Monetary and Implicit Incentives of Patent Examiners

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Abstract

Often accused of granting questionable patents, examiners might lack proper incentives to carefully scrutinize patent applications. We analyze their examination and granting behavior in presence of different incentive schemes that reward examiners based on rejected and/or accepted patents. Our findings suggest that, for a given probability of random audit by the PTO, a dual regime (based on both accepted and rejected patents) does not provide more incentive than a salary based on rejected patents. An optimal probability of random audit chosen by the PTO is often too high compared to the first best, while the examiner chooses a suboptimal examination effort's level. Lastly, we study the effect of career concerns on the granting behavior of examiners. We find that monetary and implicit incentives induce patent examiners to intensify their search effort. Furthermore, a marginal increase of the random audit might reduce examiners' effort in the presence of career concerns.

Keywords: Patent Examiners; Career Concerns

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1 Introduction

Patent applications and grants have increased at an unprecedented pace over the last decades. This raise in the number of patents is associated with many criticisms concerning the functioning of patent offices, and examiners are often accused of granting patents of questionable validity. Even though some wrongly patented innovations have no value, others are valuable and, therefore, are costly for society and harm competitors.[1] In the U.S., the Patent and Trademark Office (PTO) is aware of these quality concerns and, in 2015, has started its "Enhanced Patent Quality Initiative," along with modernization plans.[2] Reports on the progress of these plans are regularly published by the U.S. Government Accountability Office (GAO).

Many of the proposed reforms[3] acknowledge that patent applicants and competitors may have better information than the PTO about the innovation and, therefore, should be involved in the patent granting process. However, even though applicants might be responsible for not providing (and possibly not searching) information useful to judge the novel content of their innovation,[4] patent examiners might also have their share of responsibility. Hence, it is important to have a better understanding of the mechanism by which patents are granted, and of the functioning of the internal organization of the PTO.[5] Since there are patents on obvious innovations (e.g., crustless peanut butter and jelly sandwich), it is legitimate to wonder how examiners grant patents on innovations that seem to have serious flaws. One of the explanations might be that they lack incentives to make the appropriate effort to search for information that will prove the innovation cannot be patented.

Even though the examination process is fairly standard,[6] it is highly dependent on the

[1] See, for instance, Farrell and Shapiro (2008), Gallini (2002), Hall (2006), Hall and Harhoff (2004).

[2] See https://www.uspto.gov/patent/patent-quality (last accessed September 2018).

[3] Some reforms involve competitors, as it is the case of an opposition system, which is equivalent to the European system (Hall, 2006; Lemley, Lichtman and Sampat, 2005). Other reforms involve applicants such that the two-speed examination process in which patent applicants can pay more to get a better scrutiny (Atal and Bar, 2014). This process, also called gold-plate patents by Lemley, Lichtman and Sampat (2005) or super patent by Lemley and Shapiro (2005), is currently implemented at the PTO.

[4] See Atal and Bar (2010), Langinier and Marcoul (2016) for analyses of strategic behavior of patent applicants while searching and revealing prior art information.

[5] See, for instance, Eckert and Langinier (2014), Hall and Harhoff (2012), and Langinier and Marcoul (2019).

[6] It is documented in the Manual of Patent Examining and Procedure, see the USPTO website at

examiners' skill.[7] Their salaries are tied to the number of applications they process: they have production quotas to meet, and earn bonuses when they exceed their quotas by at least 10% with few errors.[8] In order to determine the rate of error of each examiner, the PTO performs random check on about 2 to 3 percent of allowed patents to assess whether the claims in the patent meet the statutory criteria.[9] If a mistake has been made, the case is re-opened and the application is likely to be rejected (NAPA, 2005).[10] Production goals are based on the number of patent applications examiners review, their experience (position in the agency) and their field of expertise (GAO, 2007).

Importantly, examiners are never liable in case the patents they granted are invalidated in court and "there are no negative consequences for examiners who produce low-quality work" (GAO, 2005). Furthermore, according to Quillen et al. (2001) in the U.S. between 87% and 95% of applications are granted a patent.[11] This suggests that they are mainly rewarded on granted patents,[12] and do not bear the aftermath of granting questionable patents.

In a perfect patent system, examiners would never issue patents that would be later on invalided in court. They would make appropriate efforts, which would be observed by the PTO. However, this is not the case. Due to lack of resources (both in terms of examiners' effort and PTO monitoring), such a perfect system is impossible to implement. There is a moral hazard problem since the PTO does not know how much effort an examiner makes on every

http://www.uspto.gov/web/offices/pac/mpep/.

[7] The process consists of regular check of legal formalities, and search of prior art –the existing set of related inventions– which is where the expertise of examiners is the most important. According to Cockburn, Kortum and Stern (2003), there is a strong heterogeneity among patent examiners.

[8] They process an average of 87 applications per year and they devote about 19 hours per application spread over 3 years. According to the PTO, about 60% of eligible examiners get bonuses. Examiners claim that their quotas are too high even though more than half of them go over at least 10% of their quotas (GAO, 2007).

[9] According to the PTO, the error rate has been relatively constant over time (in 2000 it was 6.6%, 4.2% in 2002 and 5.3% in 2004) and it varies among technological centers: from 2.5% to 9% in different centers in 2004.

[10] Within the period 2000 and 2004, 302 to 401 applications have been reopened.

[11] Other studies find lower grant rates (Carley, Hedge and Marco, 2015). Empirical evidence suggests that, depending on the technological area, the likelihood of obtaining a patent varies (Lemley and Sampat, 2008), the number of citations varies (Popp et al., 2004) and there exists substantial variation of validity after a patent has been challenged (Cockburn et al., 2003).

[12] Recent studies show that examiners tend to grant rather than reject patents (Oh and Kim, 2017; Frakes and Wasserman, 2017).

application. The PTO only observes whether a patent has been granted or was ultimately rejected or abandoned.

Examiners should be responsible for granting invalid patents and, to some extent, be rewarded according to the social value they help create, or be punished for the social loss. Even though it might be difficult to implement such a system, at least their salary schemes should be set to better reflect these objectives.

Furthermore, the PTO has a hard time to hire and retain a skilled workforce, and has suffered from a high turnover level (GAO 2005, 2007). The reasons for which examiners leave the PTO are not clear.[13] It might be that examiners develop skills that are sought by private companies that value not only their technical skills but also their sound knowledge of the patent system. This raises the problem of career concerns, and the problem of knowing whether patent examiners behave strategically. When they process applications, they might account for the signal they send to the private job market.

Our goal is to provide answers to the following questions: How should examiners be rewarded? How much effort an examiner will make to acquire invalidating information? How will career concerns affect their incentives to search for information?

To address these questions, we design a simple model in which examiners are rewarded according to different incentive schemes. The PTO offers a salary scheme based on different observable outcomes. When an examiner receives a patent application, he must decide how much effort to put into it. He does not know the quality of the innovation and has only prior beliefs. His search of information allows him to find that the innovation should be patented (he cannot find any evidence against patenting), should not be patented (he finds strong evidence against patenting) or should not be patented as it stands, but can be patented with considerable narrowed claims. Therefore, by exerting effort he is able to reject non-patentable innovations, or to narrow the patent scope of too broad applications. A patent that has been granted can be audited by the PTO. We make the simplifying assumption that the PTO systematically invalidates patents granted on non-patentable innovations, but do not invalidate patents issued

[13] The office management pretends examiners leave for personal reasons, whereas patent examiners claim they leave because their production goals are too high, and have not changed over the last 30 years (GAO, 2007). In 2010, the quotas have been changed. In 2015, an extensive web-based survey of 3,336 patent examiners has been conducted by GAO, but there were very few questions about retention (GAO, 2016a, 2016b, 2016c).

on patentable innovations.

We propose a general setting in which the reward is based on both rejection and acceptance (dual regime). This encompasses the two extreme cases where the reward is based only on rejection of an application, and only on acceptance of the patent.

We first consider that the probability of audit is exogenous. For a small probability of audit, we find that an optimal organization would have large search effort levels while a dual regime jeopardizes the search intensity. A marginal increase of a larger probability of audit reduces the equilibrium reward paid to the examiner, while the examiner's effort increases. In equilibrium, the effort is less intense under an acceptance regime than under a rejection regime. We find that the PTO is better off under a rejection regime, while the examiner might be better off under an acceptance regime. Overall, a salary based on both rejected or accepted patents does not provide more incentives than a salary based only on rejected patents.[14]

Second, we consider that the PTO can choose both the salary and the probability of audit. Our findings suggest that the PTO tends to choose a too high random audit while the examiner will exert a suboptimal effort level.

We then study how the existence of career concerns shapes the incentives of the patent examiner. In line with the literature on career concerns (Holmstrom, 1982, 1999), we assume that the examiner can be skilled or not, and that, initially, his talent is unknown not only to the market but also to himself. The higher the examiner's talent, the higher the chances of gathering relevant information. The market does not observe the examiner's talent directly, but might learn about it when legal patent challenges are initiated. The market observes patents that are granted (which does not provide information about the quality of the examiner's scrutiny) but also, and most importantly, granted patents that are taken to court, and the outcome of the trial. From the observation of patents that have been challenged in court and not invalidated, the market updates its beliefs regarding the examiner's talent. The market offers a salary to

[14] Two recent empirical studies by Doran and Webster (2019) and Tabakovic and Wollmann (2018) emphasize the relationship between the examiner and the patent attorney who acts as an "advocate" of the applicant. Both papers stress the role and the influence that patent legal firms have in increasing the likelihood of a patent grant decision as well as increasing the set of claims of the granted patent. Our finding that a rejection regime should be preferred by the PTO can only be reinforced if one takes into account the existence of these influence activities aimed at increasing acceptance.

the examiner based on the observation of patents challenged in court and not invalidated. In this setting, we find that examiners with career concerns attempt to influence the market's beliefs by exerting more effort. Examiners with career concerns provide higher efforts than they would absent any reputational concerns. These findings are aligned with some findings in the literature, in particular, Fama (1980) who argues that career concerns will induce efficient behavior and discipline manager's behavior. More generally, Holmstrom (1982, 1999) shows that career motives can be either beneficial or detrimental to organizations depending on the context. In terms of policy implications, our analysis implies that the self-interested behavior of career-conscious examiners reduces the granting of non-deserving patents and, therefore, their objectives are aligned with the PTO's objective.

The paper is organized as follows. In section 2, we present the model. Section 3 is devoted to the analysis of different incentive schemes offered by the PTO when the random check is exogenous. In Section 4, we analyze the case of an endogenous random check. In Section 5, we study how career concerns may affect the examiner's level of effort, and the incentive scheme offered by the PTO. Section 6 concludes.

2 The Model

We consider a sequential game with two players, the PTO and a patent examiner, in which the PTO offers a salary scheme to the examiner based on observable outcomes. If the examiner accepts the salary scheme, he receives a patent application and makes a costly search effort to assess the patentability of the innovation.[15] Based on the information gathered during the search process, the examiner decides whether to grant a patent or to reject the application (all or part of the innovation), which is observed by the PTO.

The PTO does not observe the examiner's effort and, at the outset, neither the examiner nor the PTO knows the patentability of the innovation; they only have (common) prior beliefs. The innovation is patentable with probability γ or non-patentable *as it stands* with probability $(1-\gamma)$. Among the latter innovations, some are not patentable at all with probability p, whereas others are partly patentable with probability $(1-p)$. Hence, an innovation is patentable with

[15] In the U.S. to be patentable an innovation must be new, non-obvious and useful.

probability γ, non-patentable with probability $(1-\gamma)p$, and partly patentable with probability $(1-\gamma)(1-p)$.

As part of a random patent inspection program, once a patent has been granted, the PTO challenges it with probability π. Hence, a wrongly granted patent could be rejected if challenged. Initially, we consider that the random inspection is exogenous; we then endogeneize it.

2.1 Timing

The timing of events goes as follows in the basic model, where the random check π performed by the PTO is exogenous.

- In the first period, the PTO offers an employment contract entailing a salary scheme to the examiner. The examiner accepts or refuses the contract.
- In the second period, if the examiner accepts the contract, he is assigned a patent application whose patentability is unknown, and makes a costly effort $e \in (0, 1)$ to judge whether it is patentable or partly patentable. If he discovers that the innovation is not patentable, he refuses to grant a patent. Otherwise, he grants a patent.
- In the third period, the granted patent is challenged by the PTO with probability π.
- In the fourth period, the salary scheme is paid accordingly.

In the first extension of the model, we endogenize the random check. The timing of event is similar to the previous one except that, in the first period, the PTO chooses both the salary scheme and the probability to perform a random check. In the second extension, we propose another variant of the model in which we introduce career concerns. To do so, we add a fifth period in which the market might offer a salary to the examiner.

We now detail the examination process, before presenting the salary scheme.

2.2 Patent Examination Process and Social Patent Value

The examiner receives a patent application and exerts a costly search effort $e \in (0, 1)$ to find relevant information to be able to judge the novel content of the innovation. Depending on the gathered information, he decides whether to grant a patent or not. When a patent is not issued, the examiner must provide information that proves that the innovation is not patentable. When he decides to grant a patent, he includes all the information he found in the patent description.[16] Let the cost of search effort be

$$C(e) = \frac{1}{2}\eta e^2, \tag{1}$$

where $\eta > 1$. The examiner's effort e generates a probability $q(e)$ of finding invalidating information (when the innovation is non-patentable) where $q'(e) > 0$, $q''(e) \leq 0$, $q(0) \geq 0$. To simplify we assume that $q(e) = e$. After exerting an effort e, with probability $(1-\gamma)pe$ the examiner finds that the innovation is non-patentable and rejects it, and with probability $(1-\gamma)(1-p)e$ he finds it is partly patentable, and issues a patent on the patentable part. With probability $\gamma + (1-\gamma)(1-e)$ he does not find any invalidating information (either because it does not exist with probability $\gamma e + \gamma(1-e)$ or because he did not search enough with probability $(1-\gamma)(1-e)$) and grants a patent.

When a patent has been issued, with probability π it will be randomly checked by the PTO. If the innovation is patentable, it will be discovered by the PTO and, therefore, the patent will not be invalidated. However, if the innovation has been mistakenly patented, the PTO will discover it and will reject the patent. If only part of the innovation is not patentable, the PTO will also find it, and will reduce the patent scope to its patentable part. This random check is performed by an experienced examiner and is therefore costly.

From the PTO's viewpoint, an invalidated patent provides society with a social value of zero. The social value of a patentable innovation that is granted a patent is $\overline{W}$, whereas a non-patentable innovation that is wrongly issued a patent has a social value $\underline{W}$ where $\overline{W} > 0 > \underline{W}$. Furthermore, an innovation that is non-patentable as it stands, but is partly patentable will have a social value $\overline{W}$ if it is granted a patent on the innovative part, whereas it has a social value $\underline{W}'$ if it is granted a patent on the entire innovation. As it is costly for society to grant patents on innovations that are only partially novel, we assume that $\overline{W} > 0 \geq \underline{W}' > \underline{W}$.

Two types of errors can be made by the examiner: a patent may be refused to a patentable

[16] In reality, the patent granting process takes on average three years. It is not uncommon for patent applications to be initially rejected. Then, the applicant provides more information or he narrows the claims. However, at the end of the process, many patents are granted, some with narrowed claims. Thus, here we simply consider the final outcome of a patent being granted or not at the end of the granting process.

innovation (type I error), or a patent may be granted to a non-patentable innovation (type II error). However, as many applications are issued a patent, type II error is more likely to happen than type I error. Patentable innovations that are not issued a patent have a negative impact on society, but patents granted to non-patentable innovations impose an ever higher cost on society. A high rate of type II error will not only reduce competition, it will also increase litigation costs and provide incentives to patent dubious innovations (Bessen and Meurer, 2008; Gilbert, 2011). In our setting, we therefore consider that the PTO is concerned with preventing type II errors.

To summarize, the granting decision (issue or reject a patent) and the social value created by the patented or non-patented innovation are represented in the following table. Columns represent the types of innovation (patentable with probability γ, non-patentable with probability $(1-\gamma)p$, partly patentable with probability $(1-\gamma)(1-p)$) and the rows represent the probability of finding invalidating information, e, or not, $1-e$.

	Patentable γ	Non Patentable $(1-\gamma)p$	Partly Patentable $(1-\gamma)(1-p)$
e	Issue, $\overline{W}$	Reject, 0	Issue partly, $\overline{W}$
$1-e$	Issue, $\overline{W}$	Issue, $\underline{W}$	Issue, $\underline{W'}$

2.3 PTO Salary Scheme

The PTO does not know the innovation patentability and it cannot observes the examiner's effort. It only observes that a patent has been refused or issued. In the latter case, the PTO performs a costly random check on the validity of the patent with probability π. We therefore consider that the PTO offers a salary scheme based on both rejection and issuance of a patent.[17]

[17]In fact, the PTO should be able to contract upon some characteristics of the patent, like for instance the number of citations. Note that making the examiner's incentives contingent upon the number of citations included in the patent document might create other problems such as "citation stuffing" (e.g., overcitations). To keep our model simple we assume that when the contract is based on acceptance, it does not depend on the number of citations, and the examiner receives a fixed salary.

Conditional on making an effort e, the probability that the examiner rejects a patent is[18]

$$Prob(reject \mid e) = (1-\gamma)pe. \tag{2}$$

Conditional on making an effort e, the probability that the examiner issues a patent is[19]

$$Prob(issue \mid e) = \gamma + (1-\gamma)(1-p)e + (1-\gamma)(1-e). \tag{3}$$

An examiner who exerts effort e makes a decision to patent the innovation (fully or partly) with probability $e\gamma + e(1-\gamma)(1-p)$ as he cannot find any invalidating information on a patentable innovation or on its patentable part. On the other hand, if he does not exert enough effort, he will not find invalidating information and will grant a patent (with probability $(1-e)\gamma+(1-e)(1-\gamma)$).

As we consider a salary scheme based on both issued and rejected patents, we define $\beta \in [0,1]$ as being the weight put on a rejection and $(1-\beta)$ the weight put on an issued patent. In order to reject an application, the examiner must provide information that unambiguously shows that the innovation cannot be patented (e.g., it already existed, had already been patented). In our setting, the examiner cannot refuse a patent to a patentable innovation, as it is based on hard evidence. Hence, the examiner should be rewarded for finding (hard) information that proves that the innovation cannot be patented. When the examiner issues a patent, he can make a mistake and issue a patent to a non-patentable innovation. If this happens, the patent might be randomly checked by the PTO, and the error might be corrected. The PTO needs to account for that possibility, and reward the examiner for patents that are not invalidated after the random check. In this setting, the expected salary of the examiner is

$$S(R) = \beta S_r(R) + (1-\beta)S_a(R), \tag{4}$$

with

$$S_r(R) = Prob(reject \mid e)R, \tag{5}$$

[18] However, the probability of a patent being rejected conditional on the effort e and the random check π is $Prob(reject \mid e, \pi) = (1-\gamma)pe + \pi(1-\gamma)p(1-e)$, where the second part corresponds to a patent that should have been rejected by the examiner but was not.

[19] The probability of a patent being issued is $Prob(issue \mid e, \pi) = \gamma+(1-\gamma)(1-p)e+(1-\gamma)(1-e)[(1-\pi)+\pi(1-p)]$, where the random check performed by the PTO allows to rectify some of the mistakes made by the examiner.

being the salary received by the examiner if he rejects a patent, and

$$S_a(R) = [\underbrace{(1-\pi)Prob(issue \mid e)}_{(i)} + \underbrace{\pi(\gamma + (1-\gamma)(1-p)e)}_{(ii)}]R, \tag{6}$$

the salary he receives if he grants (or partly grants) a patent, where R is the reward provided by the PTO, $Prob(reject \mid e)$ and $Prob(issue \mid e)$ are respectively defined by (2) and (3). The first part (i) of the salary (6) corresponds to the case where there is no random check with probability $(1-\pi)$, and thus the salary only depends on whether the patent is issued. The second part (ii) corresponds to a successful random check with probability π, in which case a patent will be granted to patentable or partly patentable innovation.

The parameter β in (4) lies at the core of our model: should the PTO rewards examiners for granting patents or for rejecting patent applications? If $\beta = 0$, the regime is such that the examiner is only rewarded on granted patents; if $\beta = 1$, the examiner is only rewarded on rejected patents. Thus, this general setting encompasses the two extreme cases where only rejections or allowances are rewarded.

Note that not only random checks enhance the quality of the patents awarded, they also have an incentive effect on the examiner. Indeed, when the audit reveals that the application should have been rejected, the examiner receives a lower salary as evidenced by $\partial S/\partial\pi < 0$ where $S(.)$ is defined by (4). Thus, a higher audit intensity π hurts the examiner who can then mitigate this effect by increasing his effort ($\partial S^2/\partial\pi\partial e > 0$). As it will become clear, this cross-effect plays an important role in the optimal incentive scheme when π becomes endogenous.[20]

The case where $0 < \beta < 1$ is arguably the closest to the current system in which examiners' salaries are (partly) based on both granted patents and rejected applications. It is however not clear how issued patents are weighted against rejected applications in the calculation.[21]

3 Exogenous Random Check

We first consider the case in which the probability of random check is exogenously set. In the U.S. patent system, only 2% of granted patents are subject to a random inspection. An

[20] In the absence of random check ($\pi = 0$), it seems intuitive that rewarding solely on acceptance is dominated since all accepted patents will give rise to a reward. However, for $\pi > 0$, the answer seems no longer obvious.

[21] See GAO (2007) for a description of the salary schemes.

experienced reviewer must assess whether an examiner made an error in at least one claim that was allowed in the patent, and if it is determined that an error was made, the case is reopened. According to de Rassenfosse, Jaffe and Webster (2016) more than 15% of granted patents in the U.S. are dubious patents, which suggests that the low percentage of randomly checked patents might not be optimally defined. Therefore, we initially analyze the baseline model in which the probability π is given before exploring the more complex case of an endogenous random check.

For a given incentive scheme provided by the PTO, the examiner chooses the effort level $e \in (0,1)$ that maximizes his utility function, i.e.,

$$\max_{e} U = \{S - C(e)\}, \tag{7}$$

where S is defined by (4), and $C(e)$ is the disutility of effort defined by (1). Let $e(R)$ denote the utility maximizing effort level that depends on the level of reward R provided by the PTO.

The PTO seeks to provide the optimal level of reward to the examiner by solving the following maximization program

$$\begin{cases} \max\limits_{R} G = \{W - S - C_\pi\} \\ \qquad s.t.\ U \geq 0 \end{cases} \tag{8}$$

where S is defined by (4), U is the examiner's utility, C_π is the cost of a random audit[22] and

$$W = \underbrace{\gamma\overline{W}}_{\text{patentable}} + (1-\gamma)[\underbrace{e(1-p)\overline{W}}_{\text{partly-patentable}} + \underbrace{(1-e)(1-\pi)(p\underline{W} + (1-p)\underline{W}')}_{\text{error}} + \underbrace{(1-e)\pi(1-p)\overline{W}}_{\text{successful random audit}}]. \tag{9}$$

The rational for the PTO's gross benefit (9) is the following. If the innovation is patentable, the examiner will never find otherwise and the expected social value is $\gamma\overline{W}$. If the innovation is partly patentable, the examiner can issue a patent on its patentable part with probability $e(1-p)$. In this case, the social value of a patent granted to the patentable part of the innovation is equivalent to the social value of a patent granted to a patentable innovation. If the innovation is non-patentable, the examiner can make a mistake with probability $(1-e)$ and grant a patent to a non-patentable innovation, which will not be invalidated by the PTO with probability $(1-\pi)$. If the innovation is partly patentable, the examiner can also make a mistake and issue a patent on the entire innovation that will not be invalidated later on by the PTO with probability

[22] This cost will be defined more precisely when the random check is endogenized in Section 4.

$(1-e)(1-p)(1-\pi)$. Lastly, if the PTO finds that a partly patentable innovation has been entirely granted a patent, it can reject the non-patentable part of the innovation and then grant a patent to the patentable part.

We first consider the benchmark case in which outcome and effort are perfectly observable by the PTO. Then, we consider the incentive scheme defined above.

3.1 First Best Outcome

As a benchmark, we consider the first-best outcome in which the PTO observes the examiner's effort and, therefore, chooses it optimally. The PTO maximizes the social welfare by considering its expected benefit function plus the examiner's utility function, which is equivalent to solving

$$\max_e \{W - C(e) - C_\pi\},$$

where W is defined by (9), and $C(e)$ by (1). It follows that the first-best optimal effort of the examiner for any given π is

$$e^o = \min\{\frac{1}{\eta}\omega(1-\pi), 1\}, \tag{10}$$

where

$$\omega = (1-\gamma)\left[(1-p)(\overline{W} - \underline{W}') - p\underline{W}\right] > 0 \tag{11}$$

represents the expected gain from preventing type II errors or, in other words, from avoiding to wrongly grant a patent to a non-patentable (or partly patentable) innovation.

When the intensity of a random check increases, the examiner's first-best effort level is reduced ($\partial e^o/\partial\pi \leq 0$) as the random check and the examiner's effort are substitutes. If the expected gain from avoiding to wrongly grant a patent is large ($\omega \geq \eta$), in absence of random check ($\pi = 0$), the PTO wants to pick the maximal effort's level, $e^o = 1$. In that case, the random check cannot be used as a substitute to the effort, and it is worth trying to avoid to wrongly grant patents. As the probability of audit increases, an optimal effort strictly smaller than 1 is implemented. Formally, there exists a cutoff value

$$\pi_o \equiv \frac{\omega - \eta}{\omega}, \tag{12}$$

which is positive if $\omega \geq \eta$ such that for values of $\pi > \pi_o$, the first-best effort level is not a corner solution, i.e., $e^o < 1$. However, if the gain from avoiding to wrongly grant a patent is small

($\omega < \eta$), it is too costly to have an effort $e^o = 1$ even when $\pi = 0$, and, thus, the optimal effort is always smaller that 1.

As the cost of effort increases, the PTO reduces the examiner's effort ($\partial e^o/\partial\eta < 0$). As the probability of having a patentable innovation increases, the first-best examiner's effort is reduced ($\partial e^o/\partial\gamma < 0$). Indeed, when the innovation is patentable, a higher effort level will not allow to find invalidating information, because it does not exist. As the gain from avoiding to wrongly grant a patent to a non-patentable innovation increases, so does the effort ($\partial e^o/\partial\omega > 0$). However, as the probability of having a non-patentable innovation (conditional on being non-patentable as it stands) increases, the first-best effort level increases (respectively, decreases) if $\partial\omega/\partial p > 0$ (resp., $\partial\omega/\partial p \leq 0$) or, equivalently, if $\overline{W} - \underline{W}' < -\underline{W}$ (resp., $\overline{W} - \underline{W}' \geq -\underline{W}$). In other words, if the benefit from avoiding an error for a non-patentable innovation ($-\underline{W}$) is higher than the benefit from avoiding an error for a partially patentable innovation ($\overline{W} - \underline{W}'$), the first-best optimal effort increases as it is more likely that the innovation is non-patentable.

For $e^o < 1$, the first-best benefit function of the PTO can be written as

$$G^o = \overline{W}(1 - p(1-\gamma)) - \omega(1-\pi)(1 - \frac{1}{2\eta}\omega(1-\pi)) - C_\pi.$$

An increase in the probability of having a patentable innovation or the intensity of the random audit increases the benefit of the PTO ($\partial G^o/\partial y > 0$ for $y = \gamma, \pi$). On the other hand, an increase in the cost of effort decreases the PTO's benefit ($\partial G^o/\partial\eta < 0$). As the probability of having a non-patentable innovation increases, the benefit of the PTO decreases if $\partial\omega/\partial p > 0$.

For $e^o = 1$, the PTO's benefit function becomes $G^o = \overline{W}(1 - p(1-\gamma)) - \eta/2 - C_\pi$.

This first-best outcome provides a benchmark from which we can compare and contrast our results when the effort is non-observable. We now detail the salary scheme when the PTO cannot observe the examiner's effort.

3.2 Dual Regime

The dual regime corresponds to the incentive scheme based on both issued and rejected patents, with $0 < \beta < 1$. We will also consider the two extreme cases are when the PTO only rewards the examiner on issued patents ($\beta = 0$) and on rejected patents ($\beta = 1$). The examiner gets a reward R for rejecting or granting a patent that will not be invalidated if randomly audited, and his expected salary is defined by (4). The examiner solves (7), which yields, for any given R, the following effort level

$$e(R) = \frac{1}{\eta}(1-\gamma)[\beta p + (\pi - p)(1-\beta)]R. \tag{13}$$

The examiner makes a positive effort if $\beta p + (\pi - p)(1-\beta) > 0$ or, equivalently, if $\pi(1-\beta) - p(1-2\beta) > 0$. This is always the case if $\beta \geq 1/2$ (more weight on rejected patents) or if $\beta < 1/2$ (more weight on issued patents) and $\pi \geq \underline{\pi}_e(\beta)$ (relatively high random check) where

$$\underline{\pi}_e(\beta) \equiv p\tfrac{1-2\beta}{1-\beta}.$$

In the complementary case, that is if more weight is put on issued patents ($\beta < 1/2$), and for a low value of the random check, or even in absence of random check ($0 \leq \pi < \underline{\pi}_e(\beta)$), the examiner would have no incentives to search for invalidating information, and thus his effort will be null. Random checks correct his lack of search incentives.

For a given reward R, the more likely the innovation is patentable, the lower the examiner's incentive to search for information ($\partial e(R)/\partial\gamma < 0$). Not surprisingly, the higher the reward R, the higher his search effort ($\partial e(R)/\partial R > 0$), and the higher the cost of effort, the lower his effort ($\partial e(R)/\partial\eta > 0$). Also, the higher the probability of a random check, π, the higher the effort ($\partial e(R)/\partial\pi > 0$). Furthermore, for $\pi < 2p$, the effort of the examiner is increasing with β. Thus, for a given R, the examiner intensifies his search effort when the PTO puts more weight on rejected patents rather than on issued patents.

Given $0 < \beta < 1$, the PTO seeks to provide the optimal reward level to the examiner by solving (8). For positive values of $e(R)$, we plug equation (13) into the maximization program of the PTO (8) to determine the optimal reward level R_b given to the examiner which is

$$R_b^* = \tfrac{1}{(1-\gamma)[\beta p + (\pi - p)(1-\beta)]}(\tfrac{1}{2}\omega(1-\pi) - \eta\Phi_b(\beta)), \tag{14}$$

where

$$\Phi_b(\beta) = \tfrac{\gamma + (1-\gamma)(1-\pi)}{2(1-\gamma)} \tfrac{1-\beta}{\beta p + (\pi - p)(1-\beta)}. \tag{15}$$

Notice that $\Phi_b(\beta) \geq 0$ if the conditions to have a positive (13) are satisfied, which are $\beta \geq 1/2$ or $\beta < 1/2$ and $\pi \geq \underline{\pi}_e(\beta)$. Furthermore, the reward R_b^* is strictly positive if $\frac{1}{2}\omega(1-\pi) - \eta\Phi_b(\beta) > 0$ or, equivalently, if $\pi \in [\pi_b'(\beta), \pi_b''(\beta)]$ where the cutoff values $\pi_b'(\beta)$ and $\pi_b''(\beta)$ are function of β and $\underline{\pi}_e(\beta) < \pi_b'(\beta)$ (see appendix for the calculations).

We discuss the reward R as a function of the PTO's emphasis on β. When $\beta = 1$ (reward based on rejected patents), $\pi_b'(1) < 0$ and $\pi_b''(1) > 1$ so that for any $\pi \in [0,1]$, the PTO will offer a positive reward. However, when $\beta = 0$ (reward based on acceptance), two sets emerge: for low values of π, $\pi \in [0, \pi_b'(0)]$ or larger values of π, $\pi \in [\pi_b'(0), 1]$, the PTO does not offer a reward R. For low values of π, the examiner will never respond to any level of reward as the chances to get caught in case of wrongly granting a patent are very small. Thus, the PTO does not offer any reward. For higher values of π, the PTO does not really want to induce any effort as the audit will correct any mistake.

There exists $\widetilde{\beta} > 1/2$ such that for $\beta \geq \widetilde{\beta}$, $\pi_b'(\beta)$ and $\pi_b''(\beta)$ always exist whereas for $\beta < \widetilde{\beta}$, these values exist only if $\omega > \omega''$ (see appendix for all the calculations). In words, this means that when the PTO puts more weight on rejected patents ($\beta \geq \widetilde{\beta}$), it will induce the examiner to make some effort to search for invalidating information. However, when more weight is put on accepted patents ($\beta < \widetilde{\beta}$), the PTO will induce the examiner to perform a positive effort when the gain from avoiding a mistake is large enough. Figure 1 represents the different first-best and optimal efforts areas in a graph (β, π).

Insert Figure 1

Whenever $R_b^* > 0$, in equilibrium the examiner exerts an effort

$$e_b^*(\beta) = \min\{\frac{1}{2\eta}\omega(1-\pi) - \Phi_b(\beta), 1\}, \tag{16}$$

which, in general, is suboptimal ($e_b^*(\beta) \leq e^o$), and his salary can be written as $S_b^* = \eta(e_b^*)^2 + \eta\Phi_b e_b^*$, for $e_b^*(\beta) < 1$. The examiner exerts no effort when $\beta < 1/2$ and $\pi < \underline{\pi}_e(\beta)$ and gets no salary as well. He also makes a null effort when he does not get any reward, i.e., for $\pi < \pi_b'(\beta)$ or $\pi > \pi_b''(\beta)$. Note that he can also exert an effort equals to 1 for large values of ω, for some π.

An increase in the probability of having a patentable innovation γ or in the cost of effort η reduces both the examiner's effort and his reward. As it is more likely that the innovation is patentable, the PTO reduces the reward given to the examiner who, in turn, exerts a lower level of effort. An increase in p decreases the effort (and the salary) if $\beta < 1/2$ and $\partial\omega/\partial p < 0$ (or, equivalently, $(\overline{W} - \underline{W}') > -\underline{W}$). If the benefit from avoiding an error for a non-patentable innovation is lower than the benefit from avoiding an error for a partially patentable innovation, and the PTO puts more weight on issued patents, then the effort of the examiner decreases when the likelihood of having a non-patentable innovation increases. On the other hand, if $\beta > 1/2$ and $\partial\omega/\partial p > 0$, the examiner intensifies his search effort. An increase in β increases the equilibrium effort level, which implies that a reward based only on acceptance ($\beta = 0$) induces a lower effort than a reward based only on rejected patent ($\beta = 1$).

We summarize these findings in the following Lemma.

Lemma 1 *If the incentive scheme is based on both issued and rejected patents for a given level of* π*,*

1. *the higher the probability of a patentable innovation* γ *and the higher the cost of effort* η*, the lower the search effort of the examiner;*

2. *the higher the weight on rejected patents* β*, the higher his search effort;*

3. *the higher the probability of a non-patentable innovation* p*, the lower (respectively, the higher) his effort if* $\beta < 1/2$ *and* $\partial\omega/\partial p < 0$ *(respectively,* $\beta > 1/2$ *and* $\partial\omega/\partial p > 0$*).*

Proof. All the proofs are relegated in the appendix. ■

Compared to the first-best effort level, these changes in exogenous variables have qualitatively similar impacts. The equilibrium effort (13) is always suboptimal and, for some parameter values, the first-best and optimal incentives are opposite as summarized in the following Lemma.

Lemma 2 *For* $\pi < \min\{\pi_b'(\beta), \pi_o\}$ *and* $\beta < 1$*, the first-best effort is maximum (*$e^o = 1$*) whereas no effort (*$e_b^*(\beta) = 0$*) is provided in the dual regime.*

For a small probability of audit, an optimal (first-best) organization would have large levels of search effort ($e^o = 1$). Yet, when the PTO cannot observe the examiner's effort, a dual regime jeopardizes search incentives. Hence, putting too much weight on rewarding issued patents without compensating with more random check does not provide adequate incentives to search for invalidating information.

As the probability of having an audit increases, the effort first increases and then decreases. If $\beta = 1$ (reward only based on rejected patents), the examiner's effort is affected by a change in π through the reward R. As the *ex post* screening π increases, the PTO reduces the reward given to the examiner who, in turn, reduces his effort. However, when $\beta < 1$, besides the effect through R, the examiner's effort is also directly affected by a change in π; as π increases the examiner has an incentive to intensify his search effort to avoid making a mistake.

We now analyze how the PTO's reward and the examiner's effort interact in equilibrium. For a given level of β, both effort and reward are inverted U-shape functions of π. We have seen that for small values of π ($\pi < \pi_b'(\beta)$) and for high values ($\pi > \pi_b''(\beta)$), both reward and effort are null, whereas for $\pi \in [\pi_b'(\beta), \pi_b''(\beta)]$, they are positive. We define $\pi_1 = \arg\max R_b^*$ and $\pi_2 = \arg\max e_b^*(\beta)$ and verify that $\pi_1 < \pi_2$. We show that first both reward and effort increase for $\pi \in (\pi_b'(\beta), \pi_1)$, then the reward decreases while the effort increases for $\pi_1 \leq \pi < \pi_2$, and finally both reward and effort decrease for $\pi \in (\pi_2, \pi_b''(\beta))$. As the likelihood of having a random check increases, it reduces the chances to make a mistake and wrongly grant a patent. Thus, the PTO increases the examiner's reward to induce more effort to search for invalidating information. At some point, the reward will decrease because both effort and random check are substitutes. Initially, the examiner will increase his effort before it is reduced. Therefore, for some values of π ($\pi_1 \leq \pi < \pi_2$), both reward and effort levels evolve in opposite direction (see appendix for the calculations). We summarize these findings in the following Lemma.

Lemma 3 *For a given level of β, as the probability of a random check π increases*

- *for $\pi \in [\pi_b'(\beta), \pi_b''(\beta)]$, both effort $e_b^*(\beta)$ and reward R_b^* first increase and then decrease;*
- *for $\pi \in [\pi_1, \pi_2]$, reward and effort go in opposite direction as reward decreases while effort increases.*

As evidenced by Figure 1, the first-best level of effort is weakly decreasing with π (it is initially constant and then decreases) as both effort and random check are substitutes. However, in the case of a reward based on both granted and rejected patents, search effort and audit are largely complement, and the search effort will increase when π first increases. Then, after reaching a maximum, it will decrease. In fact, even if the probability of a random check increases when its level is rather high, the examiner will reduce his effort level. Thus, if the PTO wants to induce examiner's effort through audit, then increasing π beyond π_2 will be wasteful. This result illustrates the dual role of checks performed by experienced supervisors. These checks not only enhance the average quality of issued patents by weeding out bad patents but they also keep examiners on their toes and can improve their search incentives. Note that for $\pi > \pi_2$, checking patents becomes the PTO's primary objective.

When $0 < e_b^* < 1$, the utility of the examiner can be written as

$$U_b^* = \frac{1}{2}\eta(e_b^*)^2 + 2\eta\Phi_b e_b^*, \tag{17}$$

and the benefit function of the PTO as

$$G_b^* = \overline{W}(1 - p(1-\gamma)) - \omega(1-\pi) + \eta(e_b^*)^2 - C_\pi, \tag{18}$$

which is always suboptimal.

We now consider the two extreme cases when the PTO only rewards the examiner on issued patents ($\beta = 0$) and on rejected patents ($\beta = 1$). When $\beta = 0$, we define the effort of the examiner as $e_a^* = e_b^*(0)$ while his effort is $e_r^* = e_b^*(1)$ when $\beta = 1$. Using the same notation, we define G_a^* (respectively, G_r^*) the PTO's benefit function in a regime where the examiner is rewarded on issued (resp., rejected) patents and U_a^* (resp., U_r^*) the examiner's utility.

Figure 2 represents the optimal levels of effort e^o, e_b^*, e_r^*, e_a^* as a function of π when $\omega \geq \eta$.

Insert Figure 2

A comparison of the different effort levels, the PTO's benefits and the examiner's utility leads to the following Proposition (see appendix for the calculations).

Proposition 1 *When the PTO rewards the examiner only on issued patents,*

- *the examiner exerts less effort than when he is rewarded on rejected patents, all efforts being suboptimal ($e_a^* \leq e_r^* < e^o$);*
- *the PTO's benefit is smaller than when the examiner is rewarded on rejected patents ($G_a^* < G_r^* < G^o$);*
- *if $\pi \in [\underline{\pi}, \overline{\pi}]$, the examiner can get a higher utility than when he is rewarded on rejected patents ($U_a^* > U_r^*$).*

It is important to point out that even though the salary of the examiner is always higher under a rejection regime, his utility is not necessarily higher. Indeed, for intermediate values of π, his utility under an acceptance regime is higher than under a rejection regime.

In a rejection regime, the examiner intensifies his effort compared to an acceptance regime. If he had the choice between the two regimes, the examiner would rather prefer to be rewarded on granted patents if the probability to be randomly checked is $\pi \in (\underline{\pi}, \overline{\pi})$. On the other hand, if the probability to be audited is smaller ($\pi \in (p, \underline{\pi})$) or relatively high ($\pi \in (\overline{\pi}, 1)$), his utility is higher under a rejection regime. Furthermore, the benefit of the PTO is smaller under an acceptance regime compared to a rejection regime ($G_a < G_r$). Thus, in general, incentives are more powerful under a rejection regime.

For a given level of random check π, a salary regime based on granted patents performs poorly. In fact, it is suboptimal and it is also dominated by a regime in which the PTO should reward rejected patents instead of granted patents. Therefore, the key policy message of Proposition 1 is that the PTO should be more inclined to consider salary schemes that reward rejected patents rather than accepted patents. Examiners should get rewarded on what is actually observed at the time the patent is processed, which is the invalidating information that is contained in the patent application.

In general, a rejection regime allows the PTO to extract more search effort from the examiner. As such, the latter is often worse off with this incentive regime. This is unlike an acceptance regime that usually leaves a larger rent to the examiner.

Interestingly, the optimal effort of the examiner in the rejection regime is strictly decreasing with π, as it is the case with the first-best effort level, while it is not the case for a dual regime or an acceptance regime. In the case of a rejection regime, both effort and random check are substitute: a low level of random check can actually induce a relatively high effort. This is no longer the case in the acceptance regime or the dual regime. Indeed, in these two regimes, the effort is concave with π: both effort and random check are initially complement before becoming substitute. Thus, with a low π, the effort under a rejection regime will be much higher than under dual or acceptance regimes.

As the PTO benefits from a higher effort level, it should choose to reward patent examiners only based on their rejected patents as stated in the following Proposition.

Proposition 2 *A salary based on both issued and rejected patents does not provide more incentives than a salary based only on rejected patents.*

This finding relies on the fact that patentability is established by failing to find invalidating information (or to find similar inventions). In a less realistic model in which direct evidence can be discovered (and not the lack of evidence), an optimal incentive scheme based on rejection would not be obtained.

The case for adopting an incentive scheme mostly based on rejection (i.e., β close to 1) can be made even stronger if one account for the possibility of collusion between the examiner and the applicant. Indeed, if as suggested by Tabakovic and Wollmann (2018), examiners looking for lucrative positions outside the PTO are tempted to be more lenient with potential future employers, a reward based on rejection will make lenient behavior more costly for examiners.

We now consider the case in which the PTO endogeneizes the random check.

4 Endogenous Random Check

In an extension of our model, we now consider that the PTO also chooses the probability of a random check. Not only the PTO seeks to provide the optimal level of reward to the examiner, it also chooses an optimal random check level by solving the following maximization program

$$\begin{cases} \max\limits_{R,\pi} G = \{W - S - C_\pi\} \\ \qquad s.t. \; U \geq 0 \end{cases} \tag{19}$$

where

$$C_\pi = C(\pi) = \frac{1}{2} c\pi^2, \tag{20}$$

with $c > \eta$, which insures that performing a random check is costlier than an initial examination as it will be done by a more senior examiner who gets a higher salary.

As in the previous section, we first determine the first-best outcome before considering the two different incentives schemes.

4.1 First Best Outcome

To determine the first-best outcome when the examiner's effort is observed, the PTO solves

$$\begin{cases} \max\limits_{\pi} G = \{\overline{W}(1-p(1-\gamma)) - \omega(1-\pi) + e^o(1-\pi)\omega - \frac{1}{2}\eta(e^o)^2 - \frac{1}{2}c\pi^2\}, \\ s.t. \;\; 0 \leq \pi \leq 1 \end{cases} \tag{21}$$

where e^o is defined by (10). It follows that as $c > \eta$, if $\omega \geq \eta$ there exists a corner solution with $e^{oo} = 1$ and $\pi^{oo} = 0$ (see appendix for the resolution of all the cases). The intuition of this result is fairly simple: as it costs more to perform a random check, the PTO prefers to induce the maximum effort from the examiner. For $\omega < \eta$ (gain from avoiding a mistake is small), there exists an interior solution in equilibrium such that the random check is

$$\pi^{oo} = \omega\frac{\eta-\omega}{c\eta-\omega^2}, \tag{22}$$

and the effort of the examiner is

$$e^{oo} = \omega\frac{c-\omega}{c\eta-\omega^2}. \tag{23}$$

In that case, as the gain from avoiding a mistake is small, the PTO has an incentive to induce less effort and to choose a positive probability of random check. However, as the cost of random check increases, the PTO reduces the random check.

The first-best effort e^{oo} is increasing with c and decreasing with η, while the first-best random check π^{oo} is decreasing with c and increasing with η. As the examiner's effort becomes more costly (η increases), the PTO substitutes the examiner's effort with a more intense random check. On the other hand, when it becomes more costly to perform a random check (c increases), the examiner increases his effort and the PTO reduces the random check. The first-best effort is increasing with ω, while the random check π^{oo} increases with ω only for small values of ω ($\omega < c - (c^2 - c\eta)^{\frac{1}{2}}$). For higher values, it decreases. As the expected gain from avoiding to wrongly grant patents increases from very small values, both efforts and random checks are increasing; they are complement. However, after a threshold, a marginal increase in ω still induces more effort from the examiner but reduces the random check. The induced incentive is enough and both effort and random check become substitutes.

Thus, if $\omega \geq \eta$, the benefit function of the PTO is $G^{oo} = \overline{W}(1-p(1-\gamma)) - \eta/2$, while it is

$$G^{oo} = \overline{W}(1-p(1-\gamma)) - \omega\frac{c(\eta-\omega)+\eta(c-\omega)}{2(c\eta-\omega^2)}, \tag{24}$$

if $\omega < \eta$. Not surprisingly, as the cost of effort and the cost of random check increase, the benefit is reduced ($\partial G^{oo}/\partial \eta < 0$ and $\partial G^{oo}/\partial c < 0$). In what follows, we assume that $c\eta > \omega^2$. Note that $c\eta > \omega^2$ and $c > \eta$ imply that $\eta > \omega$ and thus $\pi^{oo} > 0$.

4.2 Dual Regime

As in the exogenous case, in a dual regime, the examiner solves (7) where the salary is defined by (4) which gives $e(R)$ as defined by (13). The PTO's program is thus

$$\max_{R,\pi} G = \{W(e(R),\pi) - S(e(R),R,\pi) - C(\pi)\}, \tag{25}$$

where $W(.)$ is defined by (9), $S(.)$ by (4) and $C(\pi)$ by (20) evaluated at (13). The resolution of the program provides $R(\pi,\beta)$ and $\pi(R,\beta)$ (see appendix), but we cannot find an analytic solution. We thus analyze the two extreme cases where the incentive scheme is based only on rejected patents and only on accepted patents.

4.2.1 Incentive Scheme Based on Rejected Patents

If the PTO rewards the examiner only for rejecting applications ($\beta = 1$), his salary is defined by (5) and, thus, for any given reward R and random check π, the examiner's effort is defined by $e(R) = \frac{1}{\eta}(1-\gamma)pR$, which is independent of π, as it does not affect directly the examiner's reward. Given this effort level, the PTO seeks to provide the optimal level of reward R and the optimal random check π that solve (25). The PTO offers (π_r^{**}, R_r^{**}) and the examiner makes the effort e_r^{**} such that

$$R_r^{**} = \frac{\eta\omega}{(1-\gamma)p}\frac{(c-\omega)}{(2\eta c-\omega^2)}, \tag{26}$$

$$\pi_r^{**} = \frac{2\eta-\omega}{2c\eta-\omega^2}\omega, \tag{27}$$

and

$$e_r^{**} = \frac{c-\omega}{2c\eta-\omega^2}\omega. \tag{28}$$

These solutions are interior as long as $2c\eta > \omega^2$ which is always satisfied as $c\eta > \omega^2$. Even though the examiner is not directly affected by the choice of a random check as his reward does only depend on rejected patents that will never be reevaluated by the PTO, his equilibrium effort depends on the cost of the random check, as his reward R is affected by π. The higher the cost, the higher the examiner's effort is as his reward increases ($\partial e_r^{**}/\partial c > 0$). In the mind of the PTO, when $\beta = 1$, random check and examiner's effort are substitutes and their relative cost dictates which of these two instruments the PTO will favor. When it becomes costlier to have a random check for the PTO, the PTO prefers to put more emphasis on the reward and increases the incentive to search for invalidating information. However, when the cost is lower, the PTO prefers to rely on its random audit. The higher the gain from avoiding to make type II error, the higher the reward, the probability of a random check and the effort if $c > 2\omega$. We summarize these results in the following Lemma.

Lemma 4 *If the incentive scheme is based on rejected patents,*

1. *the higher the cost of effort η, the lower the reward R_r^{**} and the examiner's effort e_r^{**}, and the higher the random check π_r^{**};*

2. *the higher the cost of random check c, the higher the reward R_r^{**} and the examiner's effort e_r^{**}, and the lower the random check π_r^{**};*

3. *the higher the gain from avoiding to make type II error ω, the higher the reward R_r^{**}, the examiner's effort e_r^{**}, and the random check π_r^{**} if $c \geq 2\omega$; if $c < 2\omega$, both effort and reward decrease, while the random check increases.*

In Figures 3 and 4, we represent the optimal levels of effort and random check when $\omega < \eta$, and $\eta \leq \omega < (c\eta)^{\frac{1}{2}}$ respectively.

Insert Figures 3 and 4

Compared to the first-best case, we obtain the following results.

Proposition 3 *Under an incentive scheme based on rejected patents, the PTO commits to a weakly higher random check ($\pi_r^{**} \geq \pi^{oo}$) which induces a lower examiner's effort ($e_r^{**} \leq e^{oo}$).*

- *If $\omega < \eta < c < 2\eta$, all solutions are interior with $0 < \pi^{oo} < e^{oo} < 1$, $0 < e_r^{**} < \pi_r^{**} < 1$;*

- *If $\omega < \eta < 2\eta < c$, all solutions are interior with $0 < \pi^{oo} < e^{oo} < 1$, $0 < \pi_r^{**} < e_r^{**} < 1$;*

- *If $\eta \leq \omega < (c\eta)^{\frac{1}{2}}$, we obtain $\pi^{oo} = 0$, $e^{oo} = 1$, but $0 < e_r^{**} < 1$ and $0 < \pi_r^{**} < 1$.*

Proposition 3 illustrates the main problems faced by the PTO when a rejection regime is chosen. First, with perfect information (i.e., first best), both examiner's effort and random check represent similar successive screening technologies. Yet, because $c > \eta$, the PTO naturally puts more emphasis on the effort and hence chooses $e^{oo} > \pi^{oo}$. Second, with unobservable effort, the PTO needs to leave an information rent to the examiner to induce effort, which raises the cost of using examination. Thus, compared to the first-best case, the PTO distorts downward the examination effort ($e_r^{**} < e^{oo}$) and adjusts upward the intensity of random check ($\pi_r^{**} \geq \pi^{oo}$). The three cases of Proposition 3 show that when the cost of effort η is decreased progressively, the examination effort will be raised above the intensity check despite the agency rent.

The PTO obtains the following benefit

$$G_r^{**} = \overline{W}(1 - p(1 - \gamma)) - \omega \tfrac{2\eta(c-\omega)+c(2\eta-\omega)}{2(2\eta c-\omega^2)}, \tag{29}$$

which is always suboptimal ($G_r^{**} < G^{oo}$). The PTO's benefit is decreasing with η and c while it is increasing with ω. The examiner's utility is $U_r^{**} = \eta(e_r^{**})^2/2$, which is increasing with η and c. His salary is $S_r^{**} = \eta(e_r^{**})^2$.

4.2.2 Incentive Scheme Based on Accepted Patents

If the PTO rewards the examiner only for accepted patents ($\beta = 0$), his salary is defined by (6) and, thus, for any given reward R and random check π, the examiner's effort is defined by $e(R) = 1(1 - \gamma)(\pi - p)R/\eta$, which is positive as long as $\pi \geq p$. To solve the PTO's program (25), we process in two steps as we already have calculated R, which is

$$R_a(\pi) = \tfrac{1}{(1-\gamma)(\pi-p)}(\frac{1}{2}\omega(1 - \pi) - \eta\Phi_a),$$

with

$$\Phi_a = \tfrac{\gamma+(1-\gamma)(1-\pi)}{2(1-\gamma)(\pi-p)},$$

and we rewrite

$$e_a(\pi) = \frac{1}{2\eta}(1 - \pi)\omega - \Phi_a.$$

We plug $R_a(\pi)$ into (25) and, as long as $e_a(\pi) > 0$ (and $R_a(\pi) > 0$), we rewrite (25) as

$$\max_\pi G = \{\overline{W}(1 - p(1 - \gamma)) - \omega(1 - \pi) + \eta(e_a(\pi))^2 - \frac{1}{2}c\pi^2\}.$$

The first-order condition is

$$\underbrace{\omega}_{(i)} + \underbrace{2\eta e_a \frac{\partial e_a}{\partial \pi}}_{(ii)} = \underbrace{c\pi}_{(iii)} \tag{30}$$

The left-hand side of (30) represents the marginal benefit of increasing π. The first term (i) represents the direct marginal benefit from having a random check; the second term (ii), represents the incentive (indirect) effect of an increase of π on the effort e_a. When $\partial e_a/\partial \pi > 0$, (ii) is positive, and it reinforces the marginal benefit associated to having a random check. However, when $\partial e_a/\partial \pi < 0$, (ii) is negative and it goes against the marginal benefit (i). The last term (iii) on the right-hand side is the marginal cost associated to having a random check. We rewrite (30) as

$$\omega(1 - e_a) + \eta e_a \tfrac{\gamma + (1-\gamma)(1-p)}{(1-\gamma)(\pi - p)^2} = c\pi,$$

where the first two terms are always positive. Therefore, for any $e_a(\pi) \geq 0$, there exists a strictly positive value π_a^{**} that satisfies (30). Reward and effort levels are positive if $(1-\pi)\omega/2\eta - \Phi_a > 0$. Whenever $(1-\pi)\omega/2\eta - \Phi_a \leq 0$, the PTO does not offer any reward and the examiner does not exert any effort; the optimal random check is thus $\pi_a^{**} = \omega/c$. However, when $(1-\pi)\omega/2\eta - \Phi_a > 0$, the examiner exerts a positive effort level. Notice that $(1-\pi)\omega/2\eta > \Phi_a$ is equivalent to having $(1-\pi)(1-\gamma)(\omega(\pi - p) - \eta) - \gamma\eta > 0$, and thus there exist $\underline{\pi}_a$ and $\overline{\pi}_a$ such that for $\pi \in [\underline{\pi}_a, \overline{\pi}_a]$, $e_a(\pi) > 0$ (see appendix). We summarize these findings in the following Lemma.

Lemma 5 *When the incentive scheme is only based on accepted patents, in equilibrium the PTO offers the contract (R_a^{**}, π_a^{**}) such that*

- *if $c \in [\frac{\omega}{\overline{\pi}_a}, \frac{\omega}{\underline{\pi}_a}]$, $R_a^{**} = 0$ and $\pi_a^{**} = \dfrac{\omega}{c}$, which induces no effort from the examiner ($e_a^{**} = 0$);*
- *if $c < \frac{\omega}{\overline{\pi}_a}$ or $c > \frac{\omega}{\underline{\pi}_a}$, $R_a^{**} > 0$ and $0 < \pi_a^{**} < 1$, which induces a positive effort e_a^{**} from the examiner.*

From these findings, we derive the following Corollary.

Corollary 1 *When the incentive scheme is only based on accepted patents, the PTO finds it always optimal to perform a random check.*

This result shows that unlike the rejection case, the PTO cannot stop auditing, especially when the examiner stops exercising effort, as in these cases, the patent quality control exclusively depends on the audit.

Compared to the first-best case, we obtain the following results.

Proposition 4 *When the incentive scheme is based on accepted patents, the PTO commits to a higher random check ($\pi_a^{**} > \pi^{oo}$), which induces lower effort from the examiner ($e_a^{**} < e^{oo}$) if* $c \in [\frac{\omega}{\overline{\pi}_a}, \frac{\omega}{\underline{\pi}_a}]$.

The salary offered to the examiner is $S_a = \eta(e_a^{**})^2 + 2\eta\Phi_a e_a^{**}$, and his utility is $U_a = \eta(e_a^{**})^2/2 + 2\eta\Phi_a e_a^{**}$.

It is not possible to obtain analytical expressions for R_a^{**} and π_a^{**} and to compare their values with expressions (26) and (27). Yet, simulations show that examiner's incentives will be lower ($R_a^{**} < R_r^{**}$) while audit frequently will be higher, that is $\pi_a^{**} > \pi_r^{**}$. To understand this outcome, one must remember that when the examiner is rewarded on acceptance, he has very little incentives to "weed out" bad applications. Natural incentives are rather to accept them hoping that the audit will not take place. Therefore, the only source of incentives is an audit revealing non-patentable innovations. This source of incentives is visible in the second term (ii) of expression (6) where a large value of π will trigger examiner's effort as it pays to not let non-patentable innovation being patented.

5 Career Concerns of Patent Examiners

A fraction of patents are challenged in court every year. Usually, patent holders sue alleged infringers (often competitors) and judges are asked to make decisions regarding patent validity and, if validity is upheld, to award (possibly punitive) damages to the plaintiff. For our purposes, these legal proceedings are interesting because not only they reveal information about the patent's strength, but also about the awarding examiner. For instance, if validity is upheld in a patent dispute case, this may reveal that the examiner in charge did a thorough work in granting the patent. Private firms do value examiners' skills for searching for information and drafting patents. In fact, skilled examiners tend to leave the PTO to work for private companies and the PTO has a hard time to retain a skilled workforce (GAO, 2005, 2007). In this context, we wonder how the shadow of patent suits affect incentives of career conscious examiners. If, as we expect, patent disputes affect their incentives, then how is the PTO's policy affected? Are the PTO incentive schemes hindered by examiners' career concerns or, on the contrary, can the PTO "free ride" on these legal patent disputes?

To analyze that, we slightly modify our model in the following way. Once a patent has been granted it can still be challenged later on in court with probability π_c and invalidated. We also assume that the probability $q(e)$ of finding invalidating information depends on the talent of the examiner, which is unknown to both the PTO and the examiner. The timing is now as follows. The first four periods are identical to our previous setting: the PTO offers an incentive scheme to the examiner; if he accepts it, he receives a patent application, chooses a level of effort to find information that proves that the innovation is not patentable. Then, he decides to grant a patent or not, and gets paid accordingly by the PTO. In a fifth period, the (job) market observes whether a patent has been invalidated in court or not, and based on its observation, updates its beliefs about the ability of the examiner. Private companies then make an offer to an examiner who is believed to be a good examiner. To simplify, we only discount the fifth period by δ. Implicitly, we assume that what happens between the first period to the fourth period is happening during the same period, and the fifth period would be the second period of the modified model. We also consider only the case where the random audit is exogenous.

The talent of the examiner, which is unknown to all players including himself, affects the probability $q(e)$ in the following way. The examiner can be skilled with probability θ, or less skilled with probability $(1-\theta)$. The intrinsic talent of the examiner has an impact on the probability of finding information and can be valuable to private employers. A skilled examiner who exerts an effort e will find relevant information to reject the patent application with probability e. On the other hand, a less skilled examiner will find the same information with a lower probability αe, where $\alpha < 1$. Therefore, the expected talent of the examiner is $\overline{\alpha} = \theta + \alpha(1-\theta)$. The more talented the examiner, the higher the probability of invalidating the patent application. Thus, the expected probability of finding invalidating information is $q(e) = \overline{\alpha}e$. This will affect the expected payoff (9) of the PTO, which is now

$$W_{cc} = \underbrace{\gamma\overline{W}}_{\text{patentable}} + (1-\gamma)[\underbrace{\overline{\alpha}e(1-p)\overline{W}}_{\text{partly-patentable}} + \underbrace{(1-\overline{\alpha}e)(1-p)((1-\pi)\pi_c+\pi)\overline{W}}_{\text{successful random audit}}]$$

$$+\underbrace{(1-\gamma)(1-\overline{\alpha}e)(1-\pi)(1-\pi_c)(p\underline{W}+(1-p)\underline{W}')}_{\text{error}}.$$

Compared to (9), an error will arise with probability $(1-\gamma)(1-\overline{\alpha}e)(1-\pi)(1-\pi_c)$ rather than $(1-\gamma)(1-\overline{\alpha}e)(1-\pi)$, and a successful random audit on a partly patentable innovation will happen with probability $(1-\gamma)(1-\overline{\alpha}e)(1-p)((1-\pi)\pi_c+\pi)$ rather than $(1-\gamma)(1-\overline{\alpha}e)(1-p)\pi$. Indeed, even if the random audit did not correct the (mistaken) decision to entirely patent an innovation, the court will correct it with probability π_c.

Because the examiner's effort is unobservable, he might be tempted to influence the labor market *ex post* inference about his talent by increasing his effort. However, in equilibrium, the market is not fooled. To be precise, an equilibrium is an examiner effort e^{cc}, and a market belief $\widehat{\theta}$ such that the examiner's equilibrium effort is utility maximizing given the beliefs of the market, the market updates its beliefs according to Bayes' rule, and the market beliefs coincide with the examiner's effort choice.

To fully characterize the equilibrium, we first consider the second period. Using Bayes' rule, the private market updated beliefs concerning the examiner's ability are (see appendix for the calculations)

$$\widehat{\theta}(e) = \tfrac{\theta\gamma+(1-\gamma)\theta e(1-p)+\theta(1-\gamma)(1-p)(1-e)\pi}{\gamma+(1-\gamma)\overline{\alpha}e(1-p)+(1-\gamma)(1-p)(1-\overline{\alpha}e)\pi}. \tag{31}$$

By exerting more effort, the examiner attempts to manipulate the market's beliefs as $\partial\widehat{\theta}/\partial e > 0$.

We assume that the salary offered by the private market is based on its updated beliefs about the examiner's ability. The market offers a salary $\widehat{\theta}$ to examiners whose patents have not been (partly or fully) invalidated in court if challenged. For examiners whose patents have been successfully challenged, or have not been challenged, the private market does not make any offer and we assume that the examiner stays employed at the PTO.[23]

We now turn to the first period choice of effort of the examiner in the presence of career

[23] We might speculate that examiners who received outside offers might either leave the PTO or that their salary is matched through a raise or a fast track promotion to a more senior rank (e.g., supervisor).

concerns. It is solution of

$$\max_{e} \{S + \delta\pi_c[\gamma + (1-\gamma)(1-p)\overline{\alpha}e + (1-\gamma)(1-p)(1-\overline{\alpha}e)\pi]\widehat{\theta}(e) - C(e)\}, \tag{32}$$

where S the salary received by the examiner is either (6) or (5), $\gamma\pi_c$ represents the probability that a patentable innovation will be challenged in court and not invalidated, $(1-\gamma)(1-p)\overline{\alpha}e\pi_c$ represents the probability that a patented innovation that was partly patentable will be challenged and not invalidated in court, and $(1-\gamma)(1-p)(1-\overline{\alpha}e)\pi\pi_c$ represents the probability that a patented innovation that was randomly audited was discovered to be partly patentable and thus was not invalidated in court. Therefore, an examiner whose patent has been challenged but not invalidated receives a salary $\widehat{\theta}$ whereas an examiner whose patent has been challenged and invalidated or partly invalidated does not receive any offer from the market.[24] For a given level R, the examiner's effort is

$$e^{cc}(R) = e_c(R) + \delta\pi_c(1-\gamma)(1-p)(1-\pi)\theta, \tag{33}$$

where

$$e_c(R) = \frac{1}{\eta}(1-\gamma)[\beta p + (1-\beta)(\pi - p)]\overline{\alpha}R$$

is the effort level in the case of the modified model similar to effort (13).

In the presence of career concerns, for given levels R and π, the examiner intensifies his search effort no matter whether the contract is based on rejected or accepted patents ($e^{cc}(R) > e_c(R)$). For a given π, the level of reward given by the PTO is

$$R^{cc} = R^*_{cb} - \eta\tfrac{\theta(1-p)(1-\pi)\delta\pi_c}{2\overline{\alpha}[\beta p + (1-\beta)(\pi-p)]}, \tag{34}$$

where

$$R^*_{cb} = \tfrac{1}{(1-\gamma)[\beta p + (1-\beta)(\pi-p)]}(\frac{1}{2}\omega(1-\pi)(1-\pi_c) - \frac{\eta}{\overline{\alpha}}\Phi_b(\beta)),$$

is the optimal reward with the modified model similar to (14) with $\Phi_b(\beta)$ defined by (15). The optimal reward R^{cc} can be decomposed into two terms: the first term is related to monetary incentive, R^*_{cb}, and the second one is related to career concerns (non-monetary incentives). Interestingly, with career concerns, the optimal reward is reduced ($R^{cc} < R^*_{cb}$). The PTO does

[24] We make the assumption that examiners whose updated talent decreases do not see their salaries decrease. In practice, few examiners are being fired and there is downward wage rigidity (GAO, 2005).

not need to reward the examiner as much as before, as the market will give him extra incentives to search for information. The PTO can thus "free ride" on the market and save resources.

For a given π, the optimal effort level of the patent examiner in the presence of career concerns is

$$e^{cc} = e_c^* + \tfrac{1}{2}(1-p)(1-\gamma)(1-\pi)\pi_c\delta\theta, \tag{35}$$

where

$$e_c^* = \frac{1}{2\eta}\overline{\alpha}\omega(1-\pi)(1-\pi_c) - \Phi_b(\beta),$$

which implies that $e^{cc} > e_c^*$, that is, an examiner with career concerns always exerts more effort than without career concerns no matter what regime is considered.

Proposition 5 *For a given level of π, the existence of implicit incentives induce the patent examiner to intensify his search of information.*

By intensifying his effort, the examiner gets more immediate reward from non-patentable innovations that are refused, and also he increases his chances of being discovered as a skilled examiner on the job market. The existence of outside option value makes examiners more efficient.

We now perform some comparative statics and analyze how changes in different parameters affect the examiner's effort with and without career concerns. Some effects are magnified by career concerns whereas others are not. As the cost of effort η increases, both efforts with and without career concerns decrease in the same way. Similarly, when the weight β that is put on rejected patents increases, both efforts increase similarly. Not surprisingly, as the unconditional probability of having a skilled examiner θ increases, so does his effort, but at a higher rate when he has career concerns. As his probability of being skilled increases, an examiner wants to signal it to the market. As the probability of having a patentable innovation γ increases, both efforts decrease, more rapidly in the presence of career concerns. As it becomes more likely that the innovation is patentable, the examiner does not have to search for invalidating information. We summarize these findings in the following Lemma.

Lemma 6 *The optimal effort of an examiner with career concerns*

1. *increases with his talent at a higher rate than without career concerns;*

2. *decreases with the probability of having a patentable innovation at a higher rate than without career concerns.*

As the probability to go to court π_c increases, in absence of career concerns, the effort is reduced. However, in the presence of career concerns, the effort is reduced only if it is more unlikely that the examiner is skilled (i.e., for a low θ). On the contrary, when it is more likely that the examiner is skilled, he will intensify his search. We summarize these findings in the following Lemma.

Lemma 7 *The optimal effort of an examiner with career concerns increases with the probability to be challenged in court if it is more likely that he is talented.*

As π_c increases, it might be that it is easier for a patent to be challenged in court. Thus π_c can represent the strength of the requirement to go to court. In other words, if it is easier to challenge a patent to court, the examiner has more incentives to intensify his effort if it is more likely that he is talented.

When the probability of being audited π increases, in presence of career concerns, the effort decreases more rapidly than without career concerns when the examiner is rewarded on rejected patents ($\beta = 1$). When the reward is based on accepted patents ($\beta = 0$) or on a combination between accepted and rejected patents ($0 \leq \beta < 1$), without career concerns, the effort first increases and then decreases. With career concerns, it also increases and decreases, but the incentives might go in opposite direction for intermediate values of π. Let denote $\pi^*_{cc} = argmax\{e^{cc}\}$ and $\pi^*_c = argmax\{e^*_c\}$. For $\pi_c \in (\pi^*_{cc}, \pi^*_c)$, the effort e^{cc} decreases while e^*_c increases. We report this result in the following Proposition.

Proposition 6 *The optimal effort of an examiner with career concerns decreases with the probability of an audit by the PTO while without career concern it increases for $\pi_c \in (\pi^*_{cc}, \pi^*_c)$.*

This result suggests that in the presence of career concerns, the PTO could save on audit resources and induces more effort from the examiner. Indeed, by facilitating court action, the PTO introduces stronger career concerns for examiners, which allows the PTO to save on audit resources.

The analysis performed in this section offers several testable hypotheses regarding the behavior of examiners and their likelihood of leaving the PTO. Empirical implications of the career concerns model are the following.

1. *Different examiners' turnover depending on patent upholding or rejected patents.*

   An examiner whose granted patents are upheld is simply more likely to leave the PTO since there is an indication that this individual has done a satisfactory work in granting the patent. A patent attorney firm is thus more likely to hire an examiner who grants "iron-clad" patents.

2. *Different examiners' effort choices when patents cover drastic innovations or are more valuable.*

   A career conscious examiner who knows that potentially lucrative patents will be challenged in court more often will adapt his effort to avoid having patents invalidated in court, which would damage his reputation.

3. *Different turnover rates across different innovation fields.*

   In fields with strong growth where the private value of a patent is on average high, we should observe, *ceteris paribus*, more patent court challenges; that is, in these fields, more information about examiners' talent is being released. Examiners working in these fields are more likely to obtain lucrative positions in the legal services of high-tech companies.

4. *Self-selection of examiners.*

   Similarly, we expect that talented examiners will tend to migrate toward fields which have stronger opportunities for fast-track careers.

6 Conclusion

Over the last decades, the quality of patent examination has often been questioned as patents of questionable validity are issued. Having a better understanding about how patent examiners are granting patents might be helpful to contribute to the discussion on policy recommendations to improve the patent system. Nevertheless, the internal organization of patent offices has attracted little attention from economists. Only recently the process by which patents are granted has started to be empirically studied and very little has been said in terms of salary scheme and career concerns. However, there exists a significant body of literature on career concerns, and on the study of salary within public institutions. To the best of our knowledge, our paper is the first attempt at analyzing the salary scheme of patent examiners. Our aim is to consider different incentive schemes, and to investigate what is the impact of career concerns on the behavior of patent examiners.

We find that rewarding patent examiners on both rejected and issued patents does not provide more incentive to search for information and, therefore, salaries (or bonuses) could only be based on rejected patents. Furthermore, career concerns provide more incentive to search for relevant information. Indeed, in order to be discovered as being a skilled examiner, a patent examiner intensifies his search to make fewer mistakes. Explicit and implicit incentives push patent examiners to make more effort and grant patents to deserving innovations. In fact, the impact of accounting for the possibility that a patent might be challenged in court is (qualitatively) equivalent to let examiners have strong career concerns.

In terms of policy implications, our findings suggest that a salary (or bonus) scheme based on rejected patents might give more incentives to search for invalidating information, and that career concerns have a positive effect on the search effort. On the other hand, even with this rejection scheme, a tough post-patent policy (in terms of more patents being challenged in court) will induce the examiner to intensify his search effort.

As a final point, we come back to some of our main assumptions. We make the simplifying assumption that rejection does not lead to any controversy. It is obviously a strong assumption, and in reality rejection can be complex. In his Internet Patent News Service, Greg Aharonian reports frequently lawyers' complaints about the fact that patents are refused on wrong ground, and that the re-examination process is time consuming, and an appeal is never a good solution.[25] We also do not take into account the problem associated with continuation rules. According to Hall (2006), continuations accounted for more than one third of all the patent applications filed in 2004. Therefore, patent examiners spend a fair amount of their time on old patents and not on new applications. These issues are beyond the scope of the analysis of our paper, but should be addressed in future research.

[25] See the web site of Greg Aharonian at http://www.bustpatents.com/.

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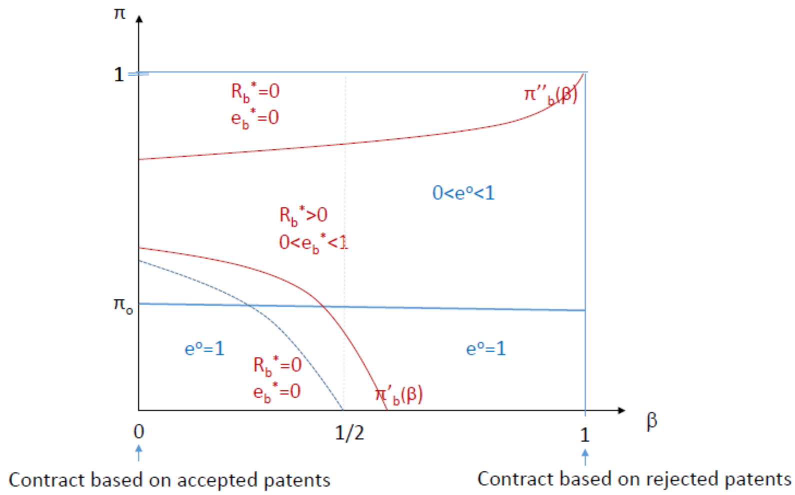

Figure 1: First-best and optimal efforts when π is exogenous

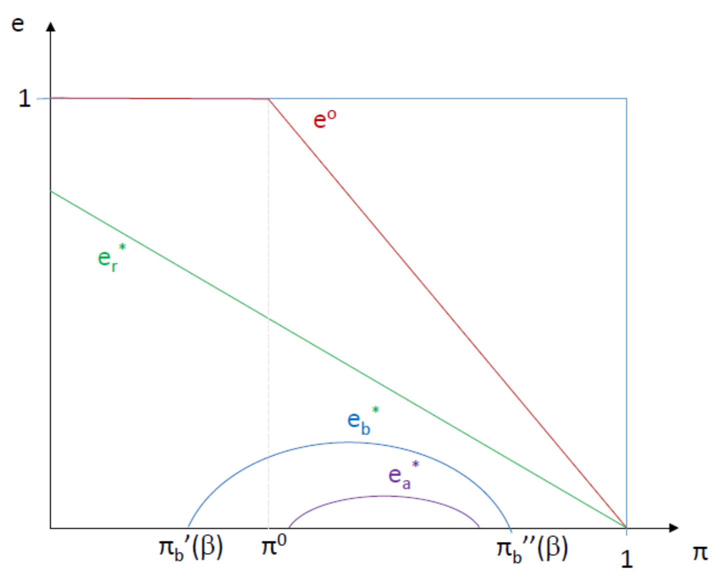

Figure 2: Examiner's optimal levels of effort when $\omega \geq \eta$

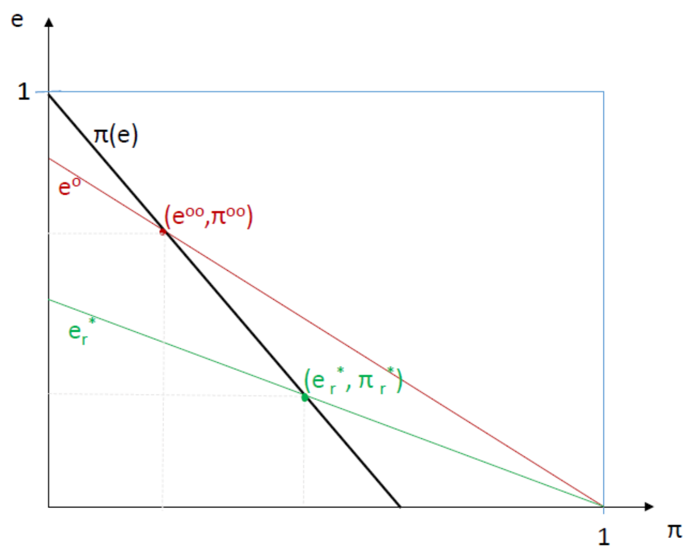

Figure 3: Optimal levels of effort and optimal random check when $\omega < \eta$

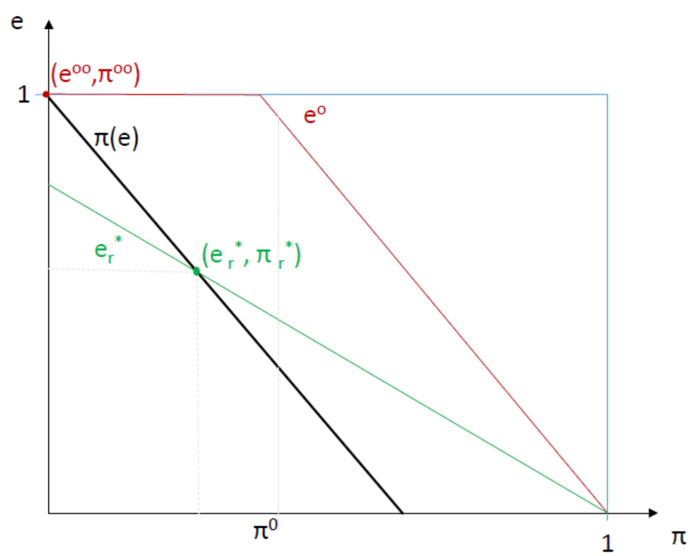

Figure 4: Optimal levels of effort and optimal random check when $\eta \leq \omega < (c\eta)^{\frac{1}{2}}$

Appendix

Exogenous Probability of Random Check

First best Outcome

The PTO solves (8) or $\max\{W - \eta e^2/2 - C_\pi\}$, where W is defined by (9), or, equivalently, $W = \overline{W}(1-p(1-\gamma)) - \omega(1-\pi) + e(1-\pi)\omega$ with ω defined by (11). The First Order Condition (FOC) yields $\omega(1-\pi) - \eta e = 0$, and the Second Order Condition (SOC) is always satisfied as $\partial^2(W - \frac{1}{2}\eta e^2)/\partial e^2 = -\eta < 0$. It follows that the first-best optimal effort of the examiner for any given π is (10). The following table provides static comparatives where $y = \gamma$, p, π, β, η.

$y = \backslash$ derivative	$\frac{\partial e^o}{\partial y}$
γ	< 0
p	< 0 if $\frac{\partial \omega}{\partial p} < 0$ > 0 if $\frac{\partial \omega}{\partial p} \geq 0$
π	< 0
β	$= 0$
η	< 0

Evaluated at e^o, the first-best benefit function of the PTO is $G^o = \overline{W}(1-p(1-\gamma)) - \omega(1-\pi) + \frac{1}{2}\eta(e^o)^2 - C_\pi$. For $e^o = 1$ it becomes $\overline{W}(1-p(1-\gamma)) - \frac{1}{2}\eta - C_\pi$, and for $e^o < 1$, it is

$$G^o = \overline{W}(1-p(1-\gamma)) - \omega(1-\pi)(1 - \frac{1}{2\eta}\omega(1-\pi)) - C_\pi.$$

Dual Regime ($0 \leq \beta \leq 1$)

Given R, the examiner solves (7) where S is defined by (4) or, equivalently, $S = (1-\gamma)pe\beta R + [\gamma + (1-\gamma)(1-p)e + (1-\gamma)(1-e)(1-\pi)](1-\beta)R$. The FOC ($\partial S/\partial e - \partial C/\partial e = 0$) is $(1-\gamma)p\beta R + (1-\gamma)[(1-p)-(1-\pi)](1-\beta)R - \eta e = 0$, which gives (13). For any $R \geq 0$, the effort $e(R) \geq 0$ if $\pi(1-\beta) - p(1-2\beta) \geq 0$. As long as $\beta \geq 1/2$ this is always satisfied. If $\beta < 1/2$, we need to insure that $\pi \geq \underline{\pi}_e(\beta) \equiv p(1-2\beta)/(1-\beta)$ to have a positive effort. The SOC is always satisfied as $\partial^2 S/\partial e^2 - \partial^2 C/\partial e^2 = -\eta < 0$.

To summarize, for any R, the effort $e(R)$ is positive if $\beta \geq 1/2$ or if $\beta < 1/2$ and $\pi < \underline{\pi}_e(\beta)$. Furthermore, the utility of the examiner is always positive for any R and π as

$$U(R,\pi) = (\gamma + (1-\gamma)(1-\pi))(1-\beta)R + \frac{1}{2}\frac{1}{\eta}(1-\gamma)^2((\pi-p)(1-\beta)+p\beta)^2R^2 \geq 0.$$

The PTO thus solves (8). By using $e(R) \geq 0$, the FOC $((\partial W/\partial e - \partial S/\partial e)\partial e/\partial R - \partial S/\partial R = 0)$ is

$$(1-\pi)\omega(1-\gamma)(p\beta + (\pi - p)(1-\beta)) - (1-\gamma)^2(p\beta + (\pi - p)(1-\beta))^2 R$$
$$-(1-\gamma)(p\beta + (\pi - p)(1-\beta))e - [\gamma + (1-\gamma)(1-\pi)](1-\beta) = 0$$

The SOC is always satisfied as $-2(1-\gamma)^2(p\beta + (\pi - p)(1-\beta))^2 < 0$. We thus plug $e(R)$ in the FOC and we obtain (14) and (16). Thus $e_b^* < e^o$, and we can rewrite

$$R_b^* = \frac{e_b(\beta)}{(1-\gamma)((\pi-p)(1-\beta)+p\beta)}.$$

As $e_b(\beta) < e^o$, $e_b(\beta) \leq 1$. Finally, we check under what conditions $R_b^* \geq 0$ (and therefore $e_b(\beta) \geq 0$). This is equivalent to checking that

$$\frac{1}{2\eta}\omega(1-\pi) - \frac{\gamma+(1-\gamma)(1-\pi)}{2(1-\gamma)} \frac{1-\beta}{\beta p + (\pi - p)(1-\beta)} \geq 0.$$

We rewrite this inequality as $-a\pi^2 + b\pi - c \geq 0$, where

$$\begin{aligned} a &= (1-\gamma)\omega(1-\beta) > 0 \\ b &= (1-\gamma)((1-\beta)(\eta+\omega) + \omega p(1-2\beta)), \\ c &= (1-\gamma)p\omega(1-2\beta) + \eta(1-\beta). \end{aligned}$$

The sign of b and c is not clearly determined. If $1-2\beta > 0$, then $b > 0$ and $c > 0$, which is no longer the case if $1-2\beta < 0$.

If the determinant $\Delta = b^2 - 4ac \geq 0$, there exists $\pi_b'(\beta)$ and $\pi_b''(\beta)$ such that $R_b^* \geq 0$ for $\pi \in [\pi_b'(\beta), \pi_b''(\beta)]$, and $R_b^* = 0$ for $\pi < \pi_b'(\beta)$ and $\pi > \pi_b''(\beta)$. These functions are

$$\begin{aligned} \pi_b'(\beta) &= \frac{(1-\gamma)((1-\beta)(\eta+\omega) + \omega p(1-2\beta)) - \sqrt{\Delta}}{2(1-\gamma)\omega(1-\beta)} > 0, \\ \pi_b''(\beta) &= \frac{(1-\gamma)((1-\beta)(\eta+\omega) + \omega p(1-2\beta)) + \sqrt{\Delta}}{2(1-\gamma)\omega(1-\beta)}. \end{aligned}$$

We also show that $\pi_b'(\beta) > \underline{\pi}_e(\beta)$ for any $\beta < 1/2$. Indeed, $\pi_b'(\beta) > p(1-2\beta)/(1-\beta)$ is equivalent to $\beta < (1-(1-\gamma)p)/(1-2p(1-\gamma))$, which is always satisfied for $\beta < 1/2$ as $(1-(1-\gamma)p)/(1-2p(1-\gamma)) > 1/2$.

If $\pi < \pi_o$ as defined by (12), the first-best optimal effort is $e^o = 1$. However, if $\pi < \pi_b'(\beta)$, $e_b(\beta) = 0$ if $\beta < \overline{\beta} < 1$ where $\overline{\beta}$ is such that $e_b(\overline{\beta}) = 0$. Therefore, for $\pi < \min\{\pi_b'(\beta), \pi_o\}$ and $\beta < \overline{\beta}$, $e^o = 1$ while $e_b(\beta) = 0$. This is the proof of **Lemma 2.**

For $R_b^* > 0$ and thus $e_b^* > 0$, the examiner's salary

$$S_b^* = (1-\gamma)p\beta R_b^* e_b^* + [\gamma + (1-\gamma)(1-p)e_b^* + (1-\gamma)(1-e_b^*)(1-\pi)](1-\beta)R_b^*.$$

can be simplified to $S_b^* = \eta(e_b^*)^2 + 2\eta\Phi_b e_b^*$, and the utility of the examiner is $U_b = \eta(e_b^*)^2/2 + 2\eta\Phi_b e_b^*$.

The following table provides static comparatives where $y = \gamma$, p, π, β, η, which represents the proof of **Lemma 1**.

$y =$\.	$\frac{\partial \Phi_b}{\partial y}$	$\frac{\partial R_b}{\partial y}$	$\frac{\partial e_b}{\partial y}$	$\frac{\partial S_b}{\partial y}$
γ	> 0	< 0	< 0	< 0
p	> 0 if $\beta < 1/2$ < 0 if $\beta > 1/2$	?	< 0 if $\frac{\partial \omega}{\partial p} < 0$ and $\beta < 1/2$ > 0 if $\frac{\partial \omega}{\partial p} \geq 0$ and $\beta > 1/2$	< 0 if $\frac{\partial \omega}{\partial p} < 0$ and $\beta < 1/2$ > 0 if $\frac{\partial \omega}{\partial p} \geq 0$ and $\beta > 1/2$
π	< 0	< 0 if $\pi < \pi_1$ > 0 if $\pi > \pi_1$	> 0 if $\pi < \pi_2$ < 0 if $\pi > \pi_2$	< 0 if $\frac{\partial e_b}{\partial p} < 0$? if $\frac{\partial e_b}{\partial p} \geq 0$
β	< 0	> 0 if $\pi > 2p$? if $\pi < 2p$	> 0	> 0
η	$= 0$	< 0	< 0	> 0

To compare R_b^* and e_b^*, we calculate the first and second derivatives of R_b^* and e_b^* with respect to π. Thus,

$$\begin{aligned}
\frac{\partial e_b(\beta)}{\partial \pi} &= -\frac{1}{2}\omega - \eta\frac{\partial \Phi_b}{\partial \pi}, \\
\frac{\partial^2 e_b(\beta)}{\partial \pi^2} &= -\eta\frac{\partial^2 \Phi_b}{\partial \pi^2} < 0,
\end{aligned}$$

as

$$\begin{aligned}
\frac{\partial \Phi_b}{\partial \pi} &= -\frac{(1-\beta)}{2(1-\gamma)\overline{\alpha}}\frac{(1-\gamma)p\beta + (1-\beta)(\gamma + (1-\gamma)(1-p))}{((\pi-p)(1-\beta)+p\beta)^2} < 0, \\
\frac{\partial^2 \Phi_b}{\partial \pi^2} &= \frac{(1-\beta)((1-\gamma)p\beta + (1-\beta)(\gamma + (1-\gamma)(1-p)))}{2(1-\gamma)\overline{\alpha}}\frac{1-\beta}{((\pi-p)(1-\beta)+p\beta)^3} > 0.
\end{aligned}$$

There exists a value π_2 such that $\partial e_b(\beta)/\partial \pi = 0$, which is

$$\pi_2 = \frac{((1-\gamma)p\beta + (1-\beta)(\gamma + (1-\gamma)(1-p)))^{\frac{1}{2}}}{(1-\beta)^{\frac{1}{2}}(1-\gamma)^{\frac{1}{2}}\omega^{\frac{1}{2}}\overline{\alpha}} + p\frac{1-2\beta}{1-\beta}.$$

We also calculate

$$\frac{\partial R_b(\beta)}{\partial \pi} = \frac{\frac{\partial e_b(\beta)}{\partial \pi}((\pi-p)(1-\beta)+p\beta) - e_b(\beta)(1-\beta)}{(1-\gamma)\overline{\alpha}((\pi-p)(1-\beta)+p\beta)^2},$$

and

$$\frac{\partial^2 R_b(\beta)}{\partial\pi^2} = \frac{1}{(1-\gamma)\overline{\alpha}}\frac{((\pi-p)(1-\beta)+p\beta)(\frac{\partial^2 e_b(\beta)}{\partial\pi^2}((\pi-p)(1-\beta)+p\beta))-2\frac{\partial e_b(\beta)}{\partial\pi}((\pi-p)(1-\beta)+p\beta)-e_b(\beta)(1-\beta)}{((\pi-p)(1-\beta)+p\beta)^3}.$$

We can set that

$$sign\frac{\partial R_b(\beta)}{\partial\pi} = sign(\frac{\partial e_b(\beta)}{\partial\pi}((\pi-p)(1-\beta)+p\beta)-e_b(\beta)(1-\beta)).$$

Thus, if $\partial e_b(\beta)/\partial\pi < 0$, then $\partial R_b(\beta)/\partial\pi < 0$. For $\partial e_b(\beta)/\partial\pi = 0$, which means evaluated at π_2, $\partial R_b(\beta)/\partial\pi < 0$. Thus, there exists $\pi_1 < \pi_2$, such that $\partial R_b(\beta)/\partial\pi > 0$ for $\pi < \pi_2$. This provides the proof of **Lemma 3**.

Let $e_b(0) = e_a$ and $e_b(1) = e_r$. If $\beta = 1$ (reward based on rejected patents)

$$U_r = \frac{1}{2}\eta(e_r)^2,$$

and if $\beta = 0$ (reward based on accepted patents)

$$U_a = \frac{1}{2}\eta(e_a)^2 + 2\eta\Phi_a e_a,$$

with $e_a = e_r - \Phi_a$. We show that $e_a \geq 0$ if $-\pi^2 a_1 + \pi b_1 - c_1 \geq 0$ where

$$\begin{aligned} a_1 &= (1-\gamma)\omega > 0, \\ b_1 &= (1-\gamma)(\eta+\omega(1+p)) > 0, \\ c_1 &= p\omega(1-\gamma)+\eta > 0. \end{aligned}$$

The determinant is $\Delta = (1-\gamma)^2(\eta+\omega(1+p))^2 - 4(1-\gamma)\omega(p\omega(1-\gamma)+\eta) > 0$, and thus for $\pi \in [\underline{\pi}, \overline{\pi}]$, $e_a \geq 0$ where

$$\begin{aligned} \underline{\pi} &= \frac{(1-\gamma)(\eta+\omega(1+p))-\sqrt{\Delta}}{2(1-\gamma)\omega} > 0, \\ \overline{\pi} &= \frac{(1-\gamma)(\eta+\omega(1+p))+\sqrt{\Delta}}{2(1-\gamma)\omega} < 1. \end{aligned}$$

Furthermore, as long as $e_a \geq 0$, then $U_r \leq U_a$. Indeed, $U_a \geq U_r$ if $e_r - (\gamma + (1-\gamma)(1-\pi))/(1-\gamma)(\pi-p) \geq 0$, which is equivalent to having $e_a \geq 0$. By taking the derivative of the PTO's benefit (18), we obtain

$$\frac{\partial G_b^*}{\partial\beta} = -2\eta\frac{\partial e_b^*}{\partial\beta} < 0,$$

as $\partial e_b^*/\partial\beta > 0$. Thus, $G_a^* < G_r^*$, both of them being suboptimal. This complete the proof of **Proposition 1.**

Endogenous Probability of Random Check

First best Case

The PTO solves (21). If $e^o = 1$, which happens when $\pi \leq (\omega - \eta)/\omega$, the program is

$$\begin{aligned} \max_{\pi} G &= \{\overline{W}(1 - p(1-\gamma)) - \frac{1}{2}\eta - \frac{1}{2}c\pi^2\}, \\ s.t.\ 0 &\leq \pi \leq 1 \end{aligned}$$

The FOC gives $-c\pi < 0$ and the SOC is always satisfied (i.e., $-c < 0$). Thus, G is (concave) strictly decreasing with π for $\pi < (\omega - \eta)/\omega$ and the solution is $\pi = 0$. The PTO's benefit is $G(0) = \overline{W}(1 - p(1-\gamma)) - \frac{1}{2}\eta$.

If $e^o < 1$, which happens when $\pi > (\omega - \eta)/\omega$, the program is

$$\begin{aligned} \underset{\pi}{Max}\ G &= \{\overline{W}(1 - p(1-\gamma)) - \omega(1-\pi) + \frac{1}{2}\eta(\frac{1}{\eta}\omega(1-\pi))^2 - \frac{1}{2}c\pi^2\}, \\ s.t.\ 0 &\leq \pi \leq 1 \end{aligned}$$

The FOC is $\omega - \frac{1}{\eta}\omega^2(1-\pi) - c\pi = 0$ and and the SOC ($\partial^2 G/\partial\pi^2 = \omega^2 - \eta c < 0$) is satisfied if $\omega^2 < \eta c$. The function is thus concave if $\omega^2 < \eta c$ and the solutions are π^{oo} and e^{oo} as defined by (22) and (23), where $\pi^{oo} > 0$ if $\eta > \omega$ and $e^{oo} >$ if $c > \omega$. Thus, if $\omega < \eta < c$, $\pi^{oo} > (\omega - \eta)/\omega$ is always satisfied. We also verify that $e^{oo} < 1$ and $\pi^{oo} < 1$.

Thus, if $\omega < \eta < c$ and $\omega^2 < \eta c$, there exists an interior solution. The benefit of the PTO G^{oo} is (24).We easily derive that $\partial G^{oo}/\partial\eta < 0$, and $\partial G^{oo}/\partial c < 0$.

Dual Regime

The examiner solves

$$\max_{e} U = \{S - C(e)\},$$

where S is defined by (4) which gives $e(R, \pi)$ as defined by (13). The PTO's program is

$$\max_{R,\pi} G = \{W(e(R,\pi),\pi) - S(e(R,\pi), R, \pi) - C(\pi)\},$$

which gives the following FOC

$$\begin{aligned} \frac{\partial G}{\partial R} &= 0 \Rightarrow (\frac{\partial W}{\partial e} - \frac{\partial S}{\partial e})\frac{\partial e}{\partial R} - \frac{\partial S}{\partial R} = 0, \\ \frac{\partial G}{\partial \pi} &= 0 \Rightarrow (\frac{\partial W}{\partial e} - \frac{\partial S}{\partial e})\frac{\partial e}{\partial \pi} + \frac{\partial W}{\partial \pi} - \frac{\partial S}{\partial \pi} - \frac{\partial C}{\partial \pi} = 0. \end{aligned}$$

The solutions are

$$\begin{aligned} R(\pi,\beta) &= \tfrac{1}{(1-\gamma)((\pi-p)(1-\beta)+p\beta)}(\frac{1}{2}\omega(1-\pi)-\eta\Phi_b), \\ \pi(R,\beta) &= \tfrac{\omega\eta+(1-\gamma)R((1-\beta)\eta+\omega((1-\beta)-p(2\beta-1))-2(1-\gamma)pR(1-\beta)(2\beta-1))}{c\eta+2(1-\gamma)R(1-\beta)(\omega+(1-\gamma)(1-\beta)R)}. \end{aligned}$$

where Φ_b is defined by(15).

Rejection Case $(\beta = 1)$

For any (π, R) the examiner solves

$$\max_e U = \{S - C(e)\},$$

where $S = (1-\gamma)peR$ which gives

$$e(R) = \frac{1}{\eta}(1-\gamma)pR.$$

This effort level is independent from π. Whatever the level of the PTO's random inspection, the examiner makes the same effort. The PTO's program becomes

$$\max_{R,\pi} G = \{\overline{W}(1-p(1-\gamma)) - \omega(1-\pi)(1-\frac{1}{\eta}(1-\gamma)pR) - (1-\gamma)p\frac{1}{\eta}(1-\gamma)pR^2 - \frac{1}{2}c\pi^2\}$$

We have already found that $R = \arg\max G$ gives

$$R(\pi) = \frac{1}{(1-\gamma)p}\frac{1}{2}\omega(1-\pi),$$

which is always positive. Thus the program of the examiner becomes

$$\max_{\pi} G = \{\overline{W}(1-p(1-\gamma)) - \omega(1-\pi) + \frac{1}{\eta}\frac{1}{2}\omega^2(1-\pi)^2 - \frac{1}{4}\frac{1}{\eta}\omega^2(1-\pi)^2 - \frac{1}{2}c\pi^2\}.$$

The FOC is $\omega(2\eta-\omega) - \pi(2\eta c - \omega^2) = 0$ and the SOC is $-(2\eta c - \omega^2) < 0$ if $2\eta c > \omega^2$, which is always satisfied as $\eta c > \omega^2$. Thus, there exists an interior solution π_r^{**} as defined by (27) and the optimal rewards R_r^{**} is (26) which is positive as $c > \omega$. The optimal effort is then e_r^{**} as defined by (28). Effort e_r^{**} and random check π_r^{**} are positive and smaller than 1. The reward R_r^{**} is also positive. In the next table, we summarize the derivatives of R_r^{**}, π_r^{**} and e_r^{**}, which represents the proof of **Lemma 4**.

$y = \backslash$ derivative	$\frac{\partial R_r^{**}}{\partial y}$	$\frac{\partial \pi_r^{**}}{\partial y}$	$\frac{\partial e_r^{**}}{\partial y}$
η	< 0	> 0	< 0
c	> 0	< 0	> 0
ω	> 0 if $c \geq 2\omega$ < 0 if $c < 2\omega$	> 0	> 0 if $c \geq 2\omega$ < 0 if $c < 2\omega$

The PTO's benefit is (29), which is always suboptimal ($G_r^{**} < G^{oo}$). and the examiner's utility is $U_r^{**} = \eta(e_r^{**})^2/2$. In the next table, we summarize some comparative statics.

$y =$ \ derivative	$\frac{\partial G_r^{**}}{\partial y}$	$\frac{\partial U_r^{**}}{\partial y}$
η	< 0	> 0
c	< 0	> 0
ω	> 0	> 0 if $c \geq 2\omega$ < 0 if $c < 2\omega$

Proof or Proposition 3

Using (27) and (28), we can write

$$e_r^{**} - \pi_r^{**} = \tfrac{\omega}{2\eta c - \omega^2}(c - 2\eta).$$

If $c > 2\eta$, $e_r^{**} > \pi_r^{**}$ and if $c < 2\eta$, $e_r^{**} < \pi_r^{**}$. We also calculate that

$$e^{oo} - \pi^{oo} = \tfrac{\omega}{\eta c - \omega^2}(c - \eta) > 0,$$

which always holds for $c > \eta$.

Acceptance Case ($\beta = 0$)

We rewrite the FOC (30) as

$$\omega - c\pi + e_a(-\omega + \tfrac{\gamma + (1-\gamma)(1-p)}{(1-\gamma)(\pi - p)^2}) = 0.$$

Once we plug e_a in the latter FOC, we obtain the following FOC for R and π

$$\begin{aligned} R(\pi) &= \tfrac{1}{(1-\gamma)(\pi-p)}(\frac{1}{2}\omega(1-\pi) - \eta\Phi_a), \\ \pi(R) &= \tfrac{\omega\eta + (1-\gamma)R(\eta + \omega(1+p) + 2p(1-\gamma)R)}{c\eta + 2(1-\gamma)R(\omega + (1-\gamma)R)}. \end{aligned}$$

We can also express the effort e as a function of π

$$e(\pi) = \frac{1}{2\eta}(1-\pi)\omega - \Phi_a.$$

Reward and effort levels are positive if $(1-\pi)\omega/2 - \eta\Phi_a > 0$. Whenever $(1-\pi)\omega/2 - \eta\Phi_a \leq 0$, the PTO does not offer any reward and the examiner does not exert any effort. However, when the parameters of the model are such that $(1-\pi)\omega/2 - \eta\Phi_a > 0$, the examiner performs a positive effort level. Notice that $(1-\pi)\omega/2 > \eta\Phi_a$ is equivalent to having $(1-\pi)(1-\gamma)(\omega(\pi-p)-\eta)-\gamma\eta > 0$. This last inequality is satisfied if at least $\omega(\pi-p)-\eta > 0$ (this is a necessary but not sufficient condition). We show that there exist $(\underline{\pi}_a, \overline{\pi}_a)$ such that for $\pi \in (\underline{\pi}_a, \overline{\pi}_a)$ both effort and reward are positive with

$$\begin{aligned}\underline{\pi}_a &= \frac{(1-\gamma)(1+(1+p)\omega)-\sqrt{(1-\gamma)((1-\gamma)(1+\omega(1+p))^2-4\omega(1+p(1-\gamma)\omega))}}{2\omega(1-\gamma)} > 0,\\ \overline{\pi}_a &= \frac{(1-\gamma)(1+(1+p)\omega)+\sqrt{(1-\gamma)((1-\gamma)(1+\omega(1+p))^2-4\omega(1+p(1-\gamma)\omega))}}{2\omega(1-\gamma)} < 1.\end{aligned}$$

If a solution exists at the equilibrium, we denote the effort e_a^{**}, the reward R_a^{**} and the random check π_a^{**}. If $\pi < \underline{\pi}_a$ or $\pi > \overline{\pi}_a$, $e_a^{**} = 0$, $R_a^{**} = 0$ and $\pi_a^{**} = \omega/c$. Thus, having $\pi_a^{**} < \underline{\pi}_a$ or $\pi_a^{**} > \overline{\pi}_a$ is equivalent to have $\omega/\underline{\pi}_a < c$ or $\omega/\overline{\pi}_a > c$. For $\omega/\underline{\pi}_a > c$ or $\omega/\overline{\pi}_a < c$, $R_a^{**} > 0$, $\pi_a^{**} > 0$ and $e_a^{**} > 0$. This complete the proof of **Lemma 5.**

For $c \in [\omega/\underline{\pi}_a, \omega/\overline{\pi}_a]$, $e_a^{**} = 0$, $R_a^{**} = 0$ and $\pi_a^{**} = \omega/c$ whereas $0 < e^{oo} < 1$ and $0 < \pi^{oo} < 1$. A comparison of π^{oo} and $\pi_a^{**} = \omega/c$ show that $\pi_a^{**} > \pi^{oo}$. This complete the proof of **Proposition 4.**

Career Concerns

Using Bayes' rule, the private market updated beliefs concerning the ability of the examiner are

$$\widehat{\theta}(e) = \Pr(talented/granted) = \frac{\Pr(granted/talented)\Pr(talented)}{\Pr(granted)}$$

which is equivalent to

$$\widehat{\theta}(e) = \theta\frac{\gamma+(1-\gamma)(1-p)e+(1-\gamma)(1-p)(1-e)\pi}{\gamma+(1-\gamma)(1-p)\overline{\alpha}e+(1-\gamma)(1-p)(1-\overline{\alpha}e)\pi}.$$

We calculate that

$$\frac{\partial\widehat{\theta}(e)}{\partial e} = \frac{\theta(1-\gamma)(1-p)(1-\pi)(1-\overline{\alpha})(\gamma+(1-\gamma)(1-p)\pi)}{(\gamma+(1-\gamma)\overline{\alpha}e(1-p)+(1-\gamma)(1-p)(1-\overline{\alpha}e)\pi)^2} > 0.$$

The first period choice of effort of the examiner in presence of career concerns is solution of (32) which is equivalent to solving

$$\max_e \{S(e) + \delta\pi_c[\gamma + (1-\gamma)e(1-p) + (1-\gamma)(1-p)(1-e)\pi]\theta - C(e)\}.$$

The FOC gives

$$\frac{\partial S(e)}{\partial e} - \frac{\partial C(e)}{\partial e} + \delta\pi_c(1-\gamma)(1-p)(1-\pi)\theta = 0$$

With the modified model setting, we obtain that

$$\frac{\partial S(e)}{\partial e} = \overline{\alpha} R(1-\gamma)[\beta p + (1-\beta)(\pi - p)],$$

and thus $e^{cc}(R)$ if defined by (33) and the reward is (34). We then calculate the optimal level of examiner's effort as (35). The derivative gives

$$\frac{\partial e^{cc}}{\partial y} = \frac{\partial e_c^*}{\partial y} + \frac{\partial}{\partial y}(\tfrac{1}{2}(1-p)(1-\gamma)(1-\pi)\pi_c \delta\theta).$$

Let's calculate

$$\frac{\partial e^{cc}}{\partial \pi} = \frac{\partial e_c^*}{\partial \pi} - \tfrac{1}{2}(1-p)(1-\gamma)\pi_c \delta\theta,$$

$\pi_c^* = \arg\max \frac{\partial e_c^*}{\partial \pi}$ and $\pi_{cc}^* = \arg\max \frac{\partial e^{cc}}{\partial \pi}$. Evaluated at π_c^*, $\frac{\partial e^{cc}}{\partial \pi}\Big|_{\pi_c^*} = -\frac{1}{2}(1-p)(1-\gamma)\pi_c\delta\theta < 0$ and, thus, $\pi_{cc}^* < \pi_c^*$. This complete the proof of **Proposition 5**.

In the following table we summarize the derivatives, which provide the proofs of **Lemmas 6 and 7 and Proposition 6**.

$y =$ \ derivative	$\frac{\partial e_c^*}{\partial y}$	$\frac{\partial e^{cc}}{\partial y}$
η	< 0	$= \frac{\partial e_c^*}{\partial \eta} < 0$
β	> 0	$= \frac{\partial e_c^*}{\partial \beta} > 0$
γ	< 0	$< \frac{\partial e_c^*}{\partial \gamma}$
θ	> 0	$> \frac{\partial e_c^*}{\partial \theta}$
p	< 0 if $\beta < 1/2$ & $\frac{\partial \omega}{\partial p} < 0$ > 0 if $\beta < 1/2$ & $\frac{\partial \omega}{\partial p} \geq 0$	$< \frac{\partial e_c^*}{\partial p}$ if $\beta < 1/2$ & $\frac{\partial \omega}{\partial p} < 0$? if $\beta < 1/2$ & $\frac{\partial \omega}{\partial p} \geq 0$
π_c	< 0	< 0 if $\theta < \frac{1}{\eta(1-\gamma)\delta(1-p)}$ > 0 if $\theta > \frac{1}{\eta(1-\gamma)\delta(1-p)}$
π	> 0 for $\pi < \pi_c^*$ < 0 if $\pi > \pi_c^*$	> 0 for $\pi < \pi_{cc}^*$ < 0 if $\pi > \pi_{cc}^*$