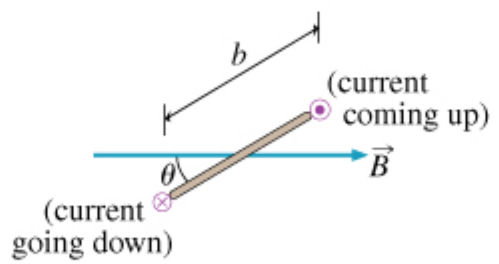
QUIZ 6, 17 MARS 2016

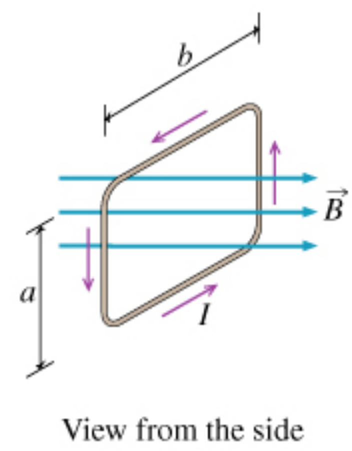
Interaction of a Current Loop with a Magnetic Field

Description: Figure out the torque acting on a rectangular current loop tilted in a uniform magnetic field at two different initial angular positions. Focus is on the direction of rotation of the loop.

The effects due to the interaction of a current-carrying loop with a magnetic field have many applications, some as common as the electric motor. This problem illustrates the basic principles of this interaction.

Consider a current I that flows in a plane rectangular current loop with height a = 4.00 cm and horizontal sides b = 2.00 cm . The loop is placed into a uniform magnetic field $\vec{B}$ in such a way that the sides of length a are perpendicular to $\vec{B}$, and there is an angle θ between the sides of length b and $\vec{B}$, as shown in the figures.





Part A

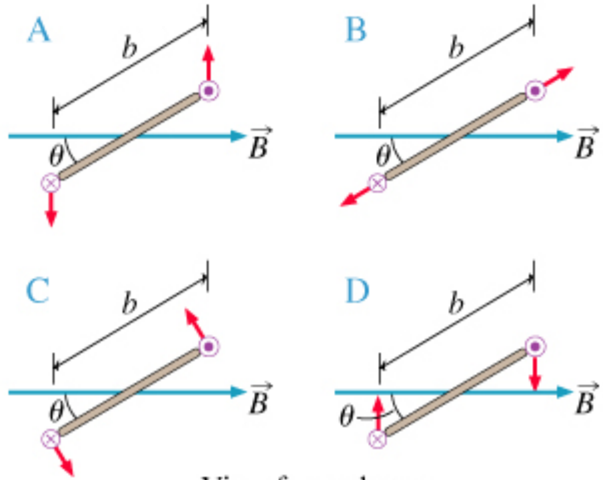
Will the interaction of the current through the loop with the magnetic field cause the loop to rotate?

Hints (2)

Hint 1. Find the direction of the forces on the parts of the loop that have length a

As current is moving through the loop, forces act on its different parts. These result from magnetic forces acting on the charges that move through the wire. What is the direction of the forces on the two pieces of wire of length a?

Select the correct diagram from the four options. The forces are symbolized by the red arrows.



View from above

Hints (1)

Hint 1. Direction of magnetic force on a straight current-carrying wire: the right-hand rule

The direction of the force $\vec{F}$ acting on a straight wire that carries a current I in a magnetic field $\vec{B}$ can be determined using the *right-hand rule*: Point the fingers of your right hand in the direction of the current; then curl your fingers toward the direction of $\vec{B}$. Your thumb will give you the direction of $\vec{F}$.

ANSWER:

- A
- B
- C
- D

The forces on these parts of the loop do not cause a motion of the center of mass, since they are equal and opposite, but they do produce a net torque, since their lines of action do not pass through the center of mass of the loop.

Hint 2. Find the direction of the forces on the parts of the loop that have length b

As current is moving through the loop, forces act on its different parts. These result from magnetic forces acting on the charges that move through the wire. What can you say about the forces on the two pieces of wire of length b?

Hints (1)

Hint 1. Direction of magnetic force on a straight current-carrying wire: the right-hand rule

The direction of the force $\vec{F}$ acting on a straight wire that carries a current I in a magnetic field $\vec{B}$ can be determined using the *right-hand rule*: Point the fingers of your right hand in the direction of the current; then curl your fingers toward the direction of $\vec{B}$. Your thumb will give you the direction of $\vec{F}$.

ANSWER:

- (selected) The forces point away from the center of the loop and cancel each other.
- The forces act as a torque about an axis through the midpoints of the two pieces of length b and make the loop turn in the counterclockwise direction.
- The forces point toward the center of the loop and cancel each other.
- Both forces point upward perpendicular to $\vec{B}$ and the piece of wire of length b.

Since the forces on these two parts of the loop cancel out, and their lines of action go through the center of mass of the loop, we need not consider them in the calculation of the torque on the loop. The key is that the forces are in line, so their torque contributions about any point cancel.

ANSWER:

- Yes, the net torque acting on the loop is negative and tends to rotate the loop in the direction of decreasing angle θ (clockwise).
- (selected) Yes, the net torque acting on the loop is positive and tends to rotate the loop in the direction of increasing angle θ (counterclockwise).
- No, the net torque acting on the loop is zero and the loop is in equilibrium.
- No, the net force acting on the loop is zero and the loop is in equilibrium.

Part B

Assume that the loop is initially positioned at $\theta = 30^\circ$ and the current flowing into the loop is 0.500 A . If the magnitude of the magnetic field is 0.300 T , what is τ, the net torque about the vertical axis of the current loop due to the interaction of the current with the magnetic field?

Express your answer in newton-meters.

Hints (3)

Hint 1. Torque on a current-carrying loop

The torque τ acting on a current-carrying loop of area A due to the interaction of the current I flowing through the loop with a magnetic field of magnitude B is given by

$$\tau = IBA\sin\phi,$$

where ϕ is the angle between the normal to the loop and the direction of the magnetic field.

Hint 2. Find the area of the loop

What is the area $A_{\rm loop}$ of a rectangular loop of wire with height 4.00 $\rm cm$ and horizontal sides 2.00 $\rm cm$?

Express your answer in square meters.

ANSWER:

$A_{\rm loop}$ = ab = 8.00×10^{-4} $\rm m^2$

Hint 3. Find the angle between the normal to the loop and the magnetic field

Consider the loop described in the introduction of this problem. Recall that θ is the angle between the sides of length b and $\vec{B}$. Now consider the normal to the loop. What is the angle ϕ between the normal and the magnetic field? Be careful to consider the correct sign for ϕ.

ANSWER:

$\phi =$

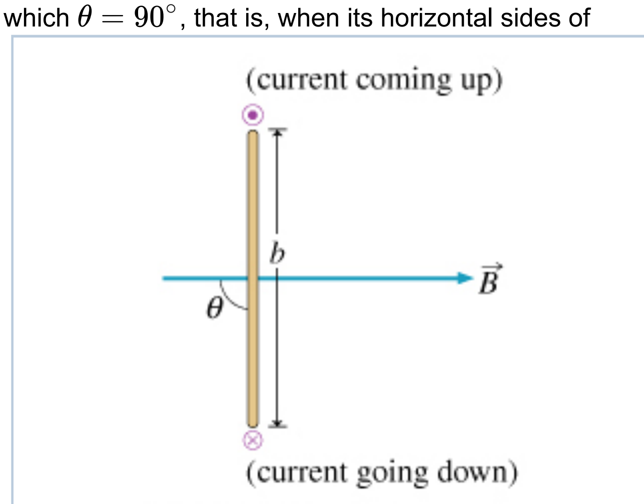
- ○ θ
- ○ $180^\circ - \theta$
- ◉ $90^\circ - \theta$
- ○ $\theta - 90^\circ$

ANSWER:

τ = $ab\sin\left(\frac{60\pi}{180}\right) B_0 I_0$ = 1.04×10^{-4} $\rm N\cdot m$

Part C

What happens to the loop when it reaches the position for which $\theta = 90^\circ$, that is, when its horizontal sides of length b are perpendicular to $\vec{B}$ (see the figure)?



Hints (1)

Hint 1. Find the net torque acting on the loop

What is τ, the net torque about the vertical axis of the current loop due to the interaction of the current with the magnetic field?

Express your answer in newton-meters.

Hints (1)

Hint 1. Find the angle between the normal to the loop and the magnetic field

What is the angle ϕ between the normal to the loop and the magnetic field when the horizontal sides of the loop of length b are perpendicular to $\vec{B}$?

Express your answer in degrees.

ANSWER:

ϕ = 0 $^\circ$

ANSWER:

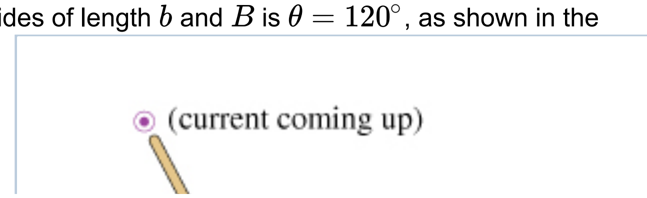
τ = 0 $\mathrm{N \cdot m}$

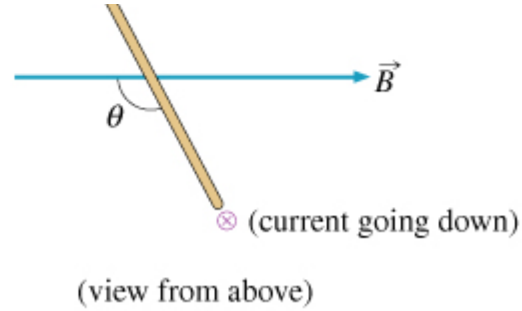
ANSWER:

- The direction of rotation changes because the net torque acting on the loop causes the loop to rotate in a clockwise direction.
- The net torque acting on the loop is zero, but the loop continues to rotate in a counterclockwise direction.
- The net torque acting on the loop is zero; therefore it stops rotating.
- The net force acting on the loop is zero, so the loop must be in equilibrium.

Part D

Now suppose that you change the initial angular position of the loop relative to $\vec{B}$, and assume that the loop is placed in such a way that initially the angle between the sides of length b and $\vec{B}$ is $\theta = 120^\circ$, as shown in the figure. Will the interaction of the current through the loop with the magnetic field cause the loop to rotate?



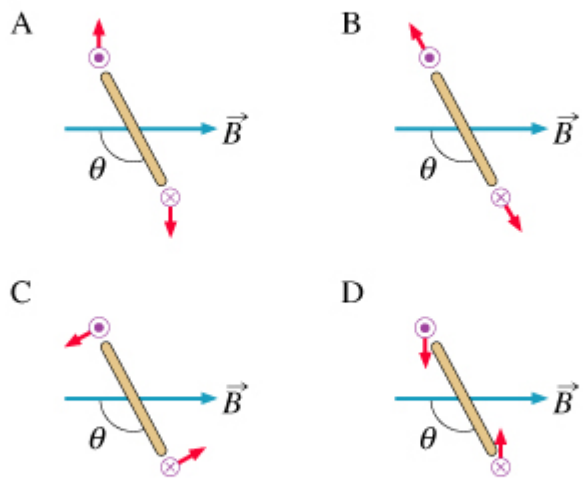


Hints (2)

Hint 1. Find the direction of the forces on the parts of the loop that have length a

As current is moving through the loop, forces act on its different parts. These result from magnetic forces acting on the charges that move through the wire. What is the direction of the forces on the two pieces of wire of length a?

Select the correct diagram from the four options. The forces are symbolized by the red arrows.



ANSWER:

- A
- B
- C
- D

Hint 2. Find the direction of the forces on the parts of the loop that have length b

As current is moving through the loop, forces act on its different parts. These result from magnetic forces acting on the charges that move through the wire. What can you say about the forces on the two pieces of wire of length b?

ANSWER:

- ◉ The forces point away from the center of the loop and cancel each other.
- ○ The forces act as a torque about an axis through the midpoints of the two pieces of length b and make the loop turn in the counterclockwise direction.
- ○ The forces point toward the center of the loop and cancel each other.
- ○ Both forces point upward perpendicular to $\vec{B}$ and the piece of wire of length b.

ANSWER:

- ◉ Yes, the net torque acting on the loop is negative and tends to rotate the loop in the direction of decreasing angle θ (clockwise).
- ○ Yes, the net torque acting on the loop is positive and tends to rotate the loop in the direction of increasing angle θ (counterclockwise).
- ○ No, the net torque acting on the loop is zero and the loop is in equilibrium.
- ○ No, the net force acting on the loop is zero and the loop is in equilibrium.

Depending on the initial position of the loop relative to $\vec{B}$, the direction of rotation of the loop will be different. If initially $0^\circ < \theta < 90^\circ$, then the net torque acting on the loop will cause the loop to rotate in the counterclockwise direction. If instead, $90^\circ < \theta < 180^\circ$, then the net torque will rotate the loop in the opposite direction.