Stockpiling and Shortages*
(the "Toilet Paper Paper")

Tilman Klumpp[†] Xuejuan Su[‡]

April 21, 2024

Abstract

Consumer stockpiling of everyday household goods, such as toilet paper, occurred in the wake of the COVID-19 pandemic, resulting in shortages of these goods in stores. Both phenomena reinforce each other: The expectation of shortages causes stockpiling behavior, which amplifies the shortages experienced by consumers, which in turn encourages more stockpiling. In this paper, we examine this feedback loop. When aggregate supply is insufficient to meet aggregate demand but prices cannot adjust to clear the market, there can be multiple stationary equilibria featuring stockpiling at different levels. Moreover, even when the supply-demand imbalances are very small, stockpiling can be large—suggesting that the incentive to stockpile is driven mostly by the stockpiling behavior of other consumers instead of the fundamental supply shortage. Transitional dynamics following a supply shock are examined as well. Stockpiling reduces overall welfare, and this effect can be particularly severe during the transitional phase when consumers build up inventories.

Keywords: Storage; consumer inventories; stockpiling; multiple equilibria; rationing; mean field games; search models.

JEL codes: C61, C62, C73, D15.

---

*We thank Yu Awaya, Lars Ehlers, Hugo Mialon, Yi Wen, Xiaohan Zhong, seminar participants at the University of Montreal, Tsinghua University, Shanghai Jiaotong University, Southwestern University of Finance and Economics, and the 2022 Canadian Economic Association Conference, as well as four anonymous referees for very helpful comments and conversations.

[†]Department of Economics, University of Alberta. 9-20 Tory Building, Edmonton, AB, Canada T6G 2H4. E-mail: klumpp@ualberta.ca.

[‡]Department of Economics, University of Alberta. 9-22 Tory Building, Edmonton, AB, Canada T6G 2H4. E-mail: xuejuan1@ualberta.ca.

There is enough in the whole country for the coming ten years. We can all poop for ten years.

Dutch prime minister Mark Rutte on the toilet paper supply in March 2020

1 Introduction

This paper examines a dynamic environment with the following properties: (i) Consumers want to consume a constant amount of a storable good in every period; (ii) the aggregate per-period supply may fall below aggregate demand due to an exogenous shock; (iii) prices are fixed and cannot adjust to balance supply and demand. In this environment, individual consumers may attempt to smooth consumption by accumulating inventories. However, stockpiling behavior has an externality, as it exacerbates the shortages experienced by other consumers. This, in turn, increases their incentive to stockpile, which reduces the availability of the good even further. The objective of this paper is to examine this feedback loop—that is, how small underlying supply shortages can be magnified via consumers' rational inventory responses.

The aforementioned market conditions were present, for example, during the early days of the COVID-19 pandemic (see, e.g., Wang *et al.* 2020; Micalizzi *et al.* 2021). As supply chains were being disrupted due to travel restrictions and hygiene measures at factories and logistics facilities, a stable supply of everyday grocery items was no longer guaranteed. Sensing that they may not be able to acquire these items in the future, consumers started buying up available quantities of storable products such as dried pasta, canned goods, and toilet paper, resulting in rows of empty supermarket shelves.[1] Stores were reluctant to implement large price increases on these everyday items, possibly due to both reputational concerns and potential legal constraints. Similarly, in May 2021 a cyberattack on Colonial Pipeline Corporation unexpectedly reduced the supply of transportation fuels in the Eastern United States and caused long queues at gas stations.[2] The emergence of these queues indicated that drivers were filling up their vehicles before their tanks were near empty (as most drivers normally would do). While gas stations did raise prices, price gauging laws prohibited them from implementing increases sufficient to fully balance supply and demand.[3] Other examples include the iodized table salt shortage in China after the 2011 Fukushima nuclear disaster and the

[1]In addition, shortages can also originate on the demand side. For example, during COVID-19 many consumers purchased and stored additional supplies of common household goods to last through a potential quarantine period. The model developed here is expressed in terms of supply reductions instead of demand increases, but can be recast to apply to the latter case as well (see Section 6).

[2]See, e.g., https://www.reuters.com/technology/colonial-pipeline-halts-all-pipeline-operations-after-cybersecurity-attack-2021-05-08/ (retrieved June 3, 2021).

[3]Currently, at least 38 states have price gauging statutes, including several with specific laws concerning the pricing of petroleum products. See https://www.ncsl.org/financial-services/price-gouging-state-statutes (retrieved September 7, 2023).

recent baby formula shortage in the United States.[4] In the situations discussed above, underlying supply disruptions were likely magnified by consumers' stockpiling responses. That is, the shortages consumers actually experienced could have been much less severe had consumers refrained from stockpiling.

To examine this two-way relationship between shortages and stockpiling behavior, we develop a dynamic model with a continuum of households, each wanting to consume one unit of a good per period, the price of which is fixed. Households can store, at a cost, up to a certain maximum quantity of the good but cannot resell it to other households. The aggregate per-period supply of the good is fixed, and if it is less than the aggregate consumption requirement, some households will be rationed. The prospect of rationing induces households to maintain inventories. A decision rule describes a household's optimal inventory behavior in each period, and a symmetric equilibrium is a decision rule that is optimal for a household if all other households adopt the same decision rule. Stockpiling arises if, in equilibrium, households buy and store more than the single unit consumed per period. Although the fixed-price and no-resale assumptions may appear restrictive, they eliminate any *speculative* motive for stockpiling and, therefore, highlight the role of stockpiling as a consumption smoothing mechanism. (More generally, as long as prices cannot adjust sufficiently to always clear the market, the same mechanism of this model—namely consumer stockpiling exacerbating shortages—still applies.)

We show that, if there is no underlying supply shortage—that is, if the aggregate supply in each period is sufficient to meet the aggregate consumption requirement of one unit per household—the unique equilibrium is for every household to obtain exactly one unit of the good per period. Thus, stockpiling cannot merely be a "self-fulfilling prophecy"—it requires an underlying, *fundamental* supply-demand imbalance. If such a fundamental imbalance exists, however, equilibria may emerge in which households store more than one unit, and may, in fact, store up to their capacity limit. These outcomes can arise even if the fundamental supply shortage is negligible. In particular, we show that the overall degree of stockpiling in any equilibrium can be decomposed into a fundamental component driven by the underlying supply shortage and an excess component caused by the reinforcing effect, and the magnitude of the second component can be multiple times that of the first component. In such stockpiling equilibria, households experience shortages with a much higher likelihood than what is indicated by fundamentals, and will buy large quantities whenever they can find the good.

We give a full characterization of all symmetric, stationary equilibria in what we call z-storage rules. A z-storage rule is a decision rule under which the household tries to maintain a target inventory level of z units; if the actual inventory falls below this threshold the household attempts to restock to an inventory of z units. Any z-storage rule with

[4] See https://www.wsj.com/articles/BL-CJB-13504 (retrieved July 28, 2023); https://www.wsj.com/articles/baby-formula-shortage-stuns-states-including-tennessee-kansas-and-delaware-11652526002 (retrieved July 28, 2023).

$z > 1$ involves stockpiling of units not immediately needed for consumption. Stationary equilibria are generally not unique, and the entire range of possible z-storage rules can be equilibria for generic parameter values. Therefore, the aforementioned positive reinforcing effect, whereby stockpiling behavior creates additional storage incentives, results in equilibrium indeterminacy. We also provide a limited characterization of non-stationary equilibria. This analysis explicitly takes into account the transitional dynamics that arise when the economy starts out in a state of balanced supply and demand, but then experiences an unexpected supply shock—e.g., because of a pandemic. In both stationary and non-stationary equilibria, welfare decreases in the degree of equilibrium stockpiling.

A classic literature in economics and finance examines price formation in competitive forward markets for storable commodities; see Telser (1958), Turnovsky (1983), Scheinkman and Schechtman (1983), Kawai (1983), Sarris (1984), and Hirshleifer (1989), among others. This line of inquiry was later extended to imperfectly competitive markets; see, e.g., Allaz (1991) and Thille (2003). The models in this literature are meant to characterize the strategies of professional traders in, e.g., agricultural markets.[5] They are, therefore, much differently motivated than the model we examine. In our model, inventories are maintained at the consumer level, and no sales from inventories can occur (which would be necessary for a commercial trader to profit from storage).

Stockpiling by consumers, on the other hand, has been studied in the marketing and industrial organization literatures. Meyer and Assunção (1990), Mela *et al.* (1998), Hong *et al.* (2002), Hendel and Nevo (2006a), Hendel and Nevo (2006b), and Ching and Osborne (2020) examine consumer's propensity to stockpile in response to temporary promotional discounts, both theoretically and empirically. An implication from these studies is that, for certain storable consumer goods, price decreases can lead to large increases in units sold even though the underlying consumption demand is relatively price inelastic. That is, demand responses to price changes often merely reflect shifts in the timing of purchases. This, in turn, has implications for firms' pricing and promotion strategies; see, e.g., Bell *et al.* (2002), Guo and Villas-Boas (2007), Su (2010), and Gangwar *et al.* (2013). Our model of consumer stockpiling differs from this literature in that prices and aggregate supply quantities are exogenously fixed—neither can households respond to price changes nor do suppliers make decisions. This choice allows us to focus on consumer stockpiling as an optimal response to other consumers' stockpiling behavior (instead of an optimal response to expected price increases).

Two other recent theoretical papers on consumer stockpiling are explicitly motivated by household behavior observed in the early days of the COVID-19 pandemic. Awaya and Krishna (2024) study a two-period mode with fixed but uncertain supply. Consumption takes place in the second period but consumers can purchase in either period. Awaya

[5]Similarly, in macroeconomics there is a literature on producers' inventory behavior driven by the stockout-avoidance motive; see, e.g., Kahn (1987) and Wen (2005, 2011).

and Krishna (2024) show that fixed prices tend to induce delayed buying while flexible prices tend to induce early buying (at a welfare loss). Moreover, with higher-order supply uncertainty, early "panic buying" becomes more likely. In our model, on the other hand, there is no uncertainty over the aggregate supply—consumers face individual consumption uncertainty, and this uncertainty is amplified in equilibrium by the behavior of other consumers. Noda and Teramoto (2022) examine a model with infinitely-lived consumers, as we do. Their model features continuous time, search costs, and a fixed positive price (whereas our model features discrete time, a storage cost, and a zero price). More importantly, Noda and Teramoto (2022) do not consider equilibrium multiplicity, which is the main focus of this paper. Their focus is on the transitional dynamics following a search cost shock, which they examine primarily using numerical methods.

Finally, our model shares similarities with some monetary search models. Berentsen (2000) shows that, in a simple extension of the Kiyotaki-Wright (1989) model, multiple stationary equilibria exist that have the same money stock and in which agents are willing to accumulate either one or two units of money. These equilibria resemble, in some ways, the 1-storage and 2-storage equilibria in our model. However, there are several important differences. First, money cannot be consumed, whereas the storable good in our model is a consumption good. Second, the supply of consumption goods in monetary search models is endogenously determined by agents' production decisions and the supply of money is (explicitly or implicitly) a policy variable. In our model, there is no money and the supply of the consumption good is exogenously fixed and cannot be increased by fiat. Third, while in both types of models inventories provide insurance against non-consumption events, the consequences of inventory accumulation are not the same. In our model, one household's inventory causes a negative externality, in that it increases the likelihood of non-consumption events faced by other households (thereby amplifying their stockpiling incentives). Money inventories, on the other hand, have a positive externality, in that they increase trading opportunities for other households.[6]

The remainder of the paper is organized as follows. In Section 2 we set up the theoretical model and define equilibrium. In Section 3 we define z-storage rules and derive properties of the dynamic system generated by these rules. Section 4 contains the main results, characterizing stationary equilibria in z-storage rules. The same Section also examines non-stationary equilibria. Section 5 contains a welfare analysis of the various equilibria of the model, along with a discussion of some policy implications. Section 6 concludes. Most proofs are in the Appendix.

[6]For example, in Berentsen (2000), the high-inventory equilibrium exhibits a higher velocity of money, which results in higher welfare compared to the lower-inventory equilibrium.

2 Model

2.1 The economy

The economy is populated by a continuum of households of measure 1. Time is divided into periods indexed by $t = 0, 1, 2, \ldots$. There is one consumption good, which can be bought and consumed in integer quantities only. In every period, a household requires one unit of the good. Each household can store up to $K \geq 2$ units of the item from one period to the next (where K is an integer).

The economy-wide supply of the good per period is a continuum of measure $m \leq 1$. If $m = 1$, the economy produces exactly as much of the item as is required to meet every household's underlying consumption need. If $m < 1$, the economy experiences an *aggregate supply shortage*.[7] There is a single store in the economy at which households can obtain the item.

Period t unfolds as follows.

1. At the beginning of the period, the store puts the entire economy-wide supply, m, on its shelves.

2. Household i enters the period with some inventory $s_i^t \in \{0, 1, \ldots, K\}$ of the good. If $s_i^t > 0$, the household consumes one unit, reducing its inventory by one. If $s_i^t = 0$, the household cannot consume the item in this period.

3. The household then makes a trip to the store. When it arrives at the store, the household is placed randomly in a queue, so that the measure of households $j \neq i$ that are ahead of i in the queue is a uniformly distributed random variable.

4. The household decides how many units of the item it wants to obtain. If the desired quantity is in stock, the household obtains it. If the desired quantity is not in stock, the household obtains the remaining stock, if positive.

If the store runs out of supply before the final shopper arrives, then some households will leave the store empty-handed. We say that these households experience an *in-store shortage*. Note that this does not mean that these households will not consume in the next period, as they may still have positive inventories in storage.

Household i receives a flow utility of 1 if it consumes the item in a given period (i.e., if $s_i > 0$), and a flow utility of 0 if it does not consume the item (i.e., if $s_i = 0$). In each period, the household also pays a storage cost proportional to its beginning-of-period inventory, λs_i, where $\lambda \in (0, 1)$. Finally, all households discount the future using a common discount factor $\beta \in (0, 1)$. To prevent trivial outcomes where not consuming is optimal, we also assume that $\lambda < \beta$.

[7] In Section 6, we discuss how the model can be adjusted in order to apply to the case of an aggregate demand increase, instead of a supply shortage.

We make the following additional assumptions. First, the price of the item is fixed and normalized to zero. Therefore, in our model, prices play no role in equating supply and demand, or in allocating the good to households.[8] If prices could adjust freely and demand or supply were not entirely inelastic, shortages could not arise. Households are nevertheless prevented from demanding an infinite amount, as they are constrained by a finite storage capacity K. In addition, the storage cost $\lambda > 0$ may reduce demand to below K. Second, a household cannot "resell" or otherwise transfer the good to another household. Third, households cannot dispose of the good they have in storage, and the only way for a household to reduce its inventory is through consumption of one unit per period. Finally, if the store has a positive amount of the good remaining after all households have shopped in a given period, the "unsold"amount is disposed of (this will not happen in any of the equilibria we examine).

2.2 State variables and decision rules

From the perspective of an individual household, it will be convenient to focus on the household's beginning-of-period-t inventory,

$$s^t \in \{0, \ldots, K\},$$

as the relevant state variable. A *decision rule* for the household in period t is a mapping that assigns to each value of s^t the household's desired beginning-of-period-$(t+1)$ inventory,

$$\sigma^t(s^t) \in \{\max\{0, s^t - 1\}, \ldots, K\}.$$

In period t, the household purchases $\sigma^t(s^t) - \max\{0, s^t - 1\}$ units of the good if the store has this many units in stock. If the store has at least one unit, but fewer than $\sigma^t(s^t) - \max\{0, s^t - 1\}$ units in stock, the household purchases the remaining stock (this will be a probability zero event for each household). If the store has zero units in stock, the household leaves empty-handed.[9]

[8] Since each household is quantitatively negligible, behavior in our model will be competitive in the sense that households (i) choose actions that are optimal given economic aggregates and (ii) ignore the impact of their actions on these aggregates. However, because there is no price system to intermediate households' behavior, the solution concept is not competitive equilibrium. Technically, our model is a mean field game, and the solution concept, formally introduced in Section 4, is (subgame perfect) Nash equilibrium.

[9] The household also observes economy-wide state variables, or has expectations about the values of such variables. These variables will be taken into account when we solve for the equilibrium, as they are obviously important in determining the optimal decision for a household in a given period. However, at this point they do not need to be explicitly listed as arguments of σ^t.

From the perspective of the aggregate economy, the relevant state in each period is the distribution of inventories across households. This state is given by the vector

$$x^t = \left(x_0^t, x_1^t, \ldots, x_{K-1}^t, x_K^t\right),$$

where x_k^t is the fraction of households that enter period t with k units in storage. The space of all such states is the K-dimensional unit simplex

$$\Delta_K \equiv \left\{x \in \mathbb{R}_+^{K+1} : \sum_{k=0}^{K} x_k = 1\right\}.$$

For $x, y \in \Delta_K$, we write $x \succsim y$ (or $y \precsim x$) if x weakly dominates y, in the sense of first-order stochastic dominance.[10]

Suppose all households use the same decision rule σ^t in period t. The aggregate measure of the good that would be purchased in period t if there was no supply constraint (i.e., if $m = \infty$) is given by

$$\theta^t = \sum_{k=0}^{K} x_k^t \left[\sigma^t(k) - \max\{0, k-1\}\right]. \tag{1}$$

Because only a measure $m \leq 1$ is available, the probability that a household arrives at the store and is able to purchase the good is

$$p^t = \min\left\{\frac{m}{\theta^t}, 1\right\}. \tag{2}$$

Therefore, the probability that a household experiences an in-store shortage is $1 - p^t$.

Households that find the good are able to execute their desired purchases and will enter period $t+1$ with $\sigma^t(s_i^t)$ units in storage. Households that experience an in-store shortage are unable to execute their desired purchases (if positive) and will enter period $t+1$ with $\max\{0, s_i^t - 1\}$ units in storage. Therefore, given aggregate state x^t and common decision rule σ^t, we can compute next period's state as follows:[11]

$$x_k^{t+1} = \begin{cases} p^t \sum_{k':\sigma^t(k')=K} x_{k'}^t & \text{if } k = K, \\ p^t \sum_{k':\sigma^t(k')=k} x_{k'}^t + (1-p^t)x_{k+1}^t & \text{if } 0 < k < K, \\ p^t \sum_{k':\sigma^t(k')=0} x_{k'}^t + (1-p^t)\left[x_1^t + x_0^t\right] & \text{if } k = 0. \end{cases} \tag{3}$$

[10] That is, $x \succsim y$ if $\sum_{s=0}^{k} x_s \leq \sum_{s=0}^{k} y_s \ \forall k = 0, \ldots, K$.

[11] Note that there can be at most one household that enters the store and finds more than zero, but fewer than $\sigma^t(s_i^t) - \max\{0, s_i^t - 1\}$, units of the item available. This household is of measure zero; hence its purchases have have no effect on the evolution of the aggregate state.

The economy is in a *stationary state* if the distribution of inventories across households is time-invariant, that is

$$x^{t+1} = x^t = x^* \quad \forall t.$$

A special case of a stationary state is a steady state, in which no individual household's inventory changes from period to period.

2.3 Equilibrium definition

Our notion of equilibrium is, essentially, that of Jovanovic and Rosenthal (1988) for anonymous sequential games. In every period, every agent chooses an action that maximizes the agent's discounted continuation payoff in that period, given state variables, and state variables in each period are determined from the previous period's state variables and the distribution of actions across individuals in the previous period. These conditions imply behavior that is the same as that in a subgame perfect Nash equilibrium.[12]

We can, however, apply two simplifications to the Jovanovic/Rosenthal definition. First, while Jovanovic and Rosenthal (1988) allow for continuous individual state and action spaces, ours are finite, and the equilibrium definition below is stated accordingly. Second, Jovanovic and Rosenthal (1988) permit mixed strategy equilibria, or—which is equivalent with a continuum of identical agents—asymmetric pure strategy equilibria. We restrict attention to symmetric pure strategy equilibria, that is, equilibria in which all agents adopt the same decision rule in a given period.

With this in mind, to define equilibrium formally we denote by V_s^t the continuation value of being in individual state $s = 0, \ldots, K$ in period t. This continuation value can be expressed recursively as follows

$$V_s^t = \begin{cases} \max\limits_{\sigma \in \{s-1,\ldots,K\}} \left\{ 1 - s\lambda + \beta \left[p^t V_\sigma^{t+1} + (1-p^t) V_{s-1}^{t+1} \right] \right\} & \text{if } s \geq 1, \\ \max\limits_{\sigma \in \{0,\ldots,K\}} \left\{ \beta \left[p^t V_\sigma^{t+1} + (1-p^t) V_0^{t+1} \right] \right\} & \text{if } s = 0. \end{cases} \tag{4}$$

Note that an individual is affected by the aggregate state x^t through the probability p^t. We then make the following definition:

[12]Note that the equilibrium conditions themselves are those of Nash equilibrium only, as they do not impose optimality at aggregate states that are not reached in equilibrium. However, no behavior that would be ruled out by the more stringent requirement of subgame perfection can emerge: With a continuum of agents, any aggregate state that is not reached in equilibrium could only be reached through a coordinated deviation by positive measure of agents. Therefore, the Nash equilibrium requirement implies subgame perfect play in the sequential game studied here.

Definition 1. Given $x^0 \in \Delta_K$, an *equilibrium* is a sequence of probabilities, states, continuation values, and decision rules

$$\Big(\big(p^t, x^{t+1}, V^t, \sigma^t\big) \in [0,1] \times \Delta_K \times \mathbb{R}^{K+1} \times \{0, \dots, K\}^{K+1}\Big)_{t=0,1,2,\dots}$$

such that for each $t \geq 0$ the following holds:

(i) p^t and x^{t+1} are determined from x^t and σ^t via (1)–(3),

(ii) $V^t(\cdot)$ and $\sigma^t(\cdot)$ are the value functions and policy functions that solve the Bellman equations (4), for $s = 0, \dots, K$.

The equilibrium is *stationary* if $x^t = x^*$ for all t.

3 z-Storage Rules

In this Section, we introduce a specific class of decision rules, called z-storage rules, defined as follows.

Definition 2. For given $z \in \{1, \dots, K\}$, the decision rule $\sigma^t(s^t) = \max\{s^t - 1, z\}$ is called the *z-storage rule.*

A household that uses a z-storage rule in period t tries to achieve a desired inventory of z units at the beginning of period $t+1$. If the household has more than z units in storage in period t, it uses one unit in period t and enters period $t+1$ with one less unit in storage. If the household has z or fewer units in period t, it tries to purchase enough so as to enter period $t+1$ with z units in storage. We call the z-storage rule the *maximum storage rule* if $z = K$, and the *minimum storage rule* if $z = 1$.[13] If a household uses a z-storage rule with $z > 1$, we say that the household *stockpiles*.

We now examine the properties of the dynamical system relating z-storage rules to the evolution of the state variable x^t. The analysis is "mechanical" insofar as it does not yet involve any optimizing on part of households, or any analysis of equilibrium. (The optimality of z-storage rules will be examined later in Section 4.) Our first result characterizes the stationary states associated with z-storage rules:

Proposition 1. *Suppose every household uses the same z-storage rule in every period, for $z \in \{1, \dots, K\}$. The unique stationary state associated with this rule is given by*

$$x^* \;=\; \big(\underbrace{(1-p^*)^z,\, p^*(1-p^*)^{z-1},\, p^*(1-p^*)^{z-2}, \dots,\, p^*(1-p^*)\,,\, p^*}_{x_0^*, \dots, x_z^*},\, \underbrace{0,\, \dots\,,\, 0}_{x_{z+1}^*, \dots, x_K^*}\big),$$

[13] Technically, a household could also follow a 0-storage rule, that is, a policy of not buying the good even if it has nothing stored. Because we assume that $\lambda < \beta$, this rule is never optimal, and we can ignore it.

where

$$p^* = 1 - (1-m)^{1/z}$$

is the probability that, in any given period, a household finds the good in the store.

Since p^* is the probability that a household is able to purchase the good, the probability that a household experiences an in-store shortage when all households use the same z-storage rule is $1 - p^* = (1-m)^{1/z}$. If $m = 1$, this probability is zero for all z; yet, if $z > 1$ every household stockpiles. These observations may appear contradictory but they are, in fact, consistent: If $m = 1$, the stationary (in fact, steady) state associated with the z-storage rule is for every household to have z units in storage, consume one unit per period, and buy one unit per period to keep the household inventory constant at z units. What Proposition 1 implies, then, is that stockpiling cannot cause permanent in-store shortages if the aggregate supply is sufficient to meet aggregate consumption needs.[14]

If $m < 1$, there must necessarily be in-store shortages: In stationary state, it is not possible to have an aggregate supply shortfall but no rationing on the individual level. The formula for p^* in Proposition 1 allows us to compute the frequency of these in-store shortages. Suppose there is a one percent aggregate shortfall in the toilet paper supply ($m = 0.99$) and households store two units ($z = 2$). In this case $p^* = 0.9$, meaning that one out of every ten trips to the store will be unsuccessful. However, in a stationary state, any household's consumption probability is independent of z. Note that in every period, the fraction of households that do not consume is

$$x_0^* = (1-p^*)^z = 1 - m.$$

Since households are symmetric, this means that the long-run probability that a household consumes the good in any given period is m, which does not depend on the z-storage rule used. Thus, while the probability that households' consumption needs are not satisfied is merely 1%, the probability that they cannot make their desired purchase is 10%, a ten-fold increase. Similarly, if households increase their storage to $z = 5$ units or $z = 10$ units, we have $p^* = 0.6019$ and $p^* = 0.3690$, respectively. In these cases, the probability that households cannot make their desired purchase increases to 40% and 63%, respectively. Therefore, in-store shortages appear more severe as households stockpile more units, even though fundamental supply conditions remain unchanged.

Finally, we turn to the question whether the economy converges to the stationary state from some initial state, if all households use the same z-storage rule. Answering this question is complicated by the fact that the mapping from one period's state to the next period's state, defined in (3), fails to be a contraction when all households use the

[14] At the same time, without in-store shortages there is clearly no need for costly storage. In Section 4, we use this observation to show that there is a unique equilibrium when $m = 1$, which is for all households to use the minimum storage rule, i.e. $z = 1$.

z-storage rule (an example is provided at the beginning of the proof of Proposition 2 in the Appendix.) Nevertheless, we can prove:

Proposition 2. *Suppose every household uses the same z-storage policy in every period. The economy converges to the stationary state x^* from any initial state x^0. Furthermore, if $x^0 \succsim (\precsim)\ x^*$ then p^t converges to p^* from above (below).*

Proposition (2) will be used later, in Section 4.3, where we examine transitional dynamics following a fundamental supply shock.

4 Equilibrium Analysis

We now examine the equilibria of the economy, as defined formally in Section 2. We consider three scenarios:

1. When there is no aggregate supply shortage, we show that there exists a unique equilibrium. In this equilibrium, all households use the minimal storage rule ($z = 1$) in every period. (See Section 4.1.)

2. When there is an aggregate supply shortage, we show that there exist one or more stationary equilibria in which all households use the same z-storage rule in every period. Equilibria with stockpiling ($z > 1$) generally exist. (See Section 4.2.)

3. We also consider the case when the economy experiences an unexpected supply shock. In this case, the economy transitions from scenario 1 to scenario 2 above. However, this transition is no immediate, and the question is: What decision rules will arise along the convergence path from the previous stationary state to the new one? We characterize conditions under which adoption of a z-storage rule following the shock is an equilibrium. (See Section 4.3.)

4.1 Equilibrium when there is no aggregate shortage

For the case where there is no aggregate supply shortage we have a clear result:

Proposition 3. *Suppose $m = 1$ and let $x^0 \in \Delta_K$ be any initial state. The unique equilibrium is for every household to use the minimum storage rule in every period.*

The main implication of Proposition 3 is that stockpiling cannot merely arise because it is a "self-fulfilling prophecy." One might imagine a situation where there is no fundamental aggregate shortage, but where in-store shortages nonetheless arise, caused by consumer stockpiling, which itself is an optimal response to the in-store shortages. As Proposition 3 shows, this situation cannot occur in our model. The reason—formally shown in the proof of Proposition 3 Appendix—is the following: For households to stockpile in period

t, they must anticipate in-store shortages in period $t+1$. Without a fundamental supply shortage, however, in-store shortages in period $t+1$ can only arise if some households stockpile in period $t+1$. This can only be optimal if households anticipate in-store shortages in period $t+2$, and so on. These dynamics would result in some households' inventories to grow without bound, which is impossible.

In order to keep the notation and analysis manageable throughout the paper, our formal equilibrium definition (see Definition 1) involves only symmetric strategy profiles, i.e., all households use the same decision rule in a given period. In principle, Proposition 3 leaves open the possibility that asymmetric equilibria exist in which some households use a decision rule that is not the minimum storage rule. However, as we show in the Appendix, no such additional equilibria can exist either if $m = 1$. Thus, when there is no aggregate supply shortage, we unambiguously conclude that all households use the minimum storage rule in all periods.

4.2 Equilibria when there is an aggregate shortage

We now assume that $m < 1$. We will be looking for equilibria in which all households use the same z-storage rule in every period. We call such profiles *z-storage equilibria.* We emphasize that we do not restrict households to use a z-storage rule, but rather identify conditions under which the use of a z-storage rule is optimal for a household if all other households use this rule.

We begin with a general result that characterizes an individual household's best response if all other households use the same z-storage rule. To do so, we first need to introduce the following notation. Let $\mathbf{p} = (p^0, p^1, \ldots)$ be a sequence of probabilities, let $y \in \{1, \ldots, K+1\}$, and define

$$\overline{\lambda}_y(\mathbf{p}) \equiv \begin{cases} 1 & \text{if } y = 1, \\ \dfrac{\beta^{y-1} \prod_{s=0}^{y-2} \left(1 - p^s\right)}{1 + \sum_{k=1}^{y-1} \beta^k \prod_{s=0}^{k-1} \left(1 - p^s\right)} & \text{if } 2 \leq y \leq K \\ 0 & \text{if } y = K+1. \end{cases} \tag{5}$$

Further, suppose $x^0 \in \Delta_K$ is the initial state and all households use the same z-storage rule in every period. For this case, let $p_z^t(x^0)$ be the probability that a household finds the good in the store in period t and let $\mathbf{p}_z^t(x^0)$ denote the sequence $\left(p_z^t(x^0), p_z^{t+1}(x^0), p_z^{t+2}(x^0), \ldots\right)$. We then have:

Lemma 4. *Suppose the initial state is x^0 and all households $j \neq i$ use the same z-storage rule in every period. For household j, it is a best response to use the y-storage rule in* *every period if and only if*

$$\overline{\lambda}_{y+1}\big(\mathbf{p}_z^t(x^0)\big) \;\leq\; \lambda \;\leq\; \overline{\lambda}_y\big(\mathbf{p}_z^t(x^0)\big) \tag{6}$$

for all $t \geq 0$.

Lemma 4 can be used to characterize the z-storage equilibria of our model, by checking condition (6) when $y = z$. (That is, the z-storage rule must be "a best response to itself.")

We begin by looking at stationary z-storage equilibria. Such equilibria can be thought of as describing long-run outcomes of the economy, in which the economy is at stationary state x^* and remains at this state permanently absent any changes in fundamentals (i.e., absent changes in m, K, β, or λ): $x^0 = x^1 = x^2 = \ldots = x^*$. The probability of being able to buy the good is then constant over time as well, and is given by value p^* in Proposition 1. After substituting p^* for all probability terms in (5), and setting $y = z$, condition (6) can be expressed as follows:

$$\frac{\gamma(z)^z\big(1-\gamma(z)\big)}{1-\gamma(z)^{z+1}} \;\leq\; \lambda \;\leq\; \frac{\gamma(z)^{z-1}\big(1-\gamma(z)\big)}{1-\gamma(z)^z}, \tag{7}$$

where

$$\gamma(z) \;\equiv\; \beta(1-m)^{1/z}.$$

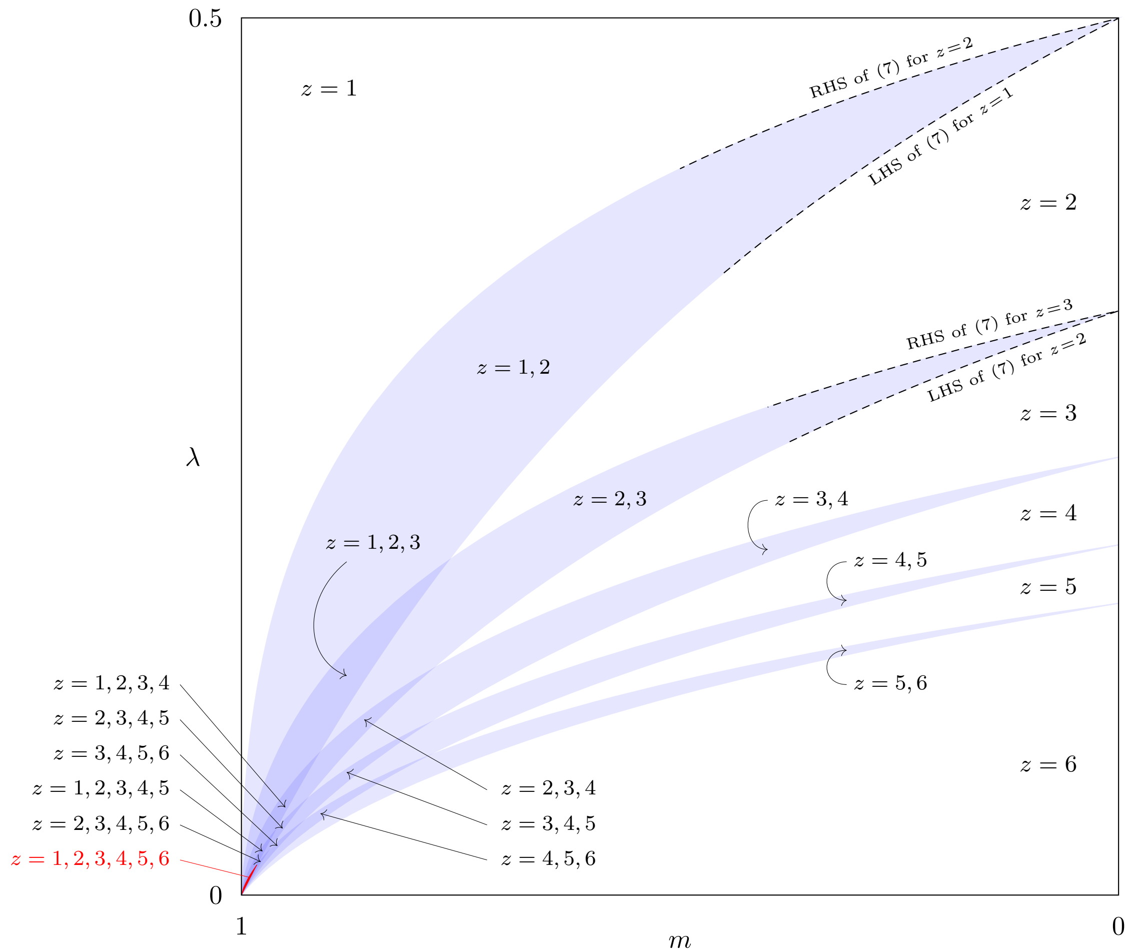
The interpretation of condition (7) is the following: The storage cost must be must be low enough for households to want to store at least z units, and also high enough for households to not want store more than z units, assuming all other households use the z-storage rule. Thus, a stationary z-storage equilibrium with $1 < z < K$ exists if condition (7) is satisfied. Likewise, a minimum storage equilibrium exists if the first inequality in (7) holds for $z = 1$, and a maximum storage equilibrium exists if the second inequality in (7) holds for $z = K$. We have:

Proposition 5. *A stationary z-storage equilibrium exists for some $z \in \{1, \ldots, K\}$.*

Figure 1 shows the stationary z-storage equilibria for the case where $\beta = 0.999$ and $K = 6$ are fixed, but varying m and λ. The regions in Figure 1 are constructed by plotting the left-hand side and right-hand side of (7), for different integer values of z. (This is indicated via the dashed curves for select z-values.) In the white parameter regions, a unique z-storage equilibrium exists; in the colored regions multiple equilibria exist.

As predicted by Proposition 3, if $m = 1$, the only equilibrium is for every household to employ the minimum storage rule. On the other hand, when $m < 1$, stationary equilibria with stockpiling can emerge. In particular, when the supply shortage is relatively small and storage is relatively cheap, a large number of different z-storage equilibria can exist simultaneously. In the small red-colored region in Figure 1, *every* z-storage rule constitutes

Figure 1: Stationary z-storage equilibria ($\beta = .999$, $K = 6$).



Note: The white regions in the figure indicate (m, λ)-combinations for which a unique stationary z-storage equilibrium exists. The shaded regions indicate (m, λ)-combinations for which two or more stationary z-storage equilibria exists. The equilibrium value(s) for z are noted in the figure. The small red region consists of (m, λ)-combinations for which stationary z-storage equilibria exists for all $z = 1, \ldots, K$.

a stationary equilibrium, including the minimum and maximum storage strategies. The following result shows that this is a generic possibility for arbitrarily small but positive aggregate shortages.

Proposition 6. *Fix $K > 1$ and $\beta < 1$. There exists $\overline{m} < 1$ such that the following is true. For every $\overline{m} < m < 1$, there exists an open interval of storage costs $\Lambda \subset (0, 1)$ such that, if $\lambda \in \Lambda$, a stationary z-storage equilibrium exists for all $z = 1, \ldots, K$.*

Figure 1 further suggests that, when multiple stationary z-storage equilibria exist, the equilibrium set is "connected" in the sense that the set of equilibrium values for z consists of consecutive integers. The following result confirms this:

Proposition 7. *If stationary z-storage and z'-storage equilibria exist, with $z < z'$, then stationary z''-storage equilibria exist for all integers $z \leq z'' \leq z'$.*

We now decompose households' stockpiling incentives into two forces. The first force is the fundamental, aggregate supply shortfall—only $m < 1$ new units of the good are available in each period despite households wanting to consume 1 unit. Without this shortfall, Proposition 3 implies that stockpiling would not arise in any equilibrium. However, if there is an aggregate supply shortage, it may be optimal for a household to maintain an inventory of more than one unit of the good in order to smooth consumption. We call this the *direct effect* of the shortage. The second force is an *indirect effect*: If every household decides to stockpile, in-store shortages become more frequent and, as a result, further stockpiling incentives are created. Thus, there exists a feedback mechanism from stockpiling to even more stockpiling. The equilibria of the model reflect the combined effect of both direct incentive and indirect feedback.

To measure the strength of the direct effect only, we can compute the rule that is optimal for an *individual household* if the aggregate supply is $m < 1$ but all other households use the minimum storage rule. In this case, the likelihood that a household finds the item in the store in any given period is then $p^* = m$. Lemma 4 implies that the y-storage rule that is a best response in this situation satisfies the condition

$$\overline{\lambda}_{y+1}(m, m, \ldots) \;\leq\; \lambda \;\leq\; \overline{\lambda}_y(m, m, \ldots)$$

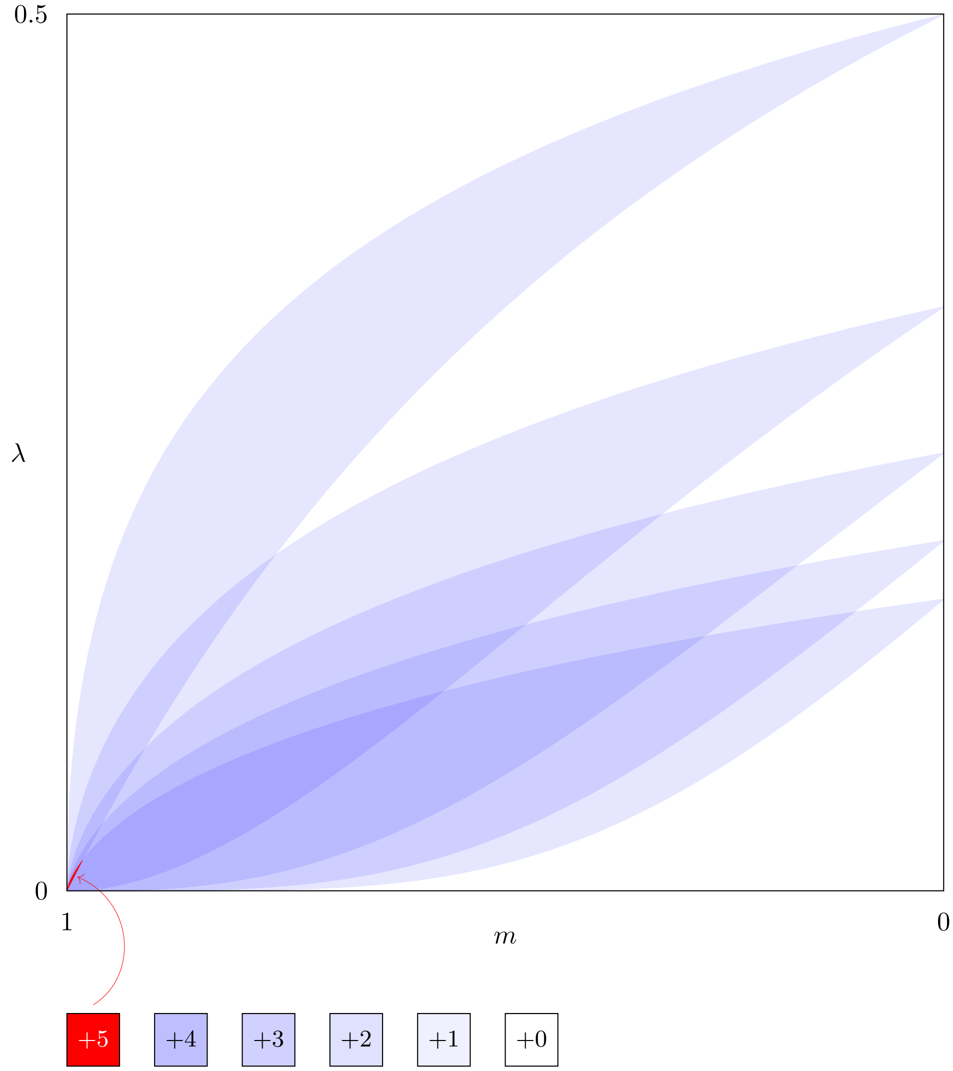
or, equivalently,

$$\frac{\gamma(1)^y\big(1-\gamma(1)\big)}{1-\gamma(1)^{y+1}} \;\leq\; \lambda \;\leq\; \frac{\gamma(1)^{y-1}\big(1-\gamma(1)\big)}{1-\gamma(1)^y}.$$

Now suppose that a z-storage equilibrium exists for some value of $z > 1$. We can then take the difference between this equilibrium value of z and the previously computed value y that would be a single household's unilateral best response to the shortage, assuming that none of the other households stockpiled. This difference can be interpreted as a measure of *excess stockpiling* arising from the aforementioned feedback loop; that is, it quantifies of the strength of the indirect stockpiling incentive.

Figure 2 depicts the amount of excess stockpiling for the same parameter values as those used to plot Figure 1. When multiple stationary z-storage equilibria exist, we selected the one with the largest z in order to measure the strength of the indirect incentive in the most extreme case. As Figure 2 shows, the feedback mechanism can generate a significant amount of excess stockpiling for certain parameter configurations. In particular, consider the set of parameter combinations for which both the minimum and maximum storage equilibrium exist (indicated in red in Figure 2 as well). If all households followed the minimum storage strategy, it would be a unilateral best response to use the minimum storage strategy as well. Therefore, if the maximum storage equilibrium is played in this case, the incentive to maintain inventories above a single unit is driven *entirely* by the fact that other households follow the same strategy. In other words, the feedback loop from stockpiling to more stockpiling accounts for $K-1$ out of the K units in each household's target inventory.

Figure 2: Excess stockpiling ($\beta = .999$, $K = 6$).



Note: Colors represent the strongest degree of excess stockpiling that can arise, measured by units stockpiled in the highest z-storage equilibrium minus units stockpiled as a unilateral best response to the minimum storage rule. In the white regions in the figure, there is no excess stockpiling; in the shaded regions, there is excess stockpiling.

4.3 Transitional dynamics

In Section 4.1 we characterized the unique outcome of the economy when there was no aggregate supply shortage; and in Section 4.2 we described the possible stationary, long-run outcomes of the economy when there was an aggregate supply shortage. We now examine what happens as the economy transitions between these two cases. Specifically, we consider the economy's response to an unanticipated supply shortage. In many applications—including the motivating examples discussed in Section 1—one may be particularly interested in this short-run response.

Consider a situation in which supply and demand are balanced (i.e., $m = 1$) in every period, until an unexpected shock reduces the supply to $m < 1$. Without loss of generality, we let period $t = 0$ be the period in which the shock occurs. In response to this shock, households may begin to accumulate inventories, thereby initiating the transition from the previous stationary state, $x^0 = (0, 1, 0, \ldots, 0)$, to a new stationary state. This transition will itself be governed by decision rules that are best responses to each other. For example, suppose that every household adopts the same z-storage rule immediately after the shock. In this case, a measure 1 of households will attempt to purchase z units of the good each, but only a measure m/z of households will be successful. Thus, next period's state is

$$x^1 = \Big(\underbrace{1 - \frac{m}{z}}_{x_0^1}, 0, \ldots, 0, \underbrace{\frac{m}{z}}_{x_z^1}, 0, \ldots, 0\Big).$$

If all households continue to follow the same z-storage rule, the state will further evolve according to the law of motion (15) and converge to the stationary state x^* described in Proposition 1. If the z-storage rule remains optimal in every period for a household that expects all other households to use the same z-storage rule in all periods, we call the resulting outcome a *transitional z-storage equilibrium.* Note that the equilibrium is non-stationary because the aggregate state x^t changes over time; however, the households' optimal strategies remain unchanged.

We now derive a condition on the storage cost λ under which a transitional z-storage equilibrium exists. It should be clear that this condition will be more stringent than the corresponding condition (7) for stationary equilibria, as the same z-storage rule must now be optimal in a larger set of circumstances. We begin with the following result:

Lemma 8. *Let $m < 0$ and fix initial state $x^0 = (0, 1, 0, \ldots, 0)$. Suppose all households use the same z-storage rule in every period $t = 0, 1, \ldots$. Let $p_z^t(x^0)$ denote the probability that a household finds the good in the store in period t. Then $p_z^{t+1}(x^0) \geq p_z^t(x^0)$ for all t, and $p_z^t(x^0) \to p^* = 1 - (1 - m)^{1/z}$.*

Note that Lemma 8 implies that $\mathbf{p}_z^0(x^0) \leq \mathbf{p}_z^1(x^0) \leq \mathbf{p}_z^2(x^0) \leq \ldots$. Furthermore, $\mathbf{p}_z^t(x^0) \to (p^*, p^*, \ldots)$ uniformly. We further have:

Lemma 9. $\overline{\lambda}_y(\cdot)$ *is decreasing:* $\mathbf{p} \geq \widehat{\mathbf{p}}$ *implies* $\overline{\lambda}_y(\mathbf{p}) \leq \overline{\lambda}_y(\widehat{\mathbf{p}})$.

Now consider whether using the z-storage rule is individually optimal in period $t = 0, 1, \ldots$. By Lemma 4, this is the case if and only if (6) holds for all t, setting $y = z$. In particular, Lemma 4 specifies two conditions. The first condition applies whenever $z > 1$ and states that the storage cost cannot be so high that the household would rather store fewer than z units:

$$\lambda \;\leq\; \overline{\lambda}_z(\mathbf{p}_z^t). \tag{8}$$

By Lemma 9, $\overline{\lambda}_z(\mathbf{p}_z^t)$ is decreasing in $\mathbf{p}_z^t$; and by Lemma 8, $\mathbf{p}_z^t$ is increasing in t. Thus, for (8) to hold for all t, it is necessary and sufficient that it holds at $\lim_{t\to\infty} \mathbf{p}^t = (p^*, p^*, p^*, \ldots)$.

$$\lambda \;\leq\; \overline{\lambda}_z(p^*, p^*, p^*, \ldots) \;=\; \frac{\gamma(z)^{z-1}\big(1-\gamma(z)\big)}{1-\gamma(z)^z}, \tag{9}$$

where the term on the right-side is the same as in (7). The second condition applies whenever $z < K$ and states that the storage cost cannot be so low that the household would rather store more than z units:

$$\lambda \;\geq\; \overline{\lambda}_{z+1}(\mathbf{p}^t). \tag{10}$$

Again applying Lemma 9 and Lemma 8, we see that (10) holds for all t if and only if it holds at $\mathbf{p}^0 = (p^0, p^1, p^2, \ldots)$:

$$\lambda \;\geq\; \overline{\lambda}_z(p^0, p^1, p^2, \ldots). \tag{11}$$

Combining (9) and (11) and making the dependence of p^t on z explicit, we get:

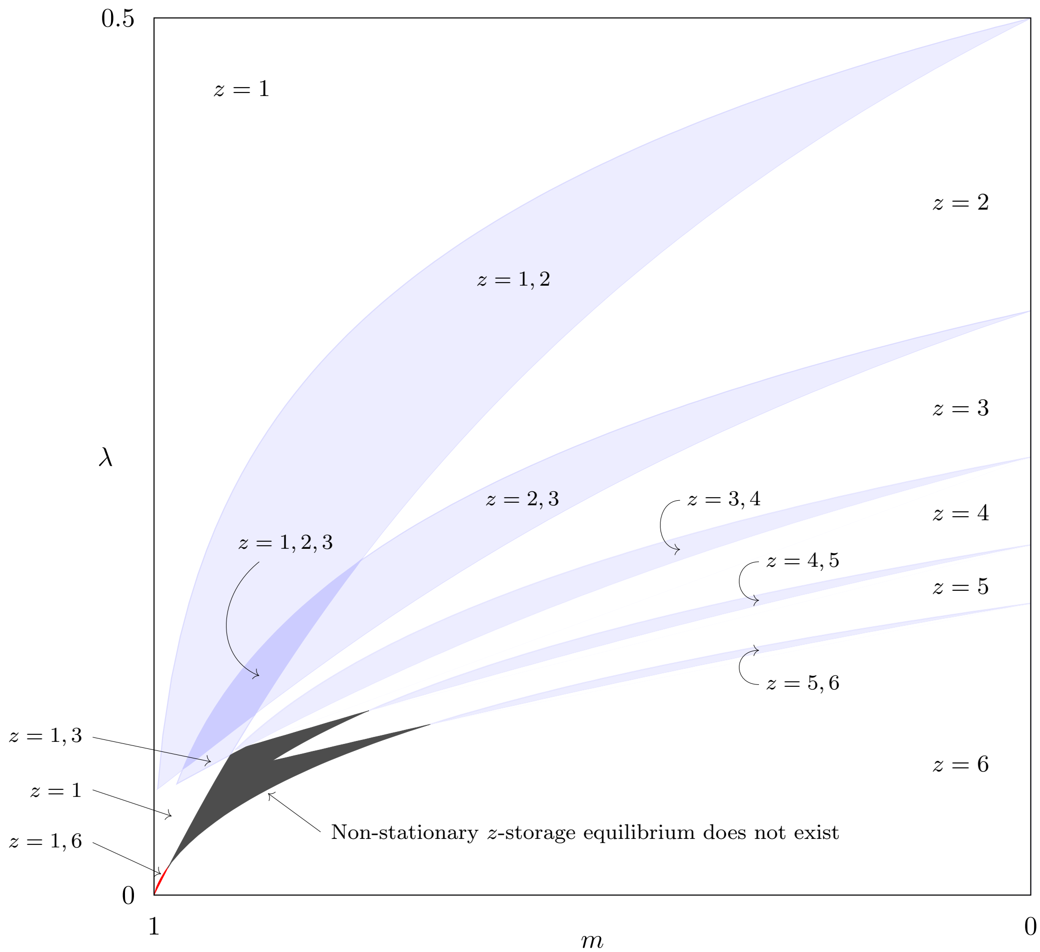
$$\overline{\lambda}_z\Big(p\big(x^0|z\big), p\big(T(x_0|z)|z\big), p\big(T^2(x_0|z)|z\big), \ldots\Big) \;\leq\; \lambda \;\leq\; \frac{\gamma(z)^{z-1}\big(1-\gamma(z)\big)}{1-\gamma(z)^z}. \tag{12}$$

A non-stationary z-storage equilibrium exists if condition (12) is satisfied (for $z = 1$, the second inequality can be ignored; and for $z = K$ the first inequality can be ignored). Note that the upper bound on the storage cost λ is the same as the previous upper bound for stationary z-storage equilibrium; however, the lower bound on the storage cost λ is larger than the previous lower bound for stationary equilibria. Therefore, a non-stationary z-storage equilibrium may fail to exist for parameter values under which a stationary z-storage equilibrium exists.[15]

[15] This means the following: Starting at the new stationary state associated with $m < 1$ and a particular z-storage rule ($z > 1$), it would be mutually optimal for all households to use this z-storage rule. However,

Figure 3 depicts the set of non-stationary z-storage equilibria for the same parameter values as were used in Figure 1 (i.e., $\beta = .999$ and $K = 6$), assuming initial state $x^0 = (0, 1, 0, 0, 0, 0, 0)$. As the new equilibrium condition (12) is more stringent than the previous condition (7), some of the z-storage rules that were stationary equilibria have disappeared. Moreover, Figure 3 demonstrates that none of the previous Propositions 5, 6, and 7 carries over to the non-stationary case: Non-stationary z-storage equilibria need not exist (see the black region in the graph), and when they exist the range of z-values for which the z-storage rule constitutes an equilibrium can have "holes."

Figure 3: Non-stationary z-storage equilibria ($\beta = .999$, $K = 6$), assuming initial state $x^0 = (0, 1, 0, 0, 0, 0, 0)$.



starting at the previous stationary state associated with $m = 1$, immediately switching to the new z-storage rule as part of the transition to the new stationary state would not be mutually optimal.

However, a non-stationary minimum storage equilibrium exists whenever a stationary minimum storage equilibria exists, and the same is true for maximum storage equilibria. For example, as shown in Figure 1 and Figure 3, in the red-colored parameter region for which the full range of stationary z-storage equilibrium exists, the minimum and maximum storage rules survive as non-stationary equilibria. To see why, note that if the initial state is $x^0 = (0, 1, 0, \ldots, 0)$ and households use the minimum storage rule, the probability of finding the item in the store is m in every period. This is the same probability as in the stationary state $x^* = (1 - m, m, 0, \ldots, 0)$. Thus, the condition for a stationary minimum storage equilibrium (i.e., the left inequality in (7), for $z = 1$) is identical to the condition for a non-stationary minimum storage equilibrium (i.e., the left inequality in (12), for $z = 1$). Similarly, the condition for a stationary maximum storage equilibrium (i.e., the right inequality in (7), for $z = K$) is identical to the condition for a non-stationary maximum storage equilibrium (i.e., the right inequality in (12), for $z = K$). We summarize this observation in the following result:

Proposition 10. *Let* $x^0 = (0, 1, 0, \ldots, 0)$. *A non-stationary minimum (maximum) storage equilibrium exists if and only if a stationary minimum (maximum) storage equilibrium exists.*

5 Welfare Comparisons

In any situation where multiple equilibria exist, it is natural to ask if these equilibria can be ranked by their welfare. And if certain equilibria result in a welfare loss, what policy interventions could improve the outcome?

Let us first compare the stationary z-storage equilibria. As shown in Section 3, in the stationary state x^* associated with any z-storage rule, the fraction of households that are able to consume the good in any period is m (and this is the maximum feasible fraction, given that m units are available in each period). Thus, welfare differs across the stationary z-storage equilibria only insofar as households pay higher total storage costs in equilibria with higher z. Specifically, in stationary z-storage equilibrium the total storage cost incurred by households in each period is

$$\lambda E\big[s_i^t\big] \;=\; \lambda\big[x_1^* + 2x_2^* + \ldots + zx_z^*\big] \;=\; \lambda \left[z - \frac{(1-m)^{1/z}}{1-(1-m)^{1/z}} m\right],$$

where x^* denotes the stationary state associated with the z-storage rule, characterized in Proposition 1. It is straightforward to verify that, if x^{**} is the stationary state associated with the z'-storage rule and $z' > z$, then $x^{**} \succ x^*$.[16] Thus the z-storage equilibrium with the lowest z is the one with the least storage cost payment. In the applications we have in mind (i.e., the stockpiling of everyday household items), this storage cost payment is minor, as the per-unit cost λ is likely insignificant in comparison to the flow utility of consumption. Therefore, any welfare differences across different stationary equilibria are relatively minor as well.[17]

These relatively small welfare losses associated with stockpiling in stationary equilibria can mask potentially larger welfare losses experienced during the transitional phase, i.e., when households begin to accumulate inventories in response to a supply shock. To explicitly account for the short-run effect of stockpiling, we now return to the non-stationary equilibria introduced in Section 4.3. Recall that these equilibria capture the transitional dynamics from the initial state $x^* = (0, 1, 0 \ldots, 0)$ to a new stationary state following a supply restriction, and it is along this transition path that the *accumulation* of inventories (and not merely the maintenance of inventories) occurs. Given a fixed per-period supply, the economy-wide inventory accumulation has a more significant impact on a household's contemporaneous utility than do storage costs alone, as inventory accumulation reduces the household's expected consumption.

For concreteness, we consider the following parameter configuration:

$$\beta = 0.999,\ K = 6,\ m = 0.9995,\ \lambda = 0.0005.$$

If the time period is one week, a discount factor of $\beta = 0.999$ implies an annual discount rate of approximately 5%. A per-unit storage cost of $\lambda = 0.0005$ means that the cost of storing a one week's worth of toilet paper is 0.05% of the utility the household obtains from the using toilet paper for a week (relative to consuming the next best substitute). And a supply of $m = 0.9995$ implies that—*absent any stockpiling by consumers*—a household would experience an in-store shortage of toilet paper during one week out of every 38 years on average. This parameterization is in the red-colored region in Figure 3, so that non-stationary minimum and maximum storage equilibria both exist. Thus, despite the negligible supply shortage, it is an equilibrium for all households to accumulate

[16] To see this, let $z' = z + 1$. From Proposition 1, $x_0^* = 1 - m = x_0^{**}$. Moreover, for $0 < s \leq z$ and $m \in (0, 1)$,

$$x_s^* \ = \ \left(1 - (1-m)^{\frac{1}{z}}\right)(1-m)^{\frac{z-s}{z}} \ > \ \left(1 - (1-m)^{\frac{1}{z+1}}\right)(1-m)^{\frac{z-s+1}{z+1}} \ = \ x_s^{**}.$$

Thus, for all $k = 0, \ldots, K$ we have $\sum_{s=0}^{k} x_s^* \geq \sum_{s=0}^{k} x_s^{**}$, with strict inequality if $k = 1, \ldots, z$, and it follows that $x^{**} \succ x^*$.

[17] In principle, an additional welfare effect arises from the fact that household inventories bind resources whose value cannot be spent on consumption of other goods. Our model abstracts from this effect because the price of the stockpiled good is zero. However, in reality this price is positive. Thus, the total value of resources tied up in inventories is, therefore, lower in an equilibrium with less stockpiling. This welfare improvement, too, should be minor, as items such as toilet paper or dried pasta do not account for significant expenditure shares in most households.

and maintain inventories lasting $K = 6$ weeks.[18] Let period $t = 0$ denotes the start of the unanticipated permanent supply shortage. Prior to period 0, we assume that the supply was $m = 1$, and the economy was in the unique minimum storage equilibrium.

Compared to the pre-shock economy, the supply shock will reduce welfare regardless of which of the non-stationary equilibria is played. Our goal is to compare the welfare reduction in the equilibrium with the least amount of stockpiling ($z = 1$) to the welfare reduction in the equilibrium with the highest amount of stockpiling ($z = 6$). To do so, we measure welfare in two different ways.

First, in any given period t we measure welfare by the expected continuation utility of a household.[19] This captures both short-run and long-run welfare effects of stockpiling (if any). Our baseline welfare measurement is the lifetime utility of a household that stores and consumes exactly one unit in every period (which is the unique equilibrium behavior before the shock). We call this value $W^{\text{Pre}} = (1 - \lambda)/(1 - \beta)$. Following the unanticipated shock, we numerically compute a households expected continuation utility in every period $t = 0, 1, 2, \ldots$, assuming that (i) all households adopt the minimum storage rule ($z = 1$); and (ii) all households adopt the maximum storage rule ($z = 6$). Both are non-stationary equilibria following the shock. We call these welfare values W_1^t and W_6^t, respectively. Figure 4 (a) plots the ratios W_1^t/W^{Pre} and W_6^t/W^{Pre}. The first ratio shows the welfare effect of the supply shock if the minimum storage equilibrium $z = 1$ was maintained. The expected welfare loss would be less than 0.05% (i.e., consumption decreases by 0.05% and so does the corresponding storage cost). Moreover, the economy reaches the new stationary equilibrium immediately. The second ratio shows the welfare effect of the supply shock if the shock were to trigger a switch to the maximum storage equilibrium. Welfare would drop by 0.89% initially and recover about half of this loss over time, converge to a level 0.47% below the baseline. Thus, stockpiling significantly amplifies the welfare loss of the supply shock.

Second, in order to focus on the short-run welfare effects we also use a 13-period rolling window to compute a household's expected discounted utility over this truncated timespan only. If the model period is one week, then a 13-period window corresponds to a welfare assessment for the leading quarter-year. In practice, policy makers often evaluate outcomes using such rolling-windows; for example, the results of a given policy or strategic plan may be assessed using monthly, quarterly, or annual time horizons.[20] Corresponding to the previous notation, we denote the corresponding truncated welfare measures by $\widehat{W}^{\text{Pre}}$, $\widehat{W}_1^t$, and $\widehat{W}_6^t$, respectively. Figure 4 (b) plots the ratios $\widehat{W}_1^t/\widehat{W}^{\text{Pre}}$ and $\widehat{W}_6^t/\widehat{W}^{\text{Pre}}$. Stockpiling now has a very large effect on short-run welfare. Compared to the baseline, if the minimum storage equilibrium was maintained after the supply shock,

[18] Similar examples can be constructed for larger values of K.

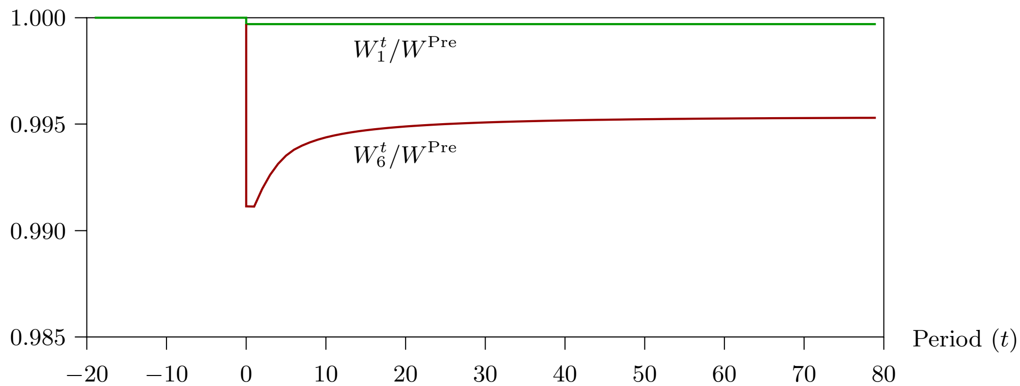
[19] Formally, we compute the expectation of V_s^t over all states s, where V_s^t is defined in (4).

[20] We also considered 4-week (monthly) and 26-week (semi-annual) rolling windows and the results are similar.

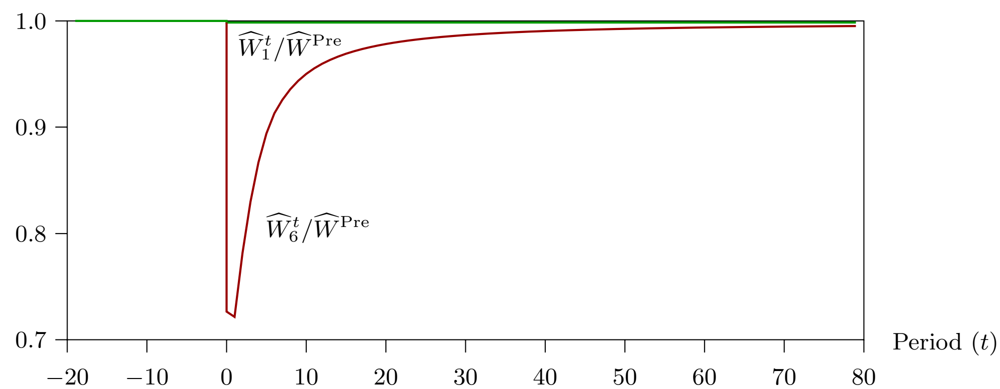
Figure 4: Welfare effects of a supply shock.

$\beta = .999$, $K = 6$, $m = 0.9995$, $\lambda = 0.0005$

(a) Relative welfare (untruncated)



(b) Relative welfare (truncated after 13 periods)



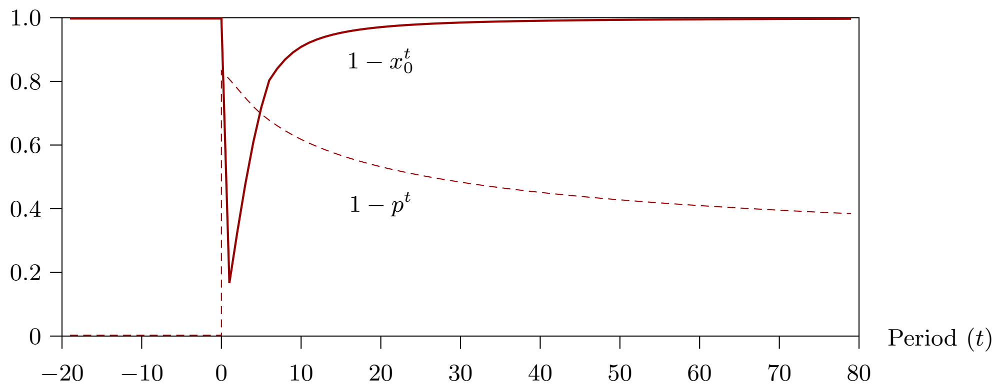
the expected welfare loss in the short-run is still only 0.05%. However, if a switch to the maximum storage equilibrium occurred, short-run welfare would drop by over 27% initially (and recover nearly all of this loss over time). Unlike in panel (a), the large loss in short-run welfare is no longer masked by the relatively small loss in long-run welfare. As expected, shorter rolling windows would make the initial drop more pronounced, while longer rolling windows would attenuate the initial drop.

For the same example, Figure 5 shows how households' consumption patterns and inventories respond to the supply shock, assuming the shock triggers a switch to the maximum storage equilibrium. Panel (a) plots a household's per-period consumption probability $(1 - x_0^t)$ and the probability that a household experiences an in-store shortage $(1 - p^t)$; panel (b) plots average household inventories $(E[s_i^t])$. As households start accumulating inventories following the shock, they experience an in-store shortage with

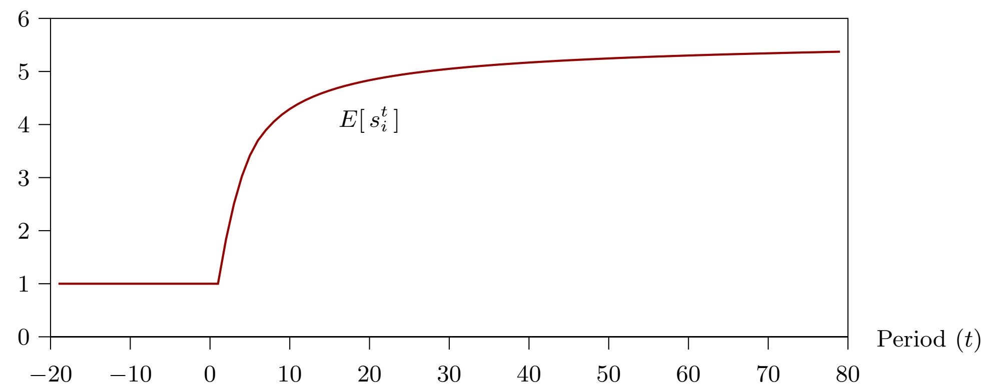
Figure 5: Consumption and inventory response to a supply shock.

$\beta = .999$, $K = 6$, $m = 0.9995$, $\lambda = 0.0005$

(a) Consumption and in-store shortage probability



(b) Inventories



probability 83.3%. Accordingly, their consumption probability next period drops to 16.7%. Over time, as inventories get built up and the speed of inventory accumulation slows, the consumption probability rises and converges to the stationary state probability, which is $m = 0.9995$. In-store shortages, however, remain stubbornly common: The probability of experiencing an in-store shortage exceeds 42% even 52 weeks after the initial shock. This does not affect long-run consumption, however, as households maintain their large inventories precisely to tide them over these shortages.

6 Discussion

6.1 Shortages due to demand increases

In our model, the supply-demand imbalances that generated the stockpiling incentives were driven by exogenous supply reductions, holding demand constant. In reality, the same imbalances can also be caused by demand increases, holding supply constant.

For example, during COVID-19 many consumers stockpiled common household items to last through a potential quarantine period. In addition, shortages of flour, yeast, and other baking supplies were primarily due to demand spikes, as many consumers took on baking as a new hobby during the lockdown. Furthermore, consumption of certain goods shifted from workplaces to homes, increasing the demand for home-use varieties of these goods. For example, the toilet paper used in many workplaces comes in large rolls that fit high-capacity dispensers, and the coffee consumed in offices is often sold in packs suitable for commercial coffee makers. Offices generally procure these items from specialized vendors that do not sell to consumers. Thus, separate markets exist for away-from-home and at-home varieties of certain goods, and the supply of away-from-home varieties cannot be redirected (in the short run) to meet increased demand for at-home varieties.

Demand-driven shortages can be accommodated in our model, by assuming that supply is always fixed $m < 1$ units per period and that each household requires one unit of the good with probability m per period. If these household-level demands are independent, the aggregate demand will be exactly equal to aggregate supply. Thus, the scenario where households require one unit with probability m is the "balanced scenario." Relative to this scenario, a demand increase happens when the consumption probability changes from m to 1, holding supply fixed at m. This setting would then be mathematically equivalent to our model with a supply shortage. Therefore, our main results carry through regardless of whether the shortages are supply driven or demand driven.

6.2 Policy implications

Given that stockpiling creates inefficiencies both in the short and long run, welfare can be improved by imposing limits on inventory accumulation. Such restrictions are not uncommon. Many countries have anti-hoarding laws in place to prevent stockpiling by businesses or households during emergencies.[21] In addition, in times of shortages retail stores often limit quantities sold per customer. These store-imposed policies are, of

[21] In the United States, for example, 50 U.S. Code § 4512 states that "[i]n order to prevent hoarding, no person shall accumulate (1) in excess of the reasonable demands of business, personal, or home consumption, or (2) for the purpose of resale at prices in excess of prevailing market prices, materials which have been designated by the President as scarce materials or materials the supply of which would be threatened by such accumulation." 50 U.S. Code § 4512 specifies a maximum fine of $10,000 and a maximum prison sentence of one year for violations.

course, not perfectly enforceable, as customers could visit more than one store, or make more than one trip to the same store. Nevertheless, to the extent that such policies can be enforced, in situations where multiple z-storage equilibria exist a limit on sales to relatively few units per household per period would result in a Pareto improvement. In particular, in cases where both a z-storage equilibrium and a z'-storage equilibrium exist, with $z' < z$, a purchasing cap of z' units is an equilibrium selection device. This implies that individual households would not "feel constrained" by the purchasing limit in equilibrium.[22] If the z'-storage rule is not an equilibrium (and no z''-storage equilibrium exists for $z'' < z'$), the purchasing cap would be experienced as binding but social welfare would still be higher than in the original z-storage equilibrium.

We note that severe in-store shortages of consumer goods are infrequent occurrences in developed market economies. As we have shown in this paper, severe in-store shortages may be *one out of many* possible equilibrium outcomes that can arise in response to small underlying supply-demand imbalances. Therefore, we conjecture that the underlying conditions that create in-store shortages could be frequent, but in only a small percentage of these instances do consumers coordinate on equilibria that involve significant in-store shortages. Our model cannot inform us why consumers coordinate on one equilibrium instead of another. However, the fact that even a small initial imbalance can tip behavior in this way has, itself, a policy implication. The true supply of a good is generally not directly observable to households, and in such cases the *mistaken* perception of even a slight shortage could trigger a switch to stockpiling strategies. One policy response is then to communicate convincingly to the public that any underlying supply-demand imbalances are negligible. For example, in the early days of the COVID-19 pandemic, Dutch prime minister Mark Rutte famously told shoppers at a grocery store that the Netherlands had sufficient toilet paper for its citizens to be able to "poop for 10 years."[23] Similarly, on March 20, 2020, Germany's federal minister for consumer protection, Julia Klöckner, posted pictures on social media that depicted her in a wholesale warehouse stocked with toilet paper and other necessities. Ms. Klöckner wrote: "There is enough for all of us!"[24] These public announcements can be interpreted as messages aimed at breaking a stockpiling equilibrium, by addressing perceptions of a severe fundamental supply shortage.

[22] Eckert *et al.* (2017) examine the use of quantity limits as equilibrium selection devices in an antitrust context. Their model, too, has the feature that the constraints will be non-binding in the equilibrium they select.

[23] See https://www.reuters.com/article/us-health-coronavirus-netherlands-toilet-idUSKBN21627A (retrieved August 28, 2023).

[24] See https://www.facebook.com/298621020195483/posts/3098208980236659 (retrieved August 28, 2023; authors' translation from German).

6.3 Open questions

Finally, our analysis of non-stationary equilibria in Section 4.3 left open the question how the transition from x^0 to some new long-run state looks like when a z-storage equilibrium does not exist. It is conceivable that the equilibrium is asymmetric, with a fraction of households using one z-storage rule and another fraction using a different z-storage rule. It is also conceivable that all households use the same z-storage rule but change the value of z over time. Yet another possibility is that the equilibrium does not involve simple decision rules like the z-storage rule, or that it does not involve convergence to a stationary state at all. A full examination of these possibilities is beyond the scope of this paper and a topic for future research.

Appendix

Proof of Proposition 1

If all households use the same z-storage rule in period t, the aggregate quantity of the good households attempt to purchase—which was defined generally in (1)—becomes

$$\theta(x^t|z) \;=\; x_z^t + 2x_{z-1}^t + 3x_{z-2}^t + \ldots + zx_1^t + zx_0^t, \tag{13}$$

and the probability that a household finds the item in period t—which was defined generally in (2)—becomes

$$p(x^t|z) \;=\; \min\left\{\frac{m}{\theta(x^t|z)}, 1\right\}. \tag{14}$$

Households that find the good in the store enter period $t+1$ with z units in storage. The remaining households enter period $t+1$ with one less unit than they had in period t; if the household already had zero units in period t they will enter period $t+1$ with zero units as well. Thus, the transition rule (3) becomes

$$x_k^{t+1} \;=\; T_k(x^t|z) \;\equiv\; \begin{cases} 0 & \text{if } k = K > z, \\ p(x^t|z) & \text{if } k = z = K, \\ (1-p(x^t|z))x_{k+1}^t + p(x^t|z) & \text{if } k = z < K, \\ (1-p(x^t|z))(x_1^t + x_0^t) & \text{if } k = 0, \\ (1-p(x^t|z))x_{k+1}^t & \text{otherwise.} \end{cases} \tag{15}$$

A stationary state of the economy is a fixed point of $T(\cdot|z) : \Delta_K \to \Delta_K$. Since T is continuous, and Δ_K is compact and convex, a fixed point x^* exists by Brouwer's Fixed Point Theorem. To characterize x^*, fix z and let $p^* \equiv p(x^*|z)$ and $\theta^* \equiv \theta(x^*|z)$. Then

(15) implies that $x_k^* = 0$ for all $k > z$ and

$$\begin{aligned}
x_z^* &= p^*, \\
x_{z-1}^* &= p^*(1-p^*), \\
x_{z-2}^* &= p^*(1-p^*)^2, \\
&\vdots \\
x_1^* &= p^*(1-p^*)^{z-1}, \\
x_0^* &= 1 - \sum_{k=1}^{z} x_k^* \;=\; 1 - p^* \sum_{k=0}^{z} (1-p^*)^k \;=\; (1-p^*)^z.
\end{aligned}$$

Thus, in the stationary state, the attempted purchase quantity, (13), becomes

$$\theta^* \;=\; \sum_{k=1}^{z} (z-k+1)p^*(1-p^*)^{z-k} \;+\; z(1-p^*)^z \;=\; \frac{1-(1-p^*)^z}{p^*} \;\geq\; 1, \tag{16}$$

and the probability that a household finds the good in the store, (14), becomes

$$p^* \;=\; \frac{m}{\theta^*} \;=\; \frac{mp^*}{1-(1-p^*)^z}. \tag{17}$$

(17) can be solved uniquely for $p^* = 1 - (1-m)^{1/z}$. □

Proof of Proposition 2

Initial remark: Note that the mapping T defined in (15) fails to be a contraction. For example, suppose $m = 1$ and $K = 5$. Consider the states

$$x = \big(0,0,0,0,0,1\big) \;\text{ and }\; y = \big(\tfrac{1}{2},0,0,0,0,\tfrac{1}{2}\big).$$

Under the standard Euclidean norm (i.e., ℓ_2), the distance between x and y is $d(x-y) = \sqrt{1/2}$. Consider now how these states change if consumers use the maximum storage rule, i.e., $z = 5$. Applying the laws of motion (13)–(15), we obtain

$$x' = T(x|5) = \big(0,0,0,0,0,1\big) \;\text{ and }\; y' = T(y|5) = \big(\tfrac{1}{3},0,0,0,\tfrac{1}{3},\tfrac{1}{3}\big).$$

This means that $d(x'-y') = \sqrt{2/3}$. Therefore, the mapping T expands the distance between certain points in Δ_K, which means that we cannot simply appeal to the Contraction Mapping Theorem to establish convergence. Instead, in the following we establish convergence "from the ground up."

Outline of proof

Take an initial state $x^0 \in \Delta_K$. For $n = 1, 2, \ldots$ define

$$x^n \equiv T^n(x^0|z) \quad \text{and} \quad p^n \equiv p(x^n),$$

where $T(\cdot|z) : \Delta_K \to \Delta_K$ and $p : \Delta_K \to [0,1]$ are defined via (13)–(15). We will construct a sequence $\underline{q}^n \to p^*$ such that $p^n \geq \underline{q}^n$ $\forall n$. We will construct a second sequence $\overline{q}^n \to p^*$ such that $p^n \leq \overline{q}^n$ $\forall n$. This implies that $p^n \to p^*$. Therefore, by definition of T in (15), we have $x^n \to x^*$.

It is sufficient to prove the result for the maximum storage rule, i.e., $z = K$. Observe that the law of motion (15) implies $z_k^n = 0$ for all $k > z$ and all $n \geq K - z$. Therefore, after at most $K - z$ iterations of $T(\cdot|z)$, we have $x^n \in \Delta_z \times \{0\}^{K-z}$, and the projection of $T(\cdot|z)$ onto Δ_z becomes

$$x_k^{n+1} = T_k(x^n|z) \equiv \begin{cases} p^n & \text{if } k = z, \\ (1-p^n)x_{k+1}^n & \text{if } 0 < k < z, \\ (1-p^n)(x_1^t + x_0^n) & \text{if } k = 0, \end{cases}$$

which is the same as (15) when $z = K$. Without loss of generality, therefore, we can restrict attention to the maximum storage rule.[25] To save on notation, for the remainder of this proof we write $T(\cdot)$ instead of $T(\cdot|z)$.

The proof is divided into a series of steps. In Step 1 we establish some preliminary results that we will apply repeatedly later on. In Step 2 we construct the sequence $\underline{q}^n$, in Step 3 we show that $\underline{q}^n \to p^*$, and in Step 4 we show that $p^n \geq \underline{q}^n$ for all n. Step 5 repeats Steps 2–4 to establish analogous results for $\overline{q}^n$. Finally, Step 6 establishes that $x^0 \succsim (\precsim)\ x^*$ implies convergence of p^n to p^* from above (below).

Step 1: Preliminaries

Define a function $f : \Delta_K \times [0,1] \to \Delta_K$ as follows:

$$f_k(x, q) = \begin{cases} q & \text{if } k = K, \\ (1-q)x_{k+1} & \text{if } 0 < k < K, \\ (1-q)(x_1 + x_0) & \text{if } k = 0. \end{cases}$$

Note that $T(x) = f(x, p(x))$. In Step 4 and Step 6, we will apply the following result:

[25] Put differently: If $z < K$, we can redefine $K := z$ and proceed with proving the result for $z = K$, thus redefined.

Lemma 11.

(a) If $x \succsim x'$ *then* $p(x) \geq p(x')$.

(b) If $x \succsim x'$ *then* $f(x,q) \succsim f(x',q)$ *for all* $q \in [0,1]$.

(c) If $q \geq q'$ *then* $f(x,q) \succsim f(x,q')$ *for all* $x \in \Delta_K$.

Proof. Part (a) is readily apparent from (13)–(14). To show part (b), suppose $x \succsim x'$, that is,

$$\sum_{s=0}^{k} x_s \;\leq\; \sum_{s=0}^{k} x'_s \quad \forall k = 0, \ldots, K.$$

Fix $q \in [0,1]$ and let $y = f(x,q)$ and $y' = f(x',q)$. Then we have

$$\sum_{s=0}^{k} y_s \;=\; (1-q)\sum_{s=0}^{k+1} x_s \;\leq\; (1-q)\sum_{s=0}^{k+1} x'_s \;=\; \sum_{s=0}^{k} y'_s$$

for all $k = 0, \ldots, K-1$, and $\sum_{s=0}^{K} y_s = 1 = \sum_{s=0}^{K} y'_s$. It follows that $y \succsim y'$. Finally, to show part (c), suppose $q \geq q'$. Fix $x \in \Delta_K$ and let $y = f(x,q)$ and $y' = f(x,q')$. Then we have

$$\sum_{s=0}^{k} y_s \;=\; (1-q)\sum_{s=0}^{k+1} x_s \;\leq\; (1-q')\sum_{s=0}^{k+1} x_s \;=\; \sum_{s=0}^{k} y'_s$$

for all $k = 0, \ldots, K-1$, and $\sum_{s=0}^{K} y_s = 1 = \sum_{s=0}^{K} y'_s$. It follows that $y \succsim y'$. □

Step 2: Construction of the sequence $\underline{q}^n$

Associated with the sequence $\underline{q}^n$ will be a sequence of states, $\underline{x}^n \in \Delta_K$, defined through $\underline{x}^0 = (1, 0, \ldots, 0)$ and $\underline{x}^{n+1} = f(\underline{x}^n, \underline{q}^n)$. For each n, define $\underline{p}^n = p(\underline{x}^n)$. Note that $\underline{p}^0 = m/K$.

We build the sequence $\underline{q}^n$ in pieces of K elements at a time. We begin by setting the first K values of $\underline{q}^n$ to

$$\underline{q}^0, \ldots, \underline{q}^{K-1} \;=\; \underline{p}^0.$$

Given the definition of f, in period K we have

$$\underline{x}^K \;=\; \left((1-\underline{p}^0)^K, (1-\underline{p}^0)^{K-1}\underline{p}^0, (1-\underline{p}^0)^{K-2}\underline{p}^0, \ldots, (1-\underline{p}^0)\underline{p}^0, \underline{p}^0\right).$$

Using the same formulas as in (16)–(17), we can write

$$\underline{p}^K \;=\; \frac{m\underline{p}^0}{1-(1-\underline{p}^0)^K}.$$

We then set the next K values of $\underline{q}^n$ to

$$\underline{q}^K, \ldots, \underline{q}^{2K-1} \;=\; \underline{p}^K.$$

Therefore, in period $2K$ we have

$$\underline{x}^{2K} \;=\; \left((1-\underline{p}^K)^K, (1-\underline{p}^K)^{K-1}\underline{p}^K, (1-\underline{p}^K)^{K-2}\underline{p}^K, \ldots, (1-\underline{p}^K)\underline{p}^K, \underline{p}^K\right)$$

and

$$\underline{p}^{2K} \;=\; \frac{m\underline{p}^K}{1-(1-\underline{p}^K)^K},$$

and we set the next K values of $\underline{q}^n$ to $\underline{q}^{2K}, \ldots, \underline{q}^{3K-1} = \underline{p}^{2K}$. Proceeding in the same fashion for $\ell = 3, 4, \ldots$, we have

$$\underline{q}^{\ell K}, \ldots, \underline{q}^{\ell K+(K-1)} \;=\; \underline{p}^{\ell K} \;=\; \frac{m\underline{p}^{(\ell-1)K}}{1-(1-\underline{p}^{(\ell-1)K})^K}. \tag{18}$$

Step 3: $\underline{q}^n \to p^*$ as $n \to \infty$

Denote the function on the right-hand side of (18) by

$$A(p) \;=\; \frac{mp}{1-(1-p)^K}.$$

The unique fixed point of $A : [0,1] \to [0,1]$ is $p^* = 1-(1-m)^{1/K}$. We will show that $0 < A'(p) < 1$ for all $p \in (0,1)$. This implies that $\underline{p}^{\ell K} \to p^*$ as $\ell \to \infty$. Since $\underline{q}^{\ell K} = \ldots = \underline{q}^{(\ell+1)K-1} = \underline{p}^{\ell K}$, it follows that $\underline{q}^n \to p^*$ as $n \to \infty$.

Note that

$$\begin{aligned}
A'(p) \;&=\; m\frac{1-(1-p)^K - pK(1-p)^{K-1}}{\left(1-(1-p)^K\right)^2} \;<\; 1 \\
&\Leftarrow\; \frac{1-(1-p)^K - pK(1-p)^{K-1}}{\left(1-(1-p)^K\right)^2} \;<\; 1 \\
&\Leftrightarrow\; K \;>\; \frac{(1-p)-(1-p)^{K+1}}{p} \;=\; \sum_{k=1}^{K}(1-p)^k,
\end{aligned}$$

which is true if $p \in (0,1)$. Likewise, note that

$$\begin{aligned}
A'(p) \;&=\; m\frac{1-(1-p)^K - pK(1-p)^{K-1}}{\left(1-(1-p)^K\right)^2} \;>\; 0 \\
&\Leftarrow\; \frac{1-(1-p)^K - pK(1-p)^{K-1}}{\left(1-(1-p)^K\right)^2} \;>\; 0
\end{aligned}$$

$$\Leftrightarrow \quad K \; < \; \frac{1-(1-p)^K}{p(1-p)^{K-1}} \; = \; \sum_{k=0}^{K-1} \frac{(1-p)^k}{(1-p)^{K-1}},$$

which, too, is true if $p \in (0,1)$. Therefore $0 < A'(p) < 1 \ \forall p \in (0,1)$.

Step 4: $p^n \geq \underline{q}^n$ for all n

It will be convenient to construct a sequence $\underline{\underline{x}}^n \in \Delta_L$ as follows: $\underline{\underline{x}}^0 = (1,0,\ldots,0)$ and $\underline{\underline{x}}^{n+1} = T(\underline{\underline{x}}^n)$. Also define $\underline{\underline{p}}^n = p(\underline{\underline{x}}^n)$. Note that $\underline{\underline{x}}^1 \succsim \underline{\underline{x}}^0$ necessarily; then using Lemma 11 (a)–(c) we can write

$$\underline{\underline{x}}^2 = T(\underline{\underline{x}}^1) = f(\underline{\underline{x}}^1, \underline{\underline{p}}^1) \succsim f(\underline{\underline{x}}^1, \underline{\underline{p}}^0) \succsim f(\underline{\underline{x}}^0, \underline{\underline{p}}^0) = T(\underline{\underline{x}}^0) = \underline{\underline{x}}^1,$$

$$\underline{\underline{x}}^3 = T(\underline{\underline{x}}^2) = f(\underline{\underline{x}}^2, \underline{\underline{p}}^2) \succsim f(\underline{\underline{x}}^2, \underline{\underline{p}}^1) \succsim f(\underline{\underline{x}}^1, \underline{\underline{p}}^1) = T(\underline{\underline{x}}^1) = \underline{\underline{x}}^2,$$

and so on. Therefore $\underline{\underline{x}}^n$ is increasing in the sense that $\underline{\underline{x}}^{n+1} \succsim \underline{\underline{x}}^n \ \forall n$. This implies that $\underline{\underline{p}}^{n+1} \geq \underline{\underline{p}}^n \ \forall n$.

We will show that $p^n \geq \underline{\underline{p}}^n \geq \underline{p}^n \geq \underline{q}^n$ for all n. All four sequences are illustrated in Figure 6 below. (The same figure also shows the corresponding sequence that will bound p^n from above; see Step 5.)

First, to show that $p^n \geq \underline{\underline{p}}^n \ \forall n$, note that $x^0 \succsim \underline{\underline{x}}^0$ necessarily. Thus, using Lemma 11 (a)–(c), we have

$$x^1 = T(x^0) = f(x^0, p(x^0)) \succsim f(x^0, \underline{\underline{p}}^0) \succsim f(\underline{\underline{x}}^0, \underline{\underline{p}}^0) = T(\underline{\underline{x}}^0) = \underline{\underline{x}}^1,$$

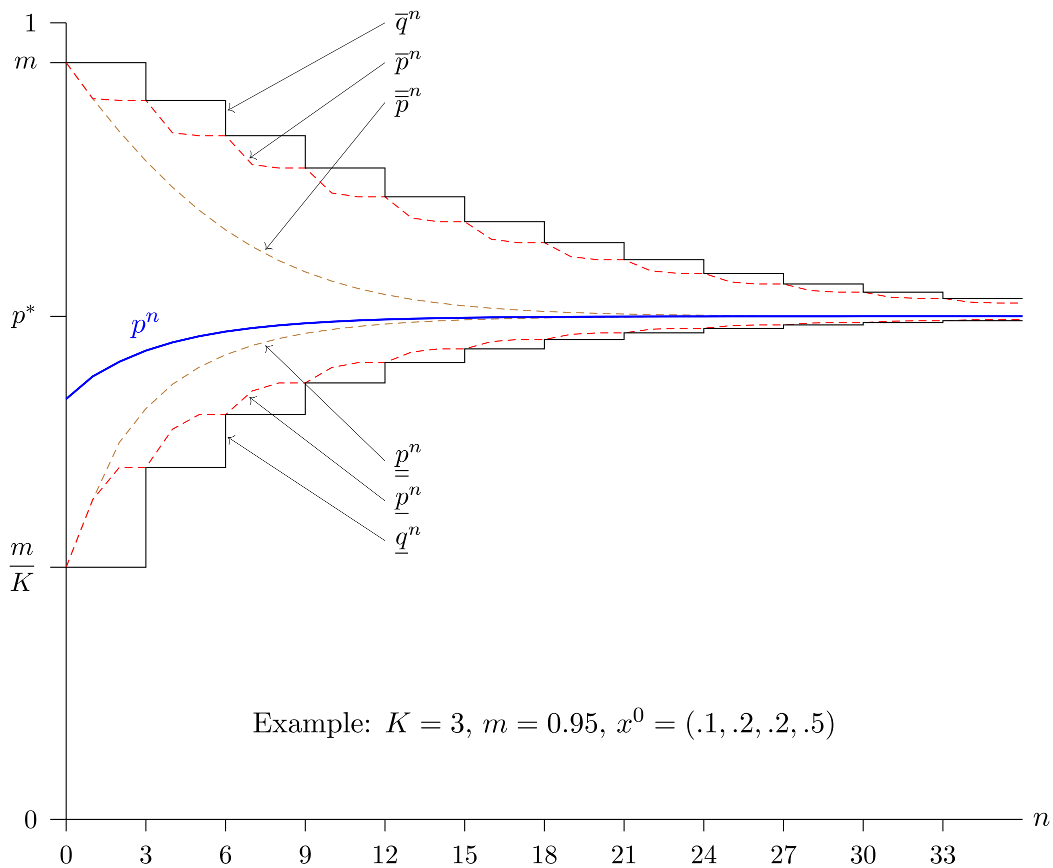
$$x^2 = T(x^1) = f(x^1, p(x^1)) \succsim f(x^1, \underline{\underline{p}}^1) \succsim f(\underline{\underline{x}}^1, \underline{\underline{p}}^1) = T(\underline{\underline{x}}^1) = \underline{\underline{x}}^2,$$

and so on. It follows that $x^n \succsim \underline{\underline{x}}^n \ \forall n$. By Lemma 11 (a), this implies $p^n \geq \underline{\underline{p}}^n \ \forall n$.

Second, to show that $\underline{\underline{p}}^n \geq \underline{p}^n \ \forall n$, observe that $\underline{\underline{x}}^0 = \underline{x}^0$ implies $\underline{\underline{p}}^{\overline{0}} = \underline{p}^0$ $(= \underline{q}^0, \ldots, \underline{q}^{K-1})$. Using Lemma 11 (a)–(c) and the fact that $\underline{\underline{x}}^n$ is increasing, it follows that

$$\begin{aligned}
\underline{\underline{x}}^1 = T(\underline{\underline{x}}^0) &= f(\underline{\underline{x}}^0, \underline{\underline{p}}^0) \\
&= f(\underline{\underline{x}}^0, \underline{p}^0) = f(\underline{x}^1, \underline{p}^0) = \underline{x}^1, \\
\underline{\underline{x}}^2 = T(\underline{\underline{x}}^1) &= f(\underline{\underline{x}}^1, \underline{\underline{p}}^1) \succsim f(\underline{\underline{x}}^1, \underline{\underline{p}}^0) \\
&= f(\underline{\underline{x}}^1, \underline{p}^0) \succsim f(\underline{x}^1, \underline{p}^0) = \underline{x}^2, \\
&\vdots \\
\underline{\underline{x}}^K = T(\underline{\underline{x}}^{K-1}) &= f(\underline{\underline{x}}^{K-1}, \underline{\underline{p}}^{K-1})) \succsim f(\underline{\underline{x}}^{K-1}, \underline{\underline{p}}^0) \\
&= f(\underline{\underline{x}}^{K-1}, \underline{p}^0) \succsim f(\underline{x}^{K-1}, \underline{p}^0) = \underline{x}^K.
\end{aligned}$$

Figure 6: Illustration of the proof of Proposition 2.



This implies that $\underline{\underline{p}}^K \geq \underline{p}^K$ $(= \underline{q}^K, \dots, \underline{q}^{2K-1})$. Using Lemma 11 (a)–(c) and the fact that $\underline{\underline{x}}^n$ is increasing, it follows that

$$\begin{aligned}
\underline{\underline{x}}^{K+1} &= T(\underline{\underline{x}}^K) = f(\underline{\underline{x}}^K, \underline{\underline{p}}^K) \\
&\quad \succsim f(\underline{\underline{x}}^K, \underline{p}^K) \succsim f(\underline{x}^K, \underline{p}^K) = \underline{x}^{K+1}, \\
\underline{\underline{x}}^{K+2} &= T(\underline{\underline{x}}^{K+1}) = f(\underline{\underline{x}}^{K+1}, \underline{\underline{p}}^{K+1}) \succsim f(\underline{\underline{x}}^{K+1}, \underline{\underline{p}}^K) \\
&\quad \succsim f(\underline{\underline{x}}^{K+1}, \underline{p}^K) \succsim f(\underline{x}^{K+1}, \underline{p}^K) = \underline{x}^{K+2}, \\
&\vdots \\
\underline{\underline{x}}^{2K} &= T(\underline{\underline{x}}^{2K-1}) = f(\underline{\underline{x}}^{2K-1}, \underline{\underline{p}}^{2K-1})) \succsim f(\underline{\underline{x}}^{2K-1}, \underline{\underline{p}}^K)) \\
&\quad \succsim f(\underline{\underline{x}}^{2K-1}, \underline{p}^K) \succsim f(\underline{x}^{2K-1}, \underline{p}^K) = \underline{x}^{2K}.
\end{aligned}$$

Proceeding in the same fashion (i.e., in blocks of K elements at a time) we can establish that $\underline{\underline{x}}^n \succsim \underline{x}^n \; \forall n$. By Lemma 11 (a), this implies $\underline{\underline{p}}^n \geq \underline{p}^n \; \forall n$.

Finally, we show $\underline{p}^n \geq \underline{q}^n \; \forall n$. As before, we proceed in blocks of K elements at a time. Note that $\underline{q}^0, \ldots, \underline{q}^{K-1} = \underline{p}^0$. Furthermore $\underline{x}^1 \succsim \underline{x}^0$ necessarily. Using Lemma 11 (b)–(c), we have

$$
\begin{aligned}
\underline{x}^2 &= f(\underline{x}^1, \underline{q}^1) \succsim f(\underline{x}^0, \underline{q}^1) = f(\underline{x}^0, \underline{p}^0) = \underline{x}^1, \\
&\vdots \\
\underline{x}^K &= f(\underline{x}^{K-1}, \underline{q}^{K-1}) \succsim f(\underline{x}^{K-2}, \underline{q}^{K-1})) = f(\underline{x}^{K-2}, \underline{p}^0) = \underline{x}^{K-1}.
\end{aligned}
$$

This implies that $\underline{x}^{K-1} \succsim \ldots \succsim \underline{x}^0$, and hence $\underline{p}^{K-1} \geq \ldots \geq \underline{p}^0 = \underline{q}^0, \ldots, \underline{q}^{K-1}$. Next, note that $\underline{q}^K, \ldots, \underline{q}^{2K} = \underline{p}^K$. Furthermore, by (18) and Step 3 we have $\underline{p}^K = A(\underline{p}^0) > \underline{p}^0$, and Lemma 11 (b)–(c) implies

$$
\underline{x}^{K+1} = f(\underline{x}^K, \underline{q}^K) \succsim f(\underline{x}^{K-1}, \underline{q}^K) \succsim f(\underline{x}^{K-1}, \underline{p}^0) = \underline{x}^K.
$$

Using Lemma 11 (b)–(c) again, we then have

$$
\begin{aligned}
\underline{x}^{K+2} &= f(\underline{x}^{K+1}, \underline{q}^{K+1}) \succsim f(\underline{x}^K, \underline{q}^{K+1}) = f(\underline{x}^K, \underline{p}^K) = \underline{x}^{K+1}, \\
&\vdots \\
\underline{x}^{2K} &= f(\underline{x}^{2K-1}, \underline{q}^{2K-1}) \succsim f(\underline{x}^{2K-2}, \underline{q}^{2K-1})) = f(\underline{x}^{K-2}, \underline{p}^K) = \underline{x}^{2K-1}.
\end{aligned}
$$

This implies that $\underline{x}^{2K-1} \succsim \ldots \succsim \underline{x}^K$, and hence $\underline{p}^{2K-1} \geq \ldots \geq \underline{p}^K = \underline{q}^K, \ldots, \underline{q}^{2K-1}$. Proceeding in the same fashion, we can establish that $\underline{p}^n \succsim \underline{q}^n \; \forall n$.

Step 5: Construction of $\overline{q}^n \to p^*$ such that $p^n \leq \overline{q}^n$ for all n

This step is analogous to Steps 2–4. In Step 2, the sequences $\underline{x}^n$, $\underline{p}^n$, $\underline{q}^n$ are replaced with $\overline{x}^n$, $\overline{p}^n$, $\overline{q}^n$, where $\overline{x}^0 = (0, \ldots, 0, 1)$, $\overline{x}^{n+1} = f(\overline{x}^n, \overline{q}^n)$, $\overline{p}^n = p(\overline{x}^n)$,

$$
\begin{aligned}
\overline{q}^0, \ldots, \overline{q}^{K-1} &= \overline{p}^0, \\
\overline{q}^K, \ldots, \overline{q}^{2K-1} &= \overline{p}^K, \\
\overline{q}^{2K}, \ldots, \overline{q}^{3K-1} &= \overline{p}^{2K},
\end{aligned}
$$

and so on. Because we can write $\overline{p}^{\ell K} = A(p^{(\ell-1)K})$, where A was defined via the right-hand side of (18), we can apply the previous Step 3 to establish that $\overline{q}^n \to p^*$. In Step 4, the sequences $\underline{\underline{x}}^n$ and $\underline{\underline{p}}^n$ are replaced with $\overline{\overline{x}}^n$ and $\overline{\overline{p}}^n$, where $\overline{\overline{x}}^0 = (0, \ldots, 0, 1)$, $\overline{\overline{x}}^{n+1} = T(\overline{\overline{x}}^n)$ and $\overline{\overline{p}}^n = p(\overline{\overline{x}}^n)$. One can then show that $p^n \leq \overline{\overline{p}}^n \leq \overline{p}^n \leq \overline{q}^n$ for all n.

Step 6: $x^0 \succsim (\precsim)\, x^*$ implies $p^n \to p^*$ from above (below)

Suppose $x^0 \succsim x^*$. Using Lemma 11 (a)–(c), we have

$$x^1 = T(x^0) = f(x^0, p(x^0)) \succsim f(x^0, p(x^*)) = f(x^0, p^*) \succsim f(x^*, p^*) = T(x^*) = x^*,$$
$$x^2 = T(x^1) = f(x^1, p(x^1)) \succsim f(x^1, p(x^*)) = f(x^0, p^*) \succsim f(x^*, p^*) = T(x^*) = x^*,$$

and so on. It follows that $x^n \succsim x^*\ \forall n$. By Lemma 11 (a) this implies $p^n \geq p^*\ \forall n$. The argument when $x^0 \precsim x^*$ is analogous. □

Proof of Proposition 3

Existence of an equilibrium follows from our Proposition 5. We need to show that the equilibrium is unique and consists of the use of the minimum storage strategy in every period.

Recall that the assumption $\lambda < \beta$ ensures that each household wants to store at least one unit in every period. Furthermore, if $\lambda > \beta^2/(1+\beta)$ then no household wants to store more than one unit in every period.[26] In this case, the 1-storage rule is strictly dominant in every period, and the proof is complete. Thus, assume $\lambda \leq \beta^2/(1+\beta)$.

Consider the value function in (4) and note that $V_0^t - V_1^t = \lambda - 1 < 0\ \forall t$. Furthermore, for $k \geq 2$,

$$V_{k-1}^t - V_k^t \;=\; \lambda + \beta\Big(p^t \overbrace{\Big[\ \max\nolimits_{s\geq k-2} V_s^{t+1} - \max\nolimits_{s\geq k-1} V_s^{t+1}\Big]}^{\geq 0} + (1-p^t)\Big[V_{k-2}^{t+1} - V_{k-1}^{t+1}\Big]\Big).$$

Proceeding recursively, we can write

$$\begin{aligned} V_{k-1}^t - V_k^t \;&\geq\; \lambda + \beta(1-p^t)\Big[V_{k-2}^{t+1} - V_{k-1}^{t+1}\Big] \\ &\geq\; \lambda + \beta(1-p^t)\Big[\lambda + \beta(1-p^{t+1})\Big[V_{k-3}^{t+2} - V_{k-2}^{t+2}\Big]\Big] \\ &\;\;\vdots \end{aligned}$$

[26]The marginal cost of storing the second unit is $\lambda + \beta\lambda$ (as the unit would be stored for two periods) and the marginal benefit is β^2 (as the unit would be consumed after it was stored for two periods); thus, if $\lambda > \beta^2/(1+\beta)$ it is not optimal to store more than one unit.

$$
\begin{aligned}
&\geq \; \lambda \left[1 + \sum_{k'=1}^{k-2} \beta^{k'} \prod_{s=0}^{k'-1} (1 - p^{t+s})\right] \\
&\qquad\qquad + \; \beta^{k-1} \prod_{s=0}^{k-2} (1 - p^{t+s}) \underbrace{\left[V_0^{t+k-1} - V_1^{t+k-1}\right]}_{= \lambda - 1} \qquad (19) \\
&= \; \lambda \left[1 + \sum_{k'=1}^{k-1} \beta^{k'} \prod_{s=0}^{k'-1} (1 - p^{t+s})\right] - \beta^{k-1} \prod_{s=0}^{k-2} (1 - p^{t+s}) \\
&\equiv \; D_k(\mathbf{p}^t),
\end{aligned}
$$

where $\mathbf{p}^t$ denotes the sequence $(p^t, p^{t+1}, p^{t+2}, \ldots)$. From (19) it is apparent that

$$
D_K(\mathbf{p}^t) \; \geq \; D_{K-1}(\mathbf{p}^t) \; \geq \; \ldots \; \geq D_2(\mathbf{p}^t) \; \geq \; D_1(\mathbf{p}^t) \; = \; \lambda - 1. \qquad (20)
$$

Now fix period $t > 0$ and consider *any* sequence of probabilities $p^t, p^{t+1}, \ldots$. Suppose that

$$
D_2(\mathbf{p}^t) > 0 \quad \Leftrightarrow \quad p^t > \delta \equiv 1 - \frac{\lambda}{\beta(1-\lambda)} \in (0,1)
$$

(where $\delta \in (0,1)$ follows from $0 < \lambda \leq \beta^2/(1+\beta) < \beta/(1+\beta)$). Together with the preceding arguments, this implies that $V_1^t > V_2^t > \ldots > V_K^t$ and $V_1^t > V_0^t$. Thus, every household strictly prefers to enter period t with one unit of the good in storage, which implies that the 1-storage rule becomes strictly optimal in period $t-1$. If every household applies the 1-storage rule in period $t-1$, the demand for the good in period $t-1$ is $\theta^{t-1} \leq 1$. Since $m = 1$, this implies $p^{t-1} = 1 > \delta$, which makes it strictly optimal for every household to apply the minimum storage rule in period $t-2$; and so on. Thus, if $p^t > \delta$ for some t, then every household follows the minimum storage rule in every period $t' < t$.

It follows that, in any equilibrium in which some household does *not* follow the minimum storage rule in every period, there must exist t^* such that $p^t \leq \delta \; \forall t \geq t^*$. Suppose this is the case. Then, in any period $t \geq t^* + K$, the fraction of households who have experienced K or more in-store shortages in a row, and hence enter period t with a zero quantity in storage, is

$$
(1 - p^{t-1})(1 - p^{t-2}) \cdots (1 - p^{t-K}) \; \geq \; (1 - \delta)^K \; > \; 0.
$$

Since households with a zero inventory cannot consume; starting in period $t^* + K$ at most a measure $1 - (1-\delta)^K$ of the good is being consumed in each period. Since $p^t \leq \delta < 1$ $\forall t \geq t^*$, the store sells the entire supply in period t (i.e., the does not dispose of any excess supply at the end of period t; if it did then $p^t = 1$). Because households cannot resell or dispose of the good, the unused amount must end up in storage. Because each household can store at most K units, at the latest in period $t^{**} = t^* + K + \lfloor K/(1-\delta)^K \rfloor$ every household must have K units in storage. But this means that $\theta^{t^{**}} \leq 1$ which implies $p^{t^{**}} = 1$, a contradiction. It follows that there cannot exist an equilibrium in which some household does not follow the minimum storage rule in every period. □

Remark: Recall that we defined equilibrium in a way that assumes all households use a common decision rule σ^t in a given period. However, the argument presented above is based solely on the sequence of probabilities p^t with which a household is able to buy the item in period t, but does not depend on the assumption that these probabilites are derived from a common decision rule applied by all households in period t. Thus, even if we extended the equilibrium definition to include the possibility that different households apply different decision rules, the current Proposition 3 would remain true.

Proof of Lemma 4

In the following, we continue to use the notation introduced in the proof of Proposition 3. Suppose that all households use the same z-storage rule ($z \in \{1, \dots, K\}$) from period t onward, and that this rule is optimal from period t onward. Then, for $k \leq z+1$, inequality (19) becomes an equality:

$$V_{k-1}^t - V_k^t \;=\; D_k(\mathbf{p}_z^t) \quad \text{for all} \quad k \leq z+1. \tag{21}$$

where $\mathbf{p}_z^t$ is the sequence of probabilities $(p_z^t, p_z^{t+1}, \dots)$ that arises when all households use the z-storage rule, given initial state x^0. Suppose

$$D_z(\mathbf{p}_z^t) \;\leq\; 0 \quad \text{and} \quad D_{z+1}(\mathbf{p}_z^t) \;\geq\; 0. \tag{22}$$

Then (20), (21), and (22) together imply that

$$V_0^t \;\leq\; V_1^t \;\leq\; \dots \;\leq\; V_z^t \quad \text{and} \quad V_z^t \;\geq\; V_{z+1}^t \;\geq\; \dots \;\geq\; V_K^t.$$

This means that the z-storage rule is also optimal in period $t-1$.

Therefore, given $z \in \{1, \dots, K\}$ we can verify if a z-storage equilibrium exists in the following way: Fix x^0 and set $x^{t+1} = T(x^t|z)$ for all $t \geq 0$. Let $(p^0, p^1, p^2, \dots)$ be the associated sequence of probabilities of finding the item in the store. If (22) holds for all $t \geq 1$, then using the z-storage rule in every period is optimal for a household if all other households do the same. If we now define $\overline{\lambda}_z(\mathbf{p}_z^t)$ as in (5), condition (22) is equivalent to

$$\overline{\lambda}_{z+1}(\mathbf{p}_z^t) \;\leq\; \lambda \;\leq\; \overline{\lambda}_z(\mathbf{p}_z^t),$$

which is the condition in the statement of 4 in the text. □

Proof of Proposition 5

For $z < K$, let $L^*(z)$ be equal to the left-hand side of (7), and set $L^*(K) = \beta$. Similarly, for $z > 1$ let $U^*(z)$ be equal to the right-hand side of (7), and set $U^*(1) = \beta$. A stationary z-storage equilibrium exists if $\lambda \in [L^*(z), U^*(z)]$ for some $z \in \{1, \ldots, K\}$.

The argument is in three parts:

(i) $L^*(z) \leq U^*(z)$ $\forall z = 1, \ldots, K$. Therefore, the interval $[L^*(z), U^*(z)]$ is well-defined and non-empty for each z.

(ii) $L^*(z+1) \leq L^*(z)$ and $U^*(z+1) \leq U^*(z)$ $\forall z = 1, \ldots, K-1$. Therefore, the interval $[L^*(z+1), U^*(z+1)]$ is "below" the interval $[L^*(z), U^*(z)]$.

(iii) $L^*(z) \leq U^*(z+1)$ $\forall z = 1, \ldots, K-1$. Therefore, $\cup_{z=1,\ldots,K}[L^*(z), U^*(z)] = [0, \beta]$, which implies $\lambda \in [L^*(z), U^*(z)]$ for at least one $z \in \{1, \ldots, K\}$.

Part (i) is immediate from (7), noting that $\gamma(z) \in (0,1)$. Part (ii) follows from Lemma 9, noting that $L^*(z) = \overline{\lambda}_{z+1}\big((1-m)^{1/z}, (1-m)^{1/z}, \ldots\big)$ and $U^*(z) = \overline{\lambda}_z\big((1-m)^{1/z}, (1-m)^{1/z}, \ldots\big)$, and $(1-m)^{1/z}$ increases in z. For part (iii), note that

$$\begin{aligned} L^*(z) \;\leq\; U^*(z+1) \quad &\Leftrightarrow \quad \frac{\gamma(z)^z\big(1-\gamma(z)\big)}{1-\gamma(z)^{z+1}} \;\leq\; \frac{\gamma(z+1)^z\big(1-\gamma(z+1)\big)}{1-\gamma(z+1)^{z+1}} \\ &\Leftrightarrow \quad \frac{1}{\gamma(z+1)^z}\frac{1-\gamma(z+1)^{z+1}}{1-\gamma(z+1)} \;\leq\; \frac{1}{\gamma(z)^z}\frac{1-\gamma(z)^{z+1}}{1-\gamma(z)} \\ &\Leftrightarrow \quad \frac{1}{\gamma(z+1)^z}\sum_{k=0}^{z}\gamma(z+1)^k \;\leq\; \frac{1}{\gamma(z)^z}\sum_{k=0}^{z}\gamma(z)^k \\ &\Leftrightarrow \quad \sum_{k=0}^{z}\gamma(z+1)^{-k} \;\leq\; \sum_{k=0}^{z}\gamma(z)^{-k}, \end{aligned}$$

which is true since $\gamma(z+1) \leq \gamma(z) \leq 1$. □

Proof of Proposition 6

By Proposition 7 (which does not depend on this result), we only need to verify the conditions for which the minimum and maximum storage equilibria exist at the same time. Using (7), the minimum storage equilibrium exists if

$$\lambda \;\geq\; \frac{\gamma(1)\big(1-\gamma(1)\big)}{1-\gamma(1)^2} \;=\; \frac{\beta(1-m)\big(1-\beta(1-m)\big)}{1-\beta^2(1-m)^2} \;>\; 0,$$

and the maximum storage equilibrum exists if

$$\lambda \;\leq\; \frac{\gamma(K)^{K-1}\big(1-\gamma(K)\big)}{1-\gamma(K)^K} \;=\; \frac{\beta^{K-1}(1-m)^{(K-1)/K}\big(1-\beta(1-m)^{1/K}\big)}{1-\beta^K(1-m)} \;<\; 1.$$

Thus, both equilibria coexist as long as

$$\frac{\beta(1-m)\big(1-\beta(1-m)\big)}{1-\beta^2(1-m)^2} \;\leq\; \lambda \;\leq\; \frac{\beta^{K-1}(1-m)^{(K-1)/K}\big(1-\beta(1-m)^{1/K}\big)}{1-\beta^K(1-m)}. \tag{23}$$

We need to show that, if m is sufficiently large (but smaller than one), the left-hand side of (23) is strictly smaller than the right-hand side. After rearranging, this condition becomes

$$\frac{1-\beta^K(1-m)}{1-\beta^2(1-m)^2} \;<\; \frac{\beta^{K-2}\big(1-\beta(1-m)^{1/K}\big)}{(1-m)^{1/K}\big(1-\beta(1-m)\big)}. \tag{24}$$

As $m \to 1$ from below, the left-hand side in (24) converges to 1 and the right-hand side converges to $+\infty$. Thus, for fixed K and b, there exists $\overline{m} < 1$ such that (24) holds for all $\overline{m} < m < 1$. (The open interval Λ is then defined by replacing the weak inequalities in (23) with strict inequalities.) □

Proof of Proposition 7

The result follows from property (ii) in the proof of Proposition 5. Suppose that a stationary z-storage equilibrium exists and that a z' stationary storage equilibrium exists, with $z < z'$. Let z'' be an integer such that $z < z'' < z'$. Because a stationary z-storage equilibrium exists, $L^*(z) \leq \lambda \leq U^*(z)$; similarly, because a stationary z'-storage equilibrium exists, $L^*(z') \leq \lambda \leq U^*(z')$. Since $z'' > z$, by property (ii) we have $L^*(z'') \leq L^*(z) \leq \lambda$. Simmilarly, since $z'' < z'$, by property (ii) we have $U^*(z'') \geq U^*(z') \leq \lambda$. Therefore, $L^*(z'') \leq \lambda \leq U^*(z'')$, which means that a stationary z''-storage equilibrium exists. □

Proof of Lemma 8

Fix $z \in \{1, \dots, K\}$ and $x^0 = (0, 1, 0, \dots, 0)$, and let x^1 be defined as in the text. Note that, from (13)–(14),

$$p_z^0 \;=\; \frac{m}{z} \;\leq\; \frac{m}{z-(m/z)(z-1)} \;=\; \frac{m}{m/z + z\,(1-m/z)} \;=\; p_z^1.$$

We will show that $x^* \succsim x^1$. Proposition 2 then implies that $p_z^1 \leq p_z^2 \leq \dots \to p^* = 1-(1-m)^{1/z}$, and the result follows.

For $k = 0, \dots, z-1$, we have

$$\sum_{s=0}^{k} x_s^* \;\leq\; \sum_{s=0}^{z-1} x_s^* \;=\; (1-m)^{1/z} \quad \text{and} \quad \sum_{s=0}^{k} x_s^1 \;=\; 1-\frac{m}{z}.$$

Thus, we need to show that

$$(1-m)^{1/z} \;\leq\; 1-\frac{m}{z}. \tag{25}$$

If $m = 0$, then (25) holds as an equality. Therefore, it is sufficient to show that

$$\frac{\partial}{\partial m}\left[(1-m)^{1/z}\right] \;=\; -(1-m)^{1/z-1} \;\leq\; -\frac{1}{z} \;=\; \frac{\partial}{\partial m}\left[1-\frac{m}{z}\right]. \tag{26}$$

The left-hand side of (26) decreases in m, and the right-hand side is indepenent of m. Therefore, it is sufficient that (26) holds at $m = 0$. This is satisfied, as $-1 \leq -1/z$. □

Proof of Lemma 9

Fix $z \in \{2, \ldots, K\}$ and fix t. Suppose $\hat{p}^{t+s} \leq p^{t+s}$ $\forall s = 0, \ldots, K-2$. For $k = 0, \ldots, z-2$ define

$$\alpha^k \;=\; \prod_{s=0}^{k} \frac{1-\hat{p}^{t+s}}{1-p^{t+s}}.$$

Note that $\alpha^{z-2} \geq \ldots \geq \alpha^0 \geq 1$. Therefore,

$$\overline{\lambda}(\hat{\mathbf{p}}^t) \;=\; \frac{\beta^{z-1}\prod_{s=0}^{z-2}(1-\hat{p}^{t+s})}{1+\sum_{k=1}^{z-1}\beta^k\prod_{s=0}^{k-1}(1-\hat{p}^{t+s})} \;=\; \frac{\beta^{z-1}\alpha^{z-2}\prod_{s=0}^{z-2}(1-p^{t+s})}{1+\sum_{k=1}^{z-1}\beta^k\alpha^{k-1}\prod_{s=0}^{k-1}(1-p^{t+s})},$$

$$\geq\; \frac{\beta^{z-1}\alpha^{z-2}\prod_{s=0}^{z-2}(1-p^{t+s})}{\alpha^{z-2}+\sum_{k=1}^{K-1}\beta^k\alpha^{z-2}\prod_{s=0}^{k-1}(1-p^{t+s})} \;=\; \frac{\beta^{z-1}\prod_{s=0}^{z-2}(1-p^{t+^s})}{1+\sum_{k=1}^{z-1}\beta^k\prod_{s=0}^{k-1}(1-p^{t+s})} \;=\; \overline{\lambda}(\mathbf{p}^t). \quad \square$$

References

[1] Allaz, B. (1991), "Duopoly, Inventories, and Futures Markets," In: L. Phlips (ed.), *Commodity, Futures, and Financial Markets. Advanced Studies in Theoretical and Applied Econometrics* 21, Springer.

[2] Awaya, Y. and V. Krishna (2024), "Panics and Prices," *Journal of Economic Theory*, 105818.

[3] Bell, D., G. Iyer, and V. Padmanabhan (2002), "Price Competition under Stockpiling and Flexible Consumption," *Journal of Marketing Research* 39 (3), 292–303.

[4] Berentsen, A. (2000), "Money Inventories in Search Equilibrium," *Journal of Money, Credit, and Banking* 32 (2), 168–178.

[5] Ching, A. and M. Osborne (2020), "Identification and Estimation of Forward-Looking Behavior," *Marketing Science* 39 (4), 707–726.

[6] Eckert, A., T. Klumpp, and X. Su (2017), "An Equilibrium Selection Theory of Monopolization," *Southern Economic Journal* 83 (4), 1012–1037.

[7] Gangwar, M., N. Kumar, and R. Rao (2013), "Consumer Stockpiling and Competitive Promotional Strategies," *Marketing Science* 33 (1), 94–113.

[8] Guo, L. and M. Villas-Boas (2007), "Consumer Stockpiling and Price Competition in Differentiated Markets," *Journal of Economics & Management Strategy* 16 (4), 827–858.

[9] Hendel, I. and A. Nevo (2006a), "Measuring the Implications of Sales and Consumer Inventory Behavior," *Econometrica* 74 (6), 1637–1673.

[10] Hendel, I. and A. Nevo (2006b), "Sales and Consumer Inventory," *The RAND Journal of Economics* 37 (3), 543–561.

[11] Hirshleifer, D. (1989), "Futures Trading, Storage, and the Division of Risk: A Multiperiod Analysis," *The Economic Journal* 99,(397)700–719.

[12] Hong, P., P. McAfee, and A. Nayyar (2002), "Equilibrium Price Dispersion with Consumer Inventories," *Journal of Economic Theory* 105 (2), 503–517.

[13] Jovanovic, B. and R. Rosenthal (1988), "Anonymous Sequential Games," *Journal of Mathematical Economics* 17 (1), 77–87.

[14] Kahn, J. (1987), "Inventories and the Volatility of Production," *American Economic Review* 77 (4), 667–679.

[15] Kawai, M. (1983), "Price Volatility of Storable Commodities under Rational Expectations in Spot and Futures Markets," *International Economic Review* 24 (2), 435–459.

[16] Kiyotaki, N. and R. Wright (1989), "On Money as Medium of Exchange," *Journal of Political Economy* 97 (4), 927–954.

[17] Meyer, R. and J. Assunção (1990), "The Optimality of Consumer Stockpiling Strategies," *Marketing Science* 9 (1), 18–41.

[18] Mela, C., K. Jedidi, and D. Bowman (1998), "The Long-Term Impact of Promotions on Consumer Stockpiling Behavior," *Journal of Marketing Research* 35 (2), 250–262.

[19] Micalizzi, L. N. Zambrotta, and M. Bernstein (2021), "Stockpiling in the Time of COVID-19," *British Journal of Health Psychology* 26 (2), 535–543.

[20] Noda, S. and K. Teramoto (2022), "A Dynamic Model of Rational 'Panic Buying,'" Working Paper, available online at https://ssrn.com/abstract=3721706.

[21] Sarris, A. (1984), "Speculative Storage, Futures Markets, and the Stability of Commodity Prices," *Economic Inquiry* 22 (1), 80–97.

[22] Scheinkman, J. and Schechtman, J. (1983), "A Simple Competitive Model with Production and Storage," *Review of Economic Studies* L, 427–441.

[23] Su, X. (2010), "Intertemporal Pricing and Consumer Stockpiling," *Operations Research* 58 (4 part 2), 1133–1147.

[24] Telser, L. (1958), "Futures Trading and the Storage of Cotton and Wheat," *Journal of Political Economy* 66 (3), 233–255.

[25] Thille, H. (2003), "Forward Trading and Storage in a Cournot Duopoly," *Journal of Economic Dynamics and Control* 27 (4), 651–665.

[26] Turnovsky, S. (1983), "The Determination of Spot and Futures Prices with Storable Commodities," *Econometrica* 51 (5), 1363–1387.

[27] Wang, E., N. An, Z. Gao, E. Kiprop, and X. Geng (2020), "Consumer Food Stockpiling Behavior and Willingness to Pay for Food Reserves in COVID-19," *Food Security* 12, 739–747.

[28] Wen, Y. (2005), "Understanding the Inventory Cycle," *Journal of Monetary Economics* 52, 1533–1555.

[29] Wen, Y. (2011), "Input and Output Inventory Dynamics," *American Economic Journal: Macroeconomics* 3 (4), 181–212.