Subjective Performance of Patent Examiners, Implicit Contracts and Self-Funded Patent Offices

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Abstract

Self-funded patent offices should be concerned with patent quality (patents should be granted to only deserving innovations) and quantity (as revenues come from fees paid by applicants). In this context, we investigate what is the impact of the self-funded constraint on different bonus contracts, and how these contracts affect the examiners' incentive to prosecute patent applications. We consider contracts in which a patent office offers bonuses on quantity quotas (explicit contract) and on quality outcome (either an implicit contract or an explicit contract based on a quality proxy). We find that a self-funded constrained agency should make different organization choices of incentives. For a low quality proxy precision, an agency facing a tight budget operates well with implicit contracts. However, by only relaxing moderately the budget constraint, the agency might be worse off simply because this will preclude implicit contracts. Only very large patenting fees might allow the agency to compensate for the loss of implicit contracts.

Keywords: Patents; Examiners; Explicit and Implicit Contracts, Self-funded Agency

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1 Introduction

The patent system is in crisis and reforms need to be implemented.[1] In order to improve the quality of the patent prosecution process, it is crucial to have a better understanding of its internal functioning and its governance and management practices. As patent examiners have a key role to play (they grant temporary monopoly rights to applicants), it is important to analyze their examination and granting behaviors, and the structure of their incentives.

In an effort to improve the quality of the patenting process, the U.S. Patent and Trademark Office (PTO) has proposed reforms and initiatives. For instance, in 2010, it has implemented a new examiner count system to evaluate examiner's work,[2] and announced the adoption of new and more comprehensive procedures for measuring the quality of patent examination. In 2016, the PTO established an "Enhanced Patent Quality Initiative," in which issues related to examination times and production quotas system were addressed.[3] Through these initiatives, the PTO signals that patent quality matters. So does quantity. Indeed, the PTO is a self-funded agency[4] that obtains revenues from applicant fees to cover part of its operating costs.[5] Therefore, when establishing examiners' rewards, the PTO should be concerned with both quantity and quality of patents.

During the patent prosecution process, while examining a patent application, an examiner must assess the novel content of the innovation. To determine whether it satisfies the patentability requirements, he has to search for prior art information, which corresponds to the set of existing innovations prior to the filing of the patent application. At any point in time, an examiner treats new cases and disposes of ongoing cases. Therefore, there is an implicit trade-off between the effort that the examiner is exerting to search for potentially invalidating information (related to quality) and the effort exerted to process more applications (related to

[1] See Bessen and Meurer (2008) and Gilbert (2011).

[2] In February 2010, the USPTO has published a press release (http://www.uspto.gov/news/pr/2010/10_08.jsp). Until then, examiners' production quotas were based on outdated criteria established in the 1970s, and the PTO has been urged to modernize evaluations of patent examiners (GAO 2007, 2008).

[3] This initiative has now been completed, but other quality initiatives are ongoing. See the USPTO website (https://www.uspto.gov).

[4] As are other patent offices in Canada and U.K., for instance.

[5] In 2012 in the U.S., patents and trademarks generated total revenue of $ 1,854 million. At the same time, the functioning costs of the PTO were $2,148 million, 70% of which were labor costs.

quantity). However, incentives for quality and for quantity are distinct and cannot be provided by the same type of instrument. As any worker, examiners will respond differently to different incentives schemes (more quality-oriented or quantity-oriented).

Our objective is to analyze the impact of different salary schemes offered by the PTO on the provision of examiners' efforts in examining patent applications. We investigate how the structure of reward of patent examiners is affected by the weight that the PTO puts on quality and quantity, and what is the impact of having a self-funded agency on the examiner's work. To do so, we propose a theoretical framework that analyzes different contracts that the PTO might offer to patent examiners. We consider multitask examiners who are offered (explicit and/or implicit) contracts based on different (objective or subjective) performance measures linked to quantity and quality of granted patents. In this setting, we investigate how contracts based on subjective and/or objective performance measures are affected by the self-funding constraint and, ultimately, how this interacts with the provision of efforts of patent examiners, and the shape of the PTO's benefit function. Our main contribution is to analyze the link between funding operations and the type of incentive regimes that an organization can offer to its employees.

Our model features a principal, the PTO, that offers a salary scheme to an examiner based on two tasks: task 1 (related to quantity) is to process more applications and task 2 (related to quality) is to search for invalidating information. Task 1 is easily contractible (e.g., quotas system) while task 2 is not. The PTO can thus either rely on implicit contracting (where the quality assessment is done by a more senior examiner but cannot be formally included in a contract) or use a second, yet imperfect, performance measure on task 2 (e.g., number of citations added by the examiner). The examiner incurs a costly effort to process more applications (task 1) and a costly effort to search for invalidating information (task 2). The PTO observes the outcomes in terms of quantity and can observe a noisy proxy of quality, but not the efforts.

The effort on task 2 also affects the second performance measure on task 2, which is a proxy for the examiner's contribution to quality. However, this proxy depends also on a random variable; before making any effort, the examiner receives private information about the patent application. For instance, imagine that the PTO decides that the number of citations included by the examiner is a proxy for his search of invalidating information. When he receives a patent application, the examiner can learn whether this will be an easy application because he has recently encountered an application on a similar topic and he knows where to search. By only exerting a small effort, he can easily reach the goal set in the proxy (i.e., provide enough citations). Thus, when the proxy's goal is achieved, the PTO does not know whether the examiner got an 'easy' application or has been working hard to find citations.

In this setting, when self-funding is not an issue for the PTO (as patenting fees are sufficiently large), it will use objective performance measures as long as the proxy for quality tracks the true performance satisfactorily. When the proxy becomes noisy in the sense that the examiner can easily take advantage of its imprecision and "game" the incentive system set up by the PTO, the latter is reluctant to use it extensively. Instead, the PTO opts for a more subjective measure of performance. While this measure cannot be included in third-party enforceable contracts, it has the obvious advantage of being unbiased. When the PTO uses implicit contracting on bonuses, the trustworthiness of its bonus promises to examiners is driven by its incentives to renege on such promises. Knowing this, examiners will not work hard if they expect that the fallback position of the PTO is unattractive. As such, the existence of good objective proxies for high quality patent work can be a mixed blessing and decrease the PTO's benefit.

However, when funding is an issue, self-funding has an impact on the PTO's incentive organization. The role played by self-funding is crucial in our analysis as it imposes a constraint on both the power and the type of incentives that the PTO can propose to examiners. For high proxy precision, we show that there is a trade-off between high examiner performance and the level of self-funding. In other words, a higher funding level allows the PTO to provide strong incentives.

Nevertheless, when the proxy precision is low, the PTO must make rather extreme organizational choices of incentives for different levels of funding. Low levels of self-funding imply that the PTO must operate on an implicit incentive regime where subjective performance (and trust) do prevail. However, in this case we show that a moderate increase in the level of funding cannot make the PTO better off simply because, although this allows for stronger explicit incentives, such increase will preclude implicit incentives. Hence, the simple logic that higher patent fees will enhance performance incentives no longer holds. Only very large funding levels can allow the PTO to compensate for the loss of implicit incentives. Arguably, this result underscores the importance of self-funding choices for the proper functioning of the PTO.

Despite decades of patent literature, only recently attention has been brought to the patent prosecution system and patent office governance (for surveys, see Hall and Harhoff, 2012 and Eckert and Langinier, 2014). Due to easier access to patent data, a growing number of empirical studies have started to open the "black box" of the process by which patents are granted (Lemley and Sampat, 2012; Frakes and Wesserman, 2016, 2017; Langinier and Lluis, 2017) and a few surveys have been conducted in patent offices (Friebel et al., 2006; Cockburn, Kortum and Stern, 2003). These empirical contributions provide evidence of the existence of heterogeneity among patent examiners and in the examination process. Furthermore, empirical findings show that questionable patents might be granted because of poor performance by experienced examiners (Lemley and Sampat, 2012), by newly promoted ones (Frakes and Wesserman, 2017), but also by examiners who despite searching very hard to find invalidating evidence eventually had to grant patents as they need to dispose of patent cases (Lei and Wright, 2017). These findings suggest that examiners are different in their examination process, but also that examiners might exert different efforts depending on the type of application received.[6] Frakes and Wasserman (2016) find that time constraints can explain the grant of some questionable patents. These results suggest that the lack of time to process patent applications tends to push examiners to reduce their prosecution time and to grant more patents. Thus, examiners exert more effort to reach their quotas, and less effort to achieve a quality level.[7]

A few theoretical contributions analyze the strategic behavior of patent applicants and examiners, with an emphasis on the strategic behavior of applicants while searching and revealing information. The granting of questionable patents may be due to the poor knowledge of relevant prior art. Applicants might not have incentives to search for relevant information (Atal and Bar, 2010), and if they do, they do not always reveal all their prior art (Langinier and Marcoul, 2016). In these contributions, the PTO and examiners are the same entity. However, they have different objective functions, as the PTO is a federal self-funded institution for which many examiners are working.

[6] Examination practices are also different by patent offices. Gimeno-Fabra and van Pottelsberghe de la Potterie (2017) show that there are differences across patent offices at different levels in the prior art search and speed of examination. Righi and Simcoe (2017) find that patent examiners tend to specialize in particular technologies.

[7] A study on Korean patents provides empirical evidence showing that examiners tend to grant (rather than to reject) more patents and their examination quality decreases when their workload increases (Oh and Kim, 2017).

While designing examiners' incentives, in a model where examiners derive utility from both monetary transfers and intrinsic non-monetary rewards, Schuett (2013) shows that intrinsic motivation (making the right decision) could be significant for the examiner to avoid making wrong decisions. Unlike Schuett (2013), our contribution considers only monetary incentives provided by the PTO to the examiners based on both quality and quantity. We also take the information provided by the applicant as given, and we focus on the examiners' incentive to prosecute patents given the incentive scheme provided by a financially constrained PTO.

As many patent offices are self-funded agencies, patent fees must be set appropriately.[8] Several contributions have studied the optimal of setting of patenting fees to encourage patent applications. It has been shown that socially optimal renewal fees should be increasing over time (Scotchmer, 1999; Cornelli and Schankerman, 1999), even though an optimal fee structure may deviate from the socially optimal scheme when the PTO is bound by a budget constraint (Gans, King and Lampe, 2004). Indeed, a self-funded PTO has an incentive to encourage too many rewards and to reduce renewal fees. Picard and van Pottelsberghe (2013) explore how patent fee setting influences the demand for patent examination, and they argue that a self-funded PTO faces a trade-off between too soft patent examination and higher quality of granted patents. Increasing patenting fees has an impact on the demand for patents (Atal and Bar, 2014), but has also an impact on the organization choice of incentives of a self-funded patent office. Most of these contributions consider the demand for patent examination, which we assume to be inelastic. Indeed, our primary focus is to investigate whether patenting fees have an impact on the internal functioning of the PTO given an inelastic demand for patents, and we analyze the impact of a self-funded PTO on the examiners' work. This is in line with Frakes and Wasserman (2014) who, by looking at the relationship between funding structure of the PTO and its examination practice, empirically show that a self-funded PTO provides incentives to process applications faster and increase grant rates.

Our paper is also linked to the literature on multitasking. In a multitask principal-agent model, Holmstrom and Milgrom (1991) consider a static model in which a principal offers an explicit contract to a risk averse agent, whose cost depends on the total effort devoted to all

[8] Patent fees vary across time and across countries (Lerner, 2000; de Rassenfosse and van Pottelsberghe, 2011). Furthermore, patent fees are divided between application fees and renewal fees where the later fees contain some uncertainty as not all patents are renewed.

tasks. They find that when one task is poorly measured (which is typically the case for quality), it might be impossible to maintain a good quality while using a 'piece rate' scheme (where the reward is directly related to the performance measure).

As objective performance measures are not always available or are imperfect, subjective performance measures have been analyzed (Baker, Gibbons and Murphy, 1994; MacLeod and Malcomson, 1989). When rewards depend only on imperfect measures, workers' incentives are not perfectly aligned with the principal's objective. This problem may be mitigated by using subjective performance measures (non-verifiable and non-contractible). However, agreements based on subjective performance evaluation have to be self-enforcing as they cannot be part of an enforceable contract in court.

In the context of a repeated principal-agent model with risk neutral agents, Baker, Gibbons and Murphy (1994) consider that a task can be objectively or subjectively assessed and, thus, explicit but also implicit contracts can be offered. When the agent's contribution to the firm's value is not objectively measurable (but can be imperfectly assessed by the principal), an implicit contract based on subjective performance assessments may increase or replace an explicit contract based on objective performance measurements. However, trust between agents and the principal is essential and an implicit contract is a self-enforcing contract. The authors find that if the objective measure is sufficiently close to perfect, no implicit contract is feasible because the firm's fallback position after reneging is too attractive. In an efficient contract, the weight given to one performance measure should be decreasing in the precision and sensitivity of the other measures. We built on Baker, Gibbons and Murphy (1994) model in which we consider that two tasks must be performed, each of which has a different weight for the patent office.

In a multitask context, Schottner (2008) considers a repeated principal agent model in which one agent can perform three tasks that can be substitute or complement whose contributions to the firm are non-verifiable. The author finds that as the bonus on explicit contract decreases with the bonus on implicit contract, explicit and implicit contracts are substitute when the principal's fallback position is positive. Implicit contracts are more likely to exist if tasks are substitutes rather than complements. In her model, there is a unique bonus for all tasks, and the probability of achieving one task is conditional on all the efforts. In our setting, tasks are substitute, each of them contributes differently to the principal's outcome and the PTO can offer bonus on each task.

The paper is organized as follows. In Section 2 we briefly present the patent prosecution process and the incentive awards of patent examiners. The model is introduced in Section 3. Section 4 is devoted to the analysis of different incentive schemes offered by the PTO. In Section 5 we analyze under what circumstances implicit contract will be favored to explicit contracts. Section 6 concludes.

2 Patent Prosecution Process and Incentive Awards

The patent prosecution process and the incentive awards of U.S. patent examiners are well documented (NAPA, 2005). Once a patent application is received at the PTO, after completion of administrative checks it is sent to an Art Unit (group of examiners with similar expertise) where it is processed by a Supervisory Patent Examiner (SPE) who attributes the application to an examiner. This examiner becomes responsible for the application, and must determine whether the innovation satisfies the patentable requirements (new, novel and non-obvious).

The patent prosecution process can be divided into four tasks: search, examination, amendment review and post examination (NAPA, 2005). During the *search task*, the examiner reviews the application, searches for prior art information (in general, the examiner starts with a search of patented innovations closely related to the claimed invention, foreign patents and non-patented information), and analyzes the claims. The *examination task* requires that the examiner compares the invention described in the patent application to the prior art search results, and to the prior art provided by the applicant. He must determine the proper scope of the invention claimed by the inventor. Then, he prepares the 'first action letter' in which either he allows the patent to be granted or proposes a non-final rejection that is submitted to his superior for review. If there is an *amendment* to a non-final rejection, the examiner searches for more information, writes the second (generally last) action letter in which he rejects or grants the patent. In the *post examination stage*, the examiner finalizes the disposal of the patent application.[9]

The examiners award system has been in place since 1976 and is mainly based on the number of patent applications processed. New hires usually enter at around level $GS-7$ (general

[9] Examiners work closely with applicants (or attorneys) either to narrow the scope of the application or to split up the invention and file separate applications. It can take years before the final action (approval or rejection).

schedule) pay scale, whereas experienced examiners are at level $GS - 9$ or above. A production expectancy goal based on the technological field and the experience of the examiner is set for each examiner.[10] An examiner is given 'credit' at two different times during the examination process: when a patent application is first examined (new case, he gets 1.25), and when it is disposed of (allowed, rejected, abandoned, he gets 0.75). Once the quality of either of these actions is approved by a SPE, the examiner receives credit for it.[11] Thus, this quality check is done by a given SPE and is thus subjective in that sense.

As his career progresses, an examiner is expected to examine more cases. The production quotas system is measured by two different equations depending on experience. The balance disposal (BD) of experience examiners is $BD = (N + D)/2$ and a new hire $BD = (2N + D)/3$ where N corresponds to a new case (and the first action letter has been sent), and D is disposal (allowance, rejection or abandonment).

Examiners get different types of incentive awards: an annual gain-sharing award (which accounts for 1 to 6% of base salary), and a special achievement award (which accounts for 3% of base salary). The first award corresponds to the promotion to the next GS level. Examiners have noncompetitive promotion possibility at the potential rate of one GS level per year up to GS-13. In order to be promoted they need to exceed their production quotas by more than 10% on average over the fiscal year, with few errors.[12] The second award corresponds to a bonus that examiners get whenever they exceed their production goals by at least 10% on average over four consecutive quarters. If their production goals is above 110%, they get a bonus of 5% of current salary, over 120 it is a 7% and over 130% it is a 9% bonus.[13]

Examiners who consistently fail to meet their quotas get first an oral warning, then a written warning and eventually, they can be dismissed. In 2004, 329 oral warnings have been issued,

[10] For instance, for the same application (same technological field) the production expectancy goal of a $GS - 7$ examiner will be 39.3 hours, whereas it will be 27.5 hours for an examiner at the level $GS - 12$, and 20.4 hours for a $GS - 14$ examiner.

[11] Expected annual productivity is calculated by assuming that 80% of the 2,080 hours in a 52 work week per year of 40 hours per week will be spent examining applications (NAPA, 2005). For instance, an examiner with a goal of 31.6 hours per application would need to complete 53 applications or 106 counts, whereas an examiner with a goal of 14.3 hours per application needs to complete 116 applications or 232 counts. On average, 87 applications per year must be processed per examiner.

[12] In the period 1999-2003 between 60% to 73% of patent examiners got promoted (NAPA, 2005).

[13] Within the period 1999 and 2003, between 63% and 77% of patent examiners got a bonus (NAPA, 2005).

48 written warnings and 17 removals (NAPA, 2005). As part of a patent quality measurement program, the PTO performs annual quality reviews on about 2 to 3 percent of granted patents. A reviewer determines whether an examiner made an error in at least one claim that was allowed in the patent.[14] If a reviewer determines that an examiner made an error, the case is reopened.[15]

The necessary skills to be an examiner are decision-maker skills, ability to compartmentalize, writing and reading. Examiners need to be multitask,[16] as they simultaneously have to determine the patentability of an innovation for new cases, and have to make decisions on older cases.[17]

Overall, examiners evaluations are based on quotas (number of processed patent applications), errors (if the examiner is junior, there is systematically a more senior examiner who signs patent applications for him), and random check of issued patents. However, according to GAO (2016), the PTO provides examiners with incentives (bonuses) for reaching their quotas, but does not offer a bonus for producing high-quality work.

3 The Model

We consider an infinitely repeated one-stage game with two players, the PTO and a patent examiner. In each stage game, the PTO offers a salary scheme (fixed salary and bonuses) to the examiner who reviews one patent application[18] and assesses the patentability of the innovation.[19]

The PTO values both quantity and quality of granted patents. In terms of quantity, we assume that the PTO sets a quota, which consists of a number of patents that must be processed (based, for instance, on the experience of the examiner and the fields of technology).[20] This is equivalent to having the PTO set a (average) number of hours per application. We denote y_1

[14] According to the PTO, the error rate is relatively constant over time (in 2000 it was 6.6%, 4.2% in 2002 and 5.3% in 2004) and it varies among technological centers: from 2.5% to 9% in different centers in 2004.

[15] Within the period 2000 and 2004, 302 to 401 applications have been reopened.

[16] Their work is complex and highly specialized as they have to complete different tasks.

[17] They also need to be knowledgeable of the patent law: if correctly granted patents should not be invalidated in court in case of prosecution.

[18] We implicitly assume that the examiner reviews n patent applications, where we normalize $n = 1$.

[19] The patent examiner should reject an application that fails to meet the patentability requirement and grant a patent to an innovation that satisfies the patentability standards. Here, we do not consider the outcome *per se*, but only whether mistakes have been made during the examination process.

[20] If the examiner were to review n patent applications, the quota could be set at $\underline{n}$, such that the examiner must process at least $n \geq \underline{n}$.

this first task and we assume that y_1 equals zero or one; if the examiner reaches the targeted quota, $y_1 = 1$. It is also the PTO's responsibility to insure that only deserving innovations be granted patents, which is linked to quality. Let y_2 be this second task, where y_2 also equals zero or one, with $y_2 = 1$ corresponds to the case where no mistakes have been made.

Once the examiner receives a patent application, in order to complete these two tasks, he exerts costly efforts: a continuous effort $e_1 \in [0, 1]$ to perform satisfactory on the first task (to process the application in the appropriate time frame), and a continuous effort $e_2 \in [0, 1]$ to perform on the second task (to grant a patent to a patentable innovation). The second task implies that the examiner exerts effort to search for information that will prove that the innovation is not novel (i.e., the examiner will find invalidating information).

The essential inputs in the patent examination process are the examiner's efforts e_1 and e_2. When the examiner reaches the targeted quantity, the output on task 1 is $y_1 = 1$ and when he reaches some threshold measure of quality then $y_2 = 1$. Therefore, we define the probability that he performs satisfactorily on task i, $i = 1, 2$ as

$$p_i = \Pr\left(y_i = 1 \mid e_i\right) = \frac{1}{2}\left(1 + e_i\right). \tag{1}$$

For task 1, this conditional probability states that, even in the absence of effort, the examiner can reach his quota with probability 1/2. That is, by just processing applications with no effort (a "push paper" strategy), he may meet his quota. Similarly, if he does not make any effort toward quality (task 2), the examiner might get lucky and grants a patent to a deserving innovation.

The total cost incurred by the examiner when he exerts efforts (e_1, e_2) is

$$C\left(e_1, e_2\right) = \frac{e_1^2}{2} + \frac{e_2^2}{2} + \theta e_1 e_2, \tag{2}$$

where $\theta \in [0, 1[$ denotes the conflict between the two tasks as perceived by the examiner. Indeed, increasing the effort to comply with patent quality standards raises the marginal cost of 'speeding up' the patent processing rate. If $\theta = 0$, both tasks are independent, whereas if $\theta > 0$ both tasks are substitutes. Therefore, an increase in θ has two effects: it increases the overall cost incurred by the examiner but it also increases the conflict between the tasks.

We consider that the PTO rewards the examiner based on a formulaic bonus approach that takes into account whether the quota has been reached[21] and a quality measure has been

[21] Or the number of processed applications (whether rejected or accepted).

achieved. Although the quota is easy to measure, perfectly observable and contractible, the outcome of task 2 while observable by an expert is not contractible as objective measures of quality are arguably hard to assess. We assume that a senior examiner will check some of the patent applications that have been processed by an examiner, but this remains a subjective assessment. Thus, the outcome of task 2 is observable by a supervisor but not contractible. However, proxies (e.g., applicant's satisfaction surveys, number of legally disputed patents, or number of citations included in the patent) for the performance on task 2 are available for contracting purposes. Hence, when a proxy is included in the contract, the examiner works towards achieving a higher proxy score which may involve a distortion with respect to the true performance (for instance, the examiner may try to "over please" the applicant which may be different than providing quality work).

We thus consider that there is a second performance measure P_2 of task 2 that is also affected by e_2. It is an imperfect proxy for the examiner's contribution to quality. For simplicity, we also assume that P_2 takes only the value zero or one. When a proxy is used, the probability that the proxy yields a "successful" signal is

$$p_\beta = \Pr\left(P_2 = 1 \mid e_2\right) = \frac{\mu}{2}\left(1 + e_2\right) < 1, \tag{3}$$

where μ is a positive random variable that has a mean $\overline{\mu} = 1$ and a variance σ^2. We assume that, when a proxy is used, a learning process occurs in which the examiner learns some of the characteristics of the application. These characteristics may facilitate or hinder the examiner's workload and are related to the examiner's work history. For instance, consider that the proxy is (positively) related to the number of citations included by the examiner. When he receives an application, the examiner learns that his task to find relevant information will be easier to perform because he has recently granted a similar patent. To model this simple learning process, we assume that the examiner receives a private information μ (thus, the examiner observes a realization of μ) before choosing his effort levels. The interpretation is similar to Baker, Gibbons and Murphy (1994). For the PTO, using a proxy to measure quality will have a distorting effect on the examiner's behavior. For instance, if $\mu < 1$, the examiner knows that μ is small, and thus a high effort on task 2 will increase the probability to reach the quality level (i.e., $y_2 = 1$) but not the probability to get $P_2 = 1$. If $\mu > 1$, a small effort will increase the probability to get $P_2 = 1$, but not $y_2 = 1$.

The examiner's compensation w offered by the PTO includes a base salary s and different bonuses, one for each task.[22] We restrict contracts to two types: either an explicit contract on both tasks (with bonuses on task 1 and on a proxy of task 2), or an explicit contract on task 1 (with a bonus on task 1) and an implicit contract on task 2 (with a bonus on task 2). In both cases, the compensation w will be s, the base salary, if none of the tasks has been completed. If only task 1 has been successfully completed, a bonus b_1 will be paid, and the compensation will be $s + b_1$. If only task 2 has been completed, the compensation will be $s + \beta_2$ if an explicit contract is offered on an imperfect proxy for task 2 or $s + b_2$ if the PTO offers an implicit contract on the subjective performance measure. Therefore, in the case of explicit contracts on both tasks, the PTO offers a package (s, b_1, β_2) to the examiner, whose expected compensation is

$$w_{\text{exp}} = s + p_1 b_1 + p_\beta \beta_2. \tag{4}$$

In the case of an explicit contract on task 1 and an implicit contract on task 2, a package (s, b_1, b_2) is offered and the expected compensation of the examiner is

$$w_{imp} = s + p_1 b_1 + p_2 b_2. \tag{5}$$

Furthermore, we assume that the PTO is a self-funded agency, and that it cannot run a long-term deficit so that the patenting fees paid by applicants must cover the operating costs in expectation. For the PTO, while each patent application generates patenting fees, it is also costly to process.[23] We therefore add a self-funding constraint

$$F \geq w, \tag{SFC}$$

where F represents the fee paid by the patent applicant, and w the expected wage paid to the examiner which is either (4) or (5).[24]

The PTO's benefit depends on the realization of the two tasks. When the examiner reaches the targeted quantity ($y_1 = 1$) it generates a value V to the PTO and when the quality level has

[22] Having a linear contract is not necessary optimal but it allows for tractable results.

[23] Application fees should also depend on the disposal of the patent (rejected or accepted). To simplify we assume that a unique fee is paid when the applicant applies for a patent.

[24] This is similar to Picard and van Pottelsberghe (2013). It is obviously a fairly simple representation of a more complex constraint as we only consider one examiner and one patent. If an examiner were to process n patent applications, the constraint would be $F \geq w/n$.

been reached, it generates a value 1.[25] When the examiner fails on a task the PTO receives 0. If $V > 1$ (resp., $V < 1$), the PTO derives more benefit from quantity (resp., quality) than quality (resp., quantity). Even though the PTO values both quantity and quality of granted patents, it puts potentially different value on them. The PTO's expected benefit when the examiner contributes to both tasks for a compensation w is

$$B = p_1 V + p_2 - w + F, \tag{6}$$

where p_i for $i = 1, 2$ is the probability that the examiner performs satisfactorily on task i as defined by (1), V is the value of the outcome related to task 1 (the quota is achieved), and 1 the value of the outcome related to task 2 (no mistakes were found).

The timing of each stage game is the following. First, the PTO offers the examiner a salary scheme (a fixed salary and bonuses). Second, the examiner accepts or rejects it. If he rejects it, he gets his outside option value w_r (reservation salary). If he accepts the salary scheme, Nature draws a patent application that the examiner receives, he observes μ if a proxy is used, and he makes costly efforts $(e_1, e_2) \in [0, 1]^2$. Third, both the PTO and the examiner observe the realization of the examiner's contribution, and the realization of the objective performance measure. Fourth, if the examiner has successfully accomplished task 1, the PTO pays the bonus specified in the contract; if task 2 has been accomplished successfully depending on whether it is part of an implicit or an explicit contract, the PTO chooses whether to pay the bonus.

Both the PTO and the examiner are engaged in a repeated relationship, which greatly matters when the PTO agrees on an implicit contract. When everything is contractible, the PTO will always honor the contract and only the one stage game needs to be analyzed. However, in the case of an implicit contract that relies on promises and cannot be formally written in a contract, the repeated interaction does matter.

We summarize in the following table the different variables of the model.

[25] Alternatively, we could consider that if task 1 (resp., task 2) is completed it generated a value v_1 (resp., v_2). This is equivalent to having $V = v_1/v_2$, where V is the relative value of task 1 over task 2.

	Exogenous variables		Endogenous variables
$y_1 = 1$	if task 1 (quantity) is completed	e_1	effort to accomplish task 1 (quantity)
$y_2 = 1$	if task 2 (quality) is completed	e_2	effort to accomplish task 2 (quality)
$P_2 = 1$	if proxy on task 2 (quality) is completed		For $i = 1, 2$
μ	random variable (private information)	$p_i = \frac{1}{2}(1 + e_i)$	probability to complete task i
σ^2	variance; noisy proxy	$p_\beta = \frac{\mu}{2}(1 + e_2)$	probability to achieve proxy measure
w_r	reservation salary		
V	relative value of task 1 over task 2	s	based salary
θ	substitutability between tasks	b_1	bonus on task 1
F	patenting fee	b_2	bonus on task 2
		β_2	bonus on proxy of task 2

4 Different Incentive Schemes

In this section, we study different incentive schemes. We first analyze the case where the efforts are observable. Second, we analyze the case where the efforts are not observable, and the PTO offers only explicit contracts on both tasks (the PTO uses a proxy on quality). Third, we analyze the situation where the PTO offers an explicit contract on the contractible outcome (quantity) and an implicit contract on the non-contractible outcome linked to the quality.

4.1 First-best Effort Levels

As a benchmark, we first consider the case in which efforts are observable and the outcomes of both tasks are contractible. The expected benefit of the PTO is defined by (6) where p_i is defined by (1) for $i = 1, 2$. In order for the examiner to accept the contract offered by the PTO, he must get at least his reservation wage w_r by making efforts (e_1, e_2) such that the participation constraint

$$w - C(e_1, e_2) \geq w_r \tag{PC}$$

is satisfied. Thus, the PTO chooses the efforts (e_1, e_2) and the wage w that maximize its benefit (6) subject to satisfying both participation (PC) and self-funding (SFC) constraints. When the patenting fees are large such that $F > w^o$ where

$$w^o \equiv \frac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} + w_r, \tag{7}$$

(SFC) is always satisfied. However, (PC) is binding and, thus, the first-best effort levels are determined by equating the expected marginal benefit of effort $(\partial(p_1 V + p_2)/\partial e_i)$ with its marginal cost $(\partial C(e_1, e_2)/\partial e_i)$ for each task i, with $i = 1, 2$. The first-best optimal effort levels are

$$e_1^o = \frac{V-\theta}{2(1-\theta^2)}, \tag{8}$$

and

$$e_2^o = \frac{1-\theta V}{2(1-\theta^2)}, \tag{9}$$

where effort on task 1 increases with V, while effort on task 2 decreases with V. These efforts are strictly positive if there is not too much conflict between the two tasks such that $\theta < \min\{V, \frac{1}{V}\}$. Moreover, they are smaller than one if $V < \theta + 2(1-\theta^2)$. By introducing a slight restriction on the parameters, we summarize these two conditions in the following assumption

$$\theta < \min\{V, 2-V\}. \tag{A1}$$

However, if (A1) is violated (i.e., if there is too much conflict between the two tasks), depending on whether there is relatively more weight on quality ($V < 1$) or on quantity ($V > 1$), there will be no effort toward quantity or quality. Formally, if $\theta > V$, the efforts are $e_1^o = 0$ and $e_2^o = 1/2$. If $\theta > 1/V$ they are $e_1^o = V/2$ and $e_2^o = 0$. When the value of the outcome related to reaching the quota is higher than the value of the outcome related to avoiding making mistakes (i.e., for $V > 1$), the examiner exerts more effort to reach the quota rather than to achieve the quality level, $e_1^o > e_2^o$ for any $\theta \in [0, 1[$. For low values of θ, both efforts are decreasing as θ increases. If we start from a situation where the tasks are independent ($\theta = 0$), an increase in θ raises the overall cost of the examiner as well as the substitutability between the tasks. The first effect is larger than the second one and, therefore, the examiner puts less effort into searching for invalidating information and in reaching the quota. As θ is further increased, the conflict between the two tasks worsens and, after a certain cut-off value for θ, the examiner starts increasing his effort to perform task 1 (quantity) and keeps lowering his effort to perform task 2 (quality) until it reaches a point where the effort in task 1 is constant and the examiner makes no effort in searching for invalidating information.

On the other hand, for $V < 1$, $e_1^o < e_2^o$ for any $\theta \in [0, 1[$. As the conflict between the two tasks increases and is large ($\theta > V$), the examiner stops exerting any effort in performing task 1 ($e_1^o = 0$) while the effort on task 2 becomes constant. From these two different cases, we obtain the following standard results.

Lemma 1 *Too much conflict between tasks is potentially conducive to no effort at one task.*

In what follows, we assume that efforts are positive such that there is not too much conflict between the two tasks and thus (A1) is always satisfied.

Evaluated at the first-best effort levels (e_1^o, e_2^o) as defined by (8) and (9), the optimal wage is

$$w^o = C\left(e_1^o, e_2^o\right) + w_r,$$

and the PTO's first-best benefit function is

$$B^o = \frac{1}{2} + \frac{1}{2}V - w_r + F + \tfrac{(V-\theta)^2 + (1-\theta^2)}{8(1-\theta^2)}. \tag{10}$$

When patenting fees are small such that $F \in]w_r, w^o]$, the self-funding constraint binds and the effort levels are thus affected. The resolution of the program (provided in appendix) gives the constrained first-best effort levels (e_1^{oF}, e_2^{oF}) such that

$$e_i^{oF} = \Phi_s e_i^o, \tag{11}$$

for $i = 1, 2$ where

$$\Phi_s = \left[\tfrac{8(F-w_r)(1-\theta^2)}{(V-\theta)^2+(1-\theta^2)}\right]^{\frac{1}{2}} < 1. \tag{12}$$

As the PTO is a self-funded agency, lower patenting fees will induce lower efforts. As F increases, the effort levels increase. Indeed, as the patenting fee increases, the self-funding constraint is relaxed, and thus the PTO wants to induce more efforts. The effort levels decrease with an increase in w_r; as the outside option becomes more valuable, the PTO must induce less effort to reduce the cost of effort. An increase in θ has the same effect on e_i^{oF} as it has on e_i^o, where the effect is magnified as Φ_s does also depend on θ.

A comparison of the first-best effort levels is provided in the following Lemma.

Lemma 2 *For a given small patenting fee $F \in]w_r, w^o]$, the constrained first-best effort levels are suboptimal, $e_i^{oF} < e_i^o$ for $i = 1, 2$.*

When patenting fees are small, due to the self-funding constraint that the PTO faces, the constrained first-best effort levels are suboptimal as the PTO must reduce the wage offered to the examiner to

$$w^{oF} = C\left(e_1^{oF}, e_2^{oF}\right) + w_r < w^o.$$

The PTO's benefit function is also reduced

$$B^{oF} = B^o - \tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)}(1-\Phi_s)^2 < B^o. \tag{13}$$

To summarize, not surprisingly, when patenting fees of a self-funded PTO are set too low, there is a distortion in the provision of efforts: the PTO must reduce the salary of the examiner, which in turn provides less incentives to make efforts.

4.2 Explicit Contracts

We now consider the case where the efforts are no longer observable, and the PTO offers explicit contracts on both tasks. Task 1, related to quantity, is contractible and, thus, can be part of an explicit contract, whereas task 2 is not contractible. Therefore, a proxy on task 2 must be used, and is included in an explicit contract. The PTO offers a compensation package (s, b_1, β_2) to the examiner where bonuses and salary are always non-negative. The PTO must insure that not only the examiner will accept the contract but also that he will provide adequate effort levels, which implies that the incentive compatibility constraint must be satisfied. Thus, the program of the PTO becomes

$$\underset{e_1,e_2,s,b_1,\beta_2}{Max}\ B,$$

subject to

$$E_\mu[w_{\exp} - C\left(e_1, e_2\right)] \geq w_r, \tag{14}$$

$$(e_1, e_2) \in \arg\max\{w_{\exp} - C\left(e_1, e_2\right)\}, \tag{15}$$

$$F \geq E_\mu[w_{\exp}], \tag{SFC}$$

where $w_{\exp}$ is defined by (4).

We first consider that the patenting fees are large (such that the self-funding constraint is satisfied in equilibrium), i.e., $F \geq w^*(\sigma^2)$, where

$$w^*(\sigma^2) \equiv w^o - \tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)}. \tag{16}$$

To solve the optimization program of the PTO, we start by calculating the optimal effort levels solution of (15) for given b_1 and β_2,[26]

$$e_1(b_1, \beta_2) = \tfrac{b_1 - \theta\mu\beta_2}{2(1-\theta^2)}, \tag{17}$$

and

$$e_2(b_1, \beta_2) = \tfrac{\mu\beta_2 - \theta b_1}{2(1-\theta^2)}. \tag{18}$$

When the examiner chooses his effort levels, he has already learned the value μ. Therefore, when an interior solution exists, the higher μ, the lower his effort on task 1, and the higher his effort on task 2. Indeed, the examiner knows that for a small μ, it will be hard to reach the proxy, whereas for a high μ it will be easier to reach the proxy and thus gets the bonus. The higher the compensation b_1, the higher the effort on task 1 and the lower the effort on task 2. On the other hand, the higher the compensation β_2, the lower the effort on task 1 and the higher the effort on task 2.

When the examiner decides whether to accept the contract, he does not know the value of μ yet. By using the fact that $E\left(\mu^2\right) = var\mu + [E(\mu)]^2$, given the bonuses (b_1, β_2) we solve for the expected wage (see appendix for the details of the calculation) when both efforts are positive

$$w(b_1, \beta_2) = w_r + \tfrac{b_1^2 + \beta_2^2(1+\sigma^2) - 2\theta b_1\beta_2}{8(1-\theta^2)}. \tag{19}$$

The PTO now solves the following simplified program

$$\underset{s, b_1, \beta_2}{Max}\{\frac{1}{2}\left(1 + e_1(b_1, \beta_2)\right)V + \frac{1}{2}\left(1 + e_2(b_1, \beta_2)\right) - w(b_1, \beta_2)\},$$

where $e_1(b_1, \beta_2)$ and $e_2(b_1, \beta_2)$ are defined by (17) and (18) and $w(b_1, \beta_2)$ is defined by (19). The resolution of this maximization program yields the optimal bonuses

$$b_1^* = V - \theta\tfrac{\sigma^2}{1+\sigma^2-\theta^2},$$

and

$$\beta_2^* = 1 - \tfrac{\sigma^2}{1+\sigma^2-\theta^2},$$

[26] Note that depending on the realization of μ, the efforts e_1 and e_2 could be strictly greater than one or smaller than zero. However, by scaling down the variance σ^2, the probability of such event can be made arbitrary small. For convenience, we assume that this restriction is satisfied.

which are always positive, and the optimal fixed salary is

$$s^* = w_r - (\frac{b_1^* + \beta_2^*}{2} + \frac{b_1^{*2} + \beta_2^{*2}(1+\sigma^2) - 2\theta b_1^* \beta_2^*}{8(1-\theta^2)}).$$

If $V > 1$, for any value of θ and σ^2, the optimal bonus on task 1, b_1^*, is larger than the optimal bonus on task 2, β_2^*. As the PTO puts more weight on quantity, the examiner will receive a higher bonus for reaching his quota rather than for making more effort toward quality. Interestingly, when the PTO puts more weight on quality ($V < 1$), the bonus on task 2, β_2^*, is not always larger than the bonus on task 1, b_1^*. Indeed, when $\sigma^2 > \sigma_\beta^2$, where

$$\sigma_\beta^2 \equiv \frac{(1-\theta^2)(1-V)}{V-\theta}, \tag{20}$$

the PTO prefers to provide a higher bonus on task 1 (quantity) than on task 2, even though it puts more weight on quality than on quantity. In fact, the PTO is not eager to reward the examiner on a proxy that becomes too noisy ($\partial\beta_2^*/\partial\sigma^2 < 0$), but, at the same time, the PTO wants to avoid to put too much weight on task 1 so that $\partial b_1^*/\partial\sigma^2 < 0$. Both bonuses are also decreasing with the parameter of substitutability θ. Only the bonus on task 1 is increasing with V. We summarize these comparative static results in the following Lemma.

Lemma 3 *For a large patenting fee $F > w^*(\sigma^2)$, in the case of explicit contracts,*

(i) both bonuses b_1^* and β_2^* are positive and decreasing with σ^2 and θ;

(ii) bonus b_1^* increases with V, while bonus β_2^* is not affected by a change in V.

Comparative statics with respect to σ^2 gives the expected results as both bonuses decrease with σ^2. When the performance measure on task 2 becomes noisier, the PTO is less eager to rely on this performance measure and therefore it decreases β_2^*. However, to avoid too much emphasis on task 1, the PTO decreases the bonus on task 1 as well. Overall, the power of incentives tends to decrease as a noisier performance measure increases the fixed part of the agent's optimal compensation ($\partial s^*/\partial\sigma^2 > 0$). As there is more conflict between the two tasks for the examiner, both bonuses decrease ($\partial b_1^*/\partial\theta < 0$ and $\partial\beta_2^*/\partial\theta < 0$). If V increases, then the bonus on task 1 increases ($\partial b_1^*/\partial V > 0$), and the salary decreases ($\partial s^*/\partial V < 0$).

We illustrate these findings in Figure 1 in which we represent the different bonuses in a graph (θ, σ^2) assuming $V < 1$.

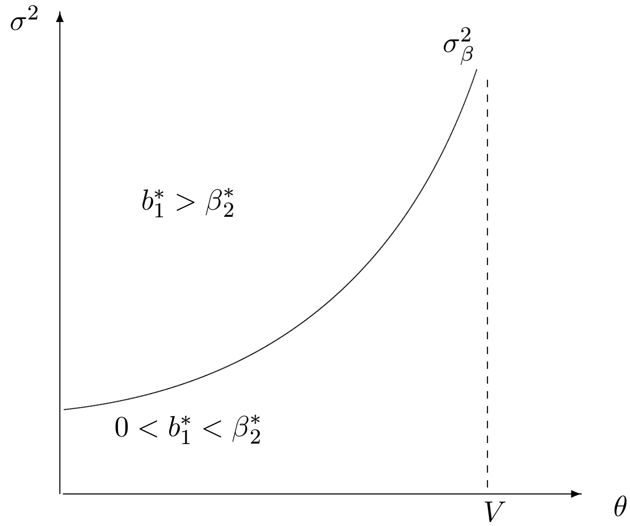

Figure 1: More weight on quality ($V < 1$)

Consider first a low level of substitutability θ. As the proxy becomes noisier (σ^2 increases), both bonus levels decrease. However, even though initially the bonus on task 1 is smaller than the bonus on task 2 (the PTO rewards quality more than quantity), as the proxy becomes noisier (for $\sigma^2 > \sigma_\beta^2$) the PTO starts offering a larger bonus on task 1 rather than on task 2. This is because the PTO does not want to reward lucky examiners. We summarize these findings in the following Proposition.

Proposition 1 *For a large patenting fee $F > w^*(\sigma^2)$, in the case of explicit contracts, when the PTO puts more weight on quality ($V < 1$), as the measure becomes noisier (i.e., σ^2 increases), the PTO is less eager to reward the examiner on a noisy proxy of quality.*

In the case of explicit contracts, the expected equilibrium effort levels $\overline{e}_i^* = E_\mu[e_i^*]$ for $i = 1, 2$ are thus $\overline{e}_1^* = e_1^o$, and

$$\overline{e}_2^* = e_2^o - \tfrac{\sigma^2}{2(1-\theta^2+\sigma^2)}.$$

These efforts are positive as long as $\sigma^2 < \sigma_e^2$ where

$$\sigma_e^2 \equiv \tfrac{(1-\theta V)(1-\theta^2)}{\theta(V-\theta)}.$$

The asymmetric information creates a distortion in the provision of effort 2 as $\overline{e}_2^* \leq e_2^o$, whereas there is no distortion in the effort level on task 1, $\overline{e}_1^* = e_1^o$. If the proxy is not too noisy, both efforts are 'almost' the first-best effort levels. However, when the proxy becomes noisier, the PTO does not want to reward as much the examiner on the second task. Overall, the effort levels are suboptimal, $\overline{e}_1^* = e_1^o$ and $\overline{e}_2^* < e_2^o$ and the examiner exerts more effort on task 1 than on task 2, $\overline{e}_1^* > \overline{e}_2^*$ if $\sigma^2 > \sigma_\beta^2$ where σ_β^2 is defined by (20). However, for $\sigma^2 < \sigma_\beta^2$, $\overline{e}_1^* < \overline{e}_2^*$. We summarize these findings in the following Lemma.

Lemma 4 *For a large patenting fee $F > w^*(\sigma^2)$, the expected effort levels under explicit contracts are (weakly) suboptimal, $\overline{e}_1^* = e_1^o$, and $\overline{e}_2^* < e_2^o$.*

The use of a proxy on task 2 introduces a distortion in the provision of efforts as the examiner has a tendency to undersupply efforts. This distortion increases when the proxy becomes noisier (i.e., when σ^2 increases). The effort on task 2 is decreasing with σ^2 while the effort on task 1 is unaffected by a change in σ^2. The impact of a change of V is the same as in the first-best case as well as the impact of a change in θ. An increase in θ raises the conflict between the tasks but also raises the overall cost of examination. When there is no conflict between the two tasks ($\theta \to 0$), the expected efforts are $\overline{e}_1^* = e_1^o = V/2$, and

$$\overline{e}_2^* = \tfrac{1}{2(1+\sigma^2)} < e_2^o = \tfrac{1}{2}.$$

Therefore, the PTO induces an undersupply of effort on task 2, which is not contractible.

The PTO's expected benefit is thus

$$B^* = B^o - \tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)}, \tag{21}$$

which is decreasing in σ^2 as the noisier the proxy, the lower the expected benefit of the PTO. Not surprisingly, the benefit is suboptimal ($B^* < B^o$).

For lower values of the patenting fees, $F \in]w_r, w^*(\sigma^2)]$, the self-funding constraint is binding. The resolution of the program (provided in appendix) gives the following bonus levels

$$b_1^{*F} = \Phi_\sigma b_1^*, \text{ and } \beta_2^{*F} = \Phi_\sigma \beta_2^*,$$

where

$$\Phi_\sigma = \left[\tfrac{8(F-w_r)(1-\theta^2)(1-\theta^2+\sigma^2)}{(V-\theta)^2(1-\theta^2+\sigma^2)+(1-\theta^2)^2}\right]^{\frac{1}{2}} < 1. \tag{22}$$

Compared to Φ_s given by (12), we show that $\Phi_\sigma > \Phi_s$ for any $\sigma^2 > 0$, and $\Phi_\sigma = \Phi_s$ for $\sigma^2 = 0$. Both bonuses are reduced compared to the case where patenting fees are large, $b_1^{*F} < b_1^*$ and

$\beta_2^{*F} < \beta_2^*$. Indeed, as now the PTO is constrained by the self-funding constraint, it has to reduce the bonuses offered on each task.

Unlike the previous result ($\partial\beta_2^*/\partial V = 0$), the binding self-funding constraint introduces a trade-off between the two bonuses, so that the bonus on task 2 is now decreasing with V ($\partial\beta_2^{*F}/\partial V < 0$) and the bonus on task 1 increases ($\partial b_1^{*F}/\partial V > 0$). As the proxy becomes noisier (i.e., σ^2 increases), the bonus on task 2 decreases as it was the case for high patenting fees, while now the bonus on task 1 increases. The PTO does not want to reward too much the examiner when the proxy is too noisy, but because of the self-funded constraint that reduces the bonuses, the PTO puts more emphasize on task 1, and thus increases the bonus on task 1. We summarize these findings in the following Lemma.

Lemma 5 *For lower patenting fees, $F < w^*(\sigma^2)$, in the case of explicit contracts,*

(i) bonus b_1^{*F} increases with σ^2 while β_2^{*F} decreasing with σ^2;

(ii) bonus b_1^* increases with V, while bonus β_2^* decreases with V.

The optimal fixed salary is also reduced

$$s^{*F} = w_r - (\tfrac{b_1^{*F} + \beta_2^{*F}}{2} + \tfrac{b_1^{*F2} + \beta_2^{*F2}(1+\sigma^2) - 2\theta b_1^{*F}\beta_2^{*F}}{8(1-\theta^2)}) < s^*.$$

The expected efforts are

$$\overline{e}_i^{*F} = \Phi_\sigma \overline{e}_i^*,$$

for $i = 1, 2$, where the effort on task 2 decreases with V, while the effort on task 1 increases with V. The effort on task 2 decreases with σ^2 as the PTO reduces the bonus on task 2, but the effort on task 1 increases with σ^2, as the bonus on task 1 increases.

The examiner exerts less effort on both tasks, $\overline{e}_1^{*F} \leq \overline{e}_1^*$ and $\overline{e}_2^{*F} < \overline{e}_2^*$ as the bonuses are lower. However, even though these effort levels are lower than the efforts made in the case where the self-funding constraint is always satisfied $(\overline{e}_1^*, \overline{e}_2^*)$, they are not necessarily suboptimal compared to (e_1^{oF}, e_2^{oF}). Indeed, the optimal effort on task 1 is higher than the constrained first-best effort level, $\overline{e}_1^{*F} \geq e_1^{oF}$ as long as $\sigma^2 > 0$ (the effort levels are equal for $\sigma^2 = 0$). On the other hand, the effort on task 2 is always smaller $\overline{e}_2^{*F} < e_2^{oF}$.

Interestingly, for intermediate values of F, such that $F \in [w^*(\sigma^2), w^o]$, the constrained first-best effort levels are constrained by the self-funding constraint (e_1^{oF}, e_2^{oF}) while the expected efforts under explicit contracts $(\overline{e}_1^*, \overline{e}_2^*)$ are not. In fact, patenting fees are too small from a first-best perspective, but this is no longer the case when the efforts are not observable. In that case, the optimal effort on task 1 is also higher than the first-best effort level, $\overline{e}_1^* > e_1^{oF}$ while the effort on task 2 is lower, $\overline{e}_2^* < e_2^{oF}$. In other words, low patenting fees with imperfect quality proxy will induce the PTO to accentuate its focus on the speed of processing patents and disregard even more the quality of patents it issues. We summarize these findings in the following Proposition.

Proposition 2 *For lower patenting fees, the effort levels under explicit contracts are not necessarily suboptimal.*

(i) For low patenting fees ($F \in]w_r, w^*(\sigma^2)[$), the effort levels are affected by the self-funding constraint such that $\overline{e}_1^{*F} \geq e_1^{oF}$ and $\overline{e}_2^{*F} < e_2^{oF}$;

(ii) For intermediate values of patenting fees ($F \in [w^*(\sigma^2), w^o]$), only the first-best efforts are affected and thus $\overline{e}_1^* > e_1^{oF}$ and $\overline{e}_2^* < e_2^{oF}$.

Without any financing problem (SFC is always satisfied as in the previous case), the introduction of a proxy on task 2 does only affect the optimal effort on task 2 (as the effort on task 1 is the first-best effort level). However, when financing becomes an issue (F is small, $F \in]w_r, w^*(\sigma^2)[$), a trade-off appears between the constrained optimal effort levels, and the introduction of a proxy reduces the effort on task 2 even more, and increases the effort on task 1 compared to the constrained first-best effort level e_1^{oF}. Furthermore, the proxy plays an important role in determining the effort levels: when the proxy becomes noisier, the distortion between the constrained first-best effort levels (e_1^{oF}, e_2^{oF}) and the constrained optimal effort levels $(\overline{e}_1^{*F}, \overline{e}_2^{*F})$ is accentuated. This result underlines the importance of taking into account the funding mechanism of an organization when designing its incentives scheme.

For intermediate patenting fees $F \in [w^*(\sigma^2), w^o]$, the self-funding constraint is no longer binding under an explicit contract, while it remains binding for the first-best case. In this case, the distortion between the constrained first-best effort levels (e_1^{oF}, e_2^{oF}) and the optimal effort levels $(\overline{e}_1^*, \overline{e}_2^*)$ is even more pronounced as the examiner chooses $\overline{e}_1^* = e_1^o > e_1^{oF}$ and $\overline{e}_2^* < e_2^{oF}$.

The expected benefit of the PTO is

$$B^{*F} = B^* - \left(\tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} - \tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)}\right)(1-\Phi_\sigma)^2, \tag{23}$$

which is smaller than the PTO's benefit if patenting fees are higher ($B^{*F} < B^{*}$), and also in the case of the (constrained) first-best PTO's benefit ($B^{*F} < B^{oF}$).

4.3 Implicit and Explicit Contracts

We now consider a situation where the PTO offers an explicit contract on contractible outcome (task 1 related to quantity) and an implicit contract on the non-contractible outcome (linked to quality). The work of an examiner is highly specialized; when he receives an application, he first goes through a series of check regarding the validity of the application, and then he searches for prior art regarding the innovation. In this searching process, each examiner develops his own search method. Arguably, this activity is hard to codify. However, the quality of the prior art search is observable for experienced examiners as they are able to reliably identify whether their peers have done a satisfactory search.

The relationship between the PTO and its examiners is a repeated one. Thus, although task 2 cannot be explicitly contracted upon, the PTO and the examiner can agree on implicit contracts where the PTO can promise bonus payments if the quality of the examiner's work is satisfactory. We now consider this possibility. These contracts are self-enforced in the sense that if one of the parties reneges on its promise, the other party will cease any informal relationship in the future. In our setting, this means that if the PTO decides not to reward an examiner for a satisfactory performance on task 2, other examiners will likely learn it and lose trust in the PTO's word to honor its promises. We assume that players use trigger strategies when one of them fails from abiding by the informal agreement.

The maximization program of the PTO is

$$\underset{e_1,e_2,s,b_1,b_2}{Max} B,$$

subject to the participation constraint (14), the incentive compatibility constraint (15) where w is defined by (5), and the self-funded constraint (SFC). However, a Non-Reneging Constraint (NRC) must be added to this program

$$\tfrac{B(b_1,b_2)-B}{r} \geq b_2, \tag{NRC}$$

where r is the common discount rate, B is PTO's explicit contract benefit that can be either B^{*} as defined by (21) or B^{*F} as defined by (23), and $B\left(b_1, b_2\right)$ is the PTO's benefit from offering the bonuses (b_1, b_2) under an implicit contract. This constraint specifies that, at any period t, the PTO has no incentive to renege as long as the benefit it gets from not reneging from that period on $(B(b_1, b_2)(1+r)/r)$ is higher than the short-term gain it will get from reneging at t and going back to explicit contracts during the next period $(B(b_1, b_2) + b_2 + B/r)$. Yet, if the PTO reneges on its bonus promise, it still has a fallback position as it can offer a formal contract by using the (contractible) proxy for task 2 performance. The PTO will not renege if the difference between the benefit derived from implicit contract and the benefit derived from explicit contract exceeds the short-term gain from reneging on its commitment (i.e., b_2).

To insure that the PTO will not always renege on its promise, we assume that

$$r < \tfrac{1}{8}. \tag{A2}$$

For large patenting fees, i.e., $F > w^o$, from (15) for given b_1 and b_2 the examiner efforts are

$$e_1(b_1, b_2) = \tfrac{b_1 - \theta b_2}{2(1-\theta^2)}, \tag{24}$$

and

$$e_2(b_1, b_2) = \tfrac{b_2 - \theta b_1}{2(1-\theta^2)}. \tag{25}$$

The solution of the PTO's maximization program depends on whether the non-reneging constraint is binding or not. There exists a cutoff value

$$\overline{\sigma}^2 \equiv \tfrac{8r(1-\theta^2)}{1-8r}, \tag{26}$$

such that for $\sigma^2 > \overline{\sigma}^2$, the fallback position of the PTO is not attractive and the non-reneging constraint is not binding (see appendix for the proof). For lower values of σ^2, $\sigma^2 \leq \overline{\sigma}^2$, the fallback position becomes attractive and thus the non-reneging constraint is binding. We also show that $\overline{\sigma}^2 > 0$ as long as r satisfies (A2).

In the case where $\sigma^2 \geq \overline{\sigma}^2$, the optimal bonuses are $b_1^{**} = V$, and $b_2^{**} = 1$. The efforts are thus the first-best effort levels (e_1^o, e_2^o), and the PTO's benefit function is defined by (10).

When $\sigma^2 < \overline{\sigma}^2$, the optimal bonuses are (see appendix for the resolution of the program)

$$b_2^{**NR} = (1-4r) + [(1-4r)^2 - \tfrac{1-\theta^2}{(1+\sigma^2-\theta^2)}]^{\frac{1}{2}},$$

and

$$b_1^{**NR} = V - \theta(1 - b_2^{**NR}). \tag{27}$$

As long as $b_2^{**NR} \geq 0$, the bonus on task 1, b_1^{**NR}, is always positive. A solution involving explicit and implicit contracts will exist provided that $\sigma^2 > \sigma^{2\prime\prime}$ where

$$\sigma^{2\prime\prime} \equiv (1-\theta^2)\tfrac{1-(1-4r)^2}{(1-4r)^2}, \tag{28}$$

which is such that $\sigma^{2\prime\prime} < \overline{\sigma}^2$. For very low values of σ^2 ($\sigma^2 < \sigma^{2\prime\prime}$), no incentive schemes involving explicit **and** implicit contracts can be offered. However, a solution exists with both implicit and explicit contracts if $\sigma^2 > \sigma^{2\prime\prime}$. These bonuses are lower than the bonus levels in case of high patenting fees, $b_1^{**NR} < b_1^{**}$ and $b_2^{**NR} < b_2^{**}$. As long as the discount rate is small such that (A2) is satisfied, the PTO needs to distort the bonus on task 2 given to the examiner to make sure that the non-reneging constraint is satisfied. By doing so, the PTO also distorts the bonus on task 1. The optimal effort levels are $e_1^{**NR} = e_1^o$,and

$$e_2^{**NR} = e_2^o - \tfrac{1}{2}(1 - b_2^{**NR}).$$

The optimal effort is suboptimal for task 2 while it is optimal for task 1. When implicit contracting corrects the distortion induced by performance measurement noise on task 2 (i.e., when the non-reneging constraint is not binding for $\sigma^2 \geq \overline{\sigma}^2$), the first-best levels of effort (e_1^o, e_2^o) are restored. However, when the constraint is binding (for $\sigma^{2\prime\prime} < \sigma^2 < \overline{\sigma}^2$), the first-best is achieved on task 1 but not on task 2. We summarize these findings in the following Proposition.

Proposition 3 *For large patenting fees, $F > w^o$, the optimal explicit and implicit bonuses offered by the PTO are as follows*:

1. *For $\sigma^2 \geq \overline{\sigma}^2$, the non-reneging constraint is not binding, and implicit contracting implements $b_1^{**} = V$ and $b_2^{**} = 1$;*
2. *For $\sigma^{2\prime\prime} < \sigma^2 < \overline{\sigma}^2$, the non-reneging constraint is binding and the bonuses are $b_1^{**NR} < b_1^{**}$ and $b_2^{**NR} < b_2^{**}$.*

In both cases (whether $\sigma^2 \geq \overline{\sigma}^2$ or $\sigma^2 < \overline{\sigma}^2$), an increase in V increases the bonus on task 1, while it does not affect the bonus on task 2. When $\sigma^2 < \overline{\sigma}^2$, both bonuses b_1^{**NR} and b_2^{**NR} decrease as r increases and increase as σ^2 increases. We summarize the comparative static results in the following Lemma.

Lemma 6 *In the case of implicit and explicit contracts, if $F > w^o$ and for $\sigma^{2\prime\prime} < \sigma^2 < \overline{\sigma}^2$,*

- *b_1^{**NR} and b_2^{**NR} are increasing with σ^2 and decreasing with r;*
- *bonus b_1^{**NR} increases with V while b_2^{**NR} is not affected by a change in V.*

When the measure of performance provided by the proxy is relatively satisfactory ($\sigma^{2\prime\prime} < \sigma^2 < \overline{\sigma}^2$), the non-reneging constraint is binding. When the proxy becomes noisier (σ^2 increases), the fallback position becomes less attractive, which relaxes the non-reneging constraint. Thus, the PTO increases the bonuses closer to the bonuses (b_1^{**}, b_2^{**}), bonuses that induce first-best efforts. Furthermore, the fact that the non-reneging constraint is binding creates a direct relationship between b_1^{**NR} and b_2^{**NR}, as shown by (27), where the bonus on task 1 is now a U-shape function of θ.

For $\sigma^{2\prime\prime} < \sigma^2 < \overline{\sigma}^2$, the PTO's benefit with explicit and implicit incentives is

$$B^{**NR} = B^o - \tfrac{1}{8} + \tfrac{1}{8} b_2^{**NR}(2 - b_2^{**NR}), \tag{29}$$

which differs markedly from the benefit obtained with a performance proxy.

Corollary 7 *The equilibrium profit B^{**NR} as defined by (29) with implicit contracting always (weakly) increases with σ^2.*

Indeed, for any $\sigma^2 \geq \overline{\sigma}^2$, the PTO's benefit is not a function of σ^2 and thus $\partial B^{**}/\partial\sigma^2 = 0$. On the other hand, for any $\sigma^2 < \overline{\sigma}^2$, $\partial B^{**NR}/\partial\sigma^2 > 0$. While the PTO's benefit with explicit contracting decreases with σ^2, the benefit with explicit and implicit contracts increases when the proxy becomes noisier. To understand this result one must recall that when σ^2 increases, $B(b_1, \beta_2)$ decreases. In other words, the fallback position of the PTO becomes less attractive and the constraint (NRC) is relaxed.

For smaller patenting fees, i.e., $F < w^o$, but not too small, $F > w_1''(\sigma^2)$, the non-reneging constraint is always satisfied (not binding) and the optimal bonuses are $b_i^{**F} = \Phi_s b_i^{**}$, for $i = 1, 2$ where Φ_s is defined by (12). These bonuses are also reduced compared to the case where the self-funding constraint is always satisfied, $b_i^{**F} < b_i^{**}$ for $i = 1, 2$. As in the case of explicit contracts, the bonus on task 1 increases with V while the bonus on task 2 decreases with V.

As the PTO reduces both bonuses, the efforts become $e_i^{**F} = \Phi_s e_i^{**} = e_i^{oF}$, for $i = 1, 2$ as defined by (11) and the PTO's benefit function is B^{oF} as defined by (13). In this case, the optimal efforts are at the first-best levels when the self-funding constraint is binding. Therefore, the self-funding constraint forces the PTO to reduce the bonuses and thus the effort levels.

For smaller patenting fees, i.e., $w^*(\sigma^2) < F < w_1''(\sigma^2)$, the non-reneging constraint is binding and the optimal bonuses are $(b_1^{**NRF}, b_2^{**NRF})$, the efforts are $(e_1^{**NRF}, e_2^{**NRF})$ and the PTO's benefit function is B^{**NRF} (see appendix for the resolution of the program, the characterization of the bonuses, efforts and benefit function).

For patenting fees such that $w_r < F < w^*(\sigma^2)$, there exists a a cutoff value $w_2''(\sigma^2)$ increasing in σ^2, such that for $w_2''(\sigma^2) < F < w^*(\sigma^2)$, the optimal bonuses are (b_1^{**F}, b_2^{**F}) as defined above, with efforts (e_1^{**F}, e_2^{**F}), and the PTO's benefit function is B^{oF}. Finally, for very low values of the patenting fees, and low σ^2, i.e., $F < w_2''(\sigma^2)$, implicit contracting is not possible.

5 Optimal choice of Contracts

In this section, we combine and compare the findings obtained with explicit contract and with explicit and implicit contracts to determine the optimal choice of contract for the PTO. In order to do so, we summarize all the feasible contracting regimes and their optimal bonuses in a graph (σ^2, F) in Figure 2 below.

In Figure 2, we represent the functions w^o, $w^*(\sigma^2)$, $w_1''(\sigma^2)$ and $w_2''(\sigma^2)$ as well as the cut-off values $\overline{\sigma}^2$, $\sigma^{2\prime\prime}$, and σ_e^2 in a graph (σ^2, F). These functions and cutoff values delimit the different areas in Figure 2. In Area I (entire hatched area), no implicit contracts can be offered. Above the curve $w^*(\sigma^2)$, the PTO offers the explicit contract (b_1^*, β_2^*), and below $w^*(\sigma^2)$, it offers the explicit contract (b_1^{*F}, β_2^{*F}). In Area II, the PTO can offer both implicit and explicit contracts; it offers the explicit contract (b_1^*, β_2^*) or the implicit contract (b_1^{**NR}, b_2^{**NR}). In area III, the same explicit contract can be offered or the following implicit contract (b_1^{**}, b_2^{**}). In Area IV, (below w^o and above $w_1''(\sigma^2)$), the PTO offers the explicit contract (b_1^*, β_2^*) or the implicit contract (b_1^{**F}, b_2^{**F}). In Area V, (below $w_1''(\sigma^2)$ and above $w^*(\sigma^2)$ for $\sigma^2 > \sigma^{2\prime\prime}$), the PTO offers the explicit contract (b_1^*, β_2^*) or the implicit contract $(b_1^{**NRF}, b_2^{**NRF})$. Finally, in Area IV (below $w^*(\sigma^2)$ and $w_2''(\sigma^2)$ and above w_r), the PTO offers the explicit contract (b_1^{*F}, β_2^{*F}) or the implicit contract (b_1^{**F}, b_2^{**F}).

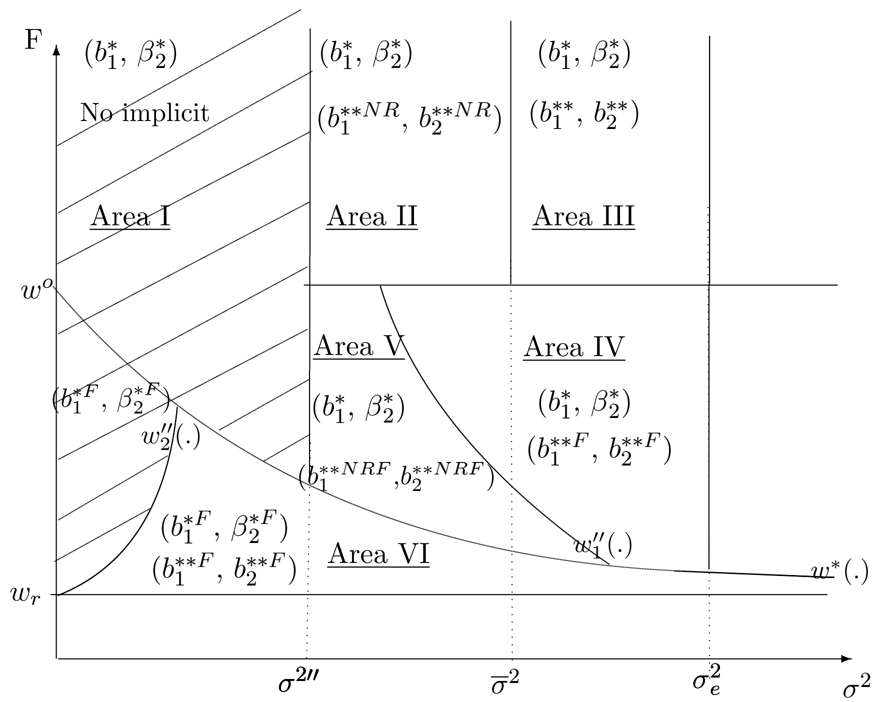

Figure 2: Feasible Contracts and Optimal Bonuses

In each of these areas, the PTO will obtain a different payoff, and might favor one type of contract over the other. To determine the contractual choice of the PTO, we first determine the optimal regime as we increase the noise of the proxy, σ^2, for given patenting fees, F. Second, we determine the optimal regime as we increase the patenting fee, F, for a given noisy proxy, σ^2.

5.1 Optimal Regime as a Function of the Noise of the Proxy

For large patenting fees, i.e., $F > w^o$, the self-funded constraint is always satisfied and, thus, no matter what type of contract the PTO offers, they are not constrained by the fact that it is a self-funding agency. For low σ^2 ($\sigma^2 \leq \sigma^{2\prime\prime}$) an implicit contract cannot be implemented, and only explicit contracts are available in which the effort on task 2 (related to a proxy on quality) is suboptimal while the effort on task 1 (quantity) is optimal.

For $\sigma^{2\prime\prime} < \sigma^2 \leq \overline{\sigma}^2$, both types of contracts can be offered. The bonuses offered in the case of explicit contracts are larger than those offered in the case of explicit and implicit contracts

($b_1^* > b_1^{**NR}$ and $\beta_2^* > b_2^{**NR}$), which induce the same effort level toward task 1, but a lower effort on task 2 in the case of explicit and implicit contracts ($\overline{e}_2^* > e_2^{**NR}$).

For larger σ^2 ($\sigma^2 > \overline{\sigma}^2$), implicit contracting restores the first-best effort levels. The PTO offers higher bonuses on both tasks, which induces a higher effort on task 2 compared to the effort on the proxy of task 2 in the case of explicit contracts ($\overline{e}_2^* < e_2^{**}$).

Depending on its benefit function, the PTO will prefer to offer one type of contract as summarized in the next Proposition.

Proposition 4 (Optimal Contract with Noisy Proxy) *If $F > w^o$, the PTO's benefit is*

1. *decreasing with σ^2 for any $\sigma^2 \leq \sigma^{2\prime\prime}$, and all contracts are explicit;*
2. *increasing for any σ^2 such that $\sigma^{2\prime\prime} < \sigma^2 \leq \overline{\sigma}^2$, and performance on task 2 is rewarded through an implicit contract;*
3. *constant and first-best for any $\sigma^2 > \overline{\sigma}^2$ and performance on task 2 is rewarded through an implicit contract.*

When the proxy used on quality is not too noisy, only explicit contracts can be offered by the PTO. When the proxy tracks well the true performance (small σ^2), the PTO will rely on it to provide incentives. Yet, when the performance of the proxy worsen (σ^2 increases), the PTO is less eager to rely on the proxy, thus it decreases the bonus, which reduces the effort and its benefit. However, when σ^2 goes beyond $\sigma^{2\prime\prime}$, the PTO is better off if it offers explicit and implicit contracts as $B^{**NR} > B^*$. As σ^2 increases, the fallback position becomes less attractive for the PTO (as B^* decreases), which allows the PTO to offer stronger implicit incentives to the examiner. For even larger values of σ^2 (i.e., $\sigma^2 > \overline{\sigma}^2$), the non-reneging constraint no longer binds, and the PTO can then offer the first-best incentives. As a result, the PTO's benefit is independent on the noise. These results are illustrated in Figure 3 below.

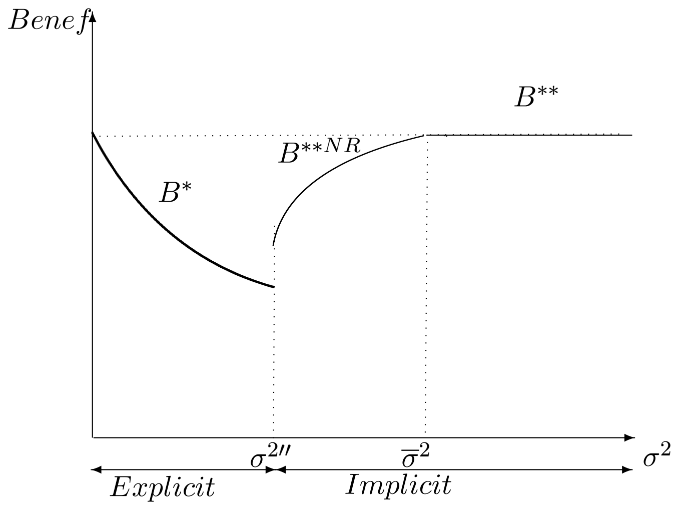

Figure 3: Optimal PTO's benefit for high patenting fees

For intermediate ($w_1''(\sigma^2) < F < w^o$) and small ($w_r < F < w_1''(\sigma^2)$) values of the patenting fees, a similar pattern is obtained with different benefit functions. For low σ^2, as in the previous case, no implicit contract can be offered, and thus the PTO's benefit decreases when the proxy becomes noisier. For values of σ^2 larger than $\sigma^{2\prime\prime}$, the PTO's gets a higher benefit with implicit contract even though the benefit is decreasing with σ^2. As patenting fees are smaller, the self-funding constraint is now binding, and unlike in the previous case, an increase in σ^2 has two effects; on the one hand, it reduces the fallback position (as in the previous case where $F > w^o$) and, on the other hand, it tightens the self-funding constraint forcing the PTO to adjust downward incentives. The second effect is always stronger than the first one and, thus, the PTO's benefit decreases with σ^2. Finally, when σ^2 is larger ($\sigma^2 > \overline{\sigma}^2$), as in the previous case, the non-reneging constraint no longer binds and the PTO's benefit is independent of σ^2.

One possible interpretation of this set of results is with respect to innovation fields (new fields versus mature fields) and their prior art contents. A low σ^2 could represent a more mature field. Indeed, any application received by an examiner of a given experience will have potentially a very similar μ (very close to 1). In a mature field where there is abundant prior art, the quotas will be set accordingly (for instance, on average an examiner has to process 110 patent applications instead of 87 applications in a new field) as well as the proxy (e.g., 15 citations on average must be added instead of 7). Quotas will be different in order to take into account the field, but the chances of getting different applications are fairly small. On the other hand, a high σ^2 is more likely to represent a new field (more spread out μ). In a new field, a given examiner can get one application that will look easy to process (high $\mu > 1$) because he has recently encountered a similar application and he is already in possession of some prior art information ready to use (or he knows where to search). However, another patent application might be more difficult to process (low μ) because it is an application for which the examiner has very little information. Therefore, our findings suggest that, in a new technological field, explicit and implicit contracts will make the PTO better off. However, in more mature fields (which represent the majority of the fields) the PTO might be better off by offering only explicit contracts based on a proxy for quality, which is a different measure of quality.

A potential broader policy implication of the results embedded in Figure 3 is with respect to the introduction of new "objective" performance measures of the quality of the examiners' work. Most patent offices have traditionally relied on a mentoring system in which experienced examiners (SPE in the U.S.) not only help build younger examiner's human capital but also serve as "quality check" for the bulk of examiners output. While modern information technologies increasingly allow for an improved access to a host of information and knowledge that was once disseminated and non-accessible, they also help to elaborate new objective performance measures of patent examination. At first sight, adopting superior measures of quality might appear quite seducing for a PTO eager to "rationalize" the measurement of examiners' performance. Our results however suggest some caution in the adoption of such measures as they can destabilize the functioning of patent offices that have traditionally relied on a culture of implicit relationships.

The next section examines the role of self-funding.

5.2 Optimal Regime as a Function of Patenting Fees

We now consider a given value of σ^2 and determine the optimal regime as a function of F. Even though the PTO's benefit is increasing with F, for any type of contracts (implicit or explicit), one contract might make the PTO better off.

For low σ^2 ($\sigma^2 \leq \sigma^{2\prime\prime}$), the PTO's benefit is higher with an implicit contract for relatively low values of F, whereas no implicit contracts can be implemented for larger values of F and, thus, only explicit contracts can be offered. This is illustrated in Figure 4 where F' is defined by

$$F' \equiv w_r + \frac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} - \frac{\sigma^2}{8(1+\sigma^2-\theta^2)}.$$

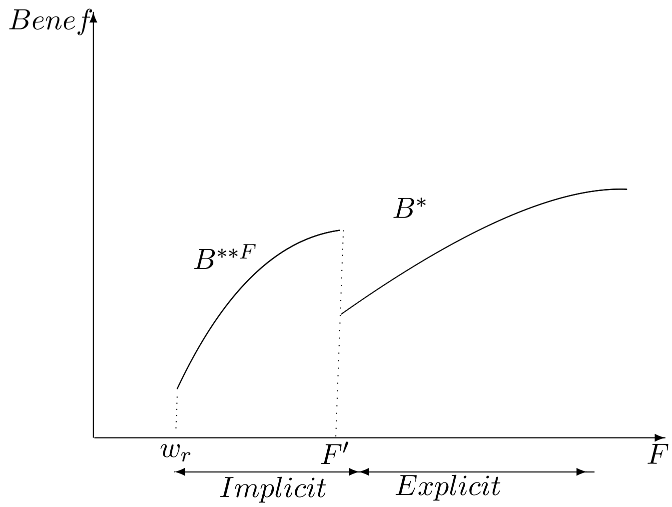

Figure 4: Optimal PTO's benefit for low noisy signal

When the proxy measure is not too noisy, implementing explicit and implicit contracts is optimal for the PTO when the patenting fees are small, i.e., when the agency is constrained by its self-funding constraint. However, when patenting fees are larger (and the PTO is not constrained by the self-funded constraint), explicit contracts should be offered. We summarize these findings in the following Proposition.

Proposition 5 (Optimal Contract with Patenting Fees) *If σ^2 is small ($\sigma^2 \leq \sigma^{2\prime\prime}$), the PTO's benefit function is discontinuous at F' where it jumps down. The PTO rewards the examiner*

- *with both implicit and explicit contracts for $F \leq F'$, and*
- *with only explicit contracts for $F > F'$.*

For low values of σ^2, the study of the optimal incentive regime reveals an interesting effect of a reduction in patenting fees. As evidenced by Figures 2 and 4, a smaller value of F has the property of making implicit incentives available to the PTO. A smaller value of F decreases the fallback benefit from B^* to B^{*F} (where $B^{*F} < B^*$) which relaxes the non-reneging constraint and makes implicit incentives available. When the PTO has access to both explicit and implicit contracts, it is more efficient (as more instruments are available) and, thus, the PTO's benefit jumps up even though it has less resources to operate.[27]

A possible interpretation of these findings is with respect to the financing structure of the PTO (tight financial situation or not). When σ^2 is small (mature field), Proposition (5) suggests that if the PTO wanted to increase the fees to get more revenue, it might be worse off. If initially the PTO offers implicit incentive schemes (for low F), it could be a curse to slightly increase F if this means that the PTO can no longer use an implicit contract. Indeed, with only explicit contracts, to be as well off as with explicit and implicit contracts, the PTO would have to drastically increase F.

For higher values of σ^2, the PTO's benefit function is increasing with F, and explicit and implicit contracts always provide a higher benefit.

Our results suggest that while considering increasing patenting fees, the PTO must be careful to not alter the structure of incentives within the organization. By overlooking that fact, the PTO might end up being worse off, even without considering any demand effect on the part of innovators. In fact, accounting for demand effects would undoubtedly magnify the jump-up and widen the set of self-funding levels for which the PTO will be worse off if it increases patenting fees. Note that the disincentive effect of an increase of patenting fees would also affect the regimes for which $\sigma^2 > \sigma^{2\prime\prime}$.

6 Conclusion

The PTO should grant patents to only new, non-obvious and useful innovations and, therefore, should be concerned with patent quality. However, being a self-funded agency, the PTO relies essentially on application fees and, thus, patent quantity matters. Furthermore, application fees are used to provide incentives to patent examiners.

In this context, we study the impact of having a self-funded PTO on different bonus contracts, and how these contracts affect the examiners' incentives to prosecute patent applications. Our framework integrates a PTO whose internal functioning is tied to the fee revenues it receives

[27] Note that Figure 4 depicts the gross PTO's benefits. The result will be magnified if instead we consider the net PTO's benefits (net of patenting fees).

and that cannot always commit to an optimal incentive scheme. We consider contracts in which the PTO offers bonuses on quantity quotas (which is part of an explicit contract) and on quality outcome (which can be part of either an implicit contract or an explicit contract based on a quality proxy). Thus, either the PTO offers explicit contracts with a bonus on quantity and a bonus on a proxy of quality, or explicit and implicit contracts with a bonus on quantity and a bonus on quality but that cannot be part of an explicit contract (and therefore is part of an implicit contract).

One of our main findings is that a self-funded constrained agency should make different organization choices of incentives. For a low quality proxy precision, an agency facing a tight budget operates well with implicit contracts. However, by only relaxing moderately the budget constraint, the agency might be worse off simply because this will preclude implicit contracts. Only very large patenting fees might allow the agency to compensate for the loss of implicit contracts.

When we consider a given fee structure, and we vary the noise of the proxy, our findings suggest that in new technological fields (where patent applications might be very different to process in terms of difficulty), explicit and implicit contracts will make the PTO better off. However, in more mature fields (where applications might be similar) the PTO might be better off by offering only explicit contracts based on a proxy for quality (as there will be less variance in the information received by the examiner), which is a different measure of quality.

On the other hand, when we consider a given noise, and we vary the patenting fees, our results suggest that while considering increasing patenting fees, the PTO must be careful not to alter the structure of incentives within the organization. By overlooking that fact, the PTO might end up being worse off, even without considering any demand effect on the part of innovators.

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Appendix

First-best effort levels

The program of the PTO is

$$\begin{cases} \underset{e_1,e_2,w}{Max}\{B = p_1 V + p_2 - w + F\} \\ s.t. \quad w - C(e_1,e_2) \geq w_r \quad \text{(PC)} \\ \qquad F \geq w \quad \text{(SFC)} \\ \qquad e_i \geq 0 \text{ for } i = 1,2 \end{cases}$$

As the Participation Constraint (PC) is binding (the PTO does not want to give more than the outside option), $w - C(e_1,e_2) = w_r$ and thus we can rewrite the program as

$$\begin{cases} \underset{e_1,e_2}{Max}\{p_1 V + p_2 - C(e_1,e_2) - w_r + F\} \\ s.t. \quad F \geq C(e_1,e_2) + w_r \\ \qquad e_i \geq 0 \text{ for } i = 1,2 \end{cases}$$

The Lagrangian of the simplified program is

$$L = \frac{1}{2}(1+e_1)V + \frac{1}{2}(1+e_2) - C(e_1,e_2) + F - w_r + \gamma_0(F - w_r - C(e_1,e_2)) + \gamma_1 e_1 + \gamma_2 e_2,$$

where γ_0, γ_1 and γ_2 are Lagrangian multipliers. The first order conditions give

$$\begin{array}{ll} \frac{\partial L}{\partial e_1} = \frac{1}{2}V - \frac{\partial C}{\partial e_1}(1+\gamma_0) + \gamma_1 = 0 & \text{(C1)} \\ \frac{\partial L}{\partial e_2} = \frac{1}{2} - \frac{\partial C}{\partial e_2}(1+\gamma_0) + \gamma_2 = 0 & \text{(C2)} \\ \frac{\partial L}{\partial \gamma_0} = F - C(e_1,e_2) - w_r \geq 0 \text{ if } \gamma_0 = 0; = 0 \text{ if } \gamma_0 > 0 & \text{(C3) (SFC)} \\ \frac{\partial L}{\partial \gamma_1} = e_1 \geq 0 \text{ if } \gamma_1 = 0; = 0 \text{ if } \gamma_1 > 0 & \text{(C4)} \\ \frac{\partial L}{\partial \gamma_2} = e_2 \geq 0 \text{ if } \gamma_2 = 0; = 0 \text{ if } \gamma_2 > 0 & \text{(C5)} \end{array}$$

We verify that the second order conditions are satisfied. We only present the cases where $\gamma_1 = \gamma_2 = 0$ as we are interested in having positive efforts (the cases where $\gamma_1 > 0$ and $\gamma_2 > 0$ are easy to derive), and we consider that $\gamma_0 > 0$ or $\gamma_0 = 0$.[28]

Case 1: When all the constraints are inactive ($\gamma_0 = \gamma_1 = \gamma_2 = 0$), from (C1) and (C2) we obtain

$$\begin{aligned} \frac{1}{2}V - e_1 - \theta e_2 &= 0, \\ \frac{1}{2} - e_2 - \theta e_1 &= 0. \end{aligned}$$

[28] The entire resolution of the maximization program is available upon request.

Solving these two equations gives the interior solution

$$e_1^o = \frac{V-\theta}{2(1-\theta^2)},$$
$$e_2^o = \frac{1-V\theta}{2(1-\theta^2)}.$$

These optimal efforts are always positive if $V-\theta>0$ and $1-V\theta>0$. To simplify, we assume that $V<2-\theta$, such that we have a simpler restriction on the parameters and only (A1) needs to be satisfied. We also verify that (SFC) is always satisfied at the optimal effort levels, or equivalently, $F>w^o$ (high patenting fees). The first-best PTO's benefit function is thus

$$B^o = \frac{1}{2} + \frac{1}{2}V + \frac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} - w_r + F. \tag{B^o}$$

<u>*Case 2*</u>: The constraint (C3) is active and both constraints (C4) and (C5) are inactive ($\gamma_0>0$, $\gamma_1=\gamma_2=0$). This case corresponds to low patenting fees ($F\le w^o$). From (C1) and (C2) we obtain the Lagrangian multiplier γ_0 that must satisfied

$$1+\gamma_0 = \frac{1}{2}\frac{V}{\frac{\partial C}{\partial e_1}} = \frac{1}{2}\frac{1}{\frac{\partial C}{\partial e_2}},$$

and thus

$$e_1 = \frac{V-\theta}{1-V\theta}e_2.$$

By solving (C3), we obtain

$$\frac{e_1^2}{2} + \frac{e_2^2}{2} + \theta e_1 e_2 = F - w_r.$$

Assuming an interior solution, we plug $e_1 = \frac{V-\theta}{1-V\theta}e_2$ into the previous equality and we obtain $e_2^{oF} = \Phi_s e_2^0 < e_2^0$, where

$$\Phi_s = \left[\frac{8(F-w_r)(1-\theta^2)}{(V-\theta)^2+(1-\theta^2)}\right]^{\frac{1}{2}} < 1,$$

as $F<w^o$. Replacing in e_1 we find that $e_1^{oF} = \Phi_s e_1^o < e_1^o$, and $e_2^{oF} = \Phi_s e_2^o < e_2^o$. The Lagrangian multiplier, which represents the marginal benefit of raising patenting fees, is strictly positive as $\gamma_0^{oF} = 1/\Phi_s - 1 > 0$. The PTO's constrained first-best benefit is

$$B^{oF} = B^o - \frac{(V-\theta)^2+(1-\theta^2)}{4(1-\theta^2)}(1-\Phi_s)^2 < B^o.$$

In the following table we provide some comparative statics.

TABLE 1: Comparative Statics

Parameter y	sign $\frac{de_1^o}{dy}$	sign $\frac{de_2^o}{dy}$	sign $\frac{dB^o}{dy}$
V	$+$	$-$	$+$
θ	$-$ if $V \leq 1$ $-$ and $+$ if $V > 1$	$-$ and $+$ if $V \leq 1$ $-$ if $V > 1$	$-$
F	.	.	$+$
w_r	.	.	$-$

Parameter y	sign $\frac{d\Phi_s}{dy}$	sign $\frac{de_1^{oF}}{dy}$	sign $\frac{de_2^{oF}}{dy}$	sign $\frac{dB^{oF}}{dy}$
V	$-$	$+$	$-$	$+$
θ	$+$	$-$ if $V \leq 1$ $-$ and $+$ if $V > 1$	$-$ and $+$ if $V \leq 1$ $-$ if $V > 1$	?
F	$+$	$+$	$+$	$+$
w_r	$-$	$-$	$-$	$-$

Explicit Contracts

The maximization program of the PTO is now

$$\begin{cases} \underset{e_1,e_2,b_1,\beta_2,s}{Max} \{E_\mu B = E_\mu[p_1 V + p_2 - C(e_1,e_2) - w_{\exp} + F]\} & \\ s.t. \quad E_\mu[w_{\exp} - C(e_1,e_2)] \geq w_r & \text{(PC)} \\ \qquad (e_1,e_2) \in \arg\max\{w - C(e_1,e_2)\} & \text{(ICC)} \\ \qquad F \geq E_\mu w & \text{(SFC)} \\ \qquad b_1 \geq 0,\ \beta_2 \geq 0 & \\ \qquad e_1 \geq 0,\ \ e_2 \geq 0 & \end{cases}$$

Following a first order approach, we first calculate the effort levels for any b_1 and β_2. From (ICC)

$$\underset{e_1,e_2}{Max}\{s + \frac{1}{2}(1+e_1)b_1 + \frac{\mu}{2}(1+e_2)\beta_2 - \tfrac{(e_1)^2}{2} - \tfrac{(e_2)^2}{2} - \theta e_1 e_2\},$$

we obtain

$$\begin{aligned} e_1 &= \frac{1}{2}b_1 - \theta e_2, \\ e_2 &= \frac{\mu}{2}\beta_2 - \theta e_1, \end{aligned}$$

and thus

$$e_1(b_1, \beta_2) = \frac{b_1 - \theta\mu\beta_2}{2(1-\theta^2)},$$

and

$$e_2(b_1, \beta_2) = \frac{\mu\beta_2 - \theta b_1}{2(1-\theta^2)}.$$

The examiner accepts the contract if he gets his reservation wage in expectation. That is, the (PC) is binding such that $E_\mu[s + p_1 b_1 + p_\beta \beta_2 - C(e_1, e_2)] = w_r$. The fixed salary must be

$$s = w_r - \frac{(b_1+\beta_2)}{2} + \frac{b_1^2 + (1+\sigma^2)\beta_2^2 - 2\beta_2 b_1 \theta}{8(1-\theta^2)}.$$

If both bonuses are positive, we can rewrite the maximization program as

$$\begin{cases} \underset{b_1,\beta_2,s}{Max} E_\mu[p_1(b_1,\beta_2)V + p_2(b_1,\beta_2) - C(e_1(b_1,\beta_2), e_2(b_1,\beta_2)) - w_r + F] \\ s.t. \qquad F \geq E_\mu C(e_1(b_1,\beta_2), e_2(b_1,\beta_2)) + w_r \quad \text{(SFC)} \\ \qquad\qquad b_1 \geq 0,\ \beta_2 \geq 0 \end{cases}$$

By replacing $e_1(b_1, \beta_2)$ and $e_2(b_1, \beta_2)$ in $E_\mu C(.)$, we obtain the following Lagrangian

$$L = \frac{1}{2} + \frac{1}{2}V - w_r + F + \frac{1}{2}\frac{b_1(V-\theta) + \beta_2(1-V\theta)}{2(1-\theta^2)} - \frac{b_1^2 + \beta_2^2(1+\sigma^2) - 2\theta b_1\beta_2}{8(1-\theta^2)} + \gamma_0(F - E_\mu C(b_1,\beta_2) - w_r) + \gamma_1 b_1 + \gamma_2\beta_2.$$

The first order conditions give

$$\begin{array}{ll} \frac{\partial L}{\partial b_1} = \frac{1}{2}\frac{V-\theta}{2(1-\theta^2)} - \frac{\partial E_\mu C(.)}{\partial b_1}(1+\gamma_0) + \gamma_1 \leq 0 & \text{(C1)} \\ \frac{\partial L}{\partial \beta_2} = \frac{1}{2}\frac{1-V\theta}{2(1-\theta^2)} - \frac{\partial E_\mu C(.)}{\partial \beta_2}(1+\gamma_0) + \gamma_2 \leq 0 & \text{(C2)} \\ \frac{\partial L}{\partial \gamma_0} = F - E_\mu C(e_1,e_2) - w_r \geq 0 \text{ if } \gamma_0 = 0; \quad = 0 \text{ if } \gamma_0 > 0 & \text{(C3) (SFC)} \\ \frac{\partial L}{\partial \gamma_1} = b_1 \geq 0 \text{ if } \gamma_1 = 0; \quad = 0 \text{ if } \gamma_1 > 0 & \text{(C4)} \\ \frac{\partial L}{\partial \gamma_2} = \beta_2 \geq 0 \text{ if } \gamma_2 = 0; \quad = 0 \text{ if } \gamma_2 > 0 & \text{(C5)} \end{array}$$

Here again we only focus on the cases where $\gamma_1 = \gamma_2 = 0$ as we consider positive bonuses and $\gamma_0 = 0$ or $\gamma_0 > 0$.

Case 1: When all the constraints are inactive ($\gamma_0 = \gamma_1 = \gamma_2 = 0$), the first order conditions becomes

$$\begin{array}{ll} \frac{\partial L}{\partial b_1} = \frac{1}{2}\frac{V-\theta}{2(1-\theta^2)} - \frac{\partial E_\mu C(.)}{\partial b_1} = 0 & \text{(C1)} \\ \frac{\partial L}{\partial \beta_2} = \frac{1}{2}\frac{1-V\theta}{2(1-\theta^2)} - \frac{\partial E_\mu C(.)}{\partial \beta_2} = 0 & \text{(C2)} \\ \frac{\partial L}{\partial \gamma_0} = F - E_\mu C(e_1,e_2) - w_r \geq 0 \text{ if } \gamma_0 = 0 & \text{(C3) (SFC)} \\ \frac{\partial L}{\partial \gamma_1} = b_1 \geq 0 \text{ if } \gamma_1 = 0 & \text{(C4)} \\ \frac{\partial L}{\partial \gamma_2} = \beta_2 \geq 0 \text{ if } \gamma_2 = 0 & \text{(C5)} \end{array}$$

From (C1) and (C2) we obtain

$$\begin{aligned} b_1^* &= V - \tfrac{\theta\sigma^2}{1+\sigma^2-\theta^2}, \\ \beta_2^* &= \tfrac{1-\theta^2}{1+\sigma^2-\theta^2} > 0. \end{aligned}$$

Note that $b_1^* > 0$ if $V(1-\theta^2) + \sigma^2(V-\theta) > 0$ which is always satisfied as $V - \theta > 0$. The expected efforts are computed as $E_\mu[e_1(b_1^*, \beta_2^*)] = \overline{e}_1^* = e_1^0$ and

$$E_\mu[e_2(b_1^*, \beta_2^*)] = \overline{e}_2^* = e_2^0 - \tfrac{\sigma^2}{2(1+\sigma^2-\theta^2)} < e_2^o.$$

We have that $e_1^0 > 0$ if $\theta < V$ and $E_\mu[e_2(b_1^*, \beta_2^*)] > 0$ if $\sigma^2 < \sigma_e^2$ where

$$\sigma_e^2 \equiv \tfrac{(1-\theta V)(1-\theta^2)}{\theta(V-\theta)}.$$

Lastly, we need to check that (SFC) is always satisfied at the expected effort levels, which is equivalent to have $F > w^*(\sigma^2)$ where

$$w^*(\sigma^2) = w^o - \tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)}.$$

The optimal fixed salary is

$$s^* = w_r - (\tfrac{b_1^* + \beta_2^*}{2} + \tfrac{b_1^{*2} + \beta_2^{*2}(1+\sigma^2) - 2\theta b_1^* \beta_2^*}{8(1-\theta^2)}),$$

and the PTO's expected benefit function is

$$B^* = B^o - \tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)}.$$

<u>*Case 2*</u>: Constraint (C3) is active and constraints (C4) and (C5) are inactive ($\gamma_0 > 0$, $\gamma_1 = \gamma_2 = 0$). The first order conditions are

$$\begin{aligned}
\tfrac{\partial L}{\partial b_1} &= \tfrac{1}{2}\tfrac{V-\theta}{2(1-\theta^2)} - \tfrac{\partial E_\mu C(.)}{\partial b_1}(1+\gamma_0) = 0 && \text{(C1)} \\
\tfrac{\partial L}{\partial \beta_2} &= \tfrac{1}{2}\tfrac{1-V\theta}{2(1-\theta^2)} - \tfrac{\partial E_\mu C(.)}{\partial \beta_2}(1+\gamma_0) = 0 && \text{(C2)} \\
\tfrac{\partial L}{\partial \gamma_0} &= F - E_\mu C\left(e_1, e_2\right) - w_r = 0 \text{ if } \gamma_0 > 0 && \text{(C3) (SFC)} \\
\tfrac{\partial L}{\partial \gamma_1} &= b_1 \geq 0 \text{ if } \gamma_1 = 0 && \text{(C4)} \\
\tfrac{\partial L}{\partial \gamma_2} &= \beta_2 \geq 0 \text{ if } \gamma_2 = 0 && \text{(C5)}
\end{aligned}$$

From (C1) and (C2) we obtain

$$1 + \gamma_0 = \frac{V-\theta}{b_1 - \theta\beta_2} = \tfrac{1-V\theta}{\beta_2(1+\sigma^2) - \theta b_1},$$

and thus

$$\beta_2 = \frac{1-\theta^2}{V(1-\theta^2+\sigma^2)-\theta\sigma^2} b_1.$$

From (C3) we have

$$\frac{b_1^2+\beta_2^2(1+\sigma^2)-2\theta b_1\beta_2}{8(1-\theta^2)} = F - w_r.$$

Thus, there exists a pair (b_1^{*F}, β_2^{*F}) solution of

$$\begin{aligned} \beta_2 &= \frac{1-\theta^2}{(1-\theta^2+\sigma^2)-\theta\sigma^2} b_1, \\ \frac{b_1^2+\beta_2^2(1+\sigma^2)-2\theta b_1\beta_2}{8(1-\theta^2)} &= F - w_r, \end{aligned}$$

which is equivalent to solving

$$\begin{aligned} &\beta_2 = A b_1, \\ &b_1^2 + \beta_2^2(1+\sigma^2) - 2\theta b_1\beta_2 - 8(1-\theta^2)(F-w_r) = 0, \end{aligned}$$

where

$$A = \frac{(1-\theta^2)}{(1-\theta^2+\sigma^2)-\theta\sigma^2}.$$

Plugging the first equation into the second one, we obtain

$$b_1^2(1 + A^2(1+\sigma^2) - 2\theta A) - 8(1-\theta^2)(F-w_r) = 0,$$

with $1 + A^2(1+\sigma^2) - 2\theta A > 0$. Thus, we derive the bonuses $b_1^{*F} = \Phi_\sigma b_1^* < b_1^*$, and $\beta_2^{*F} = \Phi_\sigma \beta_2^* < \beta_2^*$, where

$$\Phi_\sigma = \left[\frac{8(F-w_r)(1-\theta^2)(1-\theta^2+\sigma^2)}{(1-\theta^2+\sigma^2)(V-\theta)^2+(1-\theta^2)^2}\right]^{\frac{1}{2}} < 1.$$

The Lagrangian multiplier is positive as $\gamma_0 = 1/\Phi_\sigma - 1 > 0$. The expected effort levels are $\overline{e}_1^{*F} = \Phi_\sigma \overline{e}_1^* \leq \overline{e}_1^* = e_1^o$, and $\overline{e}_2^{*F} = \Phi_\sigma \overline{e}_2^* < \overline{e}_2^*$, where $e_2^{*F} > 0$ if $e_2^* > 0$. The optimal fixed salary is also reduced

$$s^{*F} = w_r - \left(\frac{b_1^{*F}+\beta_2^{*F}}{2} + \frac{b_1^{*F2}+\beta_2^{*F2}(1+\sigma^2)-2\theta b_1^{*F}\beta_2^{*F}}{8(1-\theta^2)}\right) < s^*,$$

and the PTO's expected benefit is

$$B^{*F} = B^* - \left(\frac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} - \frac{\sigma^2}{8(1+\sigma^2-\theta^2)}\right)(1-\Phi_\sigma)^2 < B^*.$$

In the following table we provide some comparative statics with respect to the equilibrium explicit contract.

TABLE 2: Comparative Statics

Parameter $y \setminus$ sign	$\frac{d\bar{e}_1^*}{dy}$	$\frac{d\bar{e}_2^*}{dy}$	$\frac{db_1^*}{dy}$	$\frac{d\beta_2^*}{dy}$	$\frac{dB^*}{dy}$
V	$+$	$-$	$+$	.	$+$
θ	$-$ if $V \leq 1$ $-$ and $+$ if $V > 1$	$-$ and $+$ if $V \leq 1$ $-$ if $V > 1$	$-$	$-$	$-$
F	.	.	.	.	$+$
w_r	.	.	.	.	$-$
σ^2	.	$-$	$-$	$-$	$-$

Parameter $y \setminus$ sign	$\frac{d\Phi_\sigma}{dy}$	$\frac{d\bar{e}_1^{*F}}{dy}$	$\frac{d\bar{e}_2^{*F}}{dy}$	$\frac{db_1^{*F}}{dy}$	$\frac{d\beta_2^{*F}}{dy}$	$\frac{dB^{*F}}{dy}$
V	$-$	$+$	$-$	$+$	$-$	$+$
θ	$+$	$-$ and $+$	$-$ and $+$	$+$ and $-$	$+$ and $-$	?
F	$+$	$+$	$+$	$+$	$+$	$+$
w_r	$-$	$-$	$-$	$-$	$-$	$-$
σ^2	$+$	$+$	$-$	$+$	$-$	$+?$

Explicit and Implicit Contracts

The maximization program of the PTO is

$$\begin{cases} \underset{e_1,e_2,b_1,b_2,s}{Max} \{B = p_1 V + p_2 - w_{imp} + F\} & \\ s.t. \quad w_{imp} - C(e_1, e_2) \geq w_r & \text{(PC)} \\ \qquad (e_1, e_2) \in \arg\max\{w_{imp} - C(e_1, e_2)\} & \text{(ICC)} \\ \qquad F \geq w_{imp} & \text{(SFC)} \\ \qquad B(b_1, b_2) \geq r b_2 + B^* & \text{(NRC)} \\ \qquad b_1 \geq 0 \text{ and } b_2 \geq 0 & \end{cases}$$

Depending on the fallback position of the PTO, the non-reneging constraint (NRC) will depend on either B^* or B^{*F}. From (ICC) we have

$$e_1(b_1, b_2) = \tfrac{b_1 - \theta b_2}{2(1-\theta^2)},$$

and

$$e_2(b_1,b_2) = \tfrac{b_2-\theta b_1}{2(1-\theta^2)}.$$

We can rewrite the program as

$$\begin{cases} \underset{b_1,b_2,s}{Max}[p_1(b_1,b_2)V + p_2(b_1,b_2) - C\left(e_1(b_1,b_2),e_2(b_1,b_2)\right) - w_r + F] \\ s.t. \qquad F \geq C\left(e_1(b_1,b_2),e_2(b_1,b_2)\right) + w_r \quad \text{(SFC)} \\ \qquad\quad B\left(b_1,b_2\right) \geq rb_2 + B^* \quad \text{(NRC)} \\ \qquad\quad b_1 \geq 0 \text{ and } b_2 \geq 0 \end{cases}$$

and the Lagrangian of this simplified program is

$$L = B\left(b_1,b_2\right) + \gamma_1(F - C\left(.\right) - w_r) + \gamma_2 b_1 + \gamma_3 b_2 + \gamma_4(B\left(b_1,b_2\right) - rb_2 - B^*).$$

The first order conditions are

$$\begin{aligned} &\tfrac{\partial L}{\partial b_1} = \tfrac{\partial B}{\partial b_1}(1+\gamma_4) - \gamma_I \tfrac{\partial C}{\partial b_1} + \gamma_2 \leq 0 && (1) \\ &\tfrac{\partial L}{\partial b_2} = \tfrac{\partial B}{\partial b_2}(1+\gamma_4) - \gamma_I \tfrac{\partial C}{\partial b_2} + \gamma_3 - \gamma_4 r \leq 0 && (2) \\ &\tfrac{\partial L}{\partial \gamma_1} = F - C\left(.\right) - w_r \geq 0 \text{ if } \gamma_1 = 0; = 0 \text{ if } \gamma_1 > 0 && (3) \\ &\tfrac{\partial L}{\partial \gamma_2} = b_1 \geq 0 \text{ if } \gamma_2 = 0; = 0 \text{ if } \gamma_2 > 0 && (4) \\ &\tfrac{\partial L}{\partial \gamma_3} = b_2 \geq 0 \text{ if } \gamma_3 = 0; = 0 \text{ if } \gamma_3 > 0 && (5) \\ &\tfrac{\partial L}{\partial \gamma_4} = B\left(b_1,b_2\right) - rb_2 - B^* \geq 0 \text{ if } \gamma_4 = 0; = 0 \text{ if } \gamma_4 > 0 && (6) \end{aligned}$$

Here again we consider only positive bonuses such that $\gamma_2 = \gamma_3 = 0$. However, we need to consider the cases where $\gamma_1 = 0$, $\gamma_1 > 0$, $\gamma_4 = 0$ and $\gamma_4 > 0$.

<u>*Case 1*</u>: All the constraints are inactive ($\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$). In that case, none of the constraints bind, and the first order conditions are

$$\begin{aligned} &\tfrac{\partial L}{\partial b_1} = \tfrac{V-\theta}{4(1-\theta^2)} - \tfrac{b_1-\theta b_2}{4(1-\theta^2)} = 0 && (1) \\ &\tfrac{\partial L}{\partial b_2} = \tfrac{1-V\theta}{4(1-\theta^2)} - \tfrac{b_2-\theta b_1}{4(1-\theta^2)} = 0 && (2) \\ &\tfrac{\partial L}{\partial \gamma_1} = F - C\left(.\right) - w_r > 0 && (3) \\ &\tfrac{\partial L}{\partial \gamma_2} = b_1 \geq 0 && (4) \\ &\tfrac{\partial L}{\partial \gamma_3} = b_2 \geq 0 && (5) \\ &\tfrac{\partial L}{\partial \gamma_4} = B\left(b_1,b_2\right) - rb_2 - B^* > 0 && (6) \end{aligned}$$

From (1) and (2) we get $b_1 - \theta b_2 = V - \theta$, and $b_2 - \theta b_1 = 1 - V\theta$. Thus the optimal bonuses are $b_1^{**} = V$, and $b_2^{**} = 1$. The optimal efforts are $e_1^{**} = e_1^0$, and $e_2^{**} = e_2^0$, and the PTO's benefit function is $B(e_1^o, e_2^o) = B^o$ where B^o is defined by (B^o).

We need to verify that conditions (3) and (6) are satisfied. Condition (3) insures that the patenting fees are large enough, $F > w^o$, and condition (6), will depend on the fallback benefit which is B^* for high patenting fees ($F > w^0$). Thus, it is satisfied if

$$\sigma^2 > \tfrac{8r(1-\theta^2)}{1-8r} \equiv \overline{\sigma}^2 > 0 \text{ if } r < \tfrac{1}{8}.$$

However, for $\sigma^2 \leq \overline{\sigma}^2$, the NRC (6) is binding.

<u>*Case 2*</u>: Constraints (1), (2), and (3) are inactive and (4) is active ($\gamma_1 = \gamma_2 = \gamma_3 = 0$ and $\gamma_4 > 0$). (NRC) is binding, the efforts are positive and (SFC) is satisfied such that

$$B(b_1, b_2) - rb_2 - B^* = 0,$$

and

$$F - C(.) - w_r > 0.$$

The first order conditions are

$$\begin{aligned}
\tfrac{\partial L}{\partial b_1} &= (\tfrac{V-\theta}{4(1-\theta^2)} - \tfrac{b_1-\theta b_2}{4(1-\theta^2)})(1+\gamma_4) = 0 && (1)\\
\tfrac{\partial L}{\partial b_2} &= (\tfrac{1-V\theta}{4(1-\theta^2)} - \tfrac{b_2-\theta b_1}{4(1-\theta^2)})(1+\gamma_4) - \gamma_4 r \leq 0 && (2)\\
\tfrac{\partial L}{\partial \gamma_1} &= F - C(.) - w_r \geq 0 \text{ if } \gamma_1 = 0 && (3)\\
\tfrac{\partial L}{\partial \gamma_2} &= b_1 \geq 0 \text{ if } \gamma_2 = 0 && (4)\\
\tfrac{\partial L}{\partial \gamma_3} &= b_2 \geq 0 \text{ if } \gamma_3 = 0 && (5)\\
\tfrac{\partial L}{\partial \gamma_4} &= B(b_1, b_2) - rb_2 - B^* = 0 \text{ if } \gamma_4 > 0 && (6)
\end{aligned}$$

From (2), we have

$$A_1 - (r - A_1)\gamma_4 \leq 0 \qquad (2)'$$

where

$$A_1 = \tfrac{1-V\theta}{4(1-\theta^2)} - \tfrac{b_2-\theta b_1}{4(1-\theta^2)}.$$

In order to have $\gamma_4 > 0$, we need to have either $A_1 > 0$ and $r > A_1$ or $A_1 < 0$ and $r - A_1 > 0$. From (1)

$$(\tfrac{V-\theta}{4(1-\theta^2)} - \tfrac{b_1-\theta b_2}{4(1-\theta^2)})(1+\gamma_4) = 0,$$

if

$$\tfrac{V-\theta}{4(1-\theta^2)} = \tfrac{b_1-\theta b_2}{4(1-\theta^2)},$$

or, equivalently, if

$$b_1 = \theta b_2 + V - \theta \quad (1')$$

From (6)

$$\frac{1}{2} + \frac{1}{2}V - w_r + F - B^* + \tfrac{b_1(V-\theta)+b_2(1-V\theta)}{4(1-\theta^2)} - \tfrac{b_1^2+b_2^2-2\theta b_1 b_2}{8(1-\theta^2)} - rb_2 = 0.$$

We plug (1') in (6) to obtain the second degree polynomial in b_2

$$b_2^2 + C_1 b_2 + C_2 = 0,$$

with

$$\begin{aligned} C_1 &= -2(1-4r) < 0, \\ C_2 &= -8(\frac{1}{2} + \frac{1}{2}V - w_r + F + \tfrac{(V-\theta)^2}{8(1-\theta^2)} - B^*) = \tfrac{1-\theta^2}{(1+\sigma^2-\theta^2)} > 0. \end{aligned}$$

A necessary condition to obtain a positive bonus b_2 is that $\Delta = C_1^2 - 4C_2 > 0$ or, equivalently,

$$\sigma^2 > \tfrac{(1-\theta^2)(1-(1-4r)^2)}{(1-4r)^2} \equiv \sigma^{2\prime\prime}.$$

Provided that $\sigma^2 > \sigma^{2\prime\prime}$, the polynomial function $b_2^2 + C_1 b_2 + C_2 = 0$ admits two real positive solution which are

$$\begin{aligned} b_2' &= (1-4r) - ((1-4r)^2 - \tfrac{1-\theta^2}{(1+\sigma^2-\theta^2)})^{\frac{1}{2}} > 0, \\ b_2'' &= (1-4r) + ((1-4r)^2 - \tfrac{1-\theta^2}{(1+\sigma^2-\theta^2)})^{\frac{1}{2}} > 0. \end{aligned}$$

Either of these solutions can be the optimal bonus.

Consider that the PTO chooses the smallest bonus b_2'. Therefore,

$$\begin{aligned} b_2^{**NR} &= (1-4r) - ((1-4r)^2 - \tfrac{1-\theta^2}{(1+\sigma^2-\theta^2)})^{\frac{1}{2}} < b_2^{**}, \\ b_1^{**NR} &= V - \theta(1 - b_2^{**NR}) < b_1^{**}. \end{aligned}$$

Yet, this candidate must also be compatible with the first order condition (2)'. We check that $A_1 > 0$ as $b_2^{**NR} < 1$, thus we need to have that $r > A_1$, which cannot hold for b_2'; a contradiction. Therefore there is no $\gamma_4 > 0$ such that $A_1 - (r - A_1)\gamma_4 \leq 0$ if $A_1 > 0$, and b_2' cannot be a solution.

The solution is therefore

$$\begin{aligned} b_2^{**NR} &= (1-4r) + ((1-4r)^2 - \tfrac{1-\theta^2}{(1+\sigma^2-\theta^2)})^{\frac{1}{2}}, \\ b_1^{**NR} &= V - \theta(1 - b_2^{**NR}) < b_1^{**}, \end{aligned}$$

where $b_2^{**NR} < b_2^{**}$ as long as $\sigma^2 < \overline{\sigma}^2$. The efforts are $e_1^{**NR} = e_1^o$, and $e_2^{**NR} = e_2^o - \frac{1}{2}(1 - b_2^{**NR}) < e_2^o$, and we need to insure that $A_1 - (r - A_1)\gamma_4 \leq 0$ is satisfied. We check that $A_1 > 0$ as $b_2^{**NR} < 1$. We further verify that when $b_2 = b_2''$, the inequality $r > A$ always hold true. Therefore, there exists $\gamma_4 > 0$ such that $A_1 - (r - A_1)\gamma_4 \leq 0$. Finally, we also need to check that (SFC) is satisfied, which is equivalent to

$$F \geq w^o + \frac{1 + (b_2^{**NR})^2}{8} > w^o \text{ for any } \sigma^2.$$

Therefore, the condition $F - C\,(.) - w_r > 0$ is always satisfied.

<u>*Case 3*</u>: Constraints (2), (3), and (4) are inactive and (1) is active ($\gamma_2 = \gamma_3 = \gamma_4 = 0$ and $\gamma_1 > 0$). (NRC) is not binding, the efforts are positive and (SFC) is binding. In that case we have $w_r < F < w^0$. The first order conditions are

$$\begin{aligned} &\tfrac{\partial L}{\partial b_1} = \tfrac{V-\theta}{4(1-\theta^2)} - \tfrac{b_1-\theta b_2}{4(1-\theta^2)} - \gamma_1 \tfrac{b_1-\theta b_2}{4(1-\theta^2)} = 0 && (1) \\ &\tfrac{\partial L}{\partial b_2} = \tfrac{1-V\theta}{4(1-\theta^2)} - \tfrac{b_2-\theta b_1}{4(1-\theta^2)} - \gamma_1 \tfrac{b_2-\theta b_1}{4(1-\theta^2)} = 0 && (2) \\ &\tfrac{\partial L}{\partial \gamma_1} = F - \tfrac{b_1^2+b_2^2-2\theta b_1 b_2}{8(1-\theta^2)} - w_r = 0 \text{ if } \gamma_1 > 0 && (3) \\ &\tfrac{\partial L}{\partial \gamma_2} = b_1 \geq 0 \text{ if } \gamma_2 = 0 && (4) \\ &\tfrac{\partial L}{\partial \gamma_3} = b_2 \geq 0 \text{ if } \gamma_3 = 0 && (5) \\ &\tfrac{\partial L}{\partial \gamma_4} = B\,(b_1, b_2) - r b_2 - B^* \geq 0 \text{ if } \gamma_4 = 0 && (6) \end{aligned}$$

From (1) and (2), we obtain

$$\gamma_1 + 1 = \tfrac{V-\theta}{b_1-\theta b_2} = \tfrac{1-V\theta}{b_2-\theta b_1},$$

and thus $b_1 = V b_2$ that we plug into (3) to obtain $b_1^{**F} = \Phi_s b_1^{**}$ and $b_2^{**F} = \Phi_s b_2^{**}$ where

$$\Phi_s = \left[\tfrac{8(1-\theta^2)(F-w_r)}{V^2+1-2\theta V}\right]^{\frac{1}{2}} \leq \Phi_\sigma < 1.$$

The efforts will then be $e_1^{**F} = \Phi_s e_1^{**} = e_1^{oF}$ and $e_2^{**F} = \Phi_s e_2^{**} = e_1^{oF}$ and the PTO's benefit is $B\left(b_1^{**F}, b_2^{**F}\right) = B^{oF}$. We check that $\gamma_1 > 0$ as $\gamma_1 = 1/\Phi_s - 1 > 0$. Finally, we need to verify that (NRC) is satisfied, where B can be either B^* (if $w^0 > F > w^*$) or B^{*F} (if $w_r < F \leq w^*$).

Consider first that the fallback benefit is B^* (for $w^0 > F > w^*$), then $B\left(b_1^{**F}, b_2^{**F}\right) - rb_2^{**F} - B^* > 0$ becomes

$$-\tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)}(\Phi_s)^2 + \Phi_s(2\tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} - r) - (\tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} - \tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)}) > 0 \quad (6')$$

where

$$\tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} - \tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)} > 0, \text{ and } 2\tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} - r > 0.$$

The determinant of (6') simplifies to

$$\Delta = r^2 + 4\tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)}(\tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)} - r),$$

which is positive as $\sigma^2 > \overline{\sigma}^2$. Thus, there are two values Φ_s' and Φ_s'' that satisfy

$$\Phi_s = \tfrac{8(1-\theta^2)}{(V-\theta)^2+(1-\theta^2)}(2\tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} - r \pm \sqrt{\Delta}) > 0,$$

where $\Phi_s' < \Phi_s''$. In order for (6)' to be satisfied, we need to have $\Phi_s > \Phi_s'$ and $\Phi_s < \Phi_s''$. The first inequality ($\Phi_s > \Phi_s'$) is satisfied if

$$F > w_r + \tfrac{8(1-\theta^2)}{(V-\theta)^2+(1-\theta^2)}(\tfrac{(V-\theta)^2+(1-\theta^2)}{4(1-\theta^2)} - r - \sqrt{r^2 + \tfrac{(V-\theta)^2+(1-\theta^2)}{2(1-\theta^2)}(\tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)} - r)})^2 \equiv w_1'',$$

while the second ($\Phi_s < \Phi_s''$) is satisfied if

$$F < w_r + \tfrac{8(1-\theta^2)}{(V-\theta)^2+(1-\theta^2)}(\tfrac{(V-\theta)^2+(1-\theta^2)}{4(1-\theta^2)} - r + \sqrt{r^2 + \tfrac{(V-\theta)^2+(1-\theta^2)}{2(1-\theta^2)}(\tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)} - r)})^2 \equiv w_1'.$$

Thus, we need to have $w_1'' < F < w_1'$. We know that $w^o > F > w^*$ where

$$\begin{aligned} w^o &= \tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} + w_r, \\ w^* &= \tfrac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} + w_r - \tfrac{\sigma^2}{8(1+\sigma^2-\theta^2)}. \end{aligned}$$

We show that $w^o > w_1''$ and $w^o < w_1'$ for the relevant range of parameters. Thus, for values of F such that $w_1'' < F < w^0$, $B\left(b_1^{**F}, b_2^{**F}\right) - rb_2^{**F} - B^* > 0$ and the solution is (b_1^{**F}, b_2^{**F}). We also verify that $\partial w_1''/\partial\sigma^2 < 0$ for the relevant range of parameters. For any F such that $w^*(\sigma^2) < F < w_1''$, the two constraints are binding and the solution is $(b_1^{**NRF}, b_2^{**NRF})$ as described in *Case 4* below.

Consider now that the fallback benefit is B^{*F} (if $w_r < F \le w^*$), we need to check that $B\left(b_1^{**F}, b_2^{**F}\right) - rb_2^{**F} - B^{*F} > 0$ is satisfied. In the same vein, we show that there exists w_2''

with $\partial w_2''/\partial\sigma^2 > 0$ such that for $F < w_2''$ and $F < w^*$, (NRC) is not binding and thus, the solution is (b_1^{**F}, b_2^{**F}). However, for $F > w_2''$, no implicit contract exists.

The PTO's benefit is

$$B^{**NR} = B^o - \tfrac{1}{8} + \tfrac{1}{8}b_2^{**NR}(2 - b_2^{**NR}).$$

Case 4: Constraints (2), and (3) are inactive and (1) and 4 is active ($\gamma_2 = \gamma_3 = 0$ and $\gamma_1 >$, $\gamma_4 > 0$). Both (NRC) and (SFC) are binding and both efforts are positive. In that case $F < w^o$ and $F < w_1''$. The first order conditions are

$$\begin{aligned}
\tfrac{\partial L}{\partial b_1} &= (\tfrac{V-\theta}{4(1-\theta^2)} - \tfrac{b_1-\theta b_2}{4(1-\theta^2)})(1+\gamma_4) - \gamma_1\tfrac{b_1-\theta b_2}{4(1-\theta^2)} \leq 0 && (1)\\
\tfrac{\partial L}{\partial b_2} &= (\tfrac{1-V\theta}{4(1-\theta^2)} - \tfrac{b_2-\theta b_1}{4(1-\theta^2)})(1+\gamma_4) - \gamma_1\tfrac{b_2-\theta b_1}{4(1-\theta^2)} - \gamma_4 r \leq 0 && (2)\\
\tfrac{\partial L}{\partial \gamma_1} &= F - \tfrac{b_1^2+b_2^2-2\theta b_1 b_2}{8(1-\theta^2)} - w_r = 0 \text{ if } \gamma_1 > 0 && (3)\\
\tfrac{\partial L}{\partial \gamma_2} &= b_1 \geq 0 \text{ if } \gamma_2 = 0 && (4)\\
\tfrac{\partial L}{\partial \gamma_3} &= b_2 \geq 0 \text{ if } \gamma_3 = 0 && (5)\\
\tfrac{\partial L}{\partial \gamma_4} &= B(b_1, b_2) - rb_2 - B^* = 0 \text{ if } \gamma_4 > 0 && (6)
\end{aligned}$$

Here again there are two cases: either (a) $w^*(\sigma^2) < F < w^o$, in which case (6) is $B(b_1, b_2) - rb_2 - B^* = 0$ and (b) $F < w^*(\sigma^2)$ and (6) is $B(b_1, b_2) - rb_2 - B^{*F} = 0$. From (1) we obtain

$$\tfrac{1+\gamma_4}{\gamma_1} = \tfrac{b_1-\theta b_2}{V-\theta-(b_1-\theta b_2)},$$

and from (2), we obtain

$$\tfrac{1-V\theta-(b_2-\theta b_1)}{4(1-\theta^2)}\frac{1+\gamma_4}{\gamma_1} = \tfrac{b_2-\theta b_1}{4(1-\theta^2)} + \frac{\gamma_4}{\gamma_1}r.$$

Thus, plugging (1) in (2), we have

$$\tfrac{b_1-Vb_2}{4(V-\theta-(b_1-\theta b_2))} = \frac{\gamma_4}{\gamma_1}r.$$

From (3), we obtain

$$\tfrac{b_1^2+b_2^2-2\theta b_1 b_2}{8(1-\theta^2)} = F - w_r.$$

Case (a). We have seen that for $w^*(\sigma^2) < F < w^o$, (NRC) is not binding for $F > w_1''$. Thus

$$\frac{1}{2} + \frac{1}{2}V - B^* + \tfrac{b_1(V-\theta)+b_2(1-V\theta)}{4(1-\theta^2)} - \tfrac{b_1^2+b_2^2-2\theta b_1 b_2}{8(1-\theta^2)} - w_r + F - rb_2 = 0.$$

Plug (3) in (6), we obtain $b_1 = -b_2 A + C$ where

$$\begin{array}{rcl} A & = & \frac{(1-V\theta)-4r(1-\theta^2)}{V-\theta} > 0 \text{ if } r < \frac{1-V\theta}{4(1-\theta^2)}, \\ C & = & \frac{B^*-(\frac{1}{2}+\frac{1}{2}V)}{V-\theta} 4(1-\theta^2) > 0 \end{array}$$

Let's plug $b_1 = -Ab_2 + C$ into (3), we obtain the following second degree polynomial in b_2

$$(A^2+1+2\theta A)b_2^2 - b_2 2C(A+\theta) + C^2 - 8(1-\theta^2)(F-w_r) = 0 \quad \text{(3)'}$$

We then calculate the values of b_2 that satisfy this equality. First, the determinant $\Delta = 4(1-\theta^2)(-C^2 + 8(F-w_r)(A^2+1+2\theta A))$ is positive only if $-C^2 + 8(F-w_r)(A^2+1+2\theta A) > 0$, which is always satisfied for $\sigma^2 > \sigma^{2\prime\prime}$. Therefore, for $F < w^*$, there is no solution with both constraints binding. For $F < w^*$, the solution of (3)' is

$$b_2^{**NRF} = \frac{2C(A+\theta)+\sqrt{\Delta}}{2(A^2+1+2\theta A)},$$

and

$$b_1^{**NRF} = -b_2^{**NRF} A + C.$$

The efforts are

$$\begin{array}{rcl} e_1^{**NRF} & = & \frac{-b_2(A+\theta)+C}{2(1-\theta^2)}, \\ e_2^{**NRF} & = & \frac{b_2(1+\theta A)-\theta C}{2(1-\theta^2)}. \end{array}$$

The PTO's benefit function is

$$B\left(b_1^{**NRF}, b_2^{**NRF}\right) = \frac{1}{2} + \frac{1}{2}V - w_r + F + \frac{(V-\theta)^2+(1-\theta^2)}{8(1-\theta^2)} + \frac{b_2(1+A\theta+(C-V)(\theta+A)) - b_2^2(A^2+1+2A\theta) - (C-V)^2 - 2\theta(C-V) - 1}{8(1-\theta^2)}.$$

In the following table we provide some comparative statics.

TABLE 3: Comparative Statics

Parameter $y \setminus$ sign	$\frac{de_1^{**}}{dy}$	$\frac{de_2^{**}}{dy}$	$\frac{db_1^{**}}{dy}$	$\frac{db_2^{**}}{dy}$	$\frac{dB^{**}}{dy}$
V	+	-	+	.	+
θ	$-$ if $V \leq 1$ $-$ and $+$ if $V > 1$	$-$ and $+$ if $V \leq 1$ $-$ if $V > 1$	.	.	-
F	.	.	.	.	+
w_r	.	.	.	.	-

TABLE 3: Comparative Statics

Parameter $y \setminus$ sign	$\frac{de_1^{**F}}{dy}$	$\frac{de_2^{**F}}{dy}$	$\frac{db_1^{**F}}{dy}$	$\frac{db_2^{**F}}{dy}$	$\frac{dB^{**F}}{dy}$
V	$+$	$-$	$+$	$-$	$+$
θ	$-$ if $V \leq 1$ $-$ and $+$ if $V > 1$	$-$ and $+$ if $V \leq 1$ $-$ if $V > 1$	?	?	?
F	$+$	$+$	$+$	$+$	$+$
w_r	$-$	$-$	$-$	$-$	$-$

$y \setminus$ sign	$\frac{de_1^{**NR}}{dy}$	$\frac{de_2^{**NR}}{dy}$	$\frac{db_1^{**NR}}{dy}$	$\frac{db_2^{**NR}}{dy}$	$\frac{dB^{**NR}}{dy}$
V	$+$	-	$+$	.	$+$
θ	$-$ if $V \leq 1$ $-$ and $+$ if $V > 1$	?	?	$+$	?
F	.	.	.	.	$+$
w_r	.	.	.	.	-
σ^2	.	$+$	$+$	$+$	$+$
r	.	-	-	-	+?