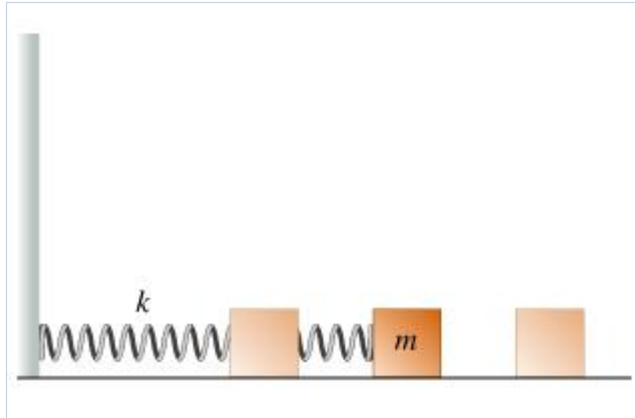
Description: ± Includes Math Remediation. Mostly conceptual questions to introduce basic relations between the restoring force, acceleration, and displacement in SHM of a mass-spring system. No previous knowledge of SHM is required.

Consider the system shown in the figure. It consists of a block of mass m attached to a spring of negligible mass and force constant k. The block is free to move on a frictionless horizontal surface, while the left end of the spring is held fixed. When the spring is neither compressed nor stretched, the block is in equilibrium. If the spring is stretched, the block is displaced to the right and when it is released, a force acts on it to pull it back toward equilibrium. By the time the block has returned to the equilibrium position, it has picked up some kinetic energy, so it overshoots, stopping somewhere on the other side, where it is again pulled back toward equilibrium. As a result, the block moves back and forth from one side of the equilibrium position to the other, undergoing *oscillations*. Since we are ignoring friction (a good approximation to many cases), the mechanical energy of the system is conserved and the oscillations repeat themselves over and over.



The motion that we have just described is typical of most systems when they are displaced from equilibrium and experience a *restoring force* that tends to bring them back to their equilibrium position. The resulting oscillations take the name of *periodic motion*. An important example of periodic motion is *simple harmonic motion* (SHM) and we will use the mass-spring system described here to introduce some of its properties.

Part A

Which of the following statements best describes the characteristic of the restoring force in the spring-mass system described in the introduction?

View Available Hint(s) (2)

ANSWER:

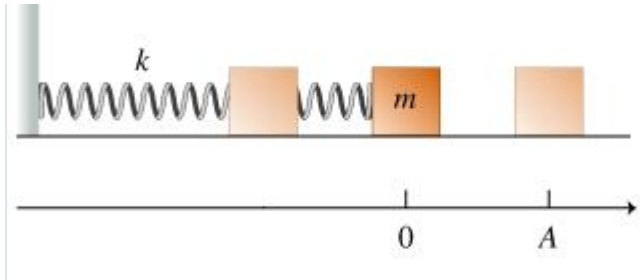
- The restoring force is constant.
- The restoring force is directly proportional to the displacement of the block.
- The restoring force is proportional to the mass of the block.
- The restoring force is maximum when the block is in the equilibrium position.

Whenever the oscillations are caused by a restoring force that is directly proportional to displacement, the resulting periodic motion is referred to as *simple harmonic motion*.

Part B

As shown in the figure, a coordinate system with the origin at the equilibrium position is chosen so that the *x* coordinate represents the displacement from the equilibrium position. (The positive direction is to the right.) What is the initial acceleration of the block, a_0, when the block is released at a distance A to the right from its equilibrium position?

Express your answer in terms of some or all of the variables A, m, and k.



View Available Hint(s) (1)

ANSWER:

a_0 = $-\frac{k}{m}A$

Part C

What is the acceleration a_1 of the block when it passes through its equilibrium position?

Express your answer in terms of some or all of the variables A, m, and k.

View Available Hint(s) (1)

ANSWER:

a_1 = 0

Your results from Parts B and C show that the acceleration of the block is negative when the block has undergone a positive displacement. Then, the acceleration's magnitude decreases to zero as the block goes through its equilibrium position. What do you expect the block's acceleration will be when the block is to the left of its equilibrium position and has undergone a negative displacement?

Part D

Select the correct expression that gives the block's acceleration at a displacement x from the equilibrium position. Note that x can be either positive or negative; that is, the block can be either to the right or left of its equilibrium position.

View Available Hint(s) (1)

ANSWER:

- ○ $a = -kx$
- ○ $a = kx$
- ○ $a = \frac{k}{m}x$
- ◉ $a = -\frac{k}{m}x$

Whether the block undergoes a positive or negative displacement, its acceleration is always opposite in sign with respect to displacement. Moreover, the block's acceleration is *not* constant; instead, it is directly proportional to displacement. This is a fundamental property of simple harmonic motion.

Using the information found so far, select the correct phrases to complete the following statements.

Part E

The magnitude of the block's acceleration reaches its maximum value when the block is

View Available Hint(s) (1)

ANSWER:

- ○ in the equilibrium position.
- ◉ at either its rightmost or leftmost position.
- ○ between its rightmost position and the equilibrium position.
- ○ between its leftmost position and the equilibrium position.

Part F

The speed of the block is zero when it is

View Available Hint(s) (1)

ANSWER:

- ○ in the equilibrium position.
- ◉ at either its rightmost or leftmost position.
- ○ between its rightmost position and the equilibrium position.
- ○ between its leftmost position and the equilibrium position.

Part G

The speed of the block reaches its maximum value when the block is

View Available Hint(s) (1)

ANSWER:

- (selected) in the equilibrium position.
- at either its rightmost or leftmost position.
- between the rightmost position and the equilibrium position.
- between the leftmost position and the equilibrium position.

Part H

Because of the periodic properties of SHM, the mathematical equations that describe this motion involve sine and cosine functions. For example, if the block is released at a distance A from its equilibrium position, its displacement x varies with time t according to the equation

$$x = A\cos\omega t,$$

where ω is a constant characteristic of the system. If time is measured is seconds, ω must be expressed in radians per second so that the quantity ωt is expressed in radians.

Use this equation and the information you now have on the acceleration and speed of the block as it moves back and forth from one side of its equilibrium position to the other to determine the correct set of equations for the block's *x* components of velocity and acceleration, v_x and a_x, respectively. In the expressions below, B and C are nonzero positive constants.

View Available Hint(s) (2)

ANSWER:

- $v_x = -B\sin\omega t$, $a_x = C\cos\omega t$
- $v_x = B\cos\omega t$, $a_x = C\sin\omega t$
- $v_x = -B\cos\omega t$, $a_x = -C\cos\omega t$
- (selected) $v_x = -B\sin\omega t$, $a_x = -C\cos\omega t$

Further calculations would show that the constants B and C can be expressed in terms of A and ω.