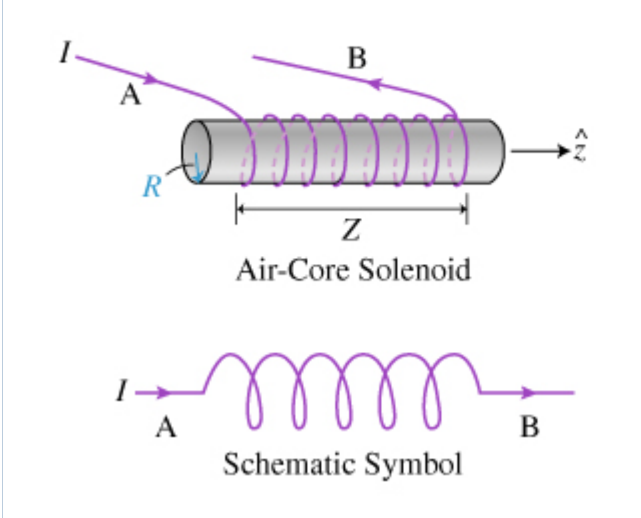
Description: Walks through calculation of self-inductance for a single solenoid with some discussion at the end. (version for algebra-based courses)

Learning Goal:
To better understand self-inductance, using the example of a long solenoid.

To understand self-inductance, it is helpful to consider the specific example of a long solenoid, as shown in the figure. This solenoid has radius R and length Z along the *z* axis, and is wound with n turns per unit length, so that the total number of turns is equal to nZ. Assume that the length of the solenoid is much greater than its radius.



As the current through the solenoid changes, the resulting magnetic flux through the solenoid will also change, and an electromotive force will be generated across the solenoid according to Faraday's law of induction:

$$\mathcal{E} = -\frac{\Delta\Phi_{\rm total}}{\Delta t},$$

where $\Phi_{\rm total}$ is the *total* magnetic flux passing through the solenoid.

The self-inductance L is defined to be $L = \Phi_{\rm total}/I$, where I is the current passing through the solenoid. Using the self-inductance, Faraday's law can be rewritten as $\mathcal{E} = -L\Delta I/\Delta t$.
The direction of the emf can be determined using Lenz's law: The induced emf always opposes any change in the current I.

Part A

Within the solenoid, but far from its ends, what is the magnetic field B due to the current I?

Express your answer in terms of some or all of the following variables I, n, Z and any relevant constants (such as μ_0).

ANSWER:

$B = \mu_0 n I$

Note that this field is independent of the radial position (the distance from the axis of symmetry) as long as it is measured at a point far from the ends of the solenoid.

Part B

What is the magnetic flux Φ_1 through a single turn of the solenoid?

Express your answer in terms of the magnetic field B, quantities given in the introduction, and any needed constants.

ANSWER:

Φ_1 = $B\pi R^2$

Also accepted: $\mu_0 n I \pi R^2$

Part C

Suppose that the current varies with time, so that $\Delta I/\Delta t \neq 0$. Find the total electromotive force $\mathcal{E}$ induced in the solenoid due to this change in current.

Express your answer in terms of $\Delta I/\Delta t$, n, Z, and R.

View Available Hint(s) (2)

Hint 1. Find the total flux in terms of the current

In Part B you found the flux Φ_1 through a single turn of the solenoid. Now find the flux $\Phi_{\rm total}$ through the entire solenoid.

Express your answer in terms of I, other quantities given in the introduction, and various constants such as μ_0.

ANSWER:

$\Phi_{\rm total}$ = $\mu_0 n^2 I \pi R^2 Z$

Hint 2. Finding the emf from the flux

Recall from the introduction that $\mathcal{E} = -\Delta\Phi_{\rm total}/\Delta t$, the *change* in the total flux per change in time. In the previous hint, you found an expression for $\Phi_{\rm total}$ that depended on I and various constants. The current I changes with time; hence, $\Delta\Phi_{\rm total}/\Delta t$ will be proportional to $\Delta I/\Delta t$.

ANSWER:

$\mathcal{E}$ = $-n^2 Z \frac{\Delta I}{\Delta t} \mu_0 \pi R^2$

Part D

The self-inductance L is related to the self-induced emf $\mathcal{E}$ by the equation $\mathcal{E} = -L\Delta I/\Delta t$. Find L for a long solenoid. (Hint: The self-inductance L will always be a positive quantity.)

Express the self-inductance in terms of the number of turns per length n, the physical dimensions R and Z, and relevant constants.

ANSWER:

L = $n^2 Z \mu_0 \pi R^2$

The direction of the induced emf can be determined using Lenz's law. In the figure, the current I flows from A to B. Suppose that this current is decreasing with time; i.e., $\Delta I/\Delta t < 0$. The induced emf will tend to *oppose* this change, and must therefore be in the direction from A to B. End B will then be at a higher potential than end A. In other words, there will be a potential (or voltage) increase from A to B. The magnitude of this potential increase will be equal to $L|\Delta I/\Delta t|$.

Part E

Which of the following is always a true statement?

ANSWER:

- ○ If I is positive, the potential at end A will necessarily be higher than that at end B.
- ◉ If $\Delta I/\Delta t$ is positive, the potential at end A will necessarily be higher than that at end B.
- ○ If I is positive, the potential at end A will necessarily be lower than that at end B.
- ○ If $\Delta I/\Delta t$ is positive, the potential at end A will necessarily be lower than that at end B.

If the current is increasing with time (i.e., $\Delta I/\Delta t > 0$), then the induced emf must point from B to A if it is to oppose this change in current. Thus, end A must be at a higher potential than end B. If, however, the current is *not* changing with time ($\Delta I/\Delta t = 0$), there will be *no* induced emf, and the potential at end A will equal the potential at end B.

Part F

Suppose a constant current I flows through the inductor, but you are not told whether this current is positive, negative, or zero. Now consider the effect that applying an *additional* voltage to the inductor will have on the current I already flowing through it; imagine that the voltage is applied to end A, while end B is grounded. Which one of the following statements is true?

ANSWER:

- ○ If V is positive, then I will necessarily be positive and $\Delta I/\Delta t$ will be negative.
- ○ If V is positive, then I will necessarily be negative and $\Delta I/\Delta t$ will be negative.
- ○ If V is positive, then I could be positive or negative, while $\Delta I/\Delta t$ will necessarily be negative.
- ○ If V is positive, then I will necessarily be positive and $\Delta I/\Delta t$ will be positive.
- ◉ If V is positive, then I could be positive or negative, while $\Delta I/\Delta t$ will necessarily be positive.
- ○ If V is positive, then I will necessarily be negative and $\Delta I/\Delta t$ will be positive.