

B Technical Appendix to “A Model of Stochastic Liquidity”

B.1 Derivation of Equation (16)

Since $N_{t-1} \in \mathcal{F}_{t-1}$, the pair of $\tilde{\delta}_t$ and \tilde{w}_{t-1} is multivariate normally distributed conditionally on \mathcal{F}_{t-1} :

$$\begin{pmatrix} \tilde{\delta}_t \\ \tilde{w}_{t-1} \end{pmatrix} \Bigg|_{\mathcal{F}_{t-1}} \sim N \begin{pmatrix} \Sigma_t & \Sigma_{\delta w, t-1} \\ \Sigma'_{\delta w, t-1} & \Sigma_{w, t-1} \end{pmatrix},$$

where, using Equation (6) in Theorem 1 and also Equation (A29) at time $t-1$,

$$\Sigma_{\delta w, t-1} = Cov_{t-1}(\tilde{\delta}_t, \tilde{w}'_{t-1}) = N_{t-1} \Sigma_t B'_{t-1}, \quad (\text{A31})$$

$$\Sigma_{w, t-1} = Var_{t-1}(\tilde{w}_{t-1}) = N_{t-1} B_{t-1} [N_{t-1} (\Sigma_t + \Gamma_{t-1}) + \Phi_{t-1}] B'_{t-1} + \Psi_{t-1}.$$

From this and Equation (9), the variance of $\Delta \tilde{P}_t$ conditional on the last two order flows is given by the usual updating formula for the multivariate normal distribution,

$$\begin{aligned} \Sigma_{\delta|w,t} &= Var(\Delta \tilde{P}_t | \tilde{w}_t, \tilde{w}_{t-1}) = Var_{t-1}(\Delta \tilde{P}_t | \tilde{w}_t, \tilde{w}_{t-1}) \\ &= Var_{t-1}(\tilde{\delta}_t | \tilde{w}_{t-1}) = \Sigma_t - \underbrace{\Sigma_{\delta w, t-1} \Sigma_{w, t-1}^{-1} \Sigma'_{\delta w, t-1}}_{=A_{t-1}} \\ &= \Sigma_t - A_{t-1} \cdot N_{t-1} B_{t-1} \Sigma_t \text{ by (A31)} \\ &= \Sigma_t - N_{t-1} J_{t-1}^{-1} \Sigma_{t-1, \xi} \text{ by Theorem 1} \\ &= \Sigma_t - N_{t-1} [2I + (N_{t-1} - 1) \Sigma_{t-1, \xi} \Sigma_t^{-1} (\Sigma_t + \Gamma_{t-1}) \Sigma_t^{-1}]^{-1} \Sigma_{t-1, \xi} \text{ by (A7)} \\ &= \Sigma_t - N_{t-1} \Sigma_t [(N_{t-1} + 1) (\Sigma_t + \Gamma_{t-1}) + 2\Phi_{t-1}]^{-1} \Sigma_t, \end{aligned}$$

which is Equation (16). ■

B.2 Note on Series Construction

The paper focuses on the 30 Dow stocks to keep the analysis manageable and, more importantly, to reliably estimate the liquidity process. This sample also makes the nonsynchronous-trading issue minimal. As stated in the paper, the sample period starts in 2002 to allow enough time after the decimalization of NYSE and NASDAQ, which presumably introduced a structural change in the way liquidity is provided in the market.²¹ From the TAQ dataset, I extract only regular quotes with positive offer and bid prices and depths. Trades must be regular-way with no correction and have a positive transaction price and a positive number of shares traded. For a given trade, the Lee and Ready (1991) algorithm matches the latest quote that is at least five seconds old. If the trade price is higher (lower) than the matched quote midpoint, the trade is classified as a buy (sell). If the trade occurs exactly at the quote midpoint, the so-called tick test is performed to determine the trade direction. The standardization described in the main text, due to Breen, Hodrick, and Korajczyk (2002), implies that a unit signed ($stov_t$) and unsigned (tov_t) share turnover corresponds to 0.1% of the outstanding shares being bought in net and traded in gross, respectively.

²¹The NYSE and NASDAQ completed the decimalization processes on January 29, 2001 and April 9, 2001, respectively.