Are Coulomb Forces Conservative?

Learning Goal:
To review the concept of conservative forces and to understand that electrostatic forces are, in fact, conservative.

As you may recall from mechanics, some forces have a very special property, namely, that the work done on an object does not depend on the object's trajectory; rather, it depends only on the initial and the final positions of the object.

Such forces are called conservative forces. If only conservative forces act within a closed system, the total amount of mechanical energy is conserved within the system (hence the term "conservative"). Such forces have a number of properties that simplify the solution of many problems.

You may also recall that a potential energy function can be defined with respect to a conservative force. This property of conservative forces will be of particular interest of us.

Not all forces that we deal with are conservative, of course. For instance, the amount of work done by a frictional force very much depends on the object's trajectory. Friction, therefore, is not a conservative force.

In contrast, the gravitational force is the most common example of a conservative force. What about electrostatic (Coulomb) forces? Are they conservative, and is there a potential energy function associated with them?

Fig 1

In this problem, you will be asked to use the given diagram (Figure 1) to calculate the work done by the electric field \( E \) on a particle of charge \( q \) and see for yourself whether that work appears to be trajectory-independent. Recall that the force acting on a charged particle in an electric field is given by \( F = E \cdot q \).

Recall that the work \( W \) done on an object by a constant force is

\[ W = F d \cos \theta, \]

where \( F \) is the magnitude of the force acting on the object, \( d \) is the magnitude of the displacement that the object undergoes, and \( \theta \) is the angle between the vectors \( F \) and \( d \).

Consider a uniform electric field \( E \) and a rectangle ABCD, as shown in the figure. Sides AB and CD are parallel to \( E \) and have length \( L \); let \( \alpha \) be angle BAC.
Part A
Calculate the work $W_{AB}$ done by the electrostatic force on a particle of charge $q$ as it moves from A to B. Express your answer in terms of some or all the variables $E$, $q$, $L$, and $\alpha$.

Ans

$W_{AB} = EqL$

Part B
Calculate the work $W_{BC}$ done by the electrostatic force on the charged particle as it moves from B to C. Express your answer in terms of some or all the variables $E$, $q$, $L$, and $\alpha$.

Ans

$W_{BC} = 0$

Part C
Calculate the total amount of work $W_{ABC}$ done by the electrostatic force on the charged particle as it moves from A to B to C. Express your answer in terms of some or all the variables $E$, $q$, $L$, and $\alpha$.

Ans

$W_{ABC} = EqL$

Part D
Now assume that the particle "chooses" a different way of traveling. Calculate the total amount of work $W_{ADC}$ done by the electrostatic force on the charged particle as it moves from A to D to C. Express your answer in terms of some or all the variables $E$, $q$, $L$ and $\alpha$.

Ans

$W_{ADC} = EqL$

Part E
Calculate the work $W_{AGC}$ done by the electrostatic force on the charged particle as it moves from A straight to C. Express your answer in terms of some or all the variables $E$, $q$, $L$, and $\alpha$.

Ans

$W_{AGC} = EqL$

With the knowledge that the Coulomb force is conservative, and again referring to the diagram, answer the following questions. These questions are meant to highlight some important properties of conservative forces.

Part F
Find the amount of work $W_{BA}$ done by the electrostatic force on the charged particle as it moves along the straight path from B to A. Express your answer in terms of some or all the variables $E$, $q$, $L$, and $\alpha$.

Ans

$W_{BA} = -EqL$
The amount of work \( W_{ABA} \) done by the electrostatic force on the charged particle as it moves from A to B to A is equal to

\[
W_{ABA} = W_{AB} + W_{BA} = W_{AB} + (-W_{AB}) = 0.
\]

**Part G**

Find the amount of work \( W_{ABCD} \) done by the electrostatic force on the charged particle as it moves from A to B to C to D to A.

Express your answer in terms of some or all the variables \( E, q, L \) and \( \alpha \).

**Ans**

\( W_{ABCD} = 0 \)