A comparison of 5D reconstruction methods
Aaron Stanton*, Nadia Kreimer, David Bonar, Mostafa Naghizadeh, and Mauricio Sacchi, Department of Physics, University of Alberta

SUMMARY
A comparison is made between three 5D reconstruction methods—Projection Onto Convex Sets (POCS), Tensor Completion (TCOM), and Minimum Weighted Norm Interpolation (MWNI). A method to measure of the quality of synthetic data reconstructions is defined and applied under various scenarios. Two different measures of performance in the case of real data reconstructions are also provided and applied to a real data example taken from a land dataset acquired in the Western Canadian Sedimentary Basin. We find that TCOM and POCS are better able to reconstruct data in the presence of low SNR. We also find that TCOM provides superior results in most synthetic data scenarios, but in the case of real data reconstruction all three methods have similar performance, with POCS giving slightly better preservation of amplitudes.

INTRODUCTION
5D Reconstruction aims to provide a fully sampled noise free estimate of the data. The majority of processing steps benefit from a fully sampled dataset such as 3DSRME (Dragoset et al., 2010) and AVA imaging (Sacchi and Liu, 2005). There are various strategies to tackle the reconstruction problem. Algorithms fall under the following two categories: those based on wave equation principles (for example Ronen (1987)), and those based on signal processing principles. Methods based on signal processing are by far the most commonly used for interpolation and denoising of seismic data because they do not require velocity information to reconstruct the data. Such methods include prediction error filter techniques (Spitz, 1991; Naghizadeh and Sacchi, 2007), transform based methods (Abma and Kabir, 2006; Liu and Sacchi, 2004), and rank reduction based methods (Trickett et al., 2010; Kreimer and Sacchi, 2011).

This paper considers three methods. Two of the methods, POCS and MWNI, are transform based methods. They reconstruct the data one frequency at a time in the frequency-wavenumber domain. The other method, TCOM, is a rank reduction based method that also reconstructs the data one frequency at a time, but in the frequency-space domain. The following sections summarize the three reconstruction methods under study.

Projection Onto Convex Sets (POCS) works by iteratively thresholding the amplitudes of the wavenumber spectrum (Abma and Kabir, 2006).

Tensor Completion (TCOM) uses the Higher-Order Singular Value Decomposition and its truncated version to reduce the rank of the core-tensor of the data stored in a tensorial form. It is an iterative procedure, where a reinsertion operation is carried out after the rank reduction (Kreimer and Sacchi, 2011).

Both POCS and TCOM use a weighted reinsertion to simulta-
EXAMPLES

Synthetic data examples

To test the quality of synthetic data reconstructions a variety of 5D synthetic datasets were generated to fall under various categories. For each category the data have dimensions of 150 time samples, and 16 samples in all four spatial dimensions. The data consist of three events with differing dip and/or curvature depending on the scenario. In all cases except when curvature was being evaluated linear events were used. In all cases when SNR was not being evaluated a value of SNR = 5 was used. All of these scenarios and their respective qualities are listed in table 1. Each of the three reconstruction methods are individually parameterized for each scenario. In the case of high SNR it is clear that all methods provide a very high quality result (the difference between Q = 12.85 dB and Q = 39.5 dB is difficult to discern visually). In this case TCOM provides the highest quality reconstruction followed by POCS and MWNI. In the case of low SNR TCOM and POCS do a much better job than MWNI. This is likely because of the power of the weighted reinsertion step used by POCS and TCOM compared to the IRLS approach used by MWNI. In the case of sparse sampling all three methods perform similarly, while in the case of highly dipping events MWNI has the best performance. In the case of low curvature (shown in Figure 1) POCS and MWNI have a similar performance of reasonable quality, while TCOM greatly outperforms these methods. In the case of high curvature both POCS and MWNI fail while TCOM reconstructs the data to a high quality. The reason for this is that POCS and MWNI reconstruct the data by imposing sparsity in the 4D wavenumber domain for each frequency. Linear or gently curved events can be reasonably reconstructed by a sparse approximation in the wavenumber domain, but this approximation can fail in the presence of strongly dipping events. For this reason Fourier reconstruction methods are often applied on small spatial windows of data where a linear approximation can be made. Conversely, TCOM is able to reconstruct highly curved events as the reconstruction is performed in the 4D spatial domain for each frequency. Curved events do not greatly increase the rank of the core tensor allowing for their accurate reconstruction.

Next the methods are applied to a dataset that has been decimated to varying degrees. The quality of the reconstruction is shown in Figure 2 for each of the three methods. The input data are composed of three linear events of moderate dip in all directions with a SNR = 5, dimensions of 150 time samples, and 16 samples in all four spatial dimensions. All three methods provide high quality reconstruction results to decimation levels approaching 80%, although it is clear that TCOM is providing a higher quality result than both POCS and MWNI. It is curious that the quality of MWNI improves gradually from a decimation level of 0% to approximately 70%. This is likely due to the difficulty the method has attenuating noise on existing traces (the method lacks a weighted reinsertion step as is used for both POCS and TCOM). As the number of input traces are reduced the ability of MWNI to estimate the noise free fully sampled data is improved (until the input data becomes too sparse).

Real data example

The real data example considers the reconstruction of a land dataset acquired over a heavy oil target in Northern Alberta, Canada. The vertical component of the data is binned into 10m CMP bins, 100m offsets, and 36 azimuth sectors. A single CMP "snail gather" is shown in Figure 4. The reconstruction results for all three methods are quite similar but have some noticeable differences. The POCS result (b) is quite smooth with the exception of several noisy traces which are present on the input data. MWNI (c) appears to have fit these traces with the interpolated data giving an overall noisier result, while TCOM (d) appears to have smoothed out the high amplitudes to the surrounding traces. The TCOM result also appears to have dimming of amplitudes at near offsets, where few traces were present on input. The measurement of fit given in equation 2 confirm these observations. POCS is found to have a fit of 6.10 dB, TCOM a fit of 1.45 dB, and MWNI a fit of 7.73 dB. This confirms that TCOM is likely underfitting the data, while MWNI is likely overfitting the data. The smooth character of the POCS result combined with its high fit suggests that it is doing a reasonable job of attenuating the noise while fitting the existing observations. Calculating the quality of the reconstructions using the stacked data we find that all three methods have a similar reconstruction quality but the POCS reconstruction has the highest quality with a value of 8.16 dB, MWNI a quality of 7.80 dB, and TCOM a quality of 6.75 dB. The stacked data after application of a 100Hz high cut fil-
Reconstruction comparison

Figure 1: Slices of 5D data shown in \(X - T\) with a low level of curvature (the 5\(^{th}\) scenario of table 1) in all four spatial directions, SNR = 5, and 60% of traces decimated. (a) noise free fully sampled data, (b) noisy decimated data, (c) POCS reconstruction, (d) MWNI reconstruction, (e) TCOM reconstruction. (f) - (j) show the corresponding \(F - K\) plots.

CONCLUSIONS

We find that in the case of synthetic data we can more easily quantify the quality of the reconstructions. In the case of real data this becomes more difficult as we lack the true data at all spatial locations necessary to measure the quality. We can define a measure of fit that compares the data at existing trace locations. Used in conjunction with a visual inspection of the pre-stack data we can use this to judge whether the reconstruction is underfitting or overfitting the input data. We also find that the stack of the input data provides an estimate of the noise free, fully sampled data at all midpoint locations which can be used to measure the quality of the reconstruction results. We find that in the case of synthetic data reconstructions TCOM provides superior results to both POCS and MWNI under most scenarios. In the case of low SNR both TCOM and POCS outperform MWNI. This is likely due to the success of the weighted reinsertion strategy that both of these methods employ. In the reconstruction of highly curved events we find that TCOM greatly outperforms the other two methods. In the reconstruction of real data we find that all three methods provide comparable results, but MWNI appears to be less proficient at denoising the data during reconstruction, and POCS is best able to preserve amplitudes during reconstruction.
Reconstruction comparison

Figure 4: 5D reconstruction of a land dataset. A single midpoint location (1,14) sorted by offset (1-8) and azimuth sector (1-36) "snail gather" is shown. The input data with missing traces (a). Reconstruction using POCS (b). Reconstruction using MWNI (c). Reconstruction using TCOM (d).

Figure 5: 5D reconstruction of a land dataset. A stacked inline is shown after application of a 100Hz high cut filter. The input data with irregular fold (a). Reconstruction using POCS (b). Reconstruction using MWNI (c). Reconstruction using TCOM (d).

ACKNOWLEDGMENTS

The authors would like to thank the sponsors of the Signal Analysis and Imaging Group (SAIG) at the University of Alberta, and Petrobank Energy and Resources Ltd for example data.
Reconstruction comparison

REFERENCES