Primaries and the New Hampshire Effect*

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Abstract

Candidates for U.S. presidential elections are determined through sequential elections in single states, the primaries. We develop a model in which candidates can influence their winning probability in electoral districts by spending money on campaigning. The equilibrium replicates several stylized facts very well: Campaigning is very intensive in the first district. The outcome of the first election then creates an asymmetry in the candidates' incentives to campaign in the next district, which endogenously increases the equilibrium probability that the first winner wins in further districts.

On the normative side, our model offers a possible explanation for the sequential organization: It leads (in expectation) to a lower level of advertising expenditures than simultaneous elections. Moreover, if one of the candidates is the more effective campaigner, sequential elections also perform better with regard to the selection of the best candidate.

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1 Introduction

Candidates for the U.S. presidential election are determined through a sequence of elections within each political party, the primaries. While the particular regulations vary between states and the two major parties, the basic system is the same in both parties, starting with the Iowa caucus and the New Hampshire primary in February, and continuing with primaries (and very few caucuses) in almost all U.S. states, in which more than 80% of the delegates to the national convention (that elects the party's candidate) are chosen.

The nomination process is one of the most controversial institutions of America's contemporary political landscape. The most common ground for attack on the modern primary system is the perception that its sequential structure is inherently "unfair" in that it shifts too much power to voters in early primary states. A notion that usually comes along with such claims is that the results of early primaries create "momentum" that carries over to later states. 1976 Democratic primary candidate Morris Udall (who eventually lost to Jimmy Carter) notes:

> "We had thirty primaries, presumably all of them equal. After three of those primaries, I'm convinced, it was all over. [...] I take a poll two weeks before the (Wisconsin) primary and he (Carter) is ahead of me, two to one, and has never been in the state except for a few quick visits. That was purely and solely and only the product of that narrow win in New Hampshire and the startling win in Florida." (Witcover, 1977)

Early primary states receive considerable attention by both the political candidates and the media. Malbin (1983) reports that in the 1980 Republican primaries George Bush and Ronald Reagan allocated roughly 3/4 of their respective total campaign budgets to states with primaries before March 31, although these states accounted for considerably less than a fifth of the delegates to the Republican convention in 1980. Among all primaries and caucuses in 1980, Iowa and New Hampshire accounted for 28% of the primary-season coverage in the CBS evening news and the United Press newswire (Robinson and Sheehan, 1983). Similarly, Adams (1987) reports that the 1984 New Hampshire primary attracted almost 20% of the season's coverage in ABC, CBS, NBC, and the New York Times. More recently, in 2004, Democratic primary candidate Howard Dean spend so much of the money he raised on campaigning in Iowa and New Hampshire that his campaign was in serious financial trouble after New Hampshire and could not even pay staff salaries. All these observations are the more surprising as New Hampshire accounts for only 0.4 percent of the U.S. population and only four out of 538 electoral votes in the presidential election, and is far from being demographically representative for the nation's electorate. Similarly, Iowa accounts for only 1.0 percent percent of the U.S population and only seven electoral votes.

The present paper has two interrelated objectives, one positive and one normative: Firstly, we address the question how the observed sequential organization can create sources for strategic momentum that can explain the stylized facts above. Why does the sequential nature of the current primary system induce candidates to campaign so heavily at early stages and the losers of early primaries to withdraw so early from the race? Secondly, we address the question how the temporal organization of elections affects a candidate's welfare, his expected campaign expenditures, and the probability of winning under alternative temporal structures. The particular comparison we make is between a sequential system, such as the current presidential primaries, and a counterfactual simultaneous system. A completely simultaneous design (a "national one-day primary") is a natural antipode as well as a prominent counterproposal to the sequential primary arrangement. Therefore, it is an important and interesting question to compare these two temporal organizations.

To this end, we develop an advertising model of political competition in which candidates have to win the majority of a number of electoral districts in order to obtain a certain prize. As in Snyder (1989), candidates can influence their probability of winning a district by their choice of campaign expenditures in that district. In the case of a sequential primary organization, campaign expenditures are very high in early districts, but decrease substantially at later stages, once one candidate has established a clear advantage. Sequential elections leave an expected rent to the candidates, which is bounded from below by a positive constant that is independent of the number of electoral districts. In contrast, simultaneous elections lead to complete rent dissipation if the number of electoral districts is sufficiently high. In other words, the expected campaign expenditures are lower when candidates face a sequential primary system. Interestingly, this cost-advantage is not so much driven by the fact that the need to go through the entire sequence of primaries rarely arises (and hence candidates can save on the spending in the last few primaries, as one might suspect). Rather, it is generated by a strategic "New Hampshire Effect" that is part of the equilibrium play: The outcome of the very first primary election creates an asymmetry between ex-ante symmetric candidates which endogenously facilitates momentum in later districts.

While there is no direct reason why parties should be concerned with candidates' expenditures and ex-ante expected level of rent, each party clearly has an interest that the candidate who wins its nomination keeps resources for the following presidential campaign against the other party's nominee. A long standing "rule" in American politics was that a candidate who lost his party's primary in New Hampshire would not become president.[1] An interpretation of this empirical fact is that a candidate who did not win the first primary, but eventually won his party's nomination, had to go through a long and costly nomination battle in his own party and lacked resources in the actual presidential campaign. If this is the case, each party has an incentive to organize its candidate selection procedure in a way that minimizes wasteful internal battles.[2]

If one candidate has an exogenous advantage over the other, in the sense that he is

[1]This "rule" was broken by Bill Clinton in 1992 and George W. Bush in 2000.

[2]The assumption that the organization of the primary system can be chosen by parties appears justified, because the primary system is not part of the constitution, and therefore is subject to fewer political and legal constraints than the following presidential election.

more likely to win if both candidates spend the same amount, a good primary system should also select the stronger candidate with a high probability. We examine two ways in which such an advantage can be formalized: First, we extend the model so that candidates have "assured districts" in which they win regardless of their opponents' campaign expenditures. For example, a candidate may be able to carry his home state's primary without much effort. Similarly, a candidate from a southern state may appeal to voters in other southern primaries, and a candidate from New England may be popular throughout the Northeast. If one player possesses a larger number of such assured districts than the other, he has an ex-ante advantage in the primaries. Our second specification assumes that one candidate is a stronger campaigner than his opponent, so that one campaign dollar spent by this candidate is more effective than one dollar spent by the other candidate. Such an efficiency advantage could reflect "soft factors" such as the candidate's campaign organization, management style, his endorsers, or key personnel within the campaign. With both specifications, the probability that the advantaged player wins the nomination is close to 1 under a sequential regime, provided that the number of primary districts is sufficiently high. A simultaneous system, on the other hand, frequently selects the weaker candidate. Furthermore, with an exogenous difference in strength between the two candidates the cost savings of the sequential design are even more dramatic than with like candidates. The reason is simple: The leverage that an early victory generates in the symmetric model does not need to be created by the candidates themselves, when one of them is already stronger. These two advantages of a sequential primary system—lower campaign expenditures and higher probability that the stronger candidate is selected—may explain why the sequential organization has been so persistent over time, even though it is often criticized as unfair.

Although our analysis compares mainly two extreme cases—completely sequential elections versus completely simultaneous elections—, the distinct results of the sequential case basically apply to a mixed temporal structure as well, as long as it involves some sequential elements at the early stages. One can argue that such an intermediate system is closer to the modern primary races, in which there are dates (such as "Super Tuesday") when several states vote simultaneously. Nevertheless, even in this case, some primary states vote in sequence at the very beginning of the nomination process. We show that this is enough to generate (and sometimes even amplify) the momentum effect and the spending pattern that arise in a completely sequential system.

Our paper is related to several strands of literature. Regarding the analysis of the temporal structure of elections, Dekel and Piccione (2000) have analyzed a model of sequential elections in which sophisticated voters try to aggregate their private information through voting. While, in principle, more information is available for voters in later elections, they show that the voting equilibria of sequential elections are essentially the same as those in the case of simultaneous elections. Consequently, the temporal organization of elections does not matter in their model.[3] We provide a complementary model to theirs,

[3]A model of the effects of sequential voting on information aggregation in which timing matters is Morton and Williams (1999). They perform an experiment and show that, in their setup, sequential

which abstracts from the information aggregation aspect of voting and focuses purely on the candidates' actions. We demonstrate that in such a framework, where candidates are modelled as economic agents trying to maximize their payoffs, the two organizational forms are no longer equivalent. From the point of view of the candidates and their parties, a sequential organization has considerable advantages over a simultaneous one.

Strumpf (2002) analyzes a sequential contest model of primaries in which the focus is on candidate behavior. His main interest is the effect of exogenous asymmetries between districts, and he finds that candidates who have advantages in late districts are more likely to win than candidates with an advantage in early districts. In an extension of his model, candidates can influence their probability of winning in a way that is similar to our model. However, Strumpf (2002) does not consider the important question how early victories generate momentum in a sequential contest, which is the main interest of our paper. He also does not compare simultaneous and sequential primaries, an issue which has important normative implications.

There is by now also a substantial body of literature that focuses on applications of contest theory to the design of sports tournaments (see, for example, Szymanski (2003) and Harbaugh and Klumpp (2005), and the references cited therein.) While the tournament structures considered in these papers often share some of the characteristics of primary races, the design issues are typically different from the ones considered here. In particular, in sporting tournaments it is often desirable to induce contestants to spend a maximal amount of effort, or to induce an effort allocation that increases the chance of a close contest, as this enhances the excitement level the tournament generates. For primary elections, on the other hand, we are interested in finding a design that minimizes wasteful campaign expenditures and avoids long, close battles as these will be very costly.

Several alternative approaches have been offered in the literature as explanation of the stylized facts concerning primary races. On the one hand, the political science literature contains theories of "psychological momentum" among primary voters in the sense that voters in later states enjoy voting for candidates that were successful in earlier states (Bartels, 1988). These behavioral theories lack a solid preference-based foundation resulting in rational decisions, and are hence unsatisfactory from an economics point of view. On the other hand, in their seminal work on informational cascades, Bikhchandani, Hirshleifer and Welch (1991) interpret momentum in primary races as evidence of rational herding on part of the later primary states. However, it is unclear whether primaries are really a valid example for herding: In herding models, people are concerned with *making the right choice*. In a standard model of voting, people do not so much care about whether *they themselves* voted for the correct candidate, but rather whether in the end the right candidate is selected.

Another strand of the literature on primaries is concerned with the question whether the institution of separate primaries for a left-wing and a right-wing party lead to the selection of extremist (vs. moderate) politicians as candidates (see Heckelman (2000),

elections help to aggregate information better than simultaneous elections.

Swank (2001) and Oak (2001)). In these papers, only one primary takes place within each party, so the effects of the sequential process characteristic for the *presidential* primaries (the focus of our model) cannot be explored in these models.

The paper proceeds as follows. In Section 2, we present a model of the primary system. Section 3 analyzes and compares the equilibria under different temporal regimes. In Section 4, we further explore these issues using numerical computations. In Section 5, we extend the model to allow for ex-ante asymmetric candidates. In Section 6, we discuss how our model can explain several stylized facts concerning primaries. Section 7 examines mixed primary systems involving both sequential and simultaneous stages. Section 8 concludes. We collect all proofs in Appendix A. Finally, Appendix B contains a theoretical analysis of the limit equilibria in large sequential primary races.

2 The Model

Candidates and Electoral Districts. There are two risk neutral candidates, 1 and 2, who compete in elections in J (odd) districts. The candidate who wins at least $J^* = \dfrac{J+1}{2}$ elections wins the prize Π, normalized to 1 (and assumed to be equal for both candidates).

The outcome of the election in district j is a random variable from the point of view of the candidates. They can influence the distribution of this random variable by committing campaign funds to each district (see below). Campaign expenditures represent advertising effort, the cost of time, etc. Let $x_j \geq 0$ be the amount spent by candidate 1 in district j, and likewise let $y_j \geq 0$ be the amount spent by candidate 2 in j. The net utility of a player is equal to the prize (if he wins) minus the campaign expenditures: If candidate 1 wins at least J^* districts, he obtains a payoff of $1 - \sum_{j=1}^{J} x_j$, otherwise he gets $-\sum_{j=1}^{J} x_j$. The payoff for candidate 2 is defined analogously. The rent dissipation rate, defined as the fraction of the prize that is spent by the two candidates together in their effort to win it, is $\sum x_j + \sum y_j$.

Campaign Technology. Given the spending profile (x_j, y_j), the probability that a candidate wins election j is determined by a campaign technology $f : \mathbb{R}_+^2 \to [0,1]$, that is, candidate 1 wins with probability

$$f(x,y) = \frac{x^\alpha}{x^\alpha + y^\alpha}, \tag{1}$$

if $x > 0$ or $y > 0$, and $f(0,0) = 1/2$. Observe that f is continuously differentiable on $\mathbb{R}_{++}^2$, homogeneous of degree 0 in (x,y), increasing and strictly concave in x, and decreasing and strictly convex in y. Candidates are symmetric: $f(x,y) = 1 - f(y,x)$ for all x, y.[4] The parameter α is a measure for the marginal effect of campaign spending, and we assume that $\alpha \in (0,1]$. If α is very low, the winning probability is close to 1/2 (as long as both candidates spend a positive amount) and largely independent of the candidates' spending. The higher is α, the higher is the marginal effect of campaign spending on the outcome

[4] In section 5, we consider the case of asymmetric candidates.

(and consequently, both candidates have a higher incentive to spend when α is large). The assumption that $\alpha \leq 1$ guarantees that f is globally concave.

Temporal Structure. There are two basic ways to organize J elections temporally: They can be held sequentially (as in the present presidential primary system), or simultaneously. In the sequential elections game G_J^{seq}, the candidates first choose campaign expenditure levels x_1 and y_1 in district 1. Then, they observe the outcome in district 1 and move on to district 2, where they choose x_2 and y_2, and so on until a candidate has accumulated the required majority of J^* districts. Within each district j, x_j and y_j are chosen simultaneously.

In the simultaneous election game G_J^{sim}, candidates choose all x_j and y_j $(j = 1, \ldots, J)$ simultaneously. Then, the outcomes in all districts are observed and the candidate who has gained at least the required majority J^* wins the prize.

Combinations of these two basic structures are also possible; for example, one could start with n sequential elections and hold the remaining $J - n$ elections simultaneously. We view the current primary structure as mainly sequential, although there are certain simultaneous elements added to it (e.g., on "Super Tuesday", where several states hold their primaries simultaneously). The focus of our analysis is on the comparison between the completely simultaneous and the completely sequential case; the issue of mixed temporal arrangements will be addressed briefly in section 7.

Related literature. Similar rent-seeking models have been used in the literature. For a single district, our approach was first formulated by Tullock (1980) as an all-pay auction model of lobbying. There, two bidders compete for a prize by submitting monetary bids (bribes) to a bureaucrat who has the power to allocate a political favor. The bureaucrat picks winning probabilities according to the exogenously given functional form and draws the winner. All bidders have to pay their submitted bids, regardless of whether they won the prize or not. Here, we use all-pay auctions as a model of political competition in elections, where campaign expenditures are naturally not recoverable.

To examine the effect of campaign spending on election outcomes, Snyder (1989) uses this model as well. He analyzes the campaign expenditure allocation game between two parties, which compete in a number of districts (e.g., Republicans and Democrats in an election for the House of Representatives). His focus is on the effect different objective functions of the parties have on the allocation of campaign resources (e.g., what happens if parties wish to maximize the expected number of seats, or the probability that they win the majority in the house?).

Our model is also related to Szentes and Rosenthal (2003a, 2003b), who study all pay majority auction games. The objective of players in their paper is to win a majority of objects in simultaneous auctions. Interpreting objects as districts, bidders as candidates, and bids as campaign expenditures, this auction resembles our simultaneous primaries game. However, Szentes and Rosenthal assume that the player who bids the most for a particular objects wins that object with certainty, which corresponds to $\alpha \to \infty$ in our model. In a rather involved proof, Szentes and Rosenthal (2003a) characterize a mixed strategy equilibrium. The equilibria of our model (where $\alpha \leq 1$) are very different from Szentes and Rosenthal's; we characterize them in Section 3.2.

3 Equilibrium in the symmetric case

3.1 An example: Three districts

Before we turn to a complete analysis of simultaneous and sequential elections, it is instructive to look at a simple example in which candidates compete in only three districts. Many of the differences between sequential and simultaneous primaries can be seen in this simple setup already. To further simplify the analysis, we also temporarily set $\alpha = 1$.

Consider the sequential case first. We begin by examining the election in the last district. Clearly, if one candidate has won the previous two elections, the race is decided and both candidates choose expenditures of zero. Otherwise, if each candidate has won a single district before, the third district is contested and candidate 1 solves

$$\max_{x_3} \frac{x_3}{x_3 + y_3} - x_3.$$

Note that expenditures in previous districts are sunk costs and can be ignored. For candidate 2 an analogous maximization problem obtains, and in the symmetric equilibrium of this subgame candidates choose $x_3 = y_3 = 1/4$ and have an expected continuation utility of $1/2 - 1/4 = 1/4$.

Going one step back to the second election, observe that the two candidates are necessarily asymmetric—one of them, say candidate 1, has won the first district while the other one, say candidate 2, has lost it. In the second election, candidate 1 solves

$$\max_{x_2} \frac{x_2}{x_2 + y_2} \cdot 1 + \frac{y_2}{x_2 + y_2} \cdot \frac{1}{4} - x_2,$$

because with probability $\frac{x_2}{x_2+y_2}$ he wins the second primary and hence the nomination of value 1, and with the opposite probability he gets the continuation utility of 1/4 calculated above. (Any campaign expenditures from the first election are sunk and do not enter the second-district problem). Candidate 2 solves

$$\max_{y} \frac{y_2}{x_2 + y_2} \cdot \frac{1}{4} - y_2.$$

The equilibrium in this round is then $x_2 = \frac{9}{64}$ and $y_2 = \frac{3}{64}$, which yields expected continuation utilities at stage 2 of $u_1 = 43/64$ and $u_2 = 1/64$, respectively.

As in the analysis of the last round above, the two candidates are symmetric in the first round and choose a campaign expenditure equal to 1/4 of the difference of the continuation utilities of the front runner and the second candidate: $x_1 = y_1 = 21/128$. To summarize, the subgame perfect equilibrium calls for expenditures of .164 initially, and going into the second election for .141 for the front-runner and .047 for the runner-up of the first district. Should the race be tied after two elections, both candidates spend .25 in the last district.

There are two interesting features of this equilibrium. First, a first-district victory generates *momentum* in the second district: Since the front-runner outspends his opponent by a margin of 3 to 1 in the second election, he wins that election with probability 75%. Although the campaign technology is still the same for both candidates in the second district, the front runner chooses a higher spending level in the second district than the other candidate and consequently has a higher probability of winning. The reason is that the difference of the continuation values is higher for the first-district winner: In the second district, he plays for the difference between winning the whole race (a value of 1) and being head-to-head in the third district (continuation value 1/4). For the first-district loser, the relevant difference is between winning the second district and hence tying (value 1/4) and losing the second district and being out of the race (value 0).

Second, spending in the first district ($2 \times .164 = .328$) is higher than in the second district, where the front runner and the second candidate spend $.141 + .047 = .188$. In the last district, each candidate spends .25, but only if this election is pivotal. As we have shown, the probability of a tied race after two election is only 25%. The expected campaign expenditures in the third district are therefore $.25 \times 2 \times .25 = .125$, and the expected sum of campaign expenditures over all districts and candidates is .64. Hence, the equilibrium has a very pronounced front-loaded spending profile: Although all districts have the same weight and all have the same, concave technology, candidates spend more to win in the first district than in the second district, and more in the second district than they do in expectation in the third district.

Now consider simultaneous elections in all three districts. To win the overall race, a candidate needs to win either 2 or 3 districts. If candidates spread their expenditures evenly over the districts (we will show later that this is indeed optimal), the objective is

$$\max_x \left(\frac{x}{x+y}\right)^2 + \frac{3x^2 y}{(x+y)^3} - 3x.$$

The first order condition is

$$\frac{(3x^2 + 6xy)(x+y) - 3(x^3 + 3x^2 y)}{(x+y)^4} - 3 = 0,$$

and using symmetry this yields $x = y = 1/8$. Total expenditures in a simultaneous election campaign (i.e. across all candidates and districts) are therefore .75, which is more than in the sequential structure. (The difference may seem modest, but we shall show below that in the general J-district case it becomes dramatic). A sequential system therefore leads to less spending (on expectation), which is an attractive feature if parties want their nominees to have resources left for the general election campaign.

3.2 Simultaneous elections with J districts

We now turn to the analysis of the general J-district case. We start with the simultaneous election game, in order to highlight the problem of excessive campaigning under this organizational form. We then analyze the sequential election game in Section 3.3.

A pure strategy for a player in G_J^{sim} is a point $x \in \mathbb{R}_+^J$. A mixed strategy is a point $p \in \Delta$, where Δ is the set of all probability distributions over $\mathbb{R}_+^N$. Depending on J, these sets can be rather high dimensional. A particularly simply form of strategies, however, is described in the following definition:

Definition 1. Candidate 1 (2) plays a *uniform campaign strategy* if he chooses $x \in \mathbb{R}_+$ ($y \geq 0$) according to some cumulative distribution Λ_1 (Λ_2), and then sets $x_j = x$ ($y_j = y$) for all j. An equilibrium of G_J^{sim} in which both players choose uniform campaign strategies is called a *uniform campaign equilibrium (UCE).*

Note that a uniform campaign strategy can be pure (if Λ is degenerate) or mixed. The word "uniform" in Definition 1 means that the total investment level is equally distributed across districts, and not that Λ_i is a uniform distribution. We will often refer to a *symmetric UCE* (SUCE), using the word "symmetric" in the usual sense to indicate that both players use the same strategy ($\Lambda_1 = \Lambda_2$).

Proposition 1. *Every equilibrium of G_J^{sim} is a uniform campaign equilibrium. Furthermore, all UCE for a given J have the same payoffs.*

The proof of Proposition 1 is in the Appendix. The advantage of uniform campaign strategies is that the dimensionality of the players' strategy spaces is reduced from J to 1. The first result of Proposition 1 therefore considerably simplifies the analysis of the simultaneous elections game. The second result shows that for each J the game has essentially a unique equilibrium, in the sense that all equilibria are payoff-equivalent.

For the time being, we focus on pure strategy equilibria. We will show that a pure strategy UCE exists only if J is sufficiently small. Given two pure uniform campaign strategies $x, y \in \mathbb{R}_+$, candidate 1's payoff is

$$u_1(x, y) = F(x, y) - Jx, \tag{2}$$

where

$$F(x, y) = \sum_{k=J^*}^{J} \binom{J}{k} \frac{x^{\alpha k} y^{\alpha(J-k)}}{(x^\alpha + y^\alpha)^J} \tag{3}$$

is the overall winning probability for candidate 1: He wins the prize if he wins in $k \geq J^*$ districts. To find a SUCE in pure strategies, differentiating (2) with respect to x, invoking symmetry ($y = x$) and simplifying yields the following first order condition:

$$\binom{J-1}{\frac{J-1}{2}} \left(\frac{1}{2}\right)^{J-1} \frac{\alpha}{4x} = 1. \tag{4}$$

(In Appendix A, we show that the second order condition for a local maximum holds.) The first two terms, $\binom{J-1}{\frac{J-1}{2}} \left(\frac{1}{2}\right)^{J-1}$ represent the probability that, given equal spending by both opponents in each district, exactly one half of the $J - 1$ districts are won by candidate 1, and the other half by candidate 2: In this case (and only in this case), the outcome in the last district is pivotal. The third term, $\frac{\alpha}{4x}$, is the marginal effect of additional spending in the last district on the winning probability there. To see this, differentiate the winning probability in that district, $\frac{x^\alpha}{x^\alpha+y^\alpha}$, with respect to x and use symmetry ($y = x$). In summary, the marginal benefit of spending in any district j is the marginal increase in probability of winning district j times the probability of j being pivotal. At the optimum, this marginal benefit of spending has to be equal to its marginal cost, which is 1. Consequently, a pure strategy UCE (if it exists) is given by

$$x = y = \frac{\alpha}{2^{J+1}} \binom{J-1}{\frac{J-1}{2}}. \tag{5}$$

Since the equilibrium payoff for each candidate in a symmetric UCE is $\frac{1}{2} - Jx$, the value for x as given by (5) must satisfy $x \leq \frac{1}{2J}$ in order for a pure strategy equilibrium to exist. Hence, (5) constitutes an equilibrium if and only if

$$\alpha \leq 2^J \bigg/ J \binom{J-1}{\frac{J-1}{2}}. \tag{6}$$

In the appendix, we show that the right hand side of this inequality is decreasing in J and goes to zero for $J \to \infty$. Thus, (6) implicitly defines for each α a maximum number of districts $K(\alpha)$ for which a pure strategy symmetric UCE exists.[5]

The intuition why a pure strategy UCE fails to exist if J is too large is as follows: Independent of J, a candidate needs to win just one more district than his opponent. Suppose that J is large and both candidates spend the same amount, which cannot be larger than 1/2. Now, if player 1 increases his total campaign spending by a small amount and distributes the additional spending equally over all districts, he wins every district with a slightly higher probability than his opponent. While player 1 wins only slightly more than half of the districts in expectation, the law of large numbers implies that he wins the majority of districts with probability close to one. Hence, for large J, a symmetric pure strategy profile cannot be an equilibrium.

If a pure strategy equilibrium exists, then by (5) each candidate's payoff is given by

$$\frac{1}{2} - J \frac{\alpha}{2^{J+1}} \binom{J-1}{\frac{J-1}{2}}, \tag{7}$$

which is generically positive for $J \leq K(\alpha)$. If the number of districts is larger than $K(\alpha)$, we will show that a mixed strategy equilibria exists, and that the candidates' expected rent is completely dissipated in equilibrium. Proposition 2 collects these results:

[5]While there is no closed form solution for $K(\alpha)$, the right hand side can be approximated by using Sterling's formula ($n! \approx (n/e)^n \sqrt{2\pi n}$) to obtain

$$\frac{2^J \left[\left(\frac{J-1}{2e}\right)^{\frac{J-1}{2}} \sqrt{2\pi \frac{J-1}{2}} \right]^2}{\left(\frac{J}{e}\right)^J \sqrt{2\pi J}} = \frac{e^{-(J-1)}(J-1)^J \sqrt{2\pi}}{J^J e^{-J} \sqrt{J}} = \sqrt{2\pi} e \left(\frac{J-1}{J}\right)^J \frac{1}{\sqrt{J}} \approx \sqrt{\frac{2\pi}{J}}.$$

Solving for J yields the approximation $K(\alpha) \approx \frac{2\pi}{\alpha^2}$.

Proposition 2. *There exists a decreasing function* $K : (0,1] \to \mathbf{R}$*, implicitly defined by equality in* (6)*, such that*

(a) if $J \leq K(\alpha)$*, a symmetric UCE in pure strategies exists. Rents are (generically) not fully dissipated in this equilibrium.*

(b) If $J > K(\alpha)$*, a pure strategy equilibrium does not exist, but symmetric UCE in mixed strategies exists. This equilibrium involves full rent dissipation:* $E(\sum x_j) = E(\sum y_j) = 1/2$*, and candidates' expected rent is* $v_J^{sim} = 0$ *for all* $J > K(\alpha)$*.*

Intuitively, players have a strong incentive to "outcampaign" their opponent, as J grows, because the marginal effect of spending on the probability to win the whole race increases. This is very similar to the effect of an increase of α for a fixed number of districts, and also leads to complete rent dissipation. In fact, for $J \to \infty$, our game approaches the standard all-pay auction in which candidates choose their overall expenditures and the candidate with the higher expenditure wins. This observation alone implies that equilibrium payoffs in the limit (for $J \to \infty$) must be zero. However, our result is stronger, as there exists a finite number $K(\alpha)$ such that $v_J^{\text{sim}} = 0$ for all $J > K(\alpha)$. The intuition why K decreases in α is as follows: A high value of α means that the marginal effect of campaigning *per district* is high. Consequently, for high α, the number of districts from which on the pure strategy UCE vanishes is smaller than for small α, so that K is decreasing.

3.3 Sequential Elections

The sequential election game with J districts, G_J^{seq}, can be analyzed using backward induction. After $j-1$ elections have been held, call a *state* for a candidate a tuple (j,k) where k is the number of elections won by the candidate so far. Consequently, the opponent is in state $(j, j-k-1)$. Let $x_{j,k}$ be the candidate's spending in state (j,k), and $v_{j,k}$ his continuation value.[6] The value $v_J^{\text{seq}} = v_{1,0}$ is then the value of the game G_J^{seq}.

The continuation value does not take account of any prior investments, because these have to be considered as sunk costs by the candidates. A useful consequence of the sunk cost property is that, if we extend the game from G_J^{seq} to G_{J+2}^{seq}, all we have to do is to introduce a number of new states and relabel. That is, $v_{j,k}$ becomes $v_{j+2,k+1}$, and likewise $x_{j,k}$ becomes $x_{j+2,k+1}$. For instance, the problem when each candidate has won exactly one election in G_{J+2}^{seq} is the same as the problem at the very beginning of G_J^{seq}.

Given (j,k), we can now set up a pair of Bellman equations, one for each player:

$$v_{j,k} = \max_{x_{j,k}} \left\{ \frac{x_{j,k}^\alpha}{x_{j,k}^\alpha + x_{j,j-k-1}^\alpha} v_{j+1,k+1} + \frac{x_{j,j-k-1}^\alpha}{x_{j,k}^\alpha + x_{j,j-k-1}^\alpha} v_{j+1,k} - x_{j,k} \right\} \tag{8}$$

[6] In this section, it is notationally more convenient to denote both candidates' spending by x. Since the subscripts indicate the respective state candidates are in, no confusion should arise.

and

$$v_{j,j-k-1} = \max_{x_{j,j-k-1}} \left\{ \frac{x^{\alpha}_{j,j-k-1}}{x^{\alpha}_{j,k} + x^{\alpha}_{j,j-k-1}} v_{j+1,j-k} \right.$$

$$\left. + \frac{x^{\alpha}_{j,k}}{x^{\alpha}_{j,k} + x^{\alpha}_{j,j-k-1}} v_{j+1,j-k-1} - x_{j,j-k-1} \right\}. \quad (9)$$

When we set $v_{j,J^*} = 1$ for $j = J^*, \ldots, J+1$ and $v_{j,j-J^*-1} = 0$ for $j = J^*+1, \ldots, J+1$, the Bellman equations become a finite horizon dynamic programming problem.

Since it is always feasible for a candidate to spend zero, we have $v_{j,k} \geq 0$ for all j, k, and, as shown in the proof of Proposition 3 in the Appendix, $v_{j,k} \geq v_{j,k-1}$ and $v_{j,k} \geq v_{j+1,k}$. Together with the fact that $0 < \alpha \leq 1$, this implies that the right-hand side of each Bellman equation is strictly concave in its respective decision variable.

Define $\Delta_{j,k} = v_{j,k} - v_{j,k-1}$ to be the difference in the continuation payoff from winning the stage election in state (j,k) and losing it, and let

$$\theta_{j,k} = \frac{\Delta_{j+1,j-k}}{\Delta_{j+1,k+1}}.$$

Taking first order conditions of (8) and (9), the ratio of the candidates' expenditures is

$$\frac{x_{j,j-k-1}}{x_{j,k}} = \theta_{j,k}.$$

Using this relation in the first order conditions, the unique solutions of the first order conditions are

$$x_{j,k} = \alpha \frac{\theta^{\alpha}_{j,k}}{(1+\theta^{\alpha}_{j,k})^2} \Delta_{j+1,k+1} \text{ and } x_{j,j-k-1} = \alpha \frac{\theta^{\alpha}_{j,k}}{(1+\theta^{\alpha}_{j,k})^2} \Delta_{j+1,j-k}. \quad (10)$$

Due to the strict concavity of the Bellman equations, the first order necessary conditions are also sufficient. Since on each stage of the game, there is a unique continuation equilibrium, the usual backwards induction argument shows that the subgame perfect equilibrium of game G^{seq}_J is unique.

It is easy to show that sequential elections leave candidates with a rent, even if there are very many districts; thus, sequential elections are at least asymptotically better than simultaneous elections. Call (j,k) *symmetric* if j is odd and $k = \frac{j-1}{2}$. (In particular, the initial state $(1,0)$ of every G^{seq}_J is symmetric.) In symmetric (j,k), $\theta_{j,k} = 1$, so that

$$x_{j,k} = x_{j,j-k-1} = \frac{1}{4}\alpha\Delta_{j+1,k+1}.$$

Since each candidate is equally likely to win the stage election, we have

$$v_{j,k} = \left(\frac{1}{2} - \frac{1}{4}\alpha\right) v_{j+1,k+1} + \left(\frac{1}{2} + \frac{1}{4}\alpha\right) v_{j+1,k}$$

$$\geq \left(\frac{1}{2} - \frac{1}{4}\alpha\right) v_{j+1,k+1} \geq \frac{1}{4} v_{j+1,k+1} \geq \frac{1}{4} v_{j+2,k+1}.$$

Using $v_{J+1,J^*+1} = 1$, we have $v^{\text{seq}}_J \geq 4^{-J^*} > 0$. The following Proposition summarizes our results:

Proposition 3. *For all J, the sequential elections game G_J^{seq} has a unique subgame perfect equilibrium in pure strategies. Furthermore, for any number of districts J, $v_J^{seq} > 0$.*

A general closed form solution for the SGPE strategies and payoffs is desirable, as it would allow as to investigate how the equilibrium strategies and the value of the game depend on the parameters J and α. Unfortunately, such a solution is difficult to obtain for all but a very small number of districts. The next Section therefore contains numerical results that shed light on a number of interesting questions we are unable to address analytically.

We are nevertheless able to offer further theoretical results concerning the limiting behavior of G_J^{seq} as J becomes large. In particular, we can show that the sequential game is closely related to a sequence of contests that aborts once a candidate is ahead in the race by a certain pre-specified margin. Although such a race is potentially infinite, it will end in finite time with probability one. It is possible to characterize equilibria in this contest and show that both payoffs and strategies in our original sequential game converge to those in the modified game. Due to the complex nature of these results, we present them in Appendix B instead of the main text.

4 Numerical Results

In this section, we present quantitative results obtained from the numerical computation of the SGPE in the sequential game and the UCE of the simultaneous game. We use the following notation: As before, v_J^{sim} is the equilibrium payoff to a candidate of the simultaneous game G_J^{seq}, and Σx^{sim} is the expected sum of expenditures per candidate. Similarly, v_J^{seq} and Σx^{seq} are the value and the expected expenditures for G_J^{sim}. For the sequential game, we also report several strategy component: $x_{1,0}$ are expenditures in the first primary district, and $x_{2,1}$ (resp. $x_{2,0}$) are expenditures in the second primary district after having won (resp. lost) the first election. Finally, P denotes the probability that a candidate wins the entire race, conditional on a victory in the first district.

In Table 1, we present subgame perfect equilibria of the sequential game and the UCE of the simultaneous game, for the case of $\alpha = 1$ and different values of J. SGPE in G_J^{seq} were computed by backward induction. Pure strategy UCE of G_J^{sim} and their values are readily obtained from (5). From Proposition 2, we know already that $v_J^{\text{sim}} = 0$ in mixed strategy equilibria, so that we don't need to calculate mixed equilibrium strategies.[7]

The ex ante expected rent of a candidate in the sequential organization, v_J^{seq}, is always substantially higher than the rent in the simultaneous organization, and converges to approximately 0.1838. Hence about 37% of the maximum possible rent is not dissipated by the candidates, even for a number of districts where rent dissipation is complete in a simultaneous organization of primaries. Consequently, the expected total campaigning expenditures in the sequential organization converge to 0.6324. In the simultaneous or-

[7]However, it is possible to confirm Proposition 2 by computing the UCE in discretized versions of G_J^{sim}, using the computer software GAMBIT.

J	1	3	5	7	9	...	19	...	49	...	99
v_J^{sim}	.250	.125	.031	0	0	...	0	...	0	...	0
Σx^{sim}	.250	.375	.469	.500	.500		.500		.500		.500
v_J^{seq}	.250	.180	.181	.184	.184	...	.184	...	.184	...	.184
Σx^{seq}	.250	.320	.319	.316	.316		.316		.316		.316
$x_{1,0}$	.250	.164	.175	.178	.178	...	.178	...	.178	...	.178
$x_{2,1}$		.141	.119	.121	.122		.122		.122		.122
$x_{2,0}$		.047	.027	.027	.028		.028		.028		.028
P	1	.875	.909	.907	.905		.905		.905		.905

Table 1: Campaign equilibria with $\alpha = 1$ ($K(1) = 5$)

ganization, these expected expenditures are 1 for $J \geq 7$. In the very first district of the sequential primary race, both candidates spend 0.1775, respectively, which is about 56% of their total expected expenditures. Note that the proportion of campaign expenditures in the first district to total expenditures is essentially the same, whether there are 19, 49 or 99 districts. Hence, campaigning in the first district is fierce even if this district is seemingly unimportant in terms of its weight in the overall race.[8]

In the second district, the situation is necessarily asymmetric and second district expenditures for both candidates are considerably lower. The first-district winner spends 0.1218, while the runner-up reduces his expenditures to 0.0279. The probability that the candidate who won the first district wins the second district as well is therefore about 81.4%. Should this candidate win the second district as well, a similar asymmetry effect can be observed again, endogenously furthering the advantage of the winner: Relative to his opponent, the front runner outspends his opponent so that yet another victory is almost guaranteed. For example, if one candidate has won the first 3 elections, the candidate who has fallen behind spends about 1/5000 of the front runner's expenditures in that district, virtually conceding the race to his opponent. Yet, this increase in *relative* spending by the race leader occurs at much lower *absolute* levels, keeping overall campaign expenditures low. Thus, once a candidate has managed to establish an early lead, he will in all likelihood win the entire race, and with little use of further campaign resources.

This result sheds light on the question why sequential elections are cheaper for the candidates than simultaneous ones. A superficial explanation is that, with sequential elections, one candidate may collect enough electoral districts even before the end of the primary sequence, in which case candidates can save on expenditures in the last districts. However, if this explanation were all there is, then doubling the number of districts should result in a substantial cost increase, because the number of districts required for victory

[8]Note that convergence is effectively completed for $J = 19$. For large J, the SGPE strategies in G_J^{seq} become stationary, so that the SGPE in these games converges to a Markov perfect equilibrium (MPE), in which strategies are measurable with respect to the lead one candidate has over the other. Intuitively, large games become very similar to each other, and candidates use very similar continuation strategies in G_{99}^{seq} as they do in, say, G_{97}^{seq}.

also doubles. This is not the case in our model, and the reason is that the cost savings are due to the endogenous momentum or "New Hampshire effect" that is part of the equilibrium play: The outcome of the first election creates an asymmetry between ex-ante symmetric candidates, which triggers different spending patterns in subsequent primaries. Because it is now less likely for the candidate who has fallen behind to win, the absolute level of campaign expenditures decreases sharply.[9]

While the winner of the first election always spends more than his opponent in the second election and hence will likely expand his lead, it could of course happen that the first-district runner up wins the second district. We would then see vigorous campaigning in the third district, and, more generally, in all districts in which the candidates have won the same number of districts. In light of our model, the campaign expenditures in the New Hampshire presidential primary are high not so much because it is the first primary state, but rather because no candidate has yet established a clear lead in the race. In later stages, such a head-to-head situation with high expenditures is possible, but improbable.

To examine the effect of a change in the campaign technology, we present results for $\alpha = 0.5$ in Table 2.[10] In both simultaneous and sequential primaries, a decrease in α leads to (weakly) less rent dissipation. Intuitively, as advertising becomes marginally less effective, candidates spend less aggressively.

J	1	3	5	7	9	...	19	...	49	...	99
v_J^{sim}	.375	.313	.266	.227	.192	...	.060	...	0	...	0
Σx^{sim}	.125	.188	.234	.273	.308		.441		.500		.500
v_J^{seq}	.375	.317	.279	.252	.235	...	.224	...	.226	...	.226
Σx^{seq}	.125	.183	.221	.248	.265		.277		.274		.274
$x_{1,0}$	.125	.067	.054	.049	.047	...	.051	...	.051	...	.051
$x_{2,1}$		.077	.063	.058	.056		.062		.062		.062
$x_{2,2}$		.046	.035	.030	.027		.025		.025		.025
P	1	.782	.740	.729	.729		.755		.753		.753

Table 2: Campaign equilibria with $\alpha = 0.5$ ($K(0.5) = 23$)

Also, first district spending decreases relative to total spending. For example, for 49 districts, spending in the first district as a proportion of total expected spending declines to about 19% from approximately 56% for $\alpha = 1$. While a lower value of α leads to lower total campaign expenditures in all districts, the effect on the ratio of first-district spending to total spending is more involved. Consider a 3 district race. If α is high, a large proportion of the continuation rent is dissipated in the third district *in case that district is contested*, which happens when the candidates are tied after two rounds. Anticipating costly third-round competition, the loser of the first district will choose not to campaign

[9]This result is somewhat reminiscent of Che and Gale (2000a), who—in a context of lobbying games—show that asymmetry between lobbyists increases the expected rents in equilibrium. In our model, the sequential structure of primaries is used to create this asymmetry endogenously.

[10]Further results for $\alpha = 0.25$ and $\alpha = 0.75$ are available from the authors.

very intensely in the second district. Consequently, once an advantage is established, the front runner will likely be the eventual winner. This strategic effect makes it vital to win the first district. If, instead, α is low, a fixed advantage over the opponent becomes less valuable: In this case, the winning probability is close to $1/2$ for both candidates in every district, and so winning the first district becomes less important, because it is harder (in equilibrium) to maintain this advantage after it has been established.

5 Asymmetric Candidates

In our basic model, both candidates are symmetric with respect to their ability to transform campaign expenditures into electoral victories. In practice, a crucial function of the primaries is to select the better campaigner as the party's nominee (in the hope that this increases the party's winning probability in the general election).

At first sight, it may seem that, while sequential elections are cheap, they could have a disadvantage relative to simultaneous ones in this dimension: From the last section, we know that the very few first districts are crucial in a sequential primary system. If the worse campaigner is lucky enough to win in the first districts, then an endogenous momentum may start that carries him to victory. On the other hand, in a simultaneous primary system, no "mistaken momentum" can develop in favor of the weaker campaigner.

In this section, we show that these concerns are unfounded, at least if we assume that the candidates' campaign effectiveness is common knowledge among the candidates. The intuitive reason is that the "mistaken momentum" argument above assumes that both candidates behave in the same symmetric way as they do in the basic model, that is, even a better candidate would effectively give up if he loses by chance in the first few districts. If this were true, then sequential elections would indeed generate a substantial probability for selecting the weaker candidate. However, this is not the case in the equilibrium of the sequential game when candidates are asymmetric. In equilibrium, the stronger campaigner behaves essentially already in the beginning as if he had an advantage in terms of districts won. If, by any chance, the weaker candidate should win in one of the early districts, then the effect is similar to the lagging candidate catching up on the leading candidate in the basic model: The fight gets harder as both candidates increase their expenditures, but it is not the case that the stronger candidate gives up (provided that the sequence is long enough to make up later for early losses).

In order to introduce an exogenously given advantage of one candidate over the other, we explore two different ways of modelling such an asymmetry. The first one is that candidate 1 has a number of "assured districts" that he wins irrespective of campaign expenditures; this model has the advantage of being quite tractable. The second one is that candidate 1 uses each unit of campaign resources more effectively. This appraoch is more appealing as a model of a good campaigner, but results have to be derived numerically. Our results show that, under both specifications, the sequential primary system leads to a higher probability that the better campaigner is selected than the simultaneous system.

5.1 Assured districts

One simple way to model asymmetry is to assume that candidate 1 wins a fraction $0 < \rho < 1/2$ of the districts irrespective of campaign expenditures. These districts are *assured* for candidate 1. It is common knowledge which of the J districts are assured for candidate 1, so that neither candidate will spend anything there. In the $(1-\rho)J$ other districts, the election technology is the same as in our basic model. For reasons of tractability, we restrict ourselves to the asymptotic case of $J \to \infty$.[11]

Consider first the simultaneous election system. In the ρJ uncontested districts both candidates choose a spending level of 0. In order to win a majority, candidate 1 needs to win (slightly more than) a share of $\frac{1}{2} = \rho + (1-\rho)\frac{1-2\rho}{2(1-\rho)}$ of districts, which implies that of the $(1-\rho)J$ contested districts, candidate 1 has to win a share of $\frac{1-2\rho}{2(1-\rho)}$. If both candidates play asymmetric uniform strategies in the contested districts,[12] then candidate 1 wins the nomination if

$$\frac{x^\alpha}{x^\alpha + y^\alpha} > \frac{1-2\rho}{2(1-\rho)}, \tag{11}$$

which we can simplify to $x > (1-2\rho)^{1/\alpha} y$. If the inequality sign in (11) is reversed, candidate 2 wins. For $J \to \infty$, this game is an asymmetric all-pay auction, and we can use the standard methods for solving all-pay auction games to characterize the equilibria in the simultaneous election game. We then get the following result:

Proposition 4. *In the model with assured districts and simultaneous elections, the following holds as $J \to \infty$: Candidate 1's ex ante utility goes to $1 - (1-2\rho)^{1/\alpha} < 1$, and candidate 2's ex ante utility goes to 0. Furthermore, the probability that candidate 1 wins goes to $1 - \frac{(1-2\rho)^{1/\alpha}}{2} < 1$.*

With a sequential organization of primaries, we can assume without loss of generality that the first ρJ elections are the uncontested ones. No change arises if the uncontested elections take place at any later time, because it is known from the beginning that candidate 1 will win in these districts. Candidate 1 wins the nomination if he wins more than $(\frac{1}{2} - \rho)J$ of the last $(1-\rho)J$ districts. The initial value of the game for candidate 1 is therefore equal to the continuation value $v_{\rho J+1, \rho J+1}$ in the symmetric game. With the relative advantage ρ fixed and $J \to \infty$, candidate 1's absolute advantage gets very large. As we show in Appendix B, a candidate who leads the race by a sufficiently high *absolute* margin receives a continuation payoff of approximately 1. Since the continuation utility is equal to the probability of winning the prize, minus the expected expenditures, this also shows that candidate 1 wins the nomination with probability close to 1.

[11]The assumption that only candidate 1 has assured districts is entirely innocuous. In general, if ρ_i denotes the fraction of districts assured for candidate $i = 1, 2$,with $0 < \rho_2 < \rho_1 < 1/2$, candidate 1 still has an advantage over 2. This advantage can be expressed as a single number $\rho \equiv (\rho_1 - \rho_2)/(1 - 2\rho_2)$, a renormalization that makes the general model equivalent to the one presented in this section.

[12]Using the same procedure as in section 3.2, it is straightforward to show that a uniform equilibrium exists.

Proposition 5. *In the model with assured districts and sequential elections, the following holds as $J \to \infty$: Candidate 1's ex ante utility goes to 1 and candidate 2's ex ante utility goes to 0 as J increases. Furthermore, the probability that candidate 1 wins goes to 1.*

To sum up, in both regimes, asymmetry between candidates reduces (expected) rent dissipation. This is a result similar to Che and Gale (2000a, 2000b). However, in our model the exogeneous asymmetry is reinforced in the sequential organization through the endogenous asymmetry creating a "preemption effect," and so the effect of asymmetry is more forceful in a sequential organization.

5.2 Asymmetric campaign efficiencies

An alternative approach for introducing asymmetries into the model is to assume that candidate 1 is a more effective campaigner, in the sense that each unit of campaign resources spent by him is worth as much as $\psi > 1$ units spent by candidate 2. Specifically, we assume that in each district the probability that candidate 1 wins is given by

$$\tilde{f}(x, y) = \frac{(\psi x)^\alpha}{(\psi x)^\alpha + y^\alpha}. \tag{12}$$

If $x = y > 0$, this probability is $\psi/(1+\psi) > 1/2$. We also assume that $\tilde{f}(0,0) = \psi/(1+\psi)$. As $\psi > 1$, candidate 1 again has an advantage over candidate 2. In contrast to the previous specification, however, the advantage is now in terms of the marginal effects of campaign resources, rather than in terms of the fraction of assured districts.

With simultaneous elections, one can apply the same arguments as in the basic model to show that a uniform campaign equilibrium exists. As $J \to \infty$, the law of large numbers implies that candidate 1 wins the nomination if and only if he spends just slightly more than y/ψ per district, so that the uniform equilibrium approaches the mixed strategy equilibrium of an asymmetric all-pay auction. The following proposition, proved in the appendix, shows that in simultaneous elections with asymmetric candidates, rents are not completely dissipated (even if $J \to \infty$), and the stronger candidate is selected with probability greater than one half.

Proposition 6. *In the model with asymmetric campaign efficiencies and simultaneous elections, the following holds as $J \to \infty$: Candidate 1's ex ante utility goes to $1 - \frac{1}{\psi} < 1$, and candidate 2's ex ante utility goes to 0. Furthermore, the probability that candidate 1 wins goes to $\left[1 - \frac{1}{2\psi}\right] \in (\frac{1}{2}, 1)$.*

For sequential elections, we have to compute the equilibria numerically. Table 3 compares simultaneous and sequential elections for the case $\psi = 1.2$, $\alpha = 1$, and various values for J (results do not qualitatively depend on this particular set of parameters). We report the rents left to the candidates, expected expenditures, and the probability that candidate 1 wins (P_1^{sim} and P_1^{seq}). The sequential system is clearly better, both in terms of selection probability as in terms of rent dissipation.[13]

[13] The results for the simultaneous game were computed in GAMBIT, using a discretized version of the

J	1	3	5	7	9	...	19	...	49	...	99
v_J^{sim}	.504	.280	.179	.173	.155	...	.167	...	.169	...	.169
Σx^{sim}	.496	.720	.821	.827	.845		.833		.831		.831
P_1^{sim}	.545	.568	.597	.589	.572		.584		.586		.586
v_J^{seq}	.504	.389	.440	.505	.571	...	.888	...	1.00	...	1.00
Σx^{seq}	.496	.611	.560	.495	.429		.111		0.00		0.00
P_1^{seq}	.545	.615	.692	.754	.810		.970		1.00		1.00

Table 3: Equilibrium outcomes with asymmetric campaign efficiencies ($\alpha = 1$, $\psi = 1.2$)

While the probability that the better candidate wins stays below 60% in a simultaneous system, the corresponding probability approaches 1 in the sequential system, if the sequence is long enough to make up for possible initial losses. In equilibrium, both candidates behave in the beginning as if the better candidate had an advantage in terms of districts won. Both candidates spend only a small amount, with candidate 1 spending relatively more than candidate 2. Since candidate 1 also has a campaign efficiency advantage, he is very likely to win the first election. If he wins the first election, then a momentum effect similar to the one in the symmetric case occurs. As seen in Table 3, the stronger candidate wins with probability close to one, and at a very low overall cost.

It is of course possible, albeit unlikely, that candidate 1 loses in some early districts. If this happens, it has the effect of reducing the ex-ante asymmetry of the contest: While candidate 1 is still the stronger campaigner, candidate 2 has had a headstart. In order to win the nomination now, candidate 1 must win a larger number of the remaining districts than candidate 2, offsetting his efficiency advantage. The fight gets harder as both candidates increase their expenditures, but it is not the case that the stronger candidate gives up (provided that the sequence is long enough to make up later for early losses).

For example, consider the case $J = 19$, and the same values for α and ψ used as in Table 3. In the very first district, candidate 1 spends $x_{1,0} = .052$ and candidate 2 spends $y_{1,0} = .006$, so that the chance that candidate 1 wins the first district is more than 91%. However, if candidate 2 happens to win the first district, then in the second election they will spend $x_{2,0} = .189$ and $y_{2,1} = .129$, so costly campaigning is observed once the stronger campaigner has fallen behind. Should candidate 2 win also in the second election (which now has a chance of more than 36%), the spending profile in the third election is $x_{3,0} = .056$ and $y_{3,2} = .173$. After a third win spending declines very rapidly, and in all likelihood, candidate 2 will be carried by the momentum effect to win the nomination. However, the likelihood of this path that leads to the nomination of the wrong candidate is relatively small.

Finally, a word of caution regarding the interpretation of the results is in order. We

continuous game with 51 possible values between 0 and $1/J$ for x and y. The slight discrepancy between the theoretical limit result in Proposition 6 (i.e., v_J^{sim} should be 1/6) and the results in Table 3 with a large number of districts, e.g. $J = 99$, is due to discretization.

have assumed in this section that candidates' campaign effectiveness is common knowledge, and this may very well be an important assumption. It is conceivable that, if candidates don't know their own and their opponent's effectiveness, a simultaneous election may be good in selecting the right candidate. Suppose, for instance, that uncertainty about effectiveness is so large that there is a symmetric pure strategy equilibrium in the simultaneous election game with many districts.[14] In this case, the candidate who turns out to be the better campaigner wins almost certainly in a simultaneous primary system.

On the other hand, the outcome in a sequential system is not obvious. Results in early districts lead to updating about the effectiveness of candidates, and it may be that this learning goes (at least initially) in the wrong direction: Early victories of the inefficient campaigner may lead to the belief that the inefficient campaigner is actually more efficient, and then a momentum effect may develop that carries the inefficient campaigner to victory.[15] The learning dynamics make the model very complicated to analyze, but a different result with respect to the selection properties of the simultaneous and sequential system is at least conceivable.

6 Discussion and Relation to Stylized Facts

Our model sheds light on a number of stylized facts regarding the nomination process, such as the existence of campaign momentum and the allocation of a large share of campaign funds to early primary states. In this section, we relate the predictions of our model to these stylized facts. We discuss the tendency of many states to move their primary dates up in the calendar, as well as alternative explanations of the stylized facts.

6.1 Momentum Effects

Most observers and political analysts agree that "momentum effects" exist in primary races and are important for the determination of the winner. It is less clear how exactly such momentum emerges. This paper provides an answer to this question: The candidate who is (through pure luck in our basic model) successful in the first few primary elections is very likely to be successful in later primaries, too, even though both candidates have the same technology of converting money or effort into electoral success. Thus, in our model early successes are a good predictor of who will eventually win the nomination.

Table 4 gives an overview of the ten races between 1976 and 2004 in which no competi-

[14] Assume, for example, that candidate 1's ψ is unknown and drawn from a uniform distribution on $[0, 2]$. Candidate 2's effectiveness is still normalized to 1. In this case, there is a unique pure strategy equilibrium in which both candidates spend 1/4.

[15] On the other hand, if the number of districts is sufficiently big, almost perfect information about campaign effectiveness is probably obtained relatively early, and then our results from this section apply, so that sequential elections also choose the correct candidate in the limit. However, it is at least conceivable that simultaneous elections could outperform sequential ones for an intermediate number of districts.

tor was sitting U.S. president.[16] For each race, we indicate the New Hampshire winner and the eventual nominee. The last two columns contain the percentage of primaries won by the eventual nominee for early primaries (February and March) and late primaries (April and May).

Primary	NH Winner		Nominee	Feb + Mar	Apr + May
D-1976	Carter	(28%)	Carter	0.67	0.60
R-1980	Reagan	(50%)	Reagan	0.73	0.86
D-1984	Hart	(37%)	Mondale	0.40	0.42
D-1988	Dukakis	(36%)	Dukakis	0.36	1.00
R-1988	Bush	(38%)	Bush	0.91	1.00
D-1992	Tsongas	(33%)	Clinton	0.58	1.00
R-1996	Buchanan	(27%)	Dole	0.90	1.00
D-2000	Gore	(52%)	Gore	1.00	1.00
R-2000	McCain	(49%)	Bush	0.79	1.00
D-2004	Kerry	(39%)	Kerry	0.92	0.90

Table 4: Percentage of early vs. late primaries won by nominee

One implication of our model is that winning the first primary makes it more likely to win the nomination. Out of the ten races in Table 4, six were such that the winner of the New Hampshire primary was the eventual nominee. Although this seems hardly indicative of the existence of momentum effects described in this paper, one needs to keep in mind that in most of these races there were more than two candidates in the New Hampshire primary. To provide a feel for the number of viable NH candidates, the winners' vote shares are given in brackets. Hence, the fact that about half the nominees in Table 4 also won the New Hampshire primary means that an early win is in fact a good predictor of an overall win.

There is, of course, an obvious alternative explanation, namely that one candidate is a "better" candidate than his competitors, in the sense that he is more likely to win, in New Hampshire or elsewhere. As we have shown in the preceding section, momentum effects also emerge in the sequential game between two asymmetric candidates. Asymmetries alone (i.e. not considering strategic play) can explain why New Hampshire winners are more likely to be selected as nominees, but they do not predict momentum. That is, the ex-post probability of winning a late primary, conditional on winning the entire race, would be the same as the probability of winning an early primary. On the other hand, in our model the probability of the nominee winning an early primary is lower than the probability of winning a late primary, because the advantage in terms of districts already won is bigger later in the race than in the beginning. Therefore, a prediction that is robust to whether the candidates are equally strong or not, is that that the nominee of a

[16]Sources: Cook (2000), Washington Post 2000 presidential primary coverage, and CNN.com 2004 Democratic primary coverage.

race should win a larger fraction of the later elections than of the earlier elections. The data appear to support this momentum hypothesis: In six of the ten races in Table 4, the percentage of primaries won by the eventual nominee was higher after April 1st than before.

6.2 Campaign spending profile

As mentioned in the introduction, candidates as well as the media place a disproportionate emphasis on early primary states. For example, in the 1980 Republican primaries George Bush and Ronald Reagan allocated roughly 75% of their respective total campaign budgets to states with primaries before March 31, although these states accounted for considerably less than one fifth of the delegates to the Republican convention. Our model predicts that candidates will spend a large amount of resources whenever the race is tied or very close; in particular, expenditures will be large in early primary states (assuming candidates of equal strength). For example, for $\alpha = 0.5$ and 49 districts, each candidate's total expected campaign expenditure is 0.2739, and the expenditure in the first district alone is 0.051, or about 19% of the total expected campaign expenditure. One can also compute the number of districts after which the candidates have, on expectation, spent a fraction of 75% of their total expenditures. For example, for 49 districts and $\alpha = 1$, more than 75% of the total expenditures are allocated in the first two districts alone (4% of all districts). For $\alpha = 0.75$, the first three districts (6%), and for $\alpha = 0.5$, the first seven districts (14%) account for this share. Given that our model is stylized and does not capture many aspects that may be empirically important, we do not wish to overextent these quantitative results as a claim that we "match the data." Still, the disproportionate share of campaign resources allocated to early primary states is consistent with our model.

6.3 Movement of primary dates

Over the years, states have tended to move their primaries up to earlier dates. In 1976, the percentage of all primary voters who voted by the end of March was 22.6%. In 1992, this number had increased almost twofold, to 42.5%. In the 2004 election, 35 states, or 78% of the US population, voted before the end of March.[17] A common explanation for this competition among states for early primary dates is that each state would like to see those candidates being nominated which best represent the preferences of its residents. This explanation, of course, requires that early primary outcomes are in fact influential for the future direction of the race. Whether or not this can be the case in a theoretical model critically depends on whether candidates are included as decision making agents or not. In a model that abstracts from candidate behavior, Dekel and Piccione (2000) show that the equilibria of sequential elections are equivalent to those of simultaneous elections. Consequently, the position of any state in a sequence of elections is irrelevant for the election outcome. Our model of candidate behavior, on the other hand, suggests

[17] See http://www.fec.gov/pages/2004pdates.htm.

that there exists a motivation for moving up state primary dates, as the temporal position of elections does matter.

Beyond being consistent with observable data, however, our theory (through its normative aspects) also has policy implications concerning the optimal design of primary contests. In the preceding sections it has been shown that the cost and selection advantages of the sequential system are intricately linked to strategic momentum effects and the resulting front-loading of spending profiles. We argue that the observed tendency toward earlier election dates might in turn lead to a gradual change from sequential to essential simultaneous primaries, with all its associated disadvantages. A sequential organization basically requires that candidates can observe the outcome of an election before they commit resources to further campaigns. If elections are separated by short time intervals only, it seems unlikely that this is still a practical possibility. It can be expected that the disadvantages of simultaneous primaries, such as high rent dissipation rates and poor selection properties, would transpire in a formally sequential but temporally dense environment, too.

The problem of inefficient competition for early dates has spurred a discussion on alternative primary designs in the organization of the secretaries of state of the 50 U.S. states. One particular proposal is a system of regional primaries, in which the caucus and primary dates in Iowa and New Hampshire would remain unchanged; the other states are divided into four groups (Northeast, South, Midwest and West). All states in a group hold their primaries at the same day, starting with the first group one month after New Hampshire. The next group follows after an interval of one month, and so on. Which group goes first would rotate in a 16-year cycle. The stated reason for the cycle is fairness: Since the region with the first group primary is perceived as decisive, every state would enjoy this advantage once during the cycle.

6.4 Alternative models of primaries

Our model is not the only explanation for the momentum effects that are so typical of the U.S. presidential primary system, or the very fact that we observe sequential primaries. We will now discuss two popular alternative theories, but argue that neither one actually provides a satisfactory explanation for the existence of the sequential primary system.

One prominent alternative explanation for momentum effects is the *bandwagon theory*, which is related to models of herd behavior. It assumes that voters possess only incomplete information regarding the candidates' qualities and like to vote for the candidate whom they believe to be the best. Like our model, this simple setup generates momentum: An early victory by one candidate makes it more likely that this candidate is the better nominee, so voters in later districts become more willing to vote for him. Thus, the momentum prediction in itself is insufficient to distinguish between the model in this paper and the bandwagon story directly. However, there are other implications of the two models that are different. Most importantly, in the bandwagon model each party should prefer to hold a simultaneous primary. In a simultaneous primary, voters cannot condition their decision on previous election outcomes and therefore more citizens vote informatively. From the Condorcet jury theorem, we know that this raises the probability of a correct choice. In other words, every model that portrays momentum as consequence of herding on the part of the voters faces the challenge to explain why, if this is true, parties organize primaries in a way that leads to herding and hence imperfect information aggregation. On the other hand, in our model a sequential organization is preferable even as far as the selection of the best candidate is concerned.

An argument that is often used to explain why parties choose sequential primaries, and, in particular, why they start in small states like Iowa and New Hampshire, is the *grassroots theory* of primaries: According to this theory, parties use primary elections as a screening mechanism to see how candidates fare when directly interacting with voters. However, given that candidates in presidential elections rely heavily and increasingly on mass media for their campaigns, why should the parties put much emphasis on direct voter-candidate interaction in the primaries? With respect to candidate selection, it would be desirable that the campaign technologies used in primary campaigns resemble most closely those that are used later on in the presidential campaign. In many cases, the different skills tested in direct voter interaction and mass media appearances are correlated, but if the special nature of New Hampshire campaigning does in fact make a difference, then the grassroots effect is more likely to select the wrong candidate in a sequential than the simultaneous system. For this reason, we would argue that this feature is best interpreted as a drawback of the sequential system, rather than the reason for why parties chose it.

7 General Temporal Structures

While the basic structure of the U.S. primaries is sequential, it is not completely so: Some states hold their primaries on the same day as other states; the most important such day being "Super Tuesday." For example, in 1992, eight states (among them big states like Texas and Florida) held their primaries on that day. Typically, going into the Super Tuesday election, one candidate has a clearly established lead. The race is usually conceded by one candidate shortly afterwards.

Our basic contest model of primaries can be extended to allow for such temporal structures. Adding a simultaneous stage to an otherwise sequential race introduces an implicit threat of a fierce battle should the race still be close when this stage is reached. This threat can shorten the overall length of the race. With symmetric candidates, consider an organization in which the first two primaries are held sequentially and then, at a third period, the remaining "many" districts vote simultaneously. By "many", we mean more than $K(\alpha)$, such that, if both candidates go head-to-head into the third election period, only a mixed strategy equilibrium involving full rent dissipation exists. Assume further that, after every election period, the candidate who has fallen behind can withdraw from the race, and will do so, if (and only if) he has a zero continuation utility. If he concedes, then the remaining elections automatically go to his opponent.[18]

Consider the candidate's incentives after the first election has taken place. The best possible outcome in the second district for the first-round loser is to win and equalize the score. Then, both candidates are head-to-head in the final simultaneous election round. As we have shown, the complete rent will be dissipated in this final round, so that the continuation utility after a first-round loss will be as bad as giving up once the third round is reached, namely zero. This ensures that the candidate who loses the first election will give up, securing that the front runner does not have to spend any resources at the second and third stage. When both candidates follow this line of reasoning, they will know that, effectively, the only contested election is the first one, and that its winner will be the winner of the whole race. Consequently, they will spend $\alpha/4$ in the first district, and the loser of that first election will immediately give up. Thus, a mixed structure that starts with few sequential elections followed by a final, simultaneous round leaves even more expected rent to the candidates than a completely sequential structure.[19]

This result raises the question which temporal structure minimizes expected expenditures by the candidates. For the case of many districts we conjecture that the organizational form just presented is actually optimal. After all, the effective fight is reduced to the very first district, and it seems plausible that any other organization that induces a contest in more districts should be more expensive; however we do not have a formal proof for this conjecture. For small J, this scheme does not work since the lagging candidate does not give up before Super Tuesday. Here, the optimal organizational form can just be determined by brute force, namely by computing the equilibria numerically for all possible arrangements. No general pattern emerges, however. For $\alpha = 0.5$, the optimal organization for $J = 3, 5, 7, 9$ and 11 is completely sequential. However, for $\alpha = 1$ and $J = 7$, the optimal organization consists of sequential elections in the first 4 districts, followed by a simultaneous election in the remaining districts. For $\alpha = 1$ and $J = 9$, the optimal organization starts with 2 sequential single district elections, followed by three multi-district elections in 2, 3 and 2 districts, respectively.[20]

It should be noted that whether a mixed temporal structure is desirable also depends on whether the candidates are symmetric or not. If the candidates are asymmetric and there are sufficiently many districts, we have shown in Section 5 that a completely sequential organization selects the stronger player with probability close to one, and leaves an expected rent of close to one. Introducing a simultaneous stage to such a race is most likely to result in a welfare loss and a higher nomination probability for the weaker player.

[18]Note that adding such a withdrawal option to the completely sequential model would not change the equilibrium outcome, since every candidate has a positive continuation utility at each stage.

[19]In view of this result, it is a bit ironic that Super-Tuesday was introduced in 1988 by Southern states with the "hope that by holding their votes on the same day, they would increase the influence of the South in selecting presidential candidates and downplay the importance of the earlier New Hampshire primaries". (Source: BBC, http://news.bbc.co.uk/1/hi/in_depth/americas/2000/us_elections/glossary/q-s/652376.stm.) In our model, influence is actually shifted *to* the earlier primary states and *away* from the states that participate in Super-Tuesday.

[20]The complete numerical results for all cases mentioned can be obtained from the authors upon request.

8 Conclusion

We develop a campaigning model of primary elections in which candidates can influence their probability of winning in different districts. The equilibrium in this model has many features that are present in reality: Candidates spend a disproportionate amount in early districts as compared to late districts (or, more generally, spending is high whenever the race is close). The winners of early districts is endogenously more likely to win later districts than the loser, not because voters react to performance in previous elections, but rather because of equilibrium candidate spending behavior. In addition to reproducing these stylized facts from primary races, our model also provides a rationale for why political parties have chosen a sequential organization of primary elections: First, it induces lower expected expenditures and higher expected rents than a simultaneous structure. Moreover, if one candidate has an ex-ante advantage over the other, either in terms of campaign effectiveness or in the number of assured districts, a sequential organization selects the stronger candidate with probability close to one, provided there are sufficiently many districts.

Like every model, ours had to abstract from a number of issues that are certainly important in reality. First, we have assumed that there are only two candidates running for the nomination. While this simplifies the exposition, the qualitative features of the model remain intact with more than two candidates: Campaigning is intense whenever at least two candidates are tied, and front runners spend more than other candidates and therefore are more likely to win.

Second, in analyzing and explaining candidate behavior, we had to abstract from voting as a strategic decision. Our reduced form approach to capture the effects of campaign effort choice by the candidates prevents us from studying those informational aspects of elections that concern voters' uncertainty about candidates' qualifications. Other informational issues, however, concern the possibility of the candidates' uncertainty about each other's characteristics. For example, in the case of asymmetric campaign strength, candidates in our model are informed of the value of the parameter ψ. If they only possess a prior belief about this value, but do not know its realization, a sequential arrangement of elections would give rise to learning on the part of the candidates. In a simultaneous organization, on the other hand, no learning would take place. It is unclear how this kind of uncertainty would affect our results.

One simplification in our model, whose relaxation, we believe, would not alter our results in any fundamental way, is the assumption that all electoral districts are of equal size. The fact that New Hampshire, as the first primary state, is small compared to later states still has interesting implications. Because of the extensive campaign effort that arises in early primaries, placing small states at the beginning of the sequence appears to provide further evidence that the observed organization is indeed chosen in order to keep overall campaign costs as small as possible.

An interesting but analytically rather challenging extension of our model would be to consider candidates who face additional hard budget constraints. That is, they maximize the same objective function as in our model, but cannot spend more than a certain amount of resources.[21] Our result that total expected campaign spending is lower in a sequential structure than a simultaneous one is still very likely to hold in this new setting. More interesting, will expenditures be lower or higher, if players face hard budget constraints? The answer to this question might not be as obvious and clearcut as it appears. Che and Gale (2000) have shown that spending limits in lobbying models might actually increase the players' equilibrium spending if their valuations are asymmetric. Also, if players are asymmetric with respect to their hard budget constraint (and otherwise equal), will the candidate who has the advantage win "almost always" provided that there are many districts, as in section 5? Primary candidates often differ substantially in their spending possibilities—an example is the 2000 Republican nomination race, in which John McCain was victorious in New Hampshire, but eventually lost the race to George W. Bush who had considerably higher campaign funds. These interesting questions are left for further research.

[21]The reason why the sequential case is difficult to analyze with this extension is that the number of victories up to a certain district does not uniquely capture the state of the game, but rather the sequence, in which victories were achieved, matters. Consider the 3 district case. With symmetric candidates, both of them will spend the same in the first district, while in the second district they will spend different amounts. If the budget constraint is binding in the third district for the candidate who has less money, then candidates will spend different amounts in the third district. Hence, it matters for the spending in the third district whether candidate 1 or 2 won the first district.

Appendix A: Proofs

The order of proofs is as follows. First, we establish some technical results, to be used later. Then we prove Propositions 2, 1, and 3. The reason for this order is that the proof of Proposition 1 depends on Proposition 2.

Preliminaries

Lemma 1. *Suppose player 2 plays a (pure or mixed) uniform strategy. Then, any non-uniform pure strategy $x = (x_1, x_2, \ldots, x_J)$ is dominated for player 1 by the uniform strategy with the same total expenditure, $X = (\frac{\sum x_i}{J}, \frac{\sum x_i}{J}, \ldots, \frac{\sum x_i}{J})$.*

Lemma 2. *If Λ is a symmetric UCE in G_J^{sim}, then* $\inf supp(\Lambda) > 0$.

Lemma 3. *The function F, as defined in (3) satisfies the following monotone likelihood ratio property:*

$$\frac{F(x', y')}{F(x, y')} > \frac{F(x', y)}{F(x, y)}$$

for all $x' > x$ and $y' > y$.

Lemma 4. *Suppose $J > K(\alpha)$. For each $x > 0$ there exists $\vartheta(x) > 1$ such that*

$$\frac{F(x', x)}{F(x, x)} > \frac{x'}{x},$$

for all $x' \in (x, \vartheta(x)x)$. Furthermore, let Λ be a symmetric UCE. There exists a constant $\bar{\vartheta} > 1$ such that $\vartheta(x) \geq \bar{\vartheta}$ for all $x \in supp(\Lambda)$.

Proof of Lemma 1. Fix player 2's strategy to be a uniform pure strategy with $y_j = y$ for all j. Suppose that player 1 plays, with positive probability, some pure strategy with $x_k > x_l$ for some districts k and l. Consider a deviation which leaves expenditures in all other districts unchanged and equates the campaign levels in k and l so that total expenditures do not change:

$$\widetilde{x}_k = \widetilde{x}_l = (x_k + x_l)/2 \;\equiv\; \widetilde{x}.$$

Let Q_n, $n \in \{0, 1, 2\}$, denote the probability that player 1 wins exactly n districts among the two districts k and l, when using strategy x. Similarly, let $\widetilde{Q}_n$ denote the probability of winning exactly n districts among the two districts k and l, when using strategy $\widetilde{x}$. Finally, let P_n, $n \in \{0, \ldots, J-2\}$, be the probability of winning *exactly* n out of the remaining $J - 2$ districts.

Player 1's gain from changing to the new strategy is

$$\Delta Eu_1 = E\Big\{(\widetilde{Q}_1 + \widetilde{Q}_2 - Q_1 - Q_2)P_{J^*-1} + (\widetilde{Q}_2 - Q_2)P_{J^*-2}\Big\},$$

where the expectation is taken with respect to y. Since P_{J^*-1} and P_{J^*-2} do not change when switching from strategy x to $\widetilde{x}$, a sufficient condition for ΔEu_1 to be positive is that both $E\Big\{(\widetilde{Q}_1 + \widetilde{Q}_2 - Q_1 - Q_2)\Big\} \geq 0$ and $E\Big\{\widetilde{Q}_2 - Q_2\Big\} \geq 0$.

Since x^α is concave due to $\alpha \in (0, 1]$, we have $\widetilde{x}^\alpha - x_l^\alpha > x_k^\alpha - \widetilde{x}^\alpha$ and $(\widetilde{x}^\alpha)^2 > x_k^\alpha x_l^\alpha$. Therefore,

$$E(\widetilde{Q}_1 + \widetilde{Q}_2 - Q_1 - Q_2) = E\left(y^{2\alpha}\frac{y^\alpha(\widetilde{x}^\alpha - x_k^\alpha + \widetilde{x}^\alpha - x_l^\alpha) + (\widetilde{x}^\alpha)^2 - x_k^\alpha x_l^\alpha}{(\widetilde{x}^\alpha + y^\alpha)(\widetilde{x}^\alpha + y^\alpha)(x_k^\alpha + y^\alpha)(x_l^\alpha + y^\alpha)}\right) > 0$$

and

$$E(\widetilde{Q}_2 - Q_2) = E\left(y^\alpha \widetilde{x}^\alpha \frac{x_k^\alpha(\widetilde{x}_l^\alpha - x_l^\alpha) - x_l^\alpha(x_k^\alpha - \widetilde{x}_k^\alpha)}{(\widetilde{x}_k^\alpha + y^\alpha)(\widetilde{x}_l^\alpha + y^\alpha)(x_k^\alpha + y^\alpha)(x_l^\alpha + y^\alpha)}\right) > 0.$$

This shows that strategy $\widetilde{x}$ dominates strategy x.

To prove that the uniform strategy X dominates x,consider the following algorithm: Step 1: Start with x, and select the two districts with the greatest and the smallest spending (say, k and l). Step 2: Replace the spending levels in these districts by their mean, $(x_k + x_l)/2$; by the arguments presented above, this strategy will be better for player 1 than the initial strategy. Step 3: If the new strategy profile $(\widetilde{x})$ is not yet uniform, go back to Step 1 and repeat the procedure for the two districts with the greatest and the smallest spending level under $\widetilde{x}$. Clearly, this algorithm converges to the uniform strategy X, and since utility increases with every cycle, it is proved that the uniform strategy X dominates x.

Finally, since the preceding argument was independent of y, we conclude that strategy x is dominated by the strategy X also when player 2 randomizes over pure uniform strategies. □

Proof of Lemma 2. First, observe that 0 cannot be played with positive probability, because otherwise, a player could increase his expected payoff strictly by shifting weight from 0 to a sufficiently small, but positive number a. Second, if 0 is not played with positive probability, then

$$\left.\frac{\partial Eu\left(x \mid \Lambda\right)}{\partial x}\right|_{x=0} = -J < 0,$$

so that, by continuity, positive campaign levels very close to zero are dominated by a zero bid. Hence, $\inf supp(\Lambda) > 0$. □

Proof of Lemma 3. We have

$$\frac{F\left(x',y\right)}{F\left(x,y\right)} = \frac{F(x,y) + \int_x^{x'} F_x(t,y)dt}{F(x,y)} = 1 + \int_x^{x'} \frac{F_x(t,y)}{F(x,y)}dt.$$

Similarly, $\frac{F\left(x',y'\right)}{F\left(x,y'\right)} = 1 + \int_x^{x'} \frac{F_x\left(t,y'\right)}{F\left(x,y'\right)}dt$. Thus, it is sufficient to show that F_x/F increases in y, or equivalently

$$F_{xy}F - F_xF_y > 0. \tag{13}$$

Differentiating F with respect to x, we obtain

$$\begin{aligned}
F_x &= \sum_{k=J^*}^{J} \binom{J}{k} \left[\alpha k \frac{x^{\alpha k - 1} y^{\alpha(J-k)}}{\left(x^\alpha + y^\alpha\right)^J} - J\alpha x^{\alpha - 1} \frac{x^{\alpha k} y^{\alpha(J-k)}}{\left(x^\alpha + y^\alpha\right)^{J+1}}\right] \\
&= \frac{\alpha}{\left(x^\alpha + y^\alpha\right)^{J+1}} \sum_{k=J^*}^{J} \binom{J}{k} \left[k x^{\alpha k - 1} y^{\alpha(J-k+1)} - (J-k)\, x^{\alpha(k+1)-1} y^{\alpha(J-k)}\right] \\
&= \frac{\alpha}{\left(x^\alpha + y^\alpha\right)^{J+1}} \left[\binom{J}{J^*} J^* x^{\alpha J^* - 1} y^{\alpha(J-J^*+1)} - \binom{J}{J^*} (J - J^*)\, x^{\alpha(J^*+1)-1} y^{\alpha(J-J^*)}\right. \\
&\quad + \binom{J}{J^*+1} (J^*+1)\, x^{\alpha(J^*+1)-1} y^{\alpha(J-J^*)} - \binom{J}{J^*+1} (J - J^* - 1)\, x^{\alpha(J^*+2)-1} y^{\alpha(J-J^*-1)} \\
&\quad \vdots \\
&\quad \left. + \binom{J}{J} J x^{\alpha J - 1} y^{\alpha J^*} - 0\right] \\
&= \frac{\alpha J^*}{\left(x^\alpha + y^\alpha\right)^{J+1}} \binom{J}{J^*} x^{\alpha J^* - 1} y^{\alpha J^*}.
\end{aligned} \tag{14}$$

(The second and third term in the summation cancel out, the fourth and fifth, etc.) In a similar way, we get

$$F_y = -\frac{\alpha J^*}{\left(x^\alpha + y^\alpha\right)^{J+1}} \binom{J}{J^*} x^{\alpha J^*} y^{\alpha J^* - 1}. \tag{15}$$

Differentiating (14) with respect to y yields

$$F_{xy} = \frac{(\alpha J^*)^2}{(x^\alpha + y^\alpha)^{J+2}} \binom{J}{J^*} (xy)^{\alpha J^* - 1} (x^\alpha - y^\alpha). \tag{16}$$

Using (14)–(16), we can rewrite (13) as

$$\begin{aligned} F_{xy}F - F_x F_y = {} & \frac{(\alpha J^*)^2}{(x^\alpha + y^\alpha)^{J+2}} \binom{J}{J^*} (xy)^{\alpha J^* - 1} (x^\alpha - y^\alpha) \cdot \sum_{k=J*}^{J} \binom{J}{k} \frac{x^{\alpha k} y^{\alpha(J-k)}}{(x^\alpha + y^\alpha)^J} \\ & + \frac{\alpha J^*}{(x^\alpha + y^\alpha)^{J+1}} \binom{J}{J^*} x^{\alpha J^* - 1} y^{\alpha J^*} \cdot \frac{\alpha J^*}{(x^\alpha + y^\alpha)^{J+1}} \binom{J}{J^*} x^{\alpha J^*} y^{\alpha J^* - 1}. \end{aligned}$$

Collecting common terms, this expression reduces to

$$\begin{aligned} & \frac{(\alpha J^*)^2}{(x^\alpha + y^\alpha)^{2(J+1)}} \binom{J}{J^*} (xy)^{\alpha J^* - 1} \left[(x^\alpha - y^\alpha) \cdot \sum_{k=J*}^{J} \binom{J}{k} x^{\alpha k} y^{\alpha(J-k)} + \binom{J}{J^*} x^{\alpha J^*} y^{\alpha J^*} \right] \\ = {} & \frac{(\alpha J^*)^2}{(x^\alpha + y^\alpha)^{2(J+1)}} \binom{J}{J^*} (xy)^{\alpha J^* - 1} x^{\alpha(J+1)} > 0. \end{aligned}$$

□ □

Proof of Lemma 4. To prove the first statement, rewrite the inequality in the Lemma as

$$\frac{F(x', x)}{x'} > \frac{F(x,x)}{x}.$$

We will prove that

$$\frac{\partial}{\partial x} \left. \frac{F(x,y)}{x} \right|_{y=x} > 0. \tag{17}$$

From this, the result follows because the left-hand side of (17) is continuous in x for all $x > 0$. To prove (17), we show that $xF_x(x,x) - F(x,x) > 0$. Using (14) and $F(x,x) = \frac{1}{2}$, we get

$$\begin{aligned} xF_x(x,x) - F(x,x) & = \frac{\alpha J^*}{(2x^\alpha)^{J+1}} \binom{J}{J^*} x^{2\alpha J^*} - \frac{1}{2} \\ & = 2^{-(J+1)} \frac{\alpha J!}{(J^* - 1)!^2} - \frac{1}{2} > 0, \end{aligned}$$

which is true, because $\frac{\alpha J!}{(J^*-1)!^2} > 2^J$ by Lemma 1 for $J > K(\alpha)$.

To prove the second statement, recall that $\inf supp(\Lambda) > 0$ by Lemma 1, and $x \notin supp(\Lambda)$ if $x > 1$. Hence, there exists a compact set $W \subset (0,1]$ such that $supp(\Lambda) \subseteq W$, which bounds $\frac{\partial}{\partial x} \left. \frac{F(x,y)}{x} \right|_{y=x}$ away from zero on $supp(\Lambda)$. This implies that $\vartheta(x)$ can be chosen to be bounded away from 1 for all $x \in supp(\Lambda)$. □

Proof of Proposition 2

We first show that the right hand side of inequality (6),

$$\frac{2^J}{J\binom{J-1}{(J-1)/2}} = \frac{2^J \left(\frac{J-1}{2}!\right)^2}{J!} \tag{18}$$

is decreasing in J and goes to 0 for $J \to \infty$. Going from J to $J+2$ multiplies the numerator of the right hand side of (18) by $4\left(\frac{J+1}{2}\right)^2 = (J+1)^2$, and multiplies the denominator by $(J+1)(J+2)$. Hence, the value of the fraction decreases.

For part (a) of the Proposition, the main argument for existence is in the text. It remains to be shown that no deviation from the solution in (5) is profitable. Differentiating the objective function (2) with respect to x gives

$$\frac{\partial u_1(x,y)}{\partial x} = \binom{J-1}{\frac{J-1}{2}} \left(\frac{x^\alpha y^\alpha}{(x^\alpha + y^\alpha)^2}\right)^{\frac{J-1}{2}} \cdot \frac{\alpha y^\alpha x^{\alpha-1}}{(x^\alpha + y^\alpha)^2} - 1. \tag{19}$$

(2) is not globally concave in x. However, differentiating (19) a second time yields, after simplification,

$$\frac{x^{\alpha\frac{J+1}{2}-2} y^{\alpha\frac{J+1}{2}} \left[\alpha\frac{J+1}{2} - 1 - (J+1)\frac{x^\alpha}{x^\alpha+y^\alpha}\right]}{(x^\alpha + y^\alpha)^{J-1}}.$$

The term

$$\left[\alpha\frac{J+1}{2} - 1 - (J+1)\frac{x^\alpha}{x^\alpha + y^\alpha}\right]$$

is decreasing in x. For given y, the optimal x is therefore either zero, or the unique value so that (19) vanishes. This shows that we only need to check for a deviation from (5) to zero. Since $x = 0$ yields a zero payoff against any $y > 0$, such a deviation is not profitable as long as (6) holds.

For part (b), the argument why a pure strategy equilibrium does not exist for $J > K(\alpha)$ is given in the text. We now show that a mixed strategy symmetric UCE exists. Consider first the game that arises when we restrict players to use only uniform strategies. This game has a symmetric Nash equilibrium (hence a SUCE), because v_1 and v_2 satisfy the sufficient conditions for equilibrium existence in discontinuous games in Dasgupta and Maskin (1986). It remains to be shown that this SUCE of the restricted game is also an equilibrium of the original game, in which players are free to choose non-uniform strategies as well. By Lemma 1, if player 1 (resp. 2) plays a uniform strategy, then any non-uniform strategy is dominated for player 2 (resp. 1). If there is no profitable deviation from the equilibrium candidate using a uniform strategy, there also cannot be a profitable deviation using non-uniform strategies. The SUCE is therefore also an equilibrium of the original game.

We now prove that the rent is completely dissipated in a mixed strategy SUCE. Let $\bar{x} = \inf\{\Lambda(x) > 0\} > 0$. Assume that Λ has a density λ at $\bar{x}$, i.e. $Prob(x = \bar{x}) = 0$. This is just an assumption for convenience of notation; we will indicate in footnotes how to adjust the proof if Λ has an atom at $\bar{x}$. Since the player is willing to play $\bar{x}$ in equilibrium, it must yield utility u:

$$\int_{\bar{x}}^1 F(\bar{x}, x)d\Lambda(x) - \bar{x} = u$$

Splitting the integral on the left hand side, we get[22]

$$\lambda(\bar{x})\varepsilon F(\bar{x}, \bar{x}) + \int_{\bar{x}+\varepsilon}^1 F(\bar{x}, x)d\Lambda(x) - \bar{x} + O(\varepsilon^2) = u$$

where $O(\varepsilon^2)$ is a second order term ignored in the following.[23] Hence, we can solve for

$$\lambda(\bar{x})\varepsilon = \frac{u + \bar{x} - \int_{\bar{x}+\varepsilon}^1 F(\bar{x}, x)d\Lambda(x)}{F(\bar{x}, \bar{x})} \tag{20}$$

[22]If Λ has an atom of size λ_0 at $\bar{x}$, we would have to replace $\lambda(\bar{x})\varepsilon$ by λ_0 in the following formula.

[23]When we choose ε sufficiently small, the first order effects derived in the following will dominate any second order effect.

Playing $\bar{x}+\epsilon$ cannot give a candidate a higher utility than u, so again by splitting the integral and ignoring second order effects we have

$$\lambda(\bar{x})\varepsilon F(\bar{x}+\varepsilon,\bar{x}) + \int_{\bar{x}+\varepsilon}^{1} F(\bar{x}+\varepsilon,x)d\Lambda(x) - (\bar{x}+\varepsilon) \leq u \tag{21}$$

Substituting from (20) and rearranging, we have

$$\int_{\bar{x}+\varepsilon}^{1}\left[F(\bar{x}+\varepsilon,x) - \frac{F(\bar{x}+\varepsilon,\bar{x})}{F(\bar{x},\bar{x})}F(\bar{x},x)\right]d\Lambda(x) \leq u+\bar{x}+\varepsilon - \frac{F(\bar{x}+\varepsilon,\bar{x})}{F(\bar{x},\bar{x})}(u+\bar{x}) \tag{22}$$

The integrand on the left hand side is positive by Lemma 3, and the right hand side is smaller than $-\frac{\varepsilon u}{\bar{x}}$ (using Lemma 4), which is negative and a first order term if $u > 0$. This shows that (22) cannot hold for positive u, and so $u = 0$. $\square$

Proof of Proposition 1

We organize the proof in a sequence of three steps. First, we show that the symmetric pure strategy UCE is the only pure strategy equilibrium. Next, we show that when $J > K(\alpha)$ all equilibria are mixed strategy UCE, and all have the same payoffs as the symmetric mixed strategy SUCE, namely zero. Step 2 proves the result for $J > K(\alpha)$. Finally, we show that when $J \leq K(\alpha)$, no mixed strategy equilibria exist. Together with Step 1, this establishes the result for $J \leq K(\alpha)$.

We use the following notation: Given a (not necessarily uniform) strategy $p \in \Delta$ (where Δ is the unit simplex), let $E(p)$ denote the expected expenditure of strategy p. Given a strategy profile $(p,q) \in \Delta \times \Delta$ we let $F(p,q)$ be the probability of winning when using p against q (so $F(p,q) = 1 - F(q,p)$, and $F(p,p) = \frac{1}{2}$ by symmetry). Further, we let $u(p,q) = F(p,q) - E(p)$ be the expected payoff from playing p against q (so that $u(p,q) + u(q,p) = 1 - E(p) - E(q)$).

Step 1. Consider a pure equilibrium with spending profiles (x,y); these profiles need not be uniform. Fix some district j and let the spending profile in the other districts be denoted x_{-j}, y_{-j}. Given (x_{-j}, y_{-j}), let $\widetilde{P}_j$ be the probability that district j is pivotal. Note that this probability is the same for both candidates. Since (x,y) is an equilibrium, it must be true that x_j maximizes

$$\frac{x_j^\alpha}{x_j^\alpha + y_j^\alpha}\widetilde{P}_j - x_j,$$

which yields the first-order condition

$$\frac{\alpha x_j^{\alpha-1} y_j^\alpha}{\left(x_j^\alpha + y_j^\alpha\right)^2}\widetilde{P}_j - 1 = 0. \tag{23}$$

A similar first-order condition is obtained for candidate 2:

$$\frac{\alpha y_j^{\alpha-1} x_j^\alpha}{\left(x_j^\alpha + y_j^\alpha\right)^2}\widetilde{P}_j - 1 = 0. \tag{24}$$

To satisfy (23) and (24) simultaneously, we need $x_j = y_j$, so that district j is won with equal probability by either candidate, regardless of the spending profile in the other districts. Since this reasoning can be applied to all districts, each district is won with probability 1/2, and consequently

$$\widetilde{P}_j = \widetilde{P} = \binom{J-1}{J^*-1}\left(\frac{1}{2}\right)^{J-1} \quad \forall j.$$

Equations (23) and (24) can now be solved uniquely for

$$x_j = y_j = \frac{1}{4}\alpha\binom{J-1}{J^*-1}\widetilde{P},$$

which coincides with (5). Hence, the only pure strategy equilibria must be symmetric UCE.

Step 2. Focus now on $J > K(\alpha)$. Condition (6) together with the argument in Step 1 implies that only mixed strategy equilibria are possible. Suppose $(p, q) \in \Delta \times \Delta$ is some equilibrium of G_J^{sim}, not necessarily symmetric and not necessarily in uniform campaign strategies. Notice that (q, p) must then be an equilibrium with reversed payoffs. We will show that p and q are uniform campaign strategies, and that $u(p, q) = u(q, p) = 0$.

Let $(x, x) \in \Delta \times \Delta$ be a uniform SUCE, which exists as shown in Proposition 2 (b). We also know from Proposition 2 that $u(x, x) = 0$, and since $F(x, x) = \frac{1}{2}$ it must be that $E(x) = \frac{1}{2}$. We now gather a number of facts. Since (p, q) is an equilibrium, it is not beneficial for player 1 to deviate to q, or for player 2 to deviate to p:

$$u(p,q) \geq \frac{1}{2} - E(q) = u(q,q) \quad \text{and} \quad u(q,p) \geq \frac{1}{2} - E(p) = u(p,p). \tag{25}$$

Since $u(p, q) + u(q, p) = 1 - E(p) - E(q)$, the conditions in (25) hold with equality:

$$u(p,q) = \frac{1}{2} - E(q) = u(q,q) \quad \text{and} \quad u(q,p) = \frac{1}{2} - E(p) = u(p,p). \tag{26}$$

Hence, q is an optimal response to q and p is an optimal response to p, so that (p, p) and (q, q) are also equilibria.

Also, we know that in equilibrium (p, q), it is not beneficial for player 1 or 2 to deviate to x:

$$u(p,q) \geq F(x,q) - \frac{1}{2} = u(x,q) \quad \text{and} \quad u(q,p) \geq F(x,p) - \frac{1}{2} = u(x,p). \tag{27}$$

Finally, since (x, x) is an equilibrium, it is not beneficial for either player to deviate to q or p:

$$u(x,x) = 0 \geq F(q,x) - E(q) = u(q,x) \quad \text{and} \quad u(x,x) = 0 \geq F(p,x) - E(p) = u(p,x). \tag{28}$$

Using (26) in (27), we obtain the following conditions:

$$1 - E(q) \geq F(x,q) \quad \text{and} \quad 1 - E(p) \geq F(x,p). \tag{29}$$

Using (28), together with the fact that $F(q, x) = 1 - F(x, q)$, yields

$$1 - E(q) \leq F(x,q) \quad \text{and} \quad 1 - E(p) \leq F(x,p). \tag{30}$$

Hence, the weak inequalities in (29) and (30) must hold with equality:

$$1 - E(q) = F(x,q) \quad \text{and} \quad 1 - E(p) = F(x,p). \tag{31}$$

Now consider the strategy profile (q, x). Player 1's expected payoff in (q, x) is

$$u(q,x) = F(q,x) - E(q) = 1 - F(x,q) - E(q) = 0, \tag{32}$$

by (31). Since x is a uniform campaign strategy and $u(x, x) = 0$, if q was not uniform, then $u(q, x) < 0$ by Lemma 1, so q must be uniform. Similarly, p must be uniform. The four profiles (p, q), (q, p), (p, p), and (q, q) are therefore UCE. Since (q, q) and (p, p) are symmetric UCE in mixed strategies, they must involve full rent dissipation by Proposition 2 (b): $u(p, p) = u(q, q) = 0$. By (26), $u(p, q) = 0$, and since (p, q) is an arbitrary equilibrium, we conclude that, when $J > K(\alpha)$, every equilibrium in G_J^{sim} must be a UCE in mixed strategies and involve full rent dissipation.

Step 3. Now consider the case $J \leq K(\alpha)$. We need to show that the symmetric pure strategy UCE is the only equilibrium. We already know (from Step 1) that it is the only pure equilibrium. Suppose that a mixed strategy equilibrium (p, q) exists. Repeating the arguments in Step 2, (p, p)

must then also be a mixed equilibrium (but since $J \leq K(\alpha)$ we cannot conclude that $u(p,p) = 0$.) For (x,x) and (p,p) to be equilibria, we need

$$u(x,x) = \frac{1}{2} - E(x) \geq F(p,x) - E(p) = u(p,x) \quad \text{and} \quad u(p,p) = \frac{1}{2} - E(p) \geq F(x,p) - E(x) = u(x,p). \tag{33}$$

Adding both conditions in (33), $F(p,x) = 1 - F(x,p)$ and substituting $F(x,p) = 1 - F(p,x)$, we have $E(p) + E(x) \geq E(p) + E(x)$. Thus, (33) must hold with equality. This implies that mixed strategy p is a best reply to uniform pure strategy x, and by Lemma 1, p must be uniform. But since $J \leq K(\alpha)$, the same argument as given in the proof of Proposition 2 (a) shows that p must be a pure strategy. The same holds for q, and by Step 1 $p = q = x$. □

Proof of Proposition 3

The main arguments are in the text. It remains to be shown that

(i) $v_{j,k} \geq v_{j,k-1}$

(ii) $v_{j,k} \geq v_{j+1,k}$

Let $\Delta_{j+1,k} \equiv v_{j+1,k+1} - v_{j+1,k}$. Write the continuation value at state (j,k) as

$$v_{j,k} = \max_{x \geq 0} \left(v_{j+1,k} + f(x,y)\Delta_{j+1,k} - x \right), \tag{34}$$

letting $v_{j,k} = 1$ for all $j \geq J^*$ and $k \geq J^*$, and $v_{j,k} = 0$ for all $j \geq J^*$ and $k \leq j - J^*$.

We will prove (i) by induction on j. Obviously (i) is true for $j = J+1$ and all k, as $v_{J+1,k} = 1$ for $k \geq J^*$, and $v_{J+1,k} = 0$ for $k < J^*$. Assuming (i) is true for some $j+1$, we have $\Delta_{j+1,k} \geq 0$. We will now show that (i) also holds for j. Since $x = 0$ is a feasible choice on the right hand side of (34),

$$v_{j,k} \geq v_{j+1,k} + f(0,y)\Delta_{j+1,k} \geq v_{j+1,k}. \tag{35}$$

Next, observe that for all $x \geq 0$ we have

$$v_{j+1,k-1} + f(x,y)\Delta_{j+1,k-1} - x \leq v_{j+1,k-1} + \Delta_{j+1,k-1} = v_{j+1,k}.$$

For the maximum of the left-hand side taken over $x \geq 0$, it must therefore be true that

$$v_{j,k-1} \leq v_{j+1,k} \tag{36}$$

Combining (35) and (36), we obtain that $v_{j,k} \geq v_{j+1,k} \geq v_{j,k-1}$, so that $v_{j,k} \geq v_{j,k-1}$, proving (i) for j. This completes the proof by induction for (i), and (ii) then follows immediately from inequality (35). □

Proof of Proposition 4

Let $X = (1-\psi)Jx$ and $Y = (1-\psi)Jy$ be the total expenditures of candidate 1 and 2, respectively. Let the players' equilibrium strategies be given by the distributions Φ_1 over X and Φ_2 over Y (with densities ϕ_1 and ϕ_2). The same arguments as in symmetric all-pay auctions (see Baye et al. (1990)) imply that players' equilibrium strategies cannot have atoms—except possibly at 0 for a candidate with an ex ante expected payoff of 0—, and that Φ_1 and Φ_2 must be strictly increasing on $[0, (1-2\psi)^{1/\alpha}]$ and $[0,1]$, respectively. For every $Y \in [0,1]$, player 2's expected payoff is

$$Eu_2(\Phi_1, Y) = \Phi_1((1-2\psi)^{1/\alpha} Y) - Y.$$

Taking first-order conditions, we have $\phi_1(X) = (1-2\psi)^{-1/\alpha}$ on the interval $[0, (1-2\psi)^{1/\alpha}]$. Similarly, player 1's expected payoff is

$$Eu_1(X, \Phi_2) = \Phi_2((1-2\psi)^{-1/\alpha} X) - X,$$

and taking first-order conditions we get $\phi_2(Y) = (1-2\psi)^{1/\alpha} < 1$ on $(0,1]$. Thus there must be an atom on 0 for player 2, so that $\Phi_2(Y) = (1-(1-2\psi)^{1/\alpha}) + (1-2\psi)^{1/\alpha}Y$ for $Y \in [0,1]$. Since player 2 is willing to play an atom on 0, where he is certain to lose, his ex ante rent must be 0. If player 1 chooses $X = 0$, he wins if and only if player 2 chooses $Y = 0$. This happens with probability $1-(1-2\psi)^{1/\alpha}$. Therefore, $1-(1-2\psi)^{1/\alpha}$ is player 1's expected rent in equilibrium. The equilibrium probability that player 1 wins the nomination is then

$$\begin{aligned} \text{Prob}[Y < X(1-2\psi)^{-1/\alpha}] &= \int_0^{(1-2\psi)^{1/\alpha}} \int_0^{X(1-2\psi)^{-1/\alpha}} \phi_2(Y)dY\phi_1(X)dX \\ &= 1-\frac{(1-2\psi)^{1/\alpha}}{2}. \end{aligned}$$

This probability is greater than 1/2 and increasing in ψ, but it is always smaller than 1, even with a very large number of electoral districts. □

Proof of Proposition 6

We proceed analogous to the proof of Proposition 4. Let $X = Jx$ and $Y = Jy$ be the total expenditures of candidate 1 and 2, respectively, and let Φ_1 and Φ_2 be the distributions of X and Y, with densities ϕ_1 and ϕ_2. Player 2's expected payoff is

$$Eu_2(\Phi_1, Y) = \Phi_1(Y/\psi) - Y.$$

Taking first-order conditions, we have $\phi_1(X) = \psi$ on the interval $[0, 1/\psi]$. Similarly, player 1's expected payoff is

$$Eu_1(X, \Phi_2) = \Phi_2(\psi X) - X,$$

and taking first-order conditions we get $\phi_2(Y) = 1/\psi$ on $(0,1]$, with mass $1-1/\psi$ at zero. Since player 2 is willing to play an atom on 0, where he is certain to lose, his ex ante rent must be 0. If player 1 chooses $X = 0$, he wins if and only if player 2 chooses $Y = 0$. This happens with probability $1-1/\psi$, which then must be player 1's expected rent in equilibrium, so total campaign expenditures. The equilibrium probability that player 1 wins the nomination is then

$$\begin{aligned} \text{Prob}[Y < \psi X] &= \int_0^{1/\psi} \frac{\psi-1}{\psi} + \int_0^{\psi X} \frac{1}{\psi}\phi_2(Y)dY\phi_1(X)dX \\ &= 1-\frac{1}{2\psi}. \end{aligned}$$

This probability is greater than 1/2 and increasing in ψ, but it is always smaller than 1, even with a very large number of electoral districts. □

Appendix B: Limit Behavior in Sequential Contests

In this section, we explore theoretically the equilibria in nomination races with many districts, i.e. we examine the limit behavior of primary races as J becomes large. In particular, we are interested in the issues of rent dissipation and momentum effects. As we have already argued in the main text, it is difficult to characterize in closed form the equilibrium strategies in the sequential contest when there are more than just a few elections. While a strategic momentum effect and a lower payoff bound exists in our computations, an analytic proof is surprisingly difficult.

Here, we partly resolve these issues and focus on the asymptotics of the model as $J \to \infty$. We consider only the case $\alpha = 1$. First, we need some notation. In the primary race with J districts, we let $v^J_{j,k}$ and $x^J_{j,k}$ be the continuation payoff and equilibrium expenditure in state (j,k). Consider the continuation utility of a player who has won k of the first $j-1$ elections. Since his opponent has won $j - 1 - k$ elections, the first player has won $k - (j - 1 - k) = 2k + 1 - j$ elections more than his opponent.

Our main result can now be stated:

Proposition 7. *Consider the sequential primary game for* $\alpha = 1$ *as* $J \to \infty$.

(i) The value of the game, v_J*, does not converge to zero.*

(ii) In every district, the front runner spends more than his opponent, and is thus more likely to win another district. Formally, let $i = 2k + 1 - j$*. If* $i > 0$*, then* $\lim_{J\to\infty} x_i^J > \lim_{J\to\infty} x_{-i}^J$.

The b-advantage game

To prove Proposition 7, we analyze a variant of our original game, called the b-advantage game:

Definition 2. Let b be a positive integer. A *b-advantage game* is defined as follows: The campaign technology in each district is given by (1). District elections are held sequentially, and the game ends if and only if a candidate has established a lead of b districts. The winner obtains a prize of 1, while the other candidate obtains a prize of zero.

The b-advantage game is motivated by the numerical results. As reported in the last section, most campaign expenditures take place when both candidates are close together in terms of their victories in previous districts. Once a candidate has secured an advantage of several victories, his opponent spends almost no resources, so that securing further victories is very cheap for the front runner. This is true even if there are still very many periods to go and the front runner's advantage is relatively small in comparison to J^*, the number of districts sufficient to secure overall victory. The b-advantage game takes this story to the limit: Candidates compete until one of them has secured b more victories than the other candidate. Once such a lead is established, the contest ends.

The main reason for looking at the b-advantage game is that its stationary structure makes it analytically simpler compared to the original model. A property of the b-advantage game that we exploit in the following is that it has a Markov perfect equilibrium which is relatively easy to characterize. For sufficiently large b, the equilibrium of the b-advantage game will be similar to the equilibrium of the original game with large J: At those states where both games are defined, spending levels will be similar. At states where only the original game is defined (i.e., where one candidate has a lead of more than b districts), spending levels will be close to zero in the original game, and the front runner wins the next district with a probability that is close to one.

To formalize this link between our original game and the b-advantage game, consider a stationary equilibrium in the b-advantage game, and let w_i^b be the continuation utility of a candidate who has won i elections more than his competitor. We call i the state of the game; the initial value of the game is w_0^b. Furthermore, let x_i^b be the campaign expenditure by the front runner in state i. Recall that state j, k in the original game means that a candidate has won k out of $j - 1$ elections. Thus, the corresponding state in the b-advantage game is $i = 2k + 1 - j$. We then have the following result:

Lemma 5. *Suppose* $\alpha = 1$*. Suppose further that all* $v_{j,k}^J$ *and* $x_{j,k}^J$ *converge as* $J \to \infty$*. The b-advantage game has a unique Markov perfect equilibrium such for all* j, k,

$$\lim_{J\to\infty} v_{j,k}^J = \lim_{b\to\infty} w_{2k+1-j}^b,$$

and

$$\lim_{J\to\infty} x_{j,k}^J = \lim_{b\to\infty} x_{2k+1-j}^b.$$

Using Lemma 5, the following result proves Proposition 7:

Proposition 8. *Consider the b-advantage game for $\alpha = 1$.*

(i) As $b \to \infty$, $\lim_{b\to\infty} w_0^b > \frac{2\sqrt{33}-10}{27\sqrt{33}-147} \approx 0.18377$.

(ii) If $b > i > 0$, then $x_i^b > x_{-i}^b$.

The remainder of the Appendix is devoted to proving Lemma 5 and Proposition 8. The following result will be used in both proofs. Consider a Markov perfect equilibrium of the b-advantage game.

Lemma 6. *Let $i < b' < b$. The equilibrium winning probabilities in state i are equal in the b-advantage game and the b'-advantage game.*

Proof. For given b, the continuation values in the b-advantage game are linked by

$$w_i^b = w_{i-1}^b + (w_{i+1}^b - w_{i-1}^b)\frac{(x_i^b)^\alpha}{(x_i^b)^\alpha + (y_i^b)^\alpha} - x_i^b \text{ for all } i \geq 0, \tag{37}$$

$$w_{-i}^b = w_{-(i+1)}^b + (w_{-(i-1)}^b - w_{-(i+1)}^b)\frac{(y_i^b)^\alpha}{(x_i^b)^\alpha + (y_i^b)^\alpha} - y_i^b \text{ for all } i \geq 0, \tag{38}$$

and the terminal conditions $w_b^b = 1$ and $w_{-b}^b = 0$. From the two first order conditions on stage i, we have

$$\frac{x_i^b}{y_i^b} = \frac{w_{i+1}^b - w_{i-1}^b}{w_{-(i-1)}^b - w_{-(i+1)}}. \tag{39}$$

Now suppose that $\{w_i^b\}_{i=-b\ldots b}$ are the continuation values for a b-advantage game, and let $b' < b$. We claim that $w_i^{b'} = \gamma + \delta w_i^b$, where γ and δ solve

$$\begin{aligned} w_{b'}^{b'} &= \gamma + \delta w_{b'}^b = 1 \\ w_{-b'}^{b'} &= \gamma + \delta w_{-b'}^b = 0. \end{aligned}$$

It is easy to see that $\frac{x_i^{b'}}{y_i^{b'}} = \frac{\delta x_i^b}{\delta y_i^b}$ will be unchanged by this linear transformation, and then (37) and (38) continue to hold for $w_i^{b'} = \gamma + \delta w_i^b$. $\square$

For example, the probability that the winner of the first district wins the second election is the same, whether $b = 2$ or $b = 35$. The reason for this result is that the continuation utilities of two different b-advantage games are linear transformations of each other (i.e., $w_i^{b'} = \gamma + \delta w_i^b$), and so players' expenditures in state i in the b'-advantage game are just δ times their expenditures in the b-advantage game; hence, the expenditure ratio is unchanged. For instance, suppose that we are interested in the probability that, in the original game, the winner of the first district wins in the second district as well. If J is large, then a good approximation of that probability can be obtained by solving the 2-advantage game.

Proof of Proposition 8

We first consider the issue of rent dissipation in primaries with many districts. In the following, we explicitly calculate a lower bound for candidates' ex ante rent in the b-advantage game, which turns out to be close to our numerical result for the original game. We need two intermediate results:

Claim 1. In any b-advantage game with $\alpha = 1$, $w_{i+1} > 1 - 2w_{-i}$.

Proof. Let $y_i = x_{-i}$. If player 1 is in state i, he solves

$$\max_{x_i} \frac{x_i}{x_i + y_i} w_{i+1} + \frac{y_i}{x_i + y_i} w_{i-1} + -x_i.$$

Similarly, player 2 solves

$$\max_{y_i} \frac{y_i}{x_i + y_i} w_{-(i-1)} + \frac{x_i}{x_i + y_i} w_{-(i+1)} - y_i.$$

Solving the two first order conditions, we get (39). Using (39) in the first order conditions yields

$$x_i = \frac{(w_{-(i-1)} - w_{-(i+1)})(w_{i+1} - w_{i-1})^2}{[(w_{i+1} - w_{i-1}) + (w_{-(i-1)} - w_{-(i+1)})]^2} \tag{40}$$

and

$$y_i = \frac{(w_{-(i-1)} - w_{-(i+1)})^2(w_{i+1} - w_{i-1})}{[(w_{i+1} - w_{i-1}) + (w_{-(i-1)} - w_{-(i+1)})]^2} \tag{41}$$

Substituting back in the objective function of player 1, we find

$$\begin{aligned} w_i = & \frac{w_{i+1} - w_{i-1}}{[(w_{-(i-1)} - w_{-(i+1)}) + (w_{i+1} - w_{i-1}))]} w_{i+1} + \frac{w_{-(i-1)} - w_{-(i+1)}}{[(w_{-(i-1)} - w_{-(i+1)}) + (w_{i+1} - w_{i-1}))]} w_{i-1} \\ & - \frac{(w_{-(i-1)} - w_{-(i+1)})(w_{i+1} - w_{i-1})^2}{[(w_{i+1} - w_{i-1}) + (w_{-(i-1)} - w_{-(i+1)})]^2} \end{aligned}$$

The difference between w_{i+1} and w_i is therefore

$$\begin{aligned} w_{i+1} - w_i &= \frac{(w_{i+1} - w_{i-1})(w_{-(i-1)} - w_{-(i+1)})}{[(w_{-(i-1)} - w_{-(i+1)}) + (w_{i+1} - w_{i-1}))]} + \frac{(w_{i+1} - w_{i-1})^2(w_{-(i-1)} - w_{-(i+1)})}{[(w_{-(i-1)} - w_{-(i+1)}) + (w_{i+1} - w_{i-1}))]^2} \\ &< 2(w_{-(i-1)} - w_{-(i+1)}) \end{aligned}$$

Hence,

$$\begin{aligned} w_2 - w_1 &\leq 2(w_0 - w_{-1}) \\ w_3 - w_2 &\leq 2(w_{-1} - w_{-2}) \\ &\vdots \\ w_b - w_{b-1} &\leq 2(w_{-(b-2)} - w_{-(b-1)}) \end{aligned}$$

Summing up all inequalities starting from the ith one, we have $w_b - w_i = 1 - w_i < 2(w_{-(i-1)} - w_{-(b-1)}) < 2w_{-(i-1)}$, or $w_i > 1 - 2w_{-(i-1)}$. Replacing i with $i+1$, we have the desired inequality. □

Claim 2. The 2-advantage game with $\alpha = 1$ has the following continuation values:

$$\begin{aligned} w_0^* &= \frac{\sqrt{33} - 5}{4} \\ w_{-1}^* &= \frac{(\sqrt{33} - 5)^3}{64} \\ w_1^* &= \frac{4}{27} \frac{523503\sqrt{33} - 3007199}{(9\sqrt{33} - 49)^3} \end{aligned}$$

Proof. Now observe that from (37), (38), (40), and (41), the following conditions must hold in the 2-advantage game: $w_{-2} = 0$, $w_2 = 1$, and

$$w_0 = \frac{1}{2}w_1 + \frac{1}{2}w_{-1} - \frac{w_1 - w_{-1}}{4} = \frac{1}{4}w_1 + \frac{3}{4}w_{-1},$$

$$w_1 = \frac{w_2 - w_0}{w_2 - w_{-2}}w_2 + \frac{w_0 - w_{-2}}{w_2 - w_{-2}}w_0 - \frac{(w_2 - w_0)^2(w_0 - w_{-2})}{w_2 - w_{-2}} = 1 - 2w_0 + 3(w_0)^2 - (w_0)^3,$$

and

$$w_{-1} = \frac{w_2 - w_0}{w_2 - w_{-2}}w_{-2} + \frac{w_0 - w_{-2}}{w_2 - w_{-2}}w_0 - \frac{(w_2 - w_0)(w_0 - w_{-2})^2}{w_2 - w_{-2}} = (w_0)^3$$

These equations have the solutions given in the statement of the Lemma. □

From the proof of Lemma 6, we know that for any given b, w_i^b can be written $\delta + \gamma w_i^2$, for any i (γ and δ depend on b). Since all $w_i^b \geq 0$, $w_{-2}^b = \gamma w_{-2}^2 + \delta = \delta \geq 0$, δ must be nonnegative. From Lemma 1 with $i = 1$, we have $w_2^b = \gamma w_2^2 + \delta \geq 1 - 2[\gamma w_{-1}^2 + \delta]$ and hence $\gamma(1 + 2w_{-1}^*) + 3\delta \geq 1$.

Therefore, given b, w_0^b must be at least as large as the solution of the following constrained optimization problem:

$$\min_{\gamma,\delta} \gamma w_0^* + \delta, \quad \text{s.t.} \quad \delta \geq 0, \quad \gamma(1 + 2w_{-1}^*) + 3\delta \geq 1.$$

This problem has the solution $\delta^* = 0, \gamma^* = (1 + 2w_{-1}^*)^{-1}$, and hence a lower bound for w_0^b is given by

$$w_0^b \geq \gamma^* w_0^* = \frac{2\sqrt{33} - 10}{27\sqrt{33} - 147} \approx 0.18377.$$

(Going through the same steps, but using the 3-advantage game, one gets a slightly better approximation of 0.183847, which is already virtually indistinguishable from the numerical results.)

We now turn to the "momentum" effect in the b-advantage game: We show that the front runner is more likely to achieve another victory than his opponent. We fix b and drop the b subscript. For all i, the following inequality must hold:

$$w_i + w_{-i} < w_{i+1} + w_{-(i+1)}. \tag{42}$$

The argument is as follows: $w_i + w_{-i}$ is equal to the prize, 1, minus the expected future expenditures by both candidates; the latter can be split into the expected expenditures that will be incurred until state $i + 1$ is reached for the first time, plus the expected expenditures following that event. Similarly, $w_{i+1} + w_{-(i+1)}$ is equal to the prize minus the expected expenditures by both candidates following state $i + 1$. Hence, the difference between $w_{i+1} + w_{-(i+1)}$ and $w_i + w_{-i}$ is equal to the expected expenditures to be made between the time when state i is reached and the time when state $i + 1$ is reached for the first time, and is strictly positive. Using (42) in lagged form, we have

$$w_{i-1} + w_{-(i-1)} < w_i + w_{-i},$$

and therefore $w_{i+1} - w_{i-1} > w_{-(i-1)} - w_{-(i+1)}$. From (39) we then have $x_i > y_i$, and the player who has an advantage will therefore win the next district with a probability that is greater than 1/2. □

Proof of Lemma 5

Existence of a MPE in the b-advantage is easy to see: If one player follows a stationary strategy, there exists a stationary best response for the other. Since all subgames are reached (it is never optimal to spend zero), the stationary equilibrium is also subgame perfect. Now denote $\lim_{b\to\infty} w_i^b = w_i^\infty$ and $\lim_{J\to\infty} v_{j,k}^J = v_{j,k}^\infty$ (suppose these limits exist). We claim that $v_{j,k}^\infty = w_i^\infty$ whenever $i = 2k + 1 - j$.

Suppose that $v^{\infty}_{j,k} = v^{\infty}_{j',k'}$ if $2k+1-j = 2k'+1-j'$, and call this value ν_i $(i = 2k+1-j)$. Fix any b, and consider the continuation payoffs in the initial stages of a long original game, at those nodes where no candidate has achieved an advantage of more than b districts over his opponent. These continuation payoffs can now be interpreted as those of a modified b-advantage game with payoffs ν_b for the first candidate to reach an advantage of b districts, and ν_{-b} for the loser.

We know from the proof of Lemma 6 that the continuation payoffs in all b advantage games are linear transformations of each other, so that we must have

$$\nu_i = \gamma + \delta w^{\infty}_i$$

The claim is that $\gamma = 0$ and $\delta = 1$. Note first that $\delta \geq 0$, since it must be at least weakly better to win more districts. Since $\lim_{i\to-\infty} \nu_i = 0$ (i.e., if the disadvantage becomes too large, the continuation utility goes to zero), and $\lim_{i\to-\infty} \nu_i = \gamma + \delta \cdot 0 = \gamma$, γ must be equal to zero.

To see that it is not possible that $\delta \neq 1$, note that, at each node, the expenditures in the limit of the original game are δ times the corresponding expenditures in the limit of the $b-$advantage game. Since the transition probabilities between two nodes are the same for all b-advantage games (by Lemma 6, this implies that total expected spending in the limit of the original game is δ times the spending in the limit of the b-advantage games. If $\delta < 1$, then the candidates spend, in the limit of a very large primary game, in each district only δ times what they spend in the b-advantage game (for $b \to \infty$). Since candidates are symmetric at the beginning of the game,

$$\nu_0 = \frac{1 - \text{expected total campaign spending}}{2}.$$

Hence, if $\delta < 1$, then $\nu_0 > w^{\infty}_0$, since expected campaign spending is lower than in the limit of the $b-$advantage game. However, since $\nu_0 = \delta w^{\infty}_0 < w^{\infty}_0$, this leads to a contradiction. (Similarly, assuming $\delta > 1$ also leads to a contradiction.)

Hence, $\nu_i = w^{\infty}_i$ for all i, and thus $v^{\infty}_{j,k} = w^{\infty}_i$ whenever $i = 2k+1-j$. This proves part (i) of the Proposition. Since the continuation values of the original game converge to those of the b-advantage game when both J and b become large, strategies converge as well, which establishes (ii). $\square$

References

[1] Adams, W., 1987: "As New Hampshire Goes . . . ," in: Orren, G. and N. Polsby (eds.), "Media and Momentum," Chatham House, Chatham, NJ.

[2] Bartels, L., 1988: "Presidential Primaries and the Dynamics of Public Choice," Princeton University Press, Princeton, NJ.

[3] Baye, M., D. Kovenock, and C. De Vries, 1990: "The All Pay Auction with Complete Information," CentER Discussion Paper 9051.

[4] Baye, M., D. Kovenock, and C. De Vries, 1994: "The Solution to the Tullock Rent-Seeking Game when $R > 2$: Mixed Strategy Equilibria and Mean Dissipation Rates," *Public Choice*, **81**, 363–380.

[5] Bikhchandani, S., D. Hirshleifer, D., and I. Welch, 1992: "A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades," *Journal of Political Economy*, **100**, 992–1026.

[6] Cook, R., 2000: "United States Presidential Primary Elections, 1968-1996: A Handbook of Election Statistics," CQ Press, Washington, D.C.

[7] Cook, R. and A. McGillivray, 1997: *U.S. Primary Elections 1995-1996*, CQ Press, Washington, D.C.

[8] Che, Y.-K. and I. Gale, 2000a: "Caps on Political Lobbying," *American Economic Review*, **88**, 643–651.

[9] Che, Y.-K. and I. Gale, 2000b: "Difference-Form Contests and the Robustness of All-Pay Auctions," *Games and Economic Behavior*, **30**, 22–43.

[10] Dasgupta, P. and E. Maskin, 1986: "The Existence of Equilibrium in Discontinuous Games, I: Theory," *Review of Economic Studies*, **53**, 1–26.

[11] Dekel, E. and M. Piccione, 2000: "Sequential Voting Procedures in Symmetric Binary Elections," *Journal of Political Economy*, **108**, 34–55.

[12] Erikson, R. and T. Palfrey, 2000: "Equilibria in Campaign Spending Games: Theory and Data," *American Political Science Review*, **94**, 595–609.

[13] Harbaugh, R. and T. Klumpp, 205: "Early Round Upsets and Championship Blowouts," *Economic Inquiry*, **43**, 316–329.

[14] Heckelman, J., 2000: "Sequential Elections and Overlapping Terms: Voting for U.S. Senate," *Journal of Economic Behavior and Organization*, **42**, 97–108.

[15] Malbin, M., 1985: "You Get What You Pay for, but Is That What You Want?," in: G. Grassmuck (ed.), *Before Nomination: Our Primary Problems*, American Enterprise Institute for Public Policy Research, Washington, D.C.

[16] McGillivray, A. and R. Scammon, 1994: *America at the Polls: A Handbook of American Presidential Election Statistics*, Vol. 2, Congressional Quarterly Inc., Washington, D.C.

[17] Morton, R. and K. Williams, 1999: "Information Asymmetries and Simultaneous versus Sequential Voting," *American Political Science Review*, **93**, 51-67.

[18] Oak, M., 2001: "On the Role of the Primary System in Candidate Selection," Cornell University, mimeo.

[19] Robinson, M. and M. Sheehan, 1983: "Over the Wire and on TV," Russell Sage Foundation, New York.

[20] Snyder, J., 1989: "Election Goals and the Allocation of Campaign Resources," *Econometrica*, **57**, 630–660.

[21] Snyder, J., 1990: "Campaign Contributions as Investments: The U.S. House of Representatives, 1980-1986," *Journal of Political Economy*, **98**, 1195–1227.

[22] Strumpf, K., 2002: "Strategic Competition in Sequential Election Contests," *Public Choice*, **111**, 377-397.

[23] Swank, O., 2001: "A Comment on Sequential Elections and Overlapping Terms: Voting for U.S. Senate," *Journal of Economic Behavior and Organization*, **46**, 437–440.

[24] Szentes, B. and R. Rosenthal, 2003a: "Three-Object Two-Bidder Simultaneous Auctions: Chopsticks and Tetrahedra," *Games and Economic Behavior*, **44**, 114–133.

[25] Szentes, B. and R. Rosenthal, 2003b: "Beyond Chopsticks: Symmetric Equilibria in Majority Auction Games," *Games and Economic Behavior*, **45**, 278–295.

[26] Szymanski, S., 2003: "The Economic Design of Sporting Contests," *Journal of Economic Literature*, **41**, 1137–1187.

[27] Tullock, G., 1980: "Efficient Rent-Seeking," in: J. Buchanan, R. Tollison, and G. Tullock (eds.), *Towards a Theory of the Rent Seeking Society*, 97–112, Texas A&M University Press.

[28] Witcover, J., 1977: *Marathon: The Pursuit of the Presidency 1972-1976*, New American Library, New York.