2 Project 2: The Curve Shortening Flow

During the first lecture, I displayed a webpage [1] which evolved closed curves under the so-called *curve shortening flow*

$$\frac{\partial \vec{X}}{\partial t} = H\vec{\nu} \ ,$$

where $\vec{X}(s,t) = \vec{X}_t(s) = (x_t(s), y(s))$ is a parametrized curve with parameter s at each time t. Here t is the flow time, $H = H_t(s)$ is the mean curvature of the curve at $(x_t(s), y(s))$ at time t, and $\vec{\nu} = \vec{\nu}_t(s)$ is the unit normal vector to the curve pointing in the direction of the centre of the osculating circle at $(x_t(s), y(s))$.
The project will be to write a computer program (and perhaps build a webpage) that replicates the above page and can be modified to other flows. The main example is the inverse curve shortening flow

$$\frac{\partial \vec{X}}{\partial t} = -\frac{\vec{\nu}}{H} \ ,$$

The curve shortening flow has several interesting properties. Starting from an initial closed curve at time $t = 0$ then:

- if the initial curve does not self-intersect, the flowing curve never self-intersects,
- the flow remains smooth until the curve disappears, and
- the curve always shrinks to zero arclength and disappears in finite time.

How many of these properties can you prove analytically?
The Wikipedia page on curve shortening flow has an excellent discussion [2]. For ideas about writing a computer program, see the section on *Numerical Approximations* on that page.

References

[1] http://a.carapetis.com/csf/

[2] https://en.wikipedia.org/wiki/Curve-shortening_flow