Technical Appendix to "Price Volatility and Investor Behavior in an Overlapping Generations Model with Information Asymmetry"

A. Derivation of Moment Expressions in Partial-Information Equilibria

This appendix derives expressions for various moments in a partial-information equilibrium. From (15), the price-change vector can be decomposed into independent shocks,

$$\begin{aligned}\Delta\widetilde{P}_t &\equiv \widetilde{P}_t - \widetilde{P}_{t-1}\\ &= A_1\widetilde{\eta}_{t-1} + \frac{1}{r}\widetilde{\delta}_t + B_2(\widetilde{\delta}_{t+1} - \widetilde{\delta}_t) + A_2(\widetilde{\eta}_t - \widetilde{\eta}_{t-1})\\ &= B_2\widetilde{\delta}_{t+1} + (r^{-1}I - B_2)\widetilde{\delta}_t + A_2\widetilde{\eta}_t + (A_1 - A_2)\widetilde{\eta}_{t-1}. \qquad \text{(A36)}\end{aligned}$$

Henceforth, Δ generally denotes the first difference of a variable. Then, it is straightforward to compute

$$\begin{aligned}Var(\Delta\widetilde{P}_t) = B_2\Sigma_\delta B_2^\intercal + (r^{-1}I - B_2)\Sigma_\delta(r^{-1}I - B_2)^\intercal + A_2\Sigma_\eta A_2^\intercal\\ + (A_1 - A_2)\Sigma_\eta(A_1 - A_2)^\intercal. \qquad \text{(A37)}\end{aligned}$$

From the demand function $X_{t,i} = \frac{1}{\theta_i}S_i^{-1}m_{t,i}$, the net flow of agent i can be generally decomposed into independent pieces

$$\begin{aligned}\Delta\widetilde{\Pi}_{t,i} &\equiv \Delta\widetilde{X}_{t,i} - \widetilde{\eta}_{t,i}\\ &= B_{1,i}\Delta\widetilde{\delta}_{t+1} + B_{2,i}\widetilde{\eta}_t + B_{3,i}\widetilde{\eta}_{t-1} + B_{4,i}\Delta\widetilde{\varepsilon}_{t,i} + B_{5,i}\widetilde{\zeta}_{t,i} + B_{6,i}\widetilde{\zeta}_{t-1,i}. \qquad \text{(A38)}\end{aligned}$$

Roughly, of the three trade motives discussed in the main text, the last two terms capture the endowment effect as they derive from idiosyncratic endowment noises. The fourth term represents the heterogeneous information noise that is present even if there is no information asymmetry. These three idiosyncratic components will be gone in aggregating individual demands into group net flows. Information asymmetry is present in the first three terms whose relevant shocks comprise the changes in price signals, $\Delta\widetilde{\xi}_t$. The second term also captures the trade motive due to taste (risk aversion) because it accommodates supply shocks. We will confirm these below.

$B_{1,i}$ is given by the integrand of equation (A16),

$$\begin{aligned}B_{1,i} &= (\theta_i S_i)^{-1}[(A_1A_2^{-1} - I)B_2 + G\Sigma_i(\Sigma_i^{-1} - \Sigma_\delta^{-1}) - rB_2]\\ &= (\theta_i S_i)^{-1}[A_1F^{-1} - RB_2 + G - G\Sigma_i\Sigma_\delta^{-1}] \text{ by (A3)}\\ &= (\theta_i S_i)^{-1}[\frac{R}{r}I - (\frac{R}{r}I - G\overline{\Sigma}\Sigma_\delta^{-1}) - G\Sigma_i\Sigma_\delta^{-1}] \text{ by (A18), (A19)}\\ &\equiv B_{0,i}(\overline{\Sigma} - \Sigma_i)\Sigma_\delta^{-1},\end{aligned}$$

where

$$B_{0,i} \equiv (\theta_i S_i)^{-1}G.$$

The term $\overline{\Sigma} - \Sigma_i$ captures information asymmetry that will be non-zero if agents are asymmetrically informed.

Next, we first determine $B_{3,i}$, which is given by the integrands of (A14) less (A20),

$$\begin{aligned} B_{3,i} &= (\theta_i S_i)^{-1}[-rA_1 - \{(A_1A_2^{-1} - I)B_2F + G\Sigma_i(F^{\intercal})^{-1}\Sigma_\eta^{-1}F^{-1}F - rA_2\}] \\ &= (\theta_i S_i)^{-1}[-rA_1 - \{A_1 - RA_2 + G\Sigma_i(F^{\intercal})^{-1}\Sigma_\eta^{-1}\}] \text{ by (A3)} \\ &= (\theta_i S_i)^{-1}[-rA_1 + rA_1 + G(\overline{\Sigma} - \Sigma_i)(F^{\intercal})^{-1}\Sigma_\eta^{-1}] \text{ by (A21)} \\ &= B_{0,i}(\overline{\Sigma} - \Sigma_i)(F^{\intercal})^{-1}\Sigma_\eta^{-1}. \end{aligned}$$

$B_{2,i}$ is given by the integrand of equation (A20) less the identity matrix, and hence using $B_{3,i}$ and the integrand of (A14),

$$\begin{aligned} B_{2,i} &= -B_{3,i} + (\theta_i S_i)^{-1}(-rA_1) - I \\ &= -B_{3,i} + (\theta_i S_i)^{-1}(\overline{\theta S} - \theta_i S_i) \text{ by (A15).} \\ (&= (\theta_i S_i)^{-1}[\overline{\theta S} - \theta_i S_i - G(\overline{\Sigma} - \Sigma_i)(F^{\intercal})^{-1}\Sigma_\eta^{-1}]) \end{aligned}$$

Note that the term $\overline{\theta S} - \theta_i S_i$ captures a combination of trade motives due to risk aversion and information asymmetry; for example, it will be non-zero as long as $\theta_i \neq \overline{\theta}$ even if there is no information asymmetry, $S_i = \overline{S}$. It will also be non-zero as long as $S_i \neq \overline{S}$ even if agents are equally risk averse, $\theta_i = \overline{\theta}$.

The derivation of remaining coefficients is immediate from (A6) and (A8):

$$\begin{aligned} B_{4,i} &= (\theta_i S_i)^{-1}G\Sigma_i\Sigma_{\varepsilon,i}^{-1} = B_{0,i}\Sigma_i\Sigma_{\varepsilon,i}^{-1}, \\ B_{6,i} &= -(\theta_i S_i)^{-1}G\Sigma_i(F^{\intercal})^{-1}\Sigma_\zeta^{-1}F^{-1}(-F) = B_{0,i}\Sigma_i(F^{\intercal})^{-1}\Sigma_\zeta^{-1}, \\ B_{5,i} &= -B_{6,i} - I. \end{aligned}$$

In summary,

$$\begin{aligned} &B_{0,i} \equiv (\theta_i S_i)^{-1}G, \\ &B_{1,i} \equiv B_{0,i}(\overline{\Sigma} - \Sigma_i)\Sigma_\delta^{-1}, && B_{2,i} \equiv -B_{3,i} + (\theta_i S_i)^{-1}\overline{\theta S} - I, \\ &B_{3,i} \equiv B_{0,i}(\overline{\Sigma} - \Sigma_i)(F^{\intercal})^{-1}\Sigma_\eta^{-1}, && B_{4,i} \equiv B_{0,i}\Sigma_i\Sigma_{\varepsilon,i}^{-1}, \\ &B_{5,i} \equiv -B_{6,i} - I, && B_{6,i} \equiv B_{0,i}\Sigma_i(F^{\intercal})^{-1}\Sigma_\zeta^{-1}. \end{aligned}$$

Using these coefficients, the variance of the net flow is computed as

$$\begin{aligned} Var(\Delta\widetilde{\Pi}_{t,i}) = {}& 2B_{1,i}\Sigma_\delta B_{1,i}^{\intercal} + B_{2,i}\Sigma_\eta B_{2,i}^{\intercal} + B_{3,i}\Sigma_\eta B_{3,i}^{\intercal} \\ &+ 2B_{4,i}\Sigma_{\varepsilon,i}B_{4,i}^{\intercal} + B_{5,i}\Sigma_\zeta B_{5,i}^{\intercal} + B_{6,i}\Sigma_\zeta B_{6,i}^{\intercal}. \end{aligned}$$

This expression is used in computing the expected volume V in (26).

By the law of large numbers, only the first three terms in (A38) are relevant in computing group j net flows in (23),

$$\begin{aligned} \Delta\widetilde{\Pi}_t^j &= \int_{i\in j} \Delta\widetilde{\Pi}_{t,i}\,di \\ &= C_1^j\Delta\widetilde{\delta}_{t+1} + C_2^j\widetilde{\eta}_t + C_3^j\widetilde{\eta}_{t-1}, \qquad \text{(A39)} \end{aligned}$$

where

$$C_0^j \equiv m^j(\overline{\theta}^j\overline{S}^j)^{-1}G,$$
$$C_1^j \equiv C_0^j(\overline{\Sigma} - \overline{\Sigma}^j)\Sigma_\delta^{-1}, \tag{A40}$$
$$C_2^j \equiv -C_3^j + m^j[(\overline{\theta}^j\overline{S}^j)^{-1}\overline{\theta S} - I], \tag{A41}$$
$$C_3^j \equiv C_0^j(\overline{\Sigma} - \overline{\Sigma}^j)(F^\intercal)^{-1}\Sigma_\eta^{-1}. \tag{A42}$$

Its variance is

$$Var(\Delta\widetilde{\Pi}_t^j) = 2C_1^j\Sigma_\delta C_1^{j\intercal} + C_2^j\Sigma_\eta C_2^{j\intercal} + C_3^j\Sigma_\eta C_3^{j\intercal}.$$

This is used in computing the expected absolute flows U in (25).

Using (A39) and (A36), our measure of trading behavior in Section II.D is

$$\begin{aligned}
Cov(\Delta\widetilde{\Pi}_t^j, \Delta\widetilde{P}_t^\intercal) &= C_1^j\Sigma_\delta[B_2^\intercal - (r^{-1}I - B_2^\intercal)] + C_2^j\Sigma_\eta A_2^\intercal + C_3^j\Sigma_\eta(A_1 - A_2)^\intercal \\
&= C_0^j(\overline{\Sigma} - \overline{\Sigma}^j)(2B_2^\intercal - r^{-1}I) + C_3^j\Sigma_\eta(-A_2 + A_1 - A_2)^\intercal \\
&\quad + m^j[(\overline{\theta}^j\overline{S}^j)^{-1}\overline{\theta S} - I]\Sigma_\eta A_2^\intercal \text{ by (A40) and (A41)} \\
&= C_0^j(\overline{\Sigma} - \overline{\Sigma}^j)[2B_2^\intercal - r^{-1}I + (F^\intercal)^{-1}(A_1 - 2A_2)^\intercal] \\
&\quad + m^j[(\overline{\theta}^j\overline{S}^j)^{-1}\overline{\theta S} - I]\Sigma_\eta A_2^\intercal \text{ by (A42).}
\end{aligned}$$

Here, the first square bracket in the last line is

$$\begin{aligned}
&2B_2^\intercal - r^{-1}I + (F^\intercal)^{-1}(A_1 - 2A_2)^\intercal \\
&= 2B_2^\intercal - r^{-1}I + (F^\intercal)^{-1}(A_2 - \frac{1}{R}G\overline{\Sigma}(F^\intercal)^{-1}\Sigma_\eta^{-1} - 2A_2)^\intercal \text{ by (A21)} \\
&= 2B_2^\intercal - r^{-1}I + B_2^\intercal(A_2^\intercal)^{-1}(-A_2^\intercal) - \frac{1}{R}[G\overline{\Sigma}(F^\intercal)^{-1}\Sigma_\eta^{-1}F^{-1}]^\intercal \text{ by (A3)} \\
&= B_2^\intercal - r^{-1}I - \frac{1}{R}[G\overline{\Sigma}(F^\intercal)^{-1}\Sigma_\eta^{-1}F^{-1}]^\intercal \\
&= -\frac{1}{R}(G\overline{\Sigma}\Sigma_\delta^{-1})^\intercal - \frac{1}{R}[G\overline{\Sigma}(F^\intercal)^{-1}\Sigma_\eta^{-1}F^{-1}]^\intercal \text{ by (A19)} \\
&= -\frac{1}{R}[\Sigma_\delta^{-1} + (F^\intercal)^{-1}\Sigma_\eta^{-1}F^{-1}](G\overline{\Sigma})^\intercal.
\end{aligned}$$

Therefore,

$$\begin{aligned}
Cov(\Delta\widetilde{\Pi}_t^j, \Delta\widetilde{P}_t^\intercal) = &-\frac{1}{R}C_0^j(\overline{\Sigma} - \overline{\Sigma}^j)[\Sigma_\delta^{-1} + (F^\intercal)^{-1}\Sigma_\eta^{-1}F^{-1}](G\overline{\Sigma})^\intercal \\
&+ m^j(\overline{\theta}^j\overline{S}^j)^{-1}(\overline{\theta S} - \overline{\theta}^j\overline{S}^j)\Sigma_\eta A_2^\intercal.
\end{aligned} \tag{A43}$$

Roughly, the first term captures the trade motive due to information asymmetry, and the second term additionally agents' risk aversion; notice that the second term is generally non-zero even if agents are symmetrically informed. From this, we see that groups with different risk aversion can act as trend-followers and contrarians even if there is no information asymmetry among them.[46]

[46] If we focus on equilibria in which all matrices have the same eigenvectors, we can show that A_2 is negative definite (see the proof of Proposition 4). Therefore even if agents are symmetrically informed ($\overline{S}^j = \overline{S}\ \forall j$), groups that are more risk-averse than the market average ($\overline{\theta}^j > \overline{\theta}$) will behave like trend-followers while the less risk-averse groups will act as contrarians.

Before introducing additional assumptions, we note that the following results always hold: Premultiply $(\theta_i S_i)^{-1}$ to both sides of equation (A7), integrate over all i and $i \in j$, respectively, use the definition of average matrices, and rearrange to get

$$\overline{S} = A_2 \Sigma_\eta A_2^{\mathrm{T}} + G \overline{\Sigma} G^{\mathrm{T}} + B_2 \Sigma_\delta B_2^{\mathrm{T}},$$
$$\overline{S}^j = A_2 \Sigma_\eta A_2^{\mathrm{T}} + G \overline{\Sigma}^j G^{\mathrm{T}} + B_2 \Sigma_\delta B_2^{\mathrm{T}}.$$

So,

$$\overline{S} - \overline{S}^j = G(\overline{\Sigma} - \overline{\Sigma}^j) G^{\mathrm{T}}, \tag{A44}$$
$$\overline{S} - S_i = G(\overline{\Sigma} - \Sigma_i) G^{\mathrm{T}}.$$

From this, we can sign the covariance in (A43) under some conditions including common risk aversion, which ensures that the group net flows will be driven purely by information asymmetry. The precise statement is given in the following proposition:

PROPOSITION 4 *When groups are equally risk averse ($\overline{\theta}^j = \overline{\theta} \forall j$) and all matrices share the common eigenvectors, group j agents act as trend-followers (contrarians) for all securities if and only if $\overline{\Sigma}_\varepsilon^j - \overline{\Sigma}_\varepsilon$ is positive (negative) definite.*

Proof. When $\overline{\theta}^j = \overline{\theta}$, we can write the second term in (A43) using (A44),

$$m^j (\overline{\theta}^j \overline{S}^j)^{-1} (\overline{S} - \overline{S}^j) \overline{\theta} \Sigma_\eta A_2^{\mathrm{T}} = m^j (\overline{\theta}^j \overline{S}^j)^{-1} G(\overline{\Sigma} - \overline{\Sigma}^j) G^{\mathrm{T}} \overline{\theta} \Sigma_\eta A_2^{\mathrm{T}}.$$

So we can factor out the term $\overline{\Sigma}^j - \overline{\Sigma}$ representing information asymmetry in (A43) as follows:

$$Cov(\Delta \widetilde{\Pi}_t^j, \Delta \widetilde{P}_t^{\mathrm{T}}) = C_0^j (\overline{\Sigma}^j - \overline{\Sigma}) [\frac{1}{R} \{\Sigma_\delta^{-1} + (F^{\mathrm{T}})^{-1} \Sigma_\eta^{-1} F^{-1}\} (G \overline{\Sigma})^{\mathrm{T}} - \overline{\theta} G^{\mathrm{T}} \Sigma_\eta A_2^{\mathrm{T}}]. \tag{A45}$$

We first show that both C_0^j and the matrices inside the square bracket are positive definite. Start with writing B_2 by F using (A3),

$$\begin{aligned} B_2 &= A_2 F^{-1} && \text{(A46)} \\ &= A_1 F^{-1} + \frac{1}{R} G \overline{\Sigma} (F^{\mathrm{T}})^{-1} \Sigma_\eta^{-1} F^{-1} \text{ by (A21)} && \text{(A47)} \\ &= \frac{R}{r} I - G + \frac{1}{R} G \overline{\Sigma} (F^{\mathrm{T}})^{-1} \Sigma_\eta^{-1} F^{-1} \text{ by (A18).} \end{aligned}$$

Equate this to the right-hand side of (A19) and solve for G:

$$\begin{aligned} G &= [I - \frac{1}{R} \overline{\Sigma} \{\Sigma_\delta^{-1} + (F^{\mathrm{T}})^{-1} \Sigma_\eta^{-1} F^{-1}\}]^{-1} && \text{(A48)} \\ &= [\overline{\Sigma}^{-1} - \frac{1}{R} \{\Sigma_\delta^{-1} + (F^{\mathrm{T}})^{-1} \Sigma_\eta^{-1} F^{-1}\}]^{-1} \overline{\Sigma}^{-1} \\ &= [\overline{\Sigma}_\varepsilon^{-1} + (F^{\mathrm{T}})^{-1} \Sigma_\zeta^{-1} F^{-1} + \frac{r}{R} \{\Sigma_\delta^{-1} + (F^{\mathrm{T}})^{-1} \Sigma_\eta^{-1} F^{-1}\}]^{-1} \overline{\Sigma}^{-1} \text{ by (A12).} \end{aligned}$$

Note that S_i, Σ_i^{-1}, and $\overline{S}^{-1}$ in (A10) are always positive definite. Under the assumption of common eigenvectors, we can factor out the eigenvector matrix in a way similar to equation (A28). Then it is easy to see that $\overline{\Sigma}$ defined in (A11) is positive definite without the knowledge of the positive/negative definiteness of G. This implies that $\overline{\Sigma}_\varepsilon^{-1}$ is also positive definite, and so is G in the above equation.

Finally, we show that A_2 is negative definite. By (A18),

$$
\begin{aligned}
A_1 F^{-1} &= \frac{R}{r} I - G \\
&= G[\frac{R}{r} G^{-1} - I] \\
&= G[\frac{R}{r} I - \frac{1}{r}\overline{\Sigma}\{\Sigma_\delta^{-1} + (F^{\mathsf{T}})^{-1}\Sigma_\eta^{-1}F^{-1}\} - I] \text{ by (A48)} \\
&= r^{-1} G[I - \overline{\Sigma}\{\Sigma_\delta^{-1} + (F^{\mathsf{T}})^{-1}\Sigma_\eta^{-1}F^{-1}\}] \\
&= r^{-1} G\overline{\Sigma}[\overline{\Sigma}^{-1} - \{\Sigma_\delta^{-1} + (F^{\mathsf{T}})^{-1}\Sigma_\eta^{-1}F^{-1}\}] \\
&= r^{-1} G\overline{\Sigma}[\overline{\Sigma}_\varepsilon^{-1} + (F^{\mathsf{T}})^{-1}\Sigma_\zeta^{-1}F^{-1}].
\end{aligned}
$$

Since A_1 is negative definite and the right-hand side is positive definite, we conclude that F^{-1} is negative definite. Then the right hand side of (A47) is positive definite and so is B_2. Then (A46) says A_2 is negative definite. So the square bracket in the right-hand side of (A45) is positive definite. Obviously C_0^j is positive definite. Therefore, $Cov(\Delta\widetilde{\Pi}_t^j, \Delta\widetilde{P}_t^{\mathsf{T}})$ is positive (negative) definite if and only if $\overline{\Sigma}^j - \overline{\Sigma}$ and equivalently $\overline{\Sigma}_\varepsilon^j - \overline{\Sigma}_\varepsilon$ is positive (negative) definite (see the definitions of $\overline{\Sigma}_\varepsilon^{-1}$ and $(\overline{\Sigma}_\varepsilon^j)^{-1}$ in (A12) and (A25)). Because a positive (negative) definite matrix has all its diagonal elements positive (negative), this implies that group j that is less (better) informed than the market average ($\overline{\Sigma}_\varepsilon^j - \overline{\Sigma}_\varepsilon$ is positive (negative) definite) tends to increase (decrease) its holding of a security when its price has appreciated, and vice versa. ■

B. Multisecurity Examples

This appendix provides graphs for partial-information equilibria of a two-security model. Each graph is explained in its caption.

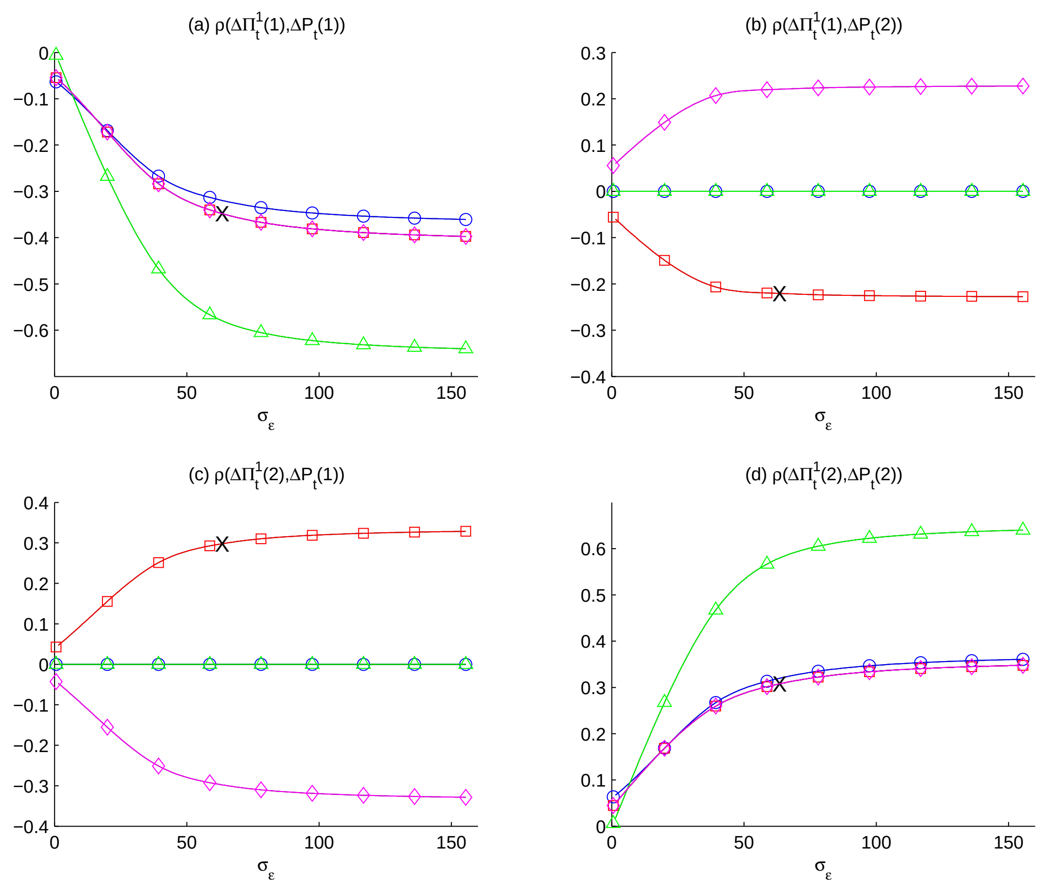

Figure 10: Correlation between the net flow of group-1 agents, $\Delta\widetilde{\Pi}_t^1(n)$, and the price change, $\Delta\widetilde{P}_t(l)$, in partial-information equilibria of a two symmetric-security model with two groups, where $n, l = 1, 2$ denote securities. Group-1 agents are on average better informed about the first security, and less informed about the second security, than group-2 agents in that $\overline{\Sigma}_\varepsilon^1 = \sigma_\varepsilon^2 \begin{pmatrix} 1/2 & 0 \\ 0 & 2 \end{pmatrix}$. The markers represent the following equilibria: stars: low volatility, low correlation. Squares: high volatility, high correlation. Circles: high volatility, low correlation. Crosses: high volatility, negative correlation. Point A gives Shiller's (1981b) aggregate volatility estimate, 69.4, at $\sigma_\varepsilon = \sigma_{\varepsilon 0} \equiv 62.2$. Parameter values: $\Sigma_\delta = 16.5^2 I$, $\Sigma_\eta = .00587^2 I$, $\overline{\Sigma}_\varepsilon = \sigma_\varepsilon^2 I$, $\Sigma_\zeta^{1/2} = 4\sigma_\eta$, $r = 5\%$ per annum or $1.05^{10} - 1$.

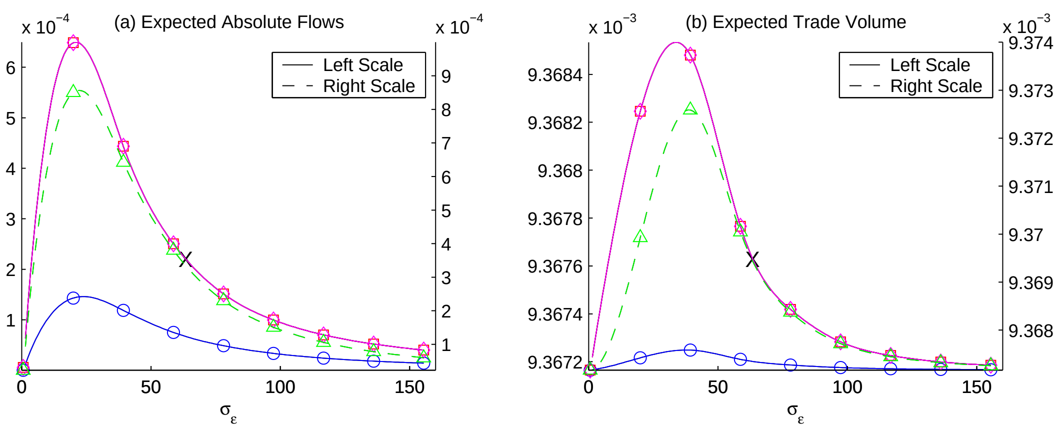

Figure 11: Trading-volume measures in partial-information equilibria of a two symmetric-security model with two groups. Panel (a): the expected absolute flow, U, and Panel (b): expected volume, V, of the first security. Group-1 agents are on average better informed about the first security, and less informed about the second security, than group-2 agents in that $\overline{\Sigma}_{\varepsilon}^{1} = \sigma_{\varepsilon}^{2} \begin{pmatrix} 1/2 & 0 \\ 0 & 2 \end{pmatrix}$. The markers represent the following equilibria: stars: low volatility, low correlation. Squares: high volatility, high correlation. Circles: high volatility, low correlation. Crosses: high volatility, negative correlation. Point A gives Shiller's (1981b) aggregate volatility estimate, 69.4, at $\sigma_{\varepsilon} = \sigma_{\varepsilon 0} \equiv 62.2$. Parameter values: $\Sigma_{\delta} = 16.5^2 I$, $\Sigma_{\eta} = .00587^2 I$, $\overline{\Sigma}_{\varepsilon} = \sigma_{\varepsilon}^2 I$, $\Sigma_{\zeta}^{1/2} = 4\sigma_{\eta}$, $r = 5\%$ per annum or $1.05^{10} - 1$.

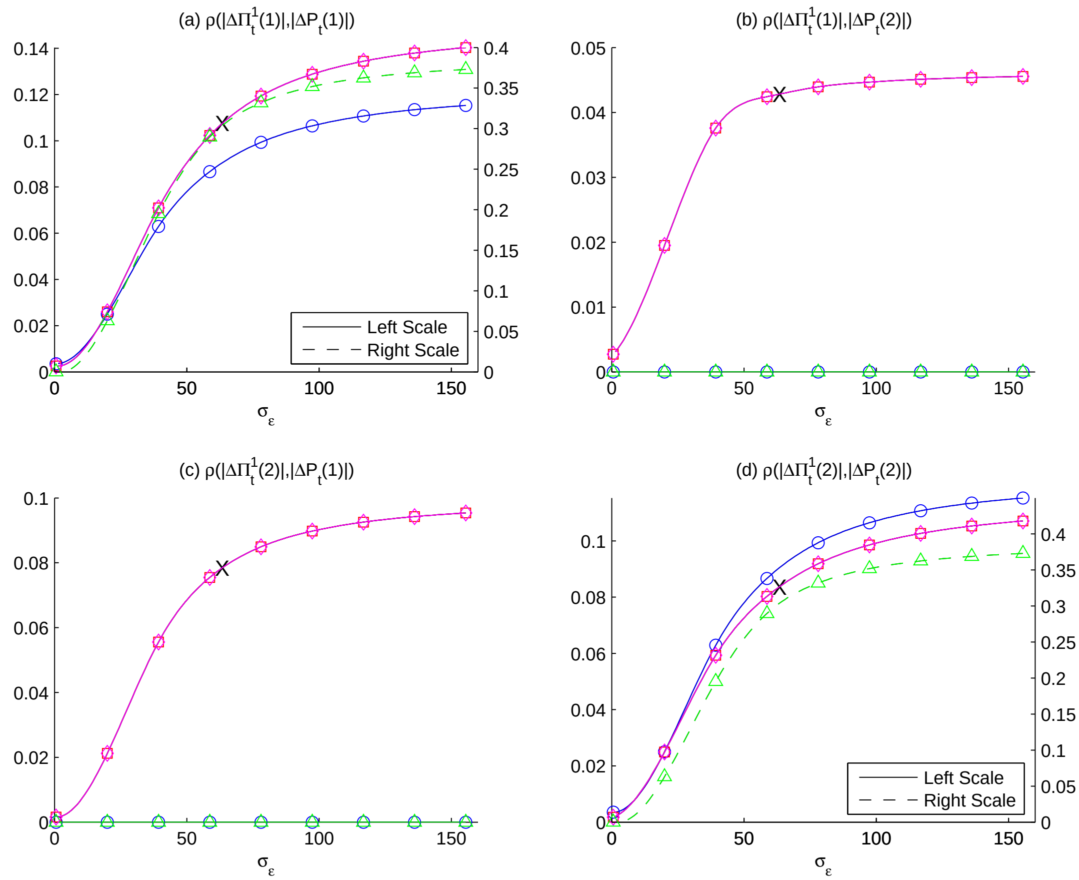

Figure 12: Correlation between the absolute flow of group-1 agents, $\widetilde{U}_t(n) = |\widetilde{\Pi}_t^1(n)|$, and the absolute price change, $|\Delta \widetilde{P}_t(l)|$, in partial-information equilibria of a two symmetric-security model with two groups, where $n, l = 1, 2$ denote securities. Group-1 agents are on average better informed about the first security, and less informed about the second security, than group-2 agents in that $\overline{\Sigma}_\varepsilon^1 = \sigma_\varepsilon^2 \begin{pmatrix} 1/2 & 0 \\ 0 & 2 \end{pmatrix}$. The markers represent the following equilibria: stars: low volatility, low correlation. Squares: high volatility, high correlation. Circles: high volatility, low correlation. Crosses: high volatility, negative correlation. Point A gives Shiller's (1981b) aggregate volatility estimate, 69.4, at $\sigma_\varepsilon = \sigma_{\varepsilon 0} \equiv 62.2$. Parameter values: $\Sigma_\delta = 16.5^2 I$, $\Sigma_\eta = .00587^2 I$, $\overline{\Sigma}_\varepsilon = \sigma_\varepsilon^2 I$, $\Sigma_\zeta^{1/2} = 4\sigma_\eta$, $r = 5\%$ per annum or $1.05^{10} - 1$.