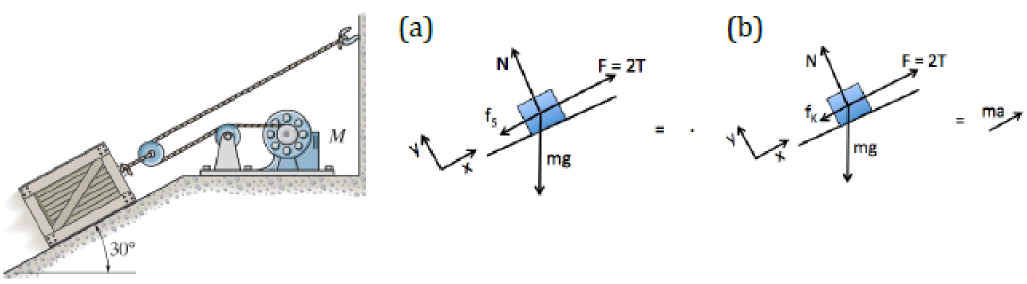
A 50-kg crate is initially at rest on an incline. Starting at $t = 0$, the motor supplies a rope tension of $T = 10t^2$ *N*, where t is in seconds. The coefficients of static and kinetic friction between the crate and the incline are 0.6 and 0.4 respectively. You may assume that the pulleys are massless and frictionless.

(a) [3 marks] Find the time t_1 when the crate begins to move.
(b) [4 marks] What is the speed of the crate when $t = 6$ *s*?
(c) [2 marks] If the power input to the motor is 6000 W when $t = 6$ *s*, what is the efficiency at this instant?

Box your answer.



Solution



(a) When the crate is on the verge of sliding (impending motion), $f_s = f_{s,max}$:

$$2T = 2(10t^2) = mg\sin\theta + \mu_s mg\cos\theta$$

$$t_1 = \sqrt{\left(\frac{(50)(9.81)}{20}\right)(\sin 30° + (0.6)\cos 30°)} = 5.00\ s$$

(b) Method 1:
For t > 5 s, the equation of motion is:

$$2T - mg\sin\theta - \mu_k mg\cos\theta = ma = m\frac{dv}{dt}$$

$$\int_0^v dv = \int_5^6 \frac{2(10t^2)}{m}dt - \int_5^6 g(\sin\theta + \mu_k\cos\theta)\,dt$$

$$v = \left(\frac{20}{50}\right)\left(\frac{6^3}{3} - \frac{5^3}{3}\right) - [(9.81)(\sin 30° + (0.4)\cos 30°)]\,(6-5)$$

$$v = 3.83\ m/s$$

Method 2: Start with the principle of linear impulse and momentum:

$$mv_2 - mv_1 = \sum\int_{t_1}^{t_2} F\,dt = \int_{t_1}^{t_2}(2T)dt - mg\,(\sin\theta - \mu_k\cos\theta)\,\Delta t$$

Taking $v_1 = 0$, $t_1 = 5$ s, $t_2 = 6$ s and $T = 10t^2$, this leads to the same result as shown above in method 1.

(c) When t = 6 s, v = 3.83 m/s.

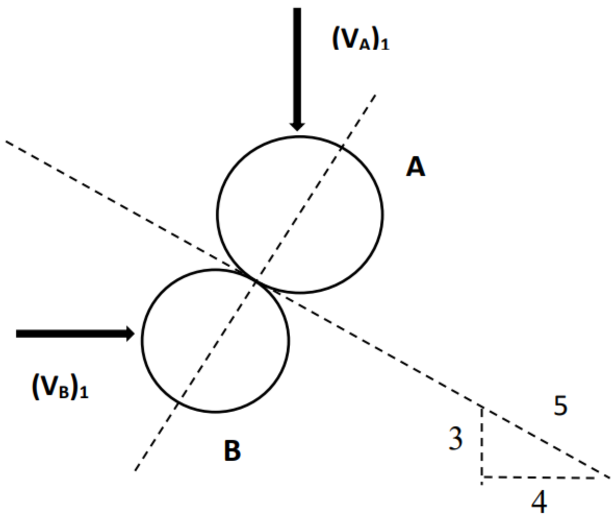
$$P_{out} = F \cdot v = 2Tv = 2(10t^2)v = (2)(10)(6^2)(3.83) = 2758\ W$$

$$\mathcal{E} = \frac{P_{out}}{P_{in}} = \frac{2758}{6000} = 0.46$$

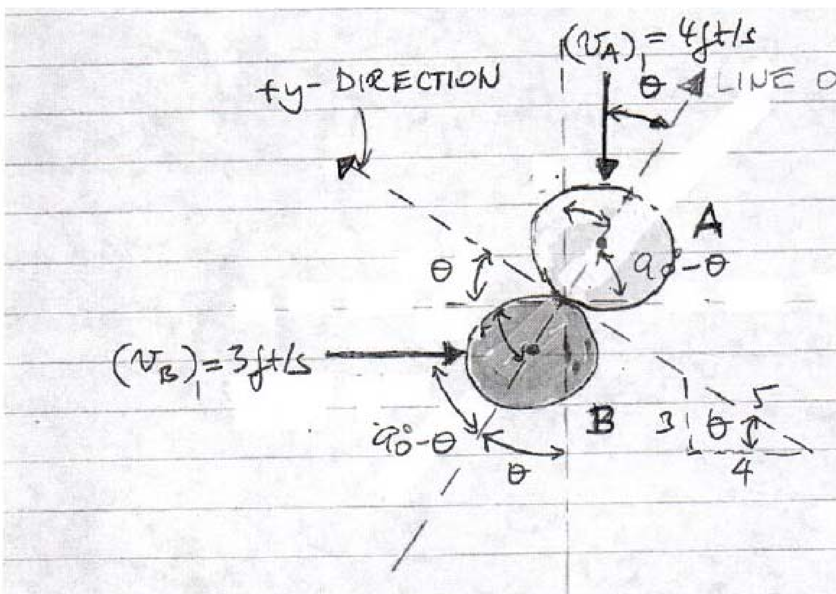
Q8. [8 marks]

Two disks **A** and **B** each have weight of 2 lb and slide on a smooth horizontal surface (with no friction). They have initial speeds $(v_A)_1 = 4$ ft/s and $(v_B)_1 = 3$ ft/s just before they collide. The initial velocity vectors are shown in the figure below. If e = 0.5 determine their velocities (magnitudes and directions) just after impact.

Box your answer.



Solution



$\sin\theta = \frac{3}{5} \Rightarrow$

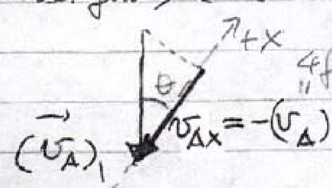
$\theta = 36.87°$

SOLUTION:

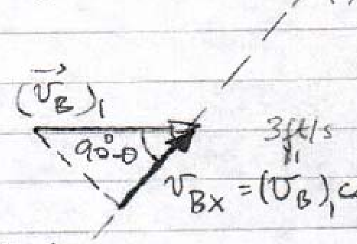
(1) SYSTEM'S MOMENTUM IS CONSERVED ALONG THE "LINE OF IMPACT"

$m_A(+\vec{v}_{Ax})_1 + m_B(+\vec{v}_{Bx})_1 = m_A(\vec{v}_{Ax})_2 + m_B(\vec{v}_{Bx})_2$

$\left(\frac{2\,lb}{32.2\,ft/s^2}\right)\left(-(4\,ft/s)\cos 36.87°\right) + \left(\frac{2\,lb}{32.2\,ft/s^2}\right)\left(+(3\,ft/s)\cos 53.13°\right) =$



$v_{Ax} = -(v_A)_1\cos(36.87°)$



$v_{Bx} = (v_B)_1\cos(53.13°)$

$= \left(\frac{2\,lb}{32.2\,ft/s^2}\right)(v_{Ax})_2 + \left(\frac{2\,lb}{32.2\,ft/s^2}\right)(v_{Bx})_2$

$$-3.20 + 1.80 = (v_{Ax})_2 + (v_{Bx})_2$$

$$\boxed{-1.40 = (v_{Ax})_2 + (v_{Bx})_2} \qquad (1)$$

COEFFICIENT OF RESTITUTION (ALONG THE LINE OF IMPACT)

(+) ↗ x

$$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$

$$0.5 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{[(-4\cos 36.87°) - (+3\cos 53.13°)]}$$

$$0.5 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{[-3.20 - 1.80]}$$

$$\boxed{-2.50 = (v_{Bx})_2 - (v_{Ax})_2} \qquad (2)$$

MATH : (1) + (2) ⇒

$$-1.40 - 2.50 = 2(v_{Bx})_2 \Rightarrow$$

$$\begin{cases} (v_{Bx})_2 = -1.95 \text{ ft/s} & \swarrow -x \\ (v_{Ax})_2 = +0.55 \text{ ft/s} & \nearrow +x \end{cases}$$

(2) PERPENDICULAR TO THE LINE OF IMPACT :

$$m_A (v_{Ay})_1 = m_A (v_{Ay})_2 \Rightarrow (v_{Ay})_1 = (v_{Ay})_2$$

$$\& \; m_B (v_{By})_1 = m_B (v_{By})_2 \Rightarrow (v_{By})_1 = (v_{By})_2$$

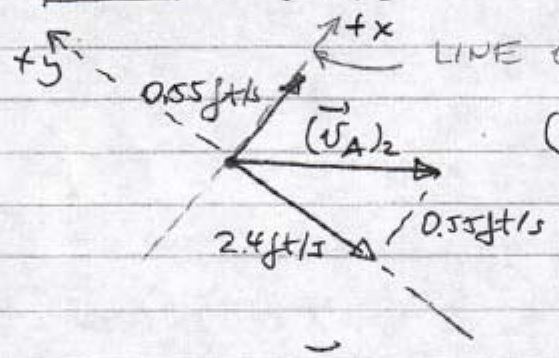
COMPONENTS OF VELOCITIES ⊥ TO LINE OF IMPACT ARE UNCHANGED (NO FORCES / IMPULSES ACT IN THIS DIRECTION)

(+)y

$$(v_{Ay})_2 = (v_{Ay})_1 = -(4\,ft/s)\sin(36.87^\circ)$$
$$= -2.40\,ft/s$$

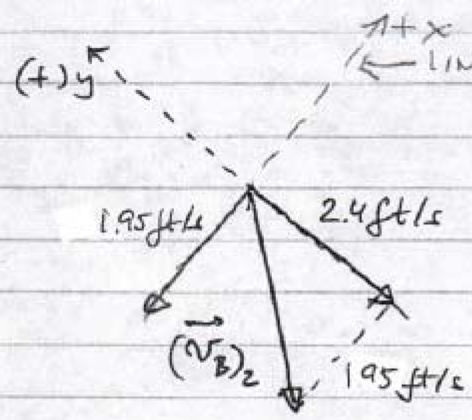
$$(v_{By})_2 = (v_{By})_1 = -(3\,ft/s)\sin(53.13^\circ)$$
$$= -2.40\,ft/s$$

VELOCITY $(\vec{v}_A)_2$:



$$(v_A)_2 = \sqrt{(0.55)^2 + (2.40)^2} =$$
$$(v_A)_2 = 2.462\,ft/s$$

VELOCITY $(\vec{v}_B)_2$:



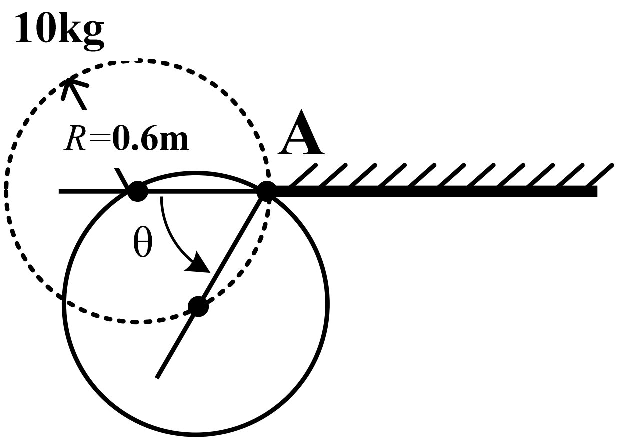
$$(v_B)_2 = \sqrt{(1.95)^2 + (2.40)^2}$$
$$(v_B)_2 = 3.09\,ft/s$$

Q9. [8 marks]

A ***uniform solid sphere***, of mass 10kg and radius 0.6m, is initially held so that its centre of mass being at the same height as point **A**, as shown by the dotted line in the figure ($\theta=0^0$). The sphere is then released from rest, allowing it to rotate downward about the frictionless hinge at **A**.

(a) [4 marks] Determine the angular velocity of the sphere when $\theta=60^0$.
(b) [4 marks] Determine the angular acceleration of the sphere when $\theta=60^0$.

Box your answer.



a) Solution

$I_G = \frac{2}{5}mR^2 = \frac{2}{5} \times 10 \times 0.6^2 = 1.44\ kgm^2$

$I_A = I_G + md^2 = 1.44 + 10 \times 0.6^2 = 5.04kgm^2$

Conservation of Energy

$mgh_1 + \frac{1}{2}I_A w_1^2 = mgh_2 + \frac{1}{2}I_A w_2^2 \rightarrow mgh_1 = \frac{1}{2}I_A w_2^2$

$h_1 = 0.6sin60^o = 0.52\ m$

$10 \times 9.81 \times 0.52 = \frac{1}{2}(5.04)w_2^2$ Therefore

$w_2 = \sqrt{\frac{98.1 \times 0.52 \times 2}{5.04}} = 4.5\frac{rad}{s}$

b) Solution

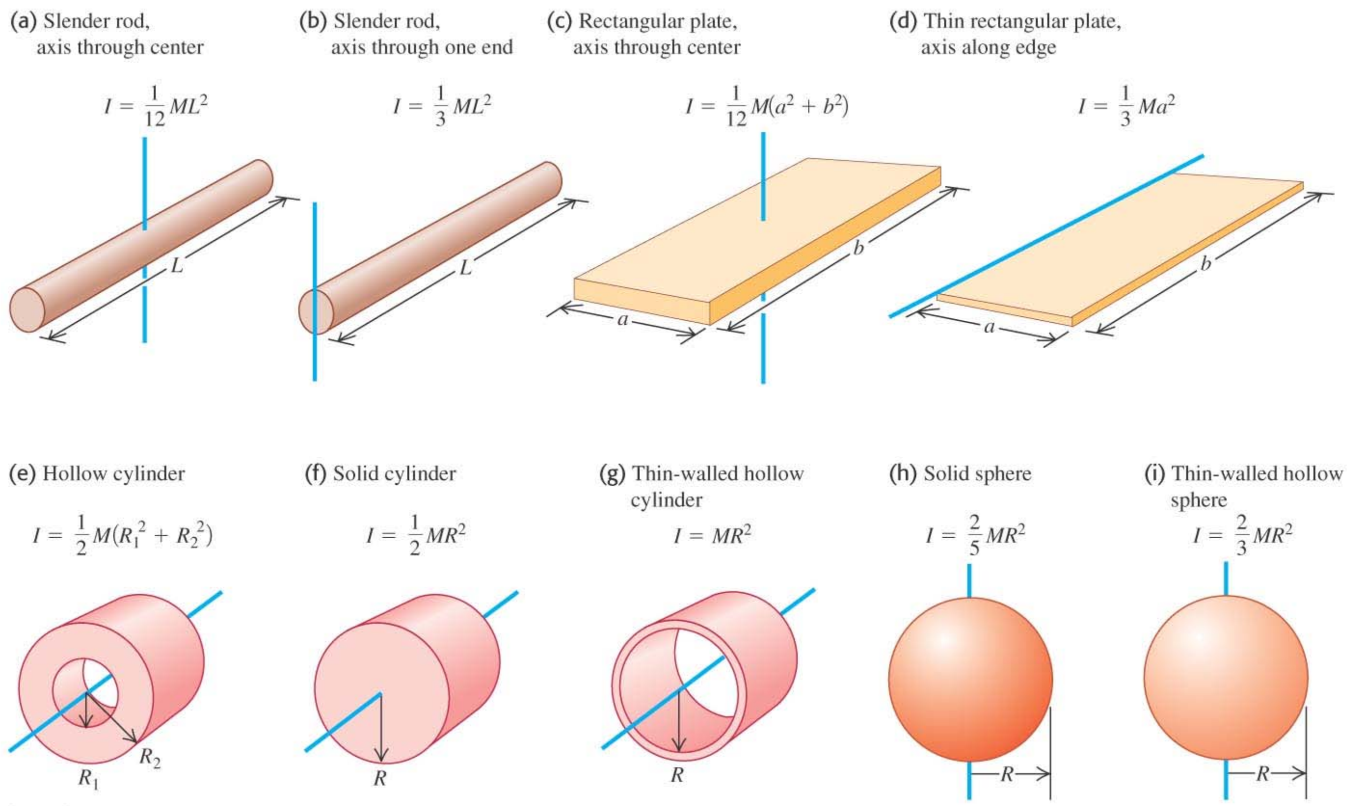
$\sum M = I_A \alpha$

$\sum M = mgcos60^o \times R = 10 \times 9.81 \times cos60^o \times 0.6 = 29.43\ Nm$ Therefore

$49.05 = 5.04\alpha$

$\alpha = \frac{29.43}{5.04} = 5.84\frac{rad}{s^2}$

Table of Moment of Inertia



Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \frac{dv}{dt}$	$v = v_0 + a_c t$
$v = \frac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$	$a_t = \dot{v} = v\frac{dv}{ds}$
	$a_n = \frac{v^2}{\rho} \quad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{\lvert d^2y/dx^2 \rvert}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant* $\alpha = \alpha_c$
$\alpha = \frac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \frac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia $I = \int r^2\,dm$

Parallel-Axis Theorem $I = I_G + md^2$

Radius of Gyration $k = \sqrt{\frac{I}{m}}$

Equations of Motion

Particle	$\Sigma\mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \frac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \quad \epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\int\mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\int\mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\int\mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\int\mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\int\mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$