Introduction

Least squares migration seeks a reflectivity model that fits the observed data when passed to a modelling operator. It is used to compensate for acquisition noise, poor sampling of sources and receivers on the surface, as well as poor illumination of the subsurface (Nemeth et al., 1999). To date least squares migration has been mainly restricted to the imaging of acoustic wavefields. We present an extension of wave equation least squares migration for elastic wavefields in isotropic media. Least squares migration is an iterative method that requires a forward and adjoint operator. The forward operator generates two component data from PP and PS images by extrapolating P and S scalar potentials independently via split-step shot-profile forward modelling. The potentials are recomposed at the receiver datum using the analytic inverse of the Helmholtz decomposition operator. Conversely, the adjoint operator separates two component data into P and S scalar potentials before extrapolating and imaging these potentials independently. Angle gathers for PP and PS images are formed using Poynting vectors calculated using the source and receiver side P-wave potentials (Higginbotham and Brown, 2009). We regularize the inversion by applying a dip filter on the depth-angle axes of each Common Image Point gather to reduce the effect of source/receiver sampling, noise, and PP/PS crosstalk artifacts.

Theory

We implement least squares migration via Conjugate Gradients (CG) (Scales, 1987) with preconditioning. Each iteration of CG involves applying the forward operator to predict data from the migrated image, computing a data residual, and migrating the residual using the adjoint operator to compute an update to the image. We form an elastic migration operator by combining Helmholtz decomposition with a wave equation migration operator that extrapolates P and S scalar potentials independently.

Isotropic elastic data can be decomposed into P and S-wave potentials by taking the divergence and curl of the wavefield components respectively:

\[
p = \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}
\]

and

\[
s = \frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}.
\]

This is commonly referred to as Helmholtz decomposition. Etgen (1988) evaluated these derivatives in the Fourier domain. Writing this as a linear operator

\[
\begin{bmatrix}
p \\
s
\end{bmatrix} = H
\begin{bmatrix}
u_x \\
u_z
\end{bmatrix}
\]

where

\[
H = F^{-1}_x \left[ \begin{array}{cc}
  ik_x & ik_z(v_s) \\
  -ik_z(v_p) & ik_x
\end{array} \right] F_x.
\]

\(F_x\) and \(F^{-1}_x\) are the forward and inverse Fourier transforms on the spatial axis respectively. In this equation the lateral wavenumber is given by \(k_x\), and the vertical wavenumbers are given by the dispersion relations:

\[
k_z(v_p) = \sqrt{\frac{\omega^2}{v_p^2} - k_x^2}
\]

and

\[
k_z(v_s) = \sqrt{\frac{\omega^2}{v_s^2} - k_x^2}
\]

where \(v_p\) and \(v_s\) are the P and S wave velocities at the receiver datum respectively and \(\omega\) is the angular frequency. Technically the receiver datum should have a constant velocity, but practically we find that the average velocity is generally a good approximation. Additionally, we may add a correction to improve the separation in laterally variant media. To recompose potentials into wavefield components we apply

\[
\begin{bmatrix}
d_x \\
d_z
\end{bmatrix} = H^{-1}
\begin{bmatrix}
p \\
s
\end{bmatrix}
\]

where

\[
H^{-1} = F^{-1}_x \left[ \frac{1}{k_x^2 + k_z(v_p)k_z(v_s)} \right]
\begin{bmatrix}
  -ik_x & ik_z(v_s) \\
  -ik_z(v_p) & -ik_x
\end{bmatrix}
F_x.
\]

Scalar extrapolation of the P and S potential is done using (for example)

\[
p = L_{pp} m_{pp},
\]

where \(L_{pp}\) is a shot profile split step forward modelling operator (Kaplan et al., 2010) that uses the P-wave velocity for...
both the source and receiver side wavefields (conversely the operator $L_{ps}$ uses the P-wave velocity for the source side wavefield and the S-wave velocity for the receiver side wavefield).

In wave equation migration a conventional zero-lag cross correlation imaging condition reduces the dimension of the data by collapsing the offset axis to zero. This leads to two negative consequences in least squares migration of elastic data. Firstly, the offset dimension provides a straightforward method to regularize the inversion: we expect this axis to be approximately flat after migration. Secondly, elastic wavefields display complicated amplitude variation as a function of offset: a conventional “migration to zero offset” imaging condition will collapse this information to a single point. More importantly, it is impossible for a modelling operator to recreate these complicated amplitude variations simply by spraying a stacked image back into data space! A subsurface offset or angle gather imaging condition is critical in least squares migration of elastic data. For our forward/adjoint operator we adopted an angle gather imaging condition using Poynting vectors. We compute the Poynting vectors using the P-component of the source and receiver wavefields. Using the source and receiver side Poynting vectors we compute incidence angle relative to reflector normal and bin both the PP and PS images using these angles. Using this strategy, the polarity reversal in PS imaging is corrected by multiplying the image amplitude by -1 whenever the computed angle is found to be positive.

Using the wavefield separation and migration operators defined above, we can combine them to form the forward and adjoint operators needed for inversion. To forward model two component data we use
\[
d = H^{-1} L m \quad \text{where} \quad d = [d_x \ d_z]^T, \quad m = [m_{pp} \ m_{ps}]^T, \quad \text{and} \quad L = \begin{bmatrix} L_{pp} & 0 \\ 0 & L_{ps} \end{bmatrix}.
\]
Conversely, to migrate two component data we take the adjoint of the forward operator to get
\[
\tilde{m} = L^T (H^{-1})^T d, \quad \text{where} \quad \tilde{m} = [\tilde{m}_{pp} \ \tilde{m}_{ps}]^T, \quad \text{and} \quad L^T = \begin{bmatrix} L_{pp}^T & 0 \\ 0 & L_{ps}^T \end{bmatrix}.
\]

Finally, for elastic least squares migration with quadratic regularization the cost function is written
\[
J = ||TH^{-1} L m - d||_2^2 + \mu ||m||_2^2. \quad (1)
\]

where $T$ is a diagonal sampling operator with 1’s in place of observations and 0’s in place of missing traces.

It is important to observe that the adjoint operator is not equivalent to Helmholtz decomposition plus migration: $L^T (H^{-1})^T \neq L^T H$ so we might expect to see PS cross-talk artifacts in the migrated PP image and vice-versa. Since the cross-talk energy is migrated at an incorrect velocity, it will appear as strongly dipping energy in angle gathers. For this reason, we regularize the inversion with an operator that weights up this energy in each angle gather
\[
J = ||TH^{-1} D m - d||_2^2 + \mu ||D m||_2^2. \quad (2)
\]

By a change of variables $z = D m$ we write
\[
J = ||TH^{-1} D^{-1} z - d||_2^2 + \mu ||z||_2^2 \quad (3)
\]

where $D^{-1}$ is a $k_z - k_\theta$ fan filter that removes strongly dipping energy in the angle gathers. We minimize equation 3 using CG with forward operator $TH^{-1} D^{-1}$ and adjoint operator $D^{-1} L^T (H^{-1})^T T$ (the $k_z - k_\theta$ fan filter is self adjoint). After the final iteration we substitute $m = D^{-1} z$.

Examples

To demonstrate our approach we generated an elastic finite difference dataset using a subset of the elastic Marmousi2 model (Martin et al., 2006). The P and S-wave velocity models for this subset of the model are shown figures 1a and 1b. Unlike the original Marmousi2 dataset, we stripped the water layer from the
model and used an absorbing boundary condition on the surface of the model. The synthetic dataset was modelled with a 40 Hz peak amplitude Ricker wavelet source with a high spatial and temporal sampling to avoid dispersion artifacts and was decimated prior to migration. The migrated dataset consists of 61 shot gathers from \( x = 2000 \) to \( x = 5000 \) in steps of 50 m. The receiver spacing is 7.494 m, and the record length is 3.492 s with 4 ms sampling. The X and Z components of a shot gather at \( x = 3500 \) are shown in figure 2. Figures 3a and 3b show PP and PS images for the migrated data at a constant incidence angle of 25°. Comparing this with the least squares migration result (figures 4a and 4b) we see that the inversion has improved the continuity of some reflectors.

**Conclusions**

Least squares migration algorithms attempt to fit recorded data with predictions generated from a migrated image. By improving the accuracy of the migration operator to include elastic wave propagation...
we expect to improve the ability of least squares migration to fit seismic amplitudes. Our example demonstrates that least squares migration can improve the imaging of multicomponent seismic data.

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References

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