A Theory of Perceived Discrimination*

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Abstract

We develop a model in which individuals compete for a fixed pool of prizes by investing effort in a contest. Individuals belong to two separate and identifiable groups. We say that the contest is discriminatory if a lower share of prizes is reserved for one group than for the other. We show that it can be difficult for an observer to detect the presence or absence of discrimination in the contest, as both regimes can be observationally equivalent. In particular, one group's belief that it is allocated a lower share of prizes than the other group can be consistent with observed data *even if no such group quotas actually exist*. Conversely, the belief that the contest does not discriminate can be consistent with data when, in fact, discrimination exists. Incorrect beliefs will therefore not be revised, as the contest generates no evidence to the contrary.

Keywords: Contests, discrimination, observational equivalence, agreeing to disagree.

JEL codes: J71, J78.

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1 Introduction

Gender, racial, religious, and other forms of discrimination are generally viewed as social ills, excluding individuals from opportunities available to others based solely on characteristics such as a person's sex or skin color. If discrimination exists, it is often not directly observable but manifests itself only indirectly, in the form of ex-post inequality across groups of the population. However, the same inequality can also arise if groups differ in relevant economic characteristics or if their members make systematically different choices. Thus, if the presence or absence of discrimination cannot be observed directly, the question becomes whether unequal outcomes constitute evidence of unequal treatment or not.

We examine this question in a model in which individuals compete for a fixed number of prizes in a contest. An individual's qualification for a prize is the sum of her effort choice and a random shock. We say that the contest is non-discriminatory, or color-blind, if the prizes are awarded to the individuals with the highest qualifications. On the other hand, the contest is discriminatory if a fixed share of prizes is reserved for one identifiable subgroup of individuals and a different share for the remaining subgroup. We show that non-discriminatory contests can be equivalent to highly discriminatory ones in observed outcome variables, such as the share of prizes received by each group, the qualifications of the winners, and individual effort decisions. In other words, to anyone who observes the outcome of the contest but not the rule by which the outcome was generated, a color-blind contest can appear discriminatory. Conversely, a discriminatory contest can appear color-blind, in that it generates no evidence to suggest otherwise.

To make our point as stark as possible, we assume away any fundamental differences across groups of the population. In addition, all relevant variables in our model are fully observable, so our results do not rely on any informational frictions. The reason why seemingly discriminatory outcomes can exist, even in the absence of discrimination, is the following. In equilibrium of a color-blind contest, some individuals will exert zero effort and have a zero chance of winning, while others exert positive effort and have a positive chance of winning. Since the identity of high-effort individuals is irrelevant, there exist equilibria in which a relatively large fraction of individuals in one group exerts effort, while a smaller fraction does so in the other group. Relative to its size, one group will then win a larger share of prizes than the other group. The crucial observation is that the same profile of efforts and outcomes is also an equilibrium of a discriminatory contest in which a larger share of prizes is reserved for one group than for the other. Thus, discriminatory and non-discriminatory contests can look alike in their respective equilibrium outcomes.

Furthermore, the problem of inferring unequal treatment from unequal representation can be embedded into our contest model itself. To do so, we depart from the assumption that the contest rules are common knowledge among the contestants. We show that it is possible that some or all individuals believe the contest discriminates, and behave accordingly, when in fact the contest does not discriminate. Conversely, individuals may believe the contest does not discriminate, and behave accordingly, when in fact it discriminates. In both situations, some or all individuals will hold incorrect beliefs about the nature of competition they are engaged in. Nevertheless, the observed data generated by the contest will not contradict these incorrect beliefs: If the contest was in fact what it is believed to be, it would generate precisely the same data. We call such outcomes *rational perceptions equilibria*. Contestants in our model can therefore "agree to disagree" about the presence of discrimination.

Labor economists have long been concerned with the problem of discrimination, and have explored two fundamentally different approaches to dealing with it. The first is market-based: The fact that employers must compete for qualified workers may be enough to prevent discriminatory employment practices (Becker, 1957). Because market remedies can be hindered by frictions arising from asymmetric information, the second approach attempts to correct this market failure through policy interventions, such as discrimination bans or affirmative action (e.g., Coate and Loury, 1993). The present paper is less concerned with whether, or how, governments should intervene in markets to correct for discrimination, but instead addresses the question whether discrimination can be inferred from market outcomes. Our equivalence results nevertheless have policy implications, which we now discuss.

First, the enforcement of anti-discrimination policies is encumbered by the very same problem we examine in our model—how the presence of discrimination, or the intent to discriminate, can be determined by government agencies or courts from observable data. For example, does a mostly white workforce constitute evidence of a company's discriminatory hiring practices against blacks? An account of these measurement problems, as they were encountered by the U.S. Equal Employment Opportunity Commission in the early days of American affirmative action, is given in Johnson and Green (2009, chapter 3). Our results suggest that, based on the observation of labor market outcomes, it may be difficult to determine whether anti-discrimination policies are needed, and whether they work if implemented.

Second, even if market forces or public policies are successful in eliminating discrimination, one should not expect that outcomes be equalized. In our model, non-discriminatory contests can have asymmetric equilibria, associated with unequal outcomes across subgroups of the population. Given the multitude of other possible equilibria, however, one may wonder why individuals would coordinate on playing an asymmetric, highly unequal one. An obvious coordination device is to look to past patterns of behavior. If the contest has actually discriminated in the past, having resulted in a particular equilibrium, then a focal equilibrium of the new, color-blind contest may simply be to continue with "business as usual," thereby perpetuating unequal outcomes.

Third, anti-discrimination policies and market forces may not only fail to equalize outcomes, but may also fail to change individual perceptions about the very nature of competition individuals are engaged in. In this regard, our "agreeing-to-disagree" result is borne out in the data, at least casually. In a highly publicized 1969 Harris Poll, 82% of black respondents stated that they thought blacks were discriminated against in obtaining white collar office jobs, while only 38% of white respondents shared this opinion. With regard to skilled labor jobs, the response rates were 83% vs. 35%, and with regard to manual labor jobs the response rates were 58% vs. 18%. One might expect that such perceptive differences would vanish over time, as more evidence becomes available. When asked the same questions in 2008, however, the response rates were 77% vs. 29%, 74% vs. 21%, and 34% vs. 10%, respectively. Thus, while perceived discrimination has decreased within both groups, large inter-group differences in perception remain.[1] In our model, even in the absence of actual discrimination, perceived discrimination by members of the disadvantaged group may persist as an equilibrium outcome. This equilibrium, again, may be focal in case of prior discrimination, and thus may further perpetuate inequality.

The remainder of the paper is organized as follows. In Section 2 we place our contribution in relation to the literature. In Section 3 we develop our contest model. In Section 4 we solve for the Nash equilibrium of this contest, both when there is no discrimination and when there is. Section 5 contains two sets of observational equivalence results. First, we show that it can be difficult to detect discrimination by looking at the outcome of a contest, assuming that the contestants are informed of the rules of the contest. Second, we show that the contestants themselves can hold false perceptions about whether or not there is discrimination. Section 6 extends our basic model by allowing for additional heterogeneity among individuals as well as externalities. Section 7 concludes. All proofs are in the Appendix.

2 Relation to the Literature

Our rational perceptions equilibrium concept is similar in spirit to, but not the same as, the *self-confirming equilibrium* pioneered by Fudenberg and Levine (1993). A self-confirming equilibrium is a profile of strategies and beliefs such that strategies are optimal given beliefs about other players' actions, and these beliefs are not contradicted by the observable actions taken by other players. However, the game itself is common knowledge among all players. Similar equilibrium concepts, in which players hold incorrect beliefs about the types of others or themselves, are also considered in Eyster and Rabin (2005) and Squintani (2006). In our model, on the other hand, possibly incorrect beliefs concern the rules of the game. This possibility is ruled out in a self-confirming equilibrium, as

[1] Source: Harris Interactive (2009). It is worth mentioning that the 2008 poll was conducted in the month of December, *after* the election of Barack Obama to the office of President of the United States, a decidedly white collar job.

the extensive form (and nature's probabilities) are known to all players.[2]

The political economy literature contains a number of contributions which explore heterogeneity in agents' beliefs about "the way the world works." In Piketty (1995), agents hold different beliefs about the relative roles of effort and predetermined factors for social mobility. Piketty (1995) shows that heterogenous beliefs—some of which must be incorrect—can survive in the long run. However, this result requires that individual effort decisions are not perfectly observable by others. In contrast, our results hold regardlesss of whether individual efforts are observable or not. Alesina and Angeletos (2005) develop a model in which individual success depends on both effort and luck. Alesina and Angeletos (2005) show that across economies with the same fundamentals, multiple equilibria may emerge which differ in the amount of effort, degree of redistributions, and in agents' beliefs about the contribution of effort for success. Within the same economy, however, all agents will have the same belief. In Saint-Paul (2010, 2011), the models used by agents to understand the world are shaped by a class of individuals called "intellectuals." These intellectuals may (knowingly or unknowingly) communicate incorrect models to the public. An incorrect model will remain undetected if the data generated by the decision of agents acting on the perceived model is consistent with the perceived model.[3]

It is also important to distinguish our framework from models of statistical discrimination in the literature. Like ours, these models rely on equilibrium indeterminacy in games where individuals invest in their qualifications. However, the two approaches are not the same. To see why, consider the result of Coate and Loury (1993) that racial stereotypes can be self-enforcing: The belief that employers discriminate induces behavior on part of the individuals which then generates the hypothesized discrimination as a best response by the employers. The belief that employers discriminate is therefore correct in equilibrium. Discrimination persists as an equilibrium outcome because, due to informational frictions, individuals are judged by employers according to the aggregate behavior of the individual's group. This aggregate judgment makes it impossible for a single individual to break out of a discriminatory equilibrium.[4] In our model, on the other hand, discrimination is exogenous. The belief that the contest discriminates

[2] In macroeconomics, the term "self-confirming equilibrium" is sometimes used to describe outcomes in which policies are based on an incorrect model (instead of the "true" model); see Sargent (2008).

[3] Outside the political economy literature, see Yildiz (2003) for a model in which players disagree on the rules of a bargaining game, and Coury and Sciubba (2010) for a model in which traders disagree on the distribution of a state-of-nature in a competitive economy, despite observing the same prices and trades.

[4] The equilibrium theory of labor market discrimination appears first in contributions by Phelps (1972) and Arrow (1973), and has recently been applied to other areas, such as crime (Verdier and Zenou, 2004; Curry and Klumpp, 2008). For a survey of this literature, see Fang and Moro (2011). Kolpin and Singell (1997) show that asymmetric information among firms can result in discrimination against groups even if they are uniformly better qualified than others. See Dal Bo (2007) for a model that can generate discrimination as a social norm without requiring informational frictions.

therefore does not, and cannot, generate discrimination. Instead, what it generates is data consistent with the hypothesis that the contest discriminates, even if the contest does not. Possibly incorrect perceptions of discrimination can persist as equilibrium beliefs because the contest generates no evidence to the contrary through which individuals might eventually learn that their perceptions are incorrect.

Finally, previous work has examined color-blind and color-conscious rules in models of college admissions. Chan and Eyster (2003) show that a color-blind admissions policy can lower the quality of admitted minority students, compared to a color-conscious policy. The mechanism through which this works relies on the interplay of an admissions officer's desire for diversity and the fact that a color-blind policy prevents the officer from using race as an admissions criterion.[5] Finally, Fu (2006) uses a contest model to study positive discrimination in college admissions. Fu (2006) shows that the preferential treatment of individuals of a group that is ex-ante handicapped (i.e., the same amount of effort produces a lower qualification than for a non-handicapped individual) can increase pre-college efforts in both groups.

3 Additive-Noise Contests with Multiple Winners

In the following, we develop a model in the spirit of the additive-noise contest of Lazear and Rosen (1981), extended to allow for a many players and many prizes.

There is a continuum I of individuals. This continuum is of size 1 and split into two identifiable groups: A fraction α^1 of the population belongs to group $I^1 \subset I$ and the remaining fraction $\alpha^2 = 1 - \alpha^1$ belongs to group $I^2 = I \backslash I^1$. Members of both groups are identical in all economically relevant aspects, except for their group label. We let $g(i) \in \{1, 2\}$ denote the group label that individual i belongs to.

All individuals compete for a measure $m \in (0, 1)$ of prizes. (For convenience, we often simply say that the individuals compete for m prizes.) Winning a prize is worth $\Pi > 0$ to each individual, and each individual can win at most one prize.

In order to win, individual i chooses effort $e_i \geq 0$ and incurs cost $c(e_i)$. Effort generates a publicly observable signal $t_i = e_i + \nu_i$, where $\nu_i \in [0, \varepsilon]$ is a random component, distributed with cumulative distribution F and density f. It will not matter whether e_i and ν_i are separately observable, or only their sum t_i is.

We make the following technical assumptions:

Assumption 1. (i) The cost function c is twice differentiable and satisfies the following: $c(0) = c'(0) = 0$, $c'(e_i) > 0$ and $c''(e_i) > 0$ for all $e_i > 0$, $c'(e_i) \to \infty$ as $e_i \to \infty$. (ii) All ν_i are distributed independently on support $[0, \varepsilon]$, and the density function f is

[5] Thus, in an environment where one cares about race but only observes qualification, qualification becomes an (imperfect) signal of race. Interestingly, this is the opposite effect of what occurs in Coate and Loury (1993): There, in an environment where one cares about qualification but only observes race, race becomes a signal of qualification.

differentiable on $(0,\varepsilon)$. (iii) $f(0) \leq c'(1/f(0))/\Pi$. (iv) For all $\nu_i \in (0,\varepsilon)$ and $e_i \geq 0$: $f'(\nu_i) > -c''(e_i)/\Pi$.

Parts (iii) and (iv) of Assumption 1 state that f cannot be too large at zero, and the slope of f cannot be too negative. These assumptions are made to guarantee that individual payoffs are sufficiently smooth and not too responsive to small changes in efforts. Conditions (iii) and (iv) are relatively mild and can be satisfied by most standard cost and density functions. For example, if the cost function is $c(e) = e^2$ and f is uniform on $[0,\varepsilon]$, then the conditions hold as long as $\varepsilon > \sqrt{\Pi/2}$.

3.1 Contest types

Let us now turn to the rules along which competition over the m prizes takes place. Specifically, we consider two different types of contests.

We say that the contest is ***non-discriminatory*** if the fraction m of individuals with the highest signals t_i win. In a non-discriminatory contest, the expected payoff to individual i is given by

$$u_i(e_i, \mathbf{e}_{-i}) = Pr\Big[\, \big|\{j \in I : e_j + \nu_j \geq e_i + \nu_i\}\big| \leq m \,\Big]\Pi - c(e_i), \tag{1}$$

where $\mathbf{e}_{-i} = (e_j)_{j\neq i}$ denotes the profile of efforts of individuals other than i, and $|\cdot|$ denotes the Lebesque measure of a set.[6]

On the other hand, the contest is said to be ***discriminatory*** if there are group-specific quotas $m^1, m^2 \in [0,1]$ with $\alpha^1 m^1 + \alpha^2 m^2 = m$, such that the top-$m^1$ (top-m^2) quantile of signals in group 1 (2) win. Thus, in a discriminatory contest the expected payoff to individual i becomes

$$u_i(e_i, \mathbf{e}_{-i}) = Pr\Big[\, \big|\{j \in I^{g(i)} : e_j + \nu_j \geq e_i + \nu_i\}\big| \leq \alpha^{g(i)} m^{g(i)} \,\Big]\Pi - c(e_i). \tag{2}$$

A ***contest type*** is then a tuple summarizing the variables pertaining to whether (and if so, how) the contest discriminates. That is, a contest type is a tuple

$$C = (\theta, m^1, m^2)$$

such that $\theta \in \{ND, D\}$ and $m^1, m^2 \in [0,1]$. The parameter θ indicates whether the contest is discriminatory or not, and if $\theta = D$ then m^1 and m^2 represent the group-specific award quotas. (If $\theta = ND$, then m^1 and m^2 are irrelevant.)

Given effort profile $\mathbf{e} = (e_i)_{i\in I}$ and a contest type $C = (\theta, m^1, m^2)$, we denote by $U_i(\mathbf{e}|C)$ the expected payoff to individual i in this contest and profile. If $\theta = ND$ then this is the payoff given in equation (1), and if $\theta = D$ then this is given in equation (2).

[6] This will always be well-defined in our equilibria.

3.2 Remarks

We will often interpret this contest model as follows, although there is nothing in the model that prevents other interpretations: We think of a prize as a job offer, and of t_i as individual i's qualification for the job. Effort e_i reflects i's investment in her qualification (e.g., the effort she spends studying at school or university), while ν_i reflects a luck component she has no control over (e.g., the quality of her teachers). In a non-discriminatory contest, the m most qualified individuals will receive a job offer, regardless of their group identity. In a discriminatory contest, on the other hand, the groups are segregated. For example, there might be $\alpha^1 m^1$ jobs reserved for whites, and $\alpha^2 m^2$ jobs reserved for blacks.

In a discriminatory contest with $m^1 \neq m^2$, one group is allocated a smaller quota of prizes than the other, relative to their representation in the population. This group is then disadvantaged in the sense that its members are, on average, less likely to win than members of the other group. Note that the disadvantaged group is not simply handicapped: Even with higher effort, they will not be able to overcome the constraint that their pool of available prizes is smaller (as a percentage of the group's size) than that of the the other group.

Notice also that a discriminatory contest with $m^1 = m^2$ is not the same as a non-discriminatory contest: It is a ***separate-but-equal*** contest in which, while neither group is disadvantaged vis-à-vis the other, individuals still cannot compete for prizes across groups (as they could in a non-discriminatory "color blind" contest). Because group-identity matters, even if $m^1 = m^2$, we will maintain the term "discriminatory" for the separate-but-equal case. Further discussion regarding the separate-but-equal contest is offered in Section 6.2

4 Equilibrium

A Nash equilibrium of the contest C is a profile $\mathbf{e}^* = (e_i^*)_{i \in I}$ of efforts such that each individual's effort e_i^* maximizes the expected payoff $U_i(e_i, \mathbf{e}_{-i}^* | C)$ with respect to e_i, given the equilibrium effort profile $\mathbf{e}_{-i}^*$ of the other individuals.

In principle, efforts can be generated by mixed strategies. Because there is a continuum of players, any mixed strategy can easily be recast in terms of fractions of the population playing different pure strategies. While both cases are equivalent, we prefer to think of pure strategies for the following reason. Any mixed strategy a player may have used to generate his or her effort is unobservable. On the other hand, effort levels themselves are, in principle, observable. Since our paper is concerned with the inferences one can draw from the observable data generated by a contest, we will focus our discussion on the pure strategy case. Our formal results include both possibilities, however.

4.1 Equilibrium of non-discriminatory contests

Let us first consider a non-discriminatory contest C_{ND}. Associated with any equilibrium of C_{ND} will be a threshold signal τ such that a fraction m of individuals obtains signals which exceed τ. These individuals will then be the winners. Since there is a continuum of individuals, no single effort choice will alter the value of τ, which individuals therefore take parametrically.

Consider the possibility of a symmetric pure strategy equilibrium in which all individuals exert the same effort, $e_i^* = \sigma$. The capacity constraint becomes $Pr[t_i \geq \tau] = Pr[\nu_i \geq \tau - \sigma] = m$, or

$$F(\tau - \sigma) = 1 - m. \tag{3}$$

To determine σ, note that each individual maximizes, with respect to e_i, the function

$$U(e_i|\tau) \equiv (1 - F(\tau - e_i))\Pi - c(e_i).$$

That is, instead of taking the opponent effort profile $\mathbf{e}_{-i}$ as given and maximizing $U_i(e_i, \mathbf{e}_{-i}|C_{ND})$, we may think of the individual as taking the cutoff signal τ as given and maximizing $U(e_i|\tau)$. The first and second derivatives of U with respect to e_i are $U'(e_i|\tau) = f(\tau - e_i)\Pi - c'(e_i)$ and $U''(e_i|\tau) = -f'(\tau - e_i)\Pi - c''(e_i)$. Notice that by Assumption 1 we have $U''(e_i|\tau) < 0$ for all $e_i \in (\tau - \varepsilon, \tau)$. Thus, there will be a unique effort level that maximizes U on $(t^* - \varepsilon, t^*)$. This effort level is characterized by the first-order condition $f(\tau - e_i)\Pi - c'(e_i) = 0$, and invoking symmetry ($e_i = \sigma$) we get

$$f(\tau - \sigma)\Pi = c'(\sigma). \tag{4}$$

Conditions (3)–(4) simultaneously pin down a cutoff signal τ and an effort level σ. In particular, (3)–(4) imply that σ is given by the implicit condition

$$c'(\sigma) = f(F^{-1}(1 - m))\Pi. \tag{5}$$

Since c is strictly convex and satisfies $c'(0) = 0$ and $c'(e_i) \to \infty$ as $e_i \to \infty$, a unique σ exists that satisfies (5). The corresponding τ is then given by plugging σ back into (3). Making the dependence on m explicit, we have:

$$\sigma(m) = [c']^{-1}\big(f(F^{-1}(1 - m))\Pi\big), \tag{6}$$

$$\tau(m) = F^{-1}(1 - m) + \sigma(m). \tag{7}$$

The effort $\sigma(m)$ is in fact the symmetric pure strategy equilibrium effort in the non-discriminatory contest with m prizes—under one condition: The number of prizes, m, must not be too small. If this condition is violated the equilibrium will become asymmetric, with some individuals exerting zero effort. (Equivalently, the equilibrium will become mixed, with individuals randomizing over positive and zero efforts.) Proposition 1 characterizes the overall equilibrium of the contest:

Proposition 1. *Consider a non-discriminatory contest. There exists a unique* $\overline{m} \in (0,1)$*, which satisfies* $\overline{m}\Pi = c(\sigma(\overline{m}))$*, such that the following holds:*

(a) If $\overline{m} < m < 1$*, there exists a symmetric pure strategy equilibrium of the contest in which every individual* $i \in I$ *spends effort* $e_i^* = \sigma(m)$*, as defined in* (6)*. The minimum signal needed for winning is* $\tau(m)$*, as defined in* (7)*.*

(b) If $0 < m \leq \overline{m}$*, there exists an asymmetric pure strategy equilibrium in which a fraction* $\lambda(m) = m/\overline{m}$ *of individuals spend effort* $\sigma(\overline{m})$ *and win if their signal is at least* $\tau(\overline{m})$*. The remaining fraction* $1 - \lambda(m)$ *spend zero effort and never win.*

(c) If $0 < m \leq \overline{m}$*, there also exists a symmetric mixed strategy equilibrium in which every individual spends effort* $\sigma(\overline{m})$ *with probability* $\lambda(m) = m/\overline{m}$*, and zero effort with the remaining probability* $1 - \lambda(m)$*.*

The proof of Proposition 1 is in the Appendix. Here we discuss why it is not an equilibrium for all individuals to invest positive effort when m is too low (specifically, when $m < \overline{m}$). If every agent invested $\sigma(m)$, and m decreases, the expected payoff to each agent eventually becomes zero at $m = \overline{m}$, and negative once m falls below $\overline{m}$. This profile clearly cannot be an equilibrium, as investing zero effort results in a zero payoff. The first-order condition (4) thus describes a local payoff optimum only. There exists a second local maximum, at zero. If $m > \overline{m}$ the interior optimum exceeds the corner one, but if $m < \overline{m}$ this is no longer the case. Thus, a solution based solely on (4) is not an equilibrium any more.

Instead, two different behaviors emerge: Some individuals exert zero effort and thereby effectively withdraw from the contest. The remaining individuals compete over the m prizes. Since m is fixed, if fewer individuals compete the relative proportion of prizes available to them increases. Thus, by setting the size of the competing pool appropriately (so that the proportion of prizes to competitors is exactly $\overline{m}$), the competing agents will earn a zero expected payoff when playing the symmetric equilibrium effort $\sigma(\overline{m})$ among themselves. Consequently, each individual is indifferent between belonging to the competing group and belonging to the non-competing group, making the asymmetric or mixed profiles an overall equilibrium (part (b) and (c) of the result).[7]

[7] The result that contestants are indifferent between different effort levels may seem unrealistic, and we will address the issue in more detail in Section 6.1 below. We note, however, that individual indifference and mixed strategies are common equilibrium features of many contest models examined in the literature. Examples include the War of Attrition (Maynard Smith, 1974), the all-pay auction with complete information (Hillman and Riley, 1989; Baye et al., 1996), difference form contests (Che and Gale, 2000), the Tullock-contest with a sufficiently convex success function (Baye et al., 1994), the Chopstick Auction (Szentes and Rosenthal, 2003), the Colonel Blotto game (Roberson, 2006), and the simultaneous primaries game (Klumpp and Polborn, 2006).

4.2 An example

Let us illustrate the equilibria described in Proposition 1 by the following example of a non-discriminatory contest. (The example will be continued in Section 5.1, where it is applied to discriminatory contests, and in Section 6.1.)

Set the prize value to $\Pi = 1$, suppose that the cost function of effort is $c(e_i) = \frac{1}{2}e^2$, and assume that ν_i follows a uniform distribution on $[0, 1]$:

$$f(\nu_i) = \begin{cases} 0 & \text{if } \nu_i < 0, \\ 1 & \text{if } \nu_i \in [0,1], \\ 0 & \text{if } \nu_i > 1, \end{cases} \qquad F(\nu_i) = \begin{cases} 0 & \text{if } \nu_i < 0, \\ \nu_i & \text{if } \nu_i \in [0,1], \\ 1 & \text{if } \nu_i > 1. \end{cases}$$

The relative group sizes α^1 and α^2 do not matter.

The first-order condition (4) implies that $\sigma = \Pi = 1$, and condition (3) implies that $\tau = 2 - m$. The expected payoff to each individual in a symmetric profile with effort $\sigma = 1$ is thus $m - \frac{1}{2}$. To check whether this interior outcome is an equilibrium, we need to consider possible deviations from $\sigma = 1$ to zero. If $\tau - 1 \geq 0$, or $m \leq 1$, then a deviation to zero guarantees a zero probability of a win, and hence a zero payoff. Thus, spending zero is strictly preferred over spending 1 if and only if $m < \frac{1}{2}$. The effort $\sigma = 1$ is therefore a symmetric equilibrium effort for $m \in [\overline{m}, 1)$ with $\overline{m} = \frac{1}{2}$. This satisfies the condition $\overline{m}\Pi = c(\sigma(\overline{m}))$ in Proposition 1.

If $m < \frac{1}{2}$, on the other hand, we can construct an equilibrium in which a fraction of the population spends effort 1 while the remainder spends zero. In this equilibrium, the payoff from both efforts must be the same. The payoff from positive effort is $(1 - F(\tau - 1)) - \frac{1}{2} = \frac{3}{2} - \tau$, while the payoff from from zero effort is zero; equality hence implies $\tau = \frac{3}{2}$. The individuals with positive effort generate signals uniformly on $[1, 2]$, so that one-half of these individuals have signals above $\tau = \frac{3}{2}$. On the other hand, none of the zero-effort individuals generate signals above τ. To fulfill the capacity constraint, therefore, the fraction of individuals exerting high effort must be $\lambda(m) = 2m$, which equals $m/\overline{m}$ as stated in Proposition 1 (b). Figure 1 plots the equilibrium variables $\lambda(m)$ and $\tau(m)$ in this example.

4.3 Equilibrium of discriminatory contests

Next, we examine discriminatory contests C_D. Observe that a discriminatory contest is simply a pair of separate non-discriminatory contests—one played by α^1 of individuals competing for $\alpha^1 m^1$ prizes, and the other played by α^2 of individuals competing for $\alpha^2 m^2$ prizes.

If m^1 and m^2 are strictly between zero and one, we can solve for an equilibrium by applying the same steps as in Section 4.1 but treating each group separately. If $m^g \in \{0, 1\}$, on the other hand, the contest is so severely biased that individuals in

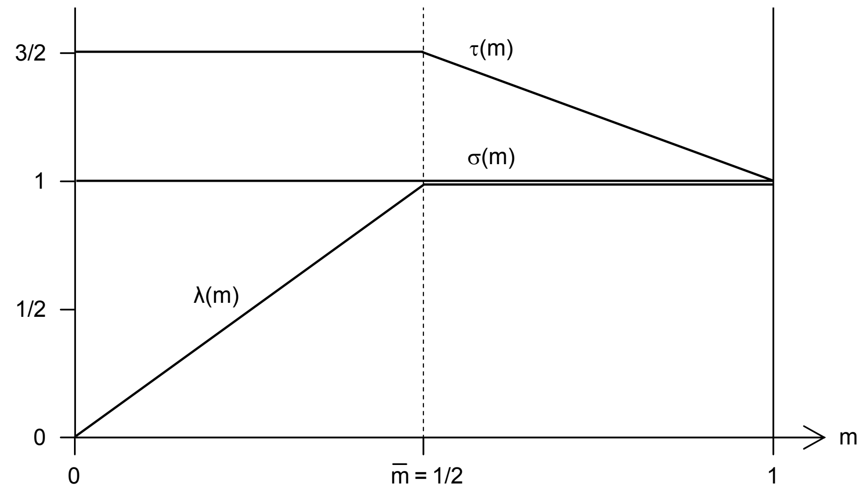

Figure 1: Equilibrium σ, λ, and τ as functions of m

group g have either a zero chance of winning or are guaranteed to win. In these extreme cases, it is clearly a dominant strategy for all individuals in group g to invest a zero effort. We thus get the following result:

Proposition 2. *Suppose the contest is discriminatory. The following holds for each group* $g = 1, 2$*:*

- *(a) If* $\overline{m} < m^g < 1$*, there exists a symmetric pure strategy equilibrium of the contest played by group* g *in which every individual* $i \in I^g$ *spends effort* $e_i^* = \sigma(m^g)$*. The minimum signal needed for an individual from group* g *to win is* $\tau^g = \tau(m^g)$*.*
- *(b) If* $0 < m^g \leq \overline{m}$*, there exists an asymmetric pure strategy equilibrium of the contest played by group* g *in which a fraction* $\lambda(m^g) = m^g/\overline{m}$ *of group-*g *individuals spend effort* $\sigma(\overline{m})$ *and win if their signal is at least* $\tau^g = \tau(\overline{m})$*. The remaining fraction* $1 - \lambda(m^g)$ *spend zero effort and never win.*
- *(c) If* $0 < m^g \leq \overline{m}$*, there also exists a symmetric mixed strategy equilibrium of the contest played by group* g*. In this equilibrium, every group-*g *individual spends effort* $\sigma(\overline{m})$ *with probability* $\lambda(m^g) = m^g/\overline{m}$*, and zero effort with the remaining probability* $1 - \lambda(m^g)$*.*
- *(d) If* $m^g \in \{0, 1\}$*, there exists a symmetric pure strategy equilibrium of the contest played by group* g *in which every individual* $i \in I^g$ *spends zero effort.*

(The proof of Proposition 2 (a)–(c) is identical to the proof of Proposition 1 and is therefore omitted. Part (d) was explained above.)

Note that in equilibrium of C_D there will be two signal thresholds, τ^1 and τ^2, such that individual $i \in I^g$ wins if $e_i^* + \nu_i \geq \tau^g$ $(g = 1, 2)$. Each τ^g is computed as a function of m^g in the same way as the single threshold $\tau(m)$ for C_{ND}.

5 Observational Equivalence and Perceptions

We have so far introduced two types of contests, non-discriminatory and discriminatory, and solved for their respective equilibria under the standard assumption that the contestants had common knowledge of the type of contest. We will now present an argument that, from the perspective of both an outside observer and that of a contestant, it can be difficult to distinguish between discriminatory and non-discriminatory contests.

To do so, we imagine an "analyst" who does not observe the contest type. However, the analyst may observe the outcome of the contest, or parts of it. That is, she may observe the following data:

1. The individual efforts,
2. the resulting qualifications (signals) of winners and losers,
3. the fraction of winners in each group,

or any combination thereof. Depending on how much one assumes regarding the information available to the analyst, only some of the above variables may be realistically observed.[8] It will not matter for our results how much outcome information is actually available to the analyst.

Before we proceed, we need to introduce some additional notation. Given $g = 1, 2$, denote by

$$T^g(\mathbf{e}, \nu|C) = \begin{cases} \inf\{\ t \ :\ \big|\{i \in I : e_i + \nu_i \geq t\}\big| \leq m \ \} & \text{if } \theta = ND, \\ \inf\{\ t^g \ :\ \big|\{i \in I^g : e_i + \nu_i \geq t^g\}\big| \leq \alpha^g m^g \ \} & \text{if } \theta = D \end{cases}$$

the *observed* cutoff signal above which members of group g win in contest C, for efforts $\mathbf{e}$ and realized noise ν. Note that, if C is non-discriminatory then $T^1(\mathbf{e}, \nu|C) = T^2(\mathbf{e}, \nu|C)$. If C is discriminatory, it is possible (but not necessary) that $T^1(\mathbf{e}, \nu|C) \neq T^2(\mathbf{e}, \nu|C)$. Because both groups contain a continuum of agents, the law of large number implies that

$$T^g(\mathbf{e}|C) \equiv E_\nu\big[T^g(\mathbf{e}, \nu|C)\big] \ = \ T^g(\mathbf{e}, \nu|C) \quad \text{almost surely.}$$

That is, for a profile given $\mathbf{e}$, the ex-post observed cutoff signal will equal the ex-ante expected threshold with probability one.

Further, denote by

$$M^g(\mathbf{e}, \nu|C) = \frac{1}{\alpha^g} \cdot \big|\{i \in I^g : e_i + \nu_i \geq T^g(\mathbf{e}, \nu|C)\}\big|$$

[8]For example, if the contest represents a labor market, statistical information about the third item (the fraction of hires in each group) can be relatively easy to obtain. The second item (the actual qualification of individuals hired and not hired) may be harder to come by, and information on the first item (the individuals' efforts while in training) may be impossible to get.

the *observed* fraction of group-g winners in contest C, again for efforts $\mathbf{e}$ and realized noise ν. Note that, if C is discriminatory with prescribed group quotas m^1 and m^2, then $M^1(\mathbf{e}, \nu|C) = m^1$ and $M^2(\mathbf{e}, \nu|C) = m^2$. If C is non-discriminatory, then it is possible (but not necessary) that $M^1(\mathbf{e}, \nu|C) = M^2(\mathbf{e}, \nu|C) = m$. As before, our continuum assumption and the law of large numbers imply that

$$M^g(\mathbf{e}|C) \equiv E_\nu\big[M^g(\mathbf{e}, \nu|C)\big] \;=\; M^g(\mathbf{e}, \nu|C) \quad \text{almost surely.}$$

That is, for a profile given $\mathbf{e}$, the ex-post observed group shares will equal the ex-ante expected shares with probability 1.

5.1 Outside observers

Our first perspective is that of an outside observer. That is, we assume that contestants possess common knowledge about the contest type, and play the contest accordingly, while the observer only observes the outcome of the contest. The following result asserts that, based on outcome information as described above, it can be impossible to tell for the observer whether the contest is discriminatory or not.

Theorem 3. *Let $\overline{m}$ and $\sigma(\cdot)$ be defined as before, and suppose $m \leq \overline{m}$. Let C_{ND} be a non-discriminatory contest with m prizes available, and let C_D be any discriminatory contest with group quotas $m^1, m^2 \leq \overline{m}$ such that $\alpha^1 m^1 + \alpha^2 m^2 = m$. There exists a profile $\mathbf{e}^*$ such that the following holds:*

(i) $\mathbf{e}^*$ is an equilibrium in both C_{ND} and C_D,

(ii) $T^1(\mathbf{e}^*|C_D) = T^2(\mathbf{e}^*|C_D) = T^1(\mathbf{e}^*|C_{ND}) = T^2(\mathbf{e}^*|C_{ND}) = \tau(\overline{m})$, and

(iii) $M^1(\mathbf{e}^*|C_D) = M^1(\mathbf{e}^*|C_{ND}) = m^1$ and $M^2(\mathbf{e}^*|C_D) = M^2(\mathbf{e}^*|C_{ND}) = m^2$.

The formal proof of Theorem 3 is in the Appendix. However, the result has a very simple intuition that can be illustrated using the example of the previous section where $\overline{m} = \frac{1}{2}$. Set the size of the pool of available prizes to $m = \frac{1}{4}$ and suppose that the two groups of the population are of size $\alpha^1 = \alpha^2 = \frac{1}{2}$. Proposition 1 (c) states that it is an equilibrium of C_{ND} if half of the population exerts effort $\sigma(\overline{m}) = 1$ while the other half exerts effort zero. Because it does not matter who the high-effort individuals are, it is an equilibrium of C_{ND} if all group-1 members exert high effort and all group-2 members exert low effort. Precisely this profile, now, is also an equilibrium of a highly discriminatory contest with $m^1 = \frac{1}{2}$ and $m^2 = 0$, as stated in Proposition 2. Observational equivalence is hence the result of the fact that the efforts of a group-1 individual and a group-2 individual can be substituted for one another while not upsetting the equilibrium of the non-discriminatory contest.

This substitution works as long as $m \leq \overline{m}$. If $m > \overline{m}$ this is no longer feasible, since all individuals exert high effort in C_{ND}. While the same profile will also be an equilibrium in C_D (provided m^1 and m^2 are high enough), if $m^1 \neq m^2$ the observed cutoff thresholds $T^1(\mathbf{e}^*|C_D), T^2(\mathbf{e}^*|C_D)$ will be different, as will be the observed win quotas $M^1(\mathbf{e}^*|C_D) = m^1$ and $M^2(\mathbf{e}^*|C_D) = m^2$. These differences in observed outcomes give away the fact that C_D discriminates and C_{ND} does not. The exception is the separate-but-equal contest:

Theorem 4. *Let $\overline{m}$ and $\sigma(\cdot)$ be defined as before, and suppose $m > \overline{m}$. Let C_{ND} be a non-discriminatory contest with m prizes available, and let C_D be a separate-but-equal contest, that is, a discriminatory contest with group quotas $m^1 = m^2 = m$. Then the following holds:*

(i) $\mathbf{e}^* = (\sigma(m))_{i\in I}$ *is an equilibrium in both C_{ND} and C_D,*

(ii) $T^1(\mathbf{e}^*|C_D) = T^2(\mathbf{e}^*|C_D) = T^1(\mathbf{e}^*|C_{ND}) = T^2(\mathbf{e}^*|C_{ND}) = \tau(m)$, *and*

(iii) $M^1(\mathbf{e}^*|C_D) = M^1(\mathbf{e}^*|C_{ND}) = M^2(\mathbf{e}^*|C_D) = M^2(\mathbf{e}^* C_{ND}) = m.$

Notice that, in real contests, much less information could be available to an outside observer than what we assume here. In particular, the assumption that one can observe all individuals' efforts and their signals need not hold. This makes our results very strong: All contests that are observationally equivalent contests according to Theorem 3 or 4 would still be equivalent if any of the equivalence requirements (i), (ii), or (iii) in the theorems were discarded.

5.2 Inside observers

We now turn to the perspective of an inside observer, by which we mean the player in the contest themselves. We will maintain the assumption that the parameters $m, \alpha^1, \alpha^2, \Pi, F$ are common knowledge across all individuals. However, we now allow individuals to differ with respect to what they believe about the potentially discriminatory nature of the contest. In other words, we do away with the assumption that the contest type is common knowledge among the players. This requires a slight change in our equilibrium concept, which until now has been Nash equilibrium.

We start with the notion of a ***perception*** of a contest, which we define in the same way as a contest type. That is, a perception for individual i is a tuple

$$\widetilde{C}_i = (\widetilde{\theta}_i, \widetilde{m}_i^1, \widetilde{m}_i^2),$$

which represents the type of contest that individual i believes she is engaged in. The collection $\widetilde{\mathbf{C}} = (\widetilde{C}_i)_{i\in I}$ is called a ***perception profile***. We can then make the following definition of an equilibrium:

Definition 1. A profile of efforts and perceptions $(\mathbf{e}^*, \widetilde{\mathbf{C}}^*) = (e_i^*, \widetilde{C}_i^*)_{i\in I}$ is a ***rational perceptions equilibrium*** of the contest C if the following holds for all $i \in I$:

(i) e_i^* maximizes the payoff $U_i(e_i^*, \mathbf{e}_{-i}^*|\widetilde{C}_i^*)$,

(ii) $\widetilde{\theta}_i^* = D$ implies $M^g(\mathbf{e}^*|C) = \widetilde{m}_i^{g*}$ for $g = 1, 2$,

(iii) $\widetilde{\theta}_i^* = ND$ implies $T^1(\mathbf{e}^*|C) = T^2(\mathbf{e}^*|C)$.

This definition has three parts. Part (i) simply says that every individual chooses her effort to maximize the expected payoff in the contest she thinks she is playing. To understand parts (ii) and (iii), our equilibrium concept is based on the idea that the contestant—just like the outside observer in Section 5.1—can see the realized outcome of the contest. That is, contestants can observe $\mathbf{e}^*$ as well as $T^g(\mathbf{e}^*|C)$ and $M^g(\mathbf{e}^*|C)$ $(g = 1, 2)$. Individuals can then utilize this information to falsify their initial perceptions about the contest in the following sense: If the realized outcome of the actual contest is not (with positive probability) the realized outcome in the individual's perceived contest, this perception cannot be part of an equilibrium. A rational perceptions equilibrium should hence be viewed as a profile of efforts and perceptions that can persist in the long run of the contest.

What specific restrictions does Definition 1 impose on perceptions to survive this possibility of falsification? First, let us look at an individual who believes that the contest discriminates with perceived group quotas $\widetilde{m}_i^1$ and $\widetilde{m}_i^2$. For this individual to maintain her perception, almost surely it must be the case that a fraction $\widetilde{m}_i^1$ of individuals from group 1 and a fraction $\widetilde{m}_i^2$ of individuals from group 2 win *in the actual contest C, given the actual efforts*. This restriction gives rise to part (ii) of Definition 1. Second, let us look at an individual who believes that the contest does not discriminate. If such an individual exists, then for her to maintain this perception it cannot happen with positive probability that a losing contestant from one group has a higher signal than a winning contestant from the other group, again in the actual contest C and given the actual efforts. This restriction gives rise to part (iii) of the definition.

We have deliberately chosen the term "rational perceptions" over the term "rational expectations." The colloquial understanding of the term "rational expectations" is that agents know, in equilibrium, the structural equations that govern the model containing these agents. This is not the case here: Our equilibrium concept permits agents to not know the contest C and instead have individual perceptions $\widetilde{C}_i$ different from C. Furthermore, individuals may or may not know that others hold perceptions that are possibly different from their own. In particular, in a rational perceptions equilibrium individuals may "agree to disagree" with one another about the true nature of the contest. All that is required is that any incorrect perceptions as to the true nature of the contest not be contradicted by the observable data generated in the correct model C, assuming behavior that is optimal given the individuals' (possibly incorrect) perceptions.

The next theorem states that individual perceptions about C do not have to agree with either C or with each other in a rational perceptions equilibrium:

Theorem 5. *Let $\overline{m}$ and $\sigma(\cdot)$ be defined as before. Suppose $m \leq \overline{m}$ and let $C = (\theta, m^1, m^2)$ be any contest with m prizes available. The profile $(\mathbf{e}^*, \widetilde{\mathbf{C}}^*)$ is a rational perceptions equilibrium of C if there exist numbers $\mu^1, \mu^2 \leq \overline{m}$ such that $\alpha^1\mu^2 + \alpha^2\mu^2 = m$ if $\theta = ND$ and $\mu^g = m^g$ $(g = 1, 2)$ if $\theta = D$, and the following holds:*

(i) A fraction $\mu^g/\overline{m}$ of individuals from group g spend effort $e_i^* = \sigma(\overline{m})$, while the rest spend $e_i^* = 0$,

(ii) $\widetilde{\theta}_i^* = D$ implies $\widetilde{m}_i^{g*} = \mu^g$ $(g = 1, 2)$.

If $m > \overline{m}$ then the following is true: $(\mathbf{e}^*, \widetilde{\mathbf{C}}^*)$ is a rational perceptions equilibrium of contest $C = (\theta, m, m)$, which either does not discriminate or is a separate-but-equal contest, if $\mathbf{e}^* = (\sigma(m))_{i \in I}$ and $\widetilde{\theta}_i^* = D$ implies $\widetilde{m}_i^{1*} = \widetilde{m}_i^{2*} = m$.

Theorem 5 states that, provided the total number of available prizes is not too large, a very large set of effort and perception profiles can be rational perceptions equilibria. As in Proposition 1 and 2, there will be some individuals who spend zero effort, and others who spend effort $\sigma(\overline{m})$. If the contest is discriminatory, the fraction of individuals choosing high effort in each group must be consistent with the equilibrium of this contest (see Proposition 2). If the contest is non-discriminatory, then the fraction of individuals choosing high effort in each group must be consistent with the equilibrium of *some* contest with the same total measure of awards. Finally, individuals who believe the contest discriminates must agree on the perceived group quotas, and if the contest is in fact discriminatory, the perceived quotas must equal the true quotas.

As before, when $m > \overline{m}$ it is easier to tell discriminatory and non-discriminatory contests apart, with the exception of the separate-but-equal case. Thus, it is possible in a rational perceptions equilibrium for some contestants to believe that the contest is non-discriminatory, while others believe it to be a separate-but-equal contest. The true contest must be one or the other.

5.3 Geometry of observational equivalence

The relationship between these observationally equivalent contests has a simple geometric representation that is depicted in Figure 2. The vertical dimension in Figure 2 represents the size m of available prizes in a non-discriminatory contest C_{ND}. The two horizontal dimensions represent the group quotas m^1 and m^2 in a discriminatory contest C_D. The shaded area in the figure and the line from $(\overline{m}, \overline{m}, \overline{m})$ to $(1, 1, 1)$ contain all points (m, m^1, m^2) such that C_D and C_{ND} are observationally equivalent (from both the outside and inside perspective).[9]

[9] The shaded area is the intersection of the plane $\{(m, m^1, m^2) : \alpha^1 m^1 + \alpha^2 m^2 = m\}$ and the cube $[0, \overline{m}]^3$. For $m > \overline{m}$, we are left with the line segment along which $m^1 = m^2 = m$.

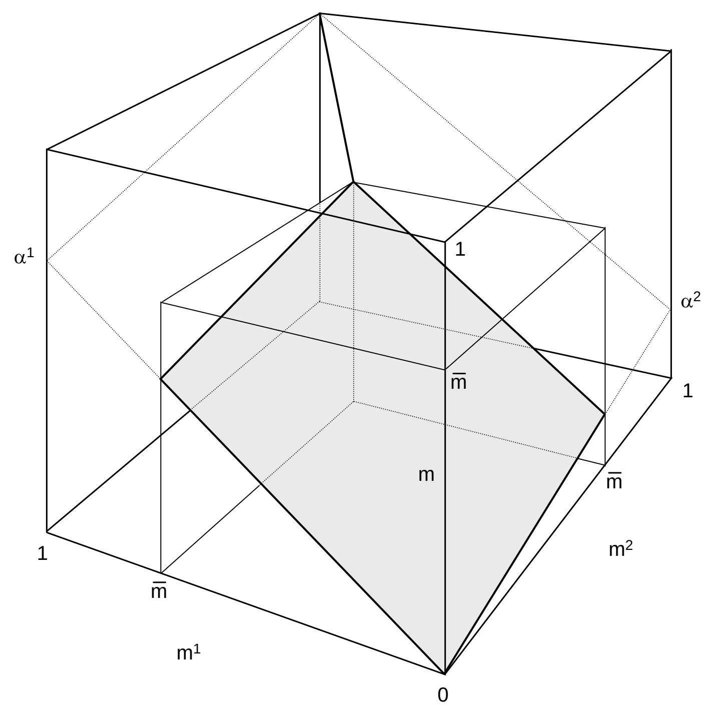

Figure 2: Observationally equivalent contests

For a given $m < \overline{m}$ there is a continuum of pairs (m^1, m^2) such that the point (m, m^1, m^2) lies in the shaded plane depicted in Figure 2. Assuming the actual contest is non-discriminatory, this implies that a continuum of rational perceptions equilibria exists in which some or all individuals believe that the contest discriminates with group quotas m^1 and m^2. We do not take a position on which of these equilibria should be regarded as more reasonable than others—with the possible exception of equilibria that arise as focal points in cases where the contests has actually discriminated in the past. (Focal points were discussed in the introduction.) Our point is that a large number of outcomes and perceptions can be supported in equilibrium of our contest model.

However, the equilibrium indeterminacy inherent in the model does seem to be consistent with the different observed patterns of inter-group differences across societies. Consider the labor market outcomes of minorities. In some societies, minority groups are under-represented in high-wage or high-prestige jobs, relative to their proportion in the population. However, in other societies minorities are over-represented in these positions. African-Americans in the United States are an example of the former, while ethnic Chinese in many parts of Southeast Asia are an example of the latter. Of course, these patterns can have many causes and explanations. In the following, we examine what our contest model has to say about the representation of minorities among the contest winners.

Suppose that group 1 is the majority and group 2 is the minority ($\alpha^1 > \frac{1}{2} > \alpha^2$). Figure 2 is drawn for this case. For each group g, denote by

$$r^g(\mathbf{e}|C) = \frac{\alpha^g M^g(\mathbf{e}|C)}{m},$$

the fraction of prizes that is obtained by members of group g in contest C and profile $\mathbf{e}$. For simplicity, consider the case $r^g \in \{0, 1\}$. That is, all prizes are won by just one group. Such severe misrepresentation occurs along the the two sides of the shaded region in Figure 2 where $m^1 = 0$ or $m^2 = 0$. For example, we have $r^1 = 1$ and $r^2 = 0$ along the line segment from $(0, 0, 0)$ to $(\alpha_1\overline{m}, \overline{m}, 0)$, and $r^1 = 0$ and $r^2 = 1$ along the line segment from $(0, 0, 0)$ to $(\alpha^2\overline{m}, 0, \overline{m})$. Observe now that for all $m \in (0, \alpha^1\overline{m})$ there exists an equilibrium of the non-discriminatory contest with m prizes that features severe over-representation of the majority group, and severe under-representation of the minority-group. Likewise, for all $m \in (0, \alpha^2\overline{m})$, there exists an equilibrium that features the reverse misrepresentation. Since $\alpha^1 > \alpha^2$, it is "easier" for the minority group to be under-represented in a non-discriminatory contest than it is for the majority group.[10]

6 Extensions

In the preceding sections, we developed a contest model with the possibility of discrimination and established two related results: The potential difficulty for an outside observer to distinguish discriminatory contests from non-discriminatory ones (Theorem 3 and 4), and the possibility that the contestants themselves may not be able to tell the two contests apart (Theorem 5).

Whenever our equivalence results apply, two conditions are satisfied: First, all individuals are indifferent between exerting effort and not exerting effort. Second, all individuals receive the same expected payoff, namely zero. The first feature may seem unrealistic. For example, there are motivated students from every race or gender, and these individuals are unlikely to be indifferent between studying and not studying. The second feature raises the question why one should be concerned with whether or not the contest is discriminatory, or perceived as such, given that everyone receives the same welfare.

In this section, we address both issues. We demonstrate that it is possible to break both individidual indifference and equality in welfare, while maintaining the model's core structure and observational equivalence results.

[10] That is, minority under-representation can arise in equilibrium of a larger set of non-discriminatory contests than majority under-representation. In particular, for intermediate values of m the minority can be severely under-represented, while the same is no longer possible for the majority.

6.1 Individual heterogeneity

To address the issue of individual indifference, we relax the assumption that individuals are homogenous in all aspects except their group label. Instead, we now let individuals differ in economically relevant characteristics, such as their valuation of winning a prize or their cost of effort. Groups are still symmetric in the sense that the distribution of individual types in group 1 is the same as in group 2.

Notice that the equivalence results in the previous section relied on the substitutability of effort across groups. This substitutability, in turn, was driven by the fact that (for a low enough m) all individuals were indifferent between exerting effort and not exerting effort. If individuals are heterogenous in economic characteristics, the indifference condition will apply to a smaller number of marginal individuals only, across which effort could be substituted. With a sufficiently high degree of heterogeneity—say, if individuals characteristics were continuously distributed in the population—these marginal individuals would be too few for their behavior to affect aggregate outcomes. In other words, if any inter-group outcome differences were observed then these could only be explained by actual discrimination.

In order to maintain the equivalence results of Theorems 3 and 5 in the presence of heterogenous individuals, therefore, some discreteness in the type distribution is required. Thus, let us assume that within each group $g = 1, 2$ there is a fraction β_H of individuals whose valuation of winning a prize is Π_H, a fraction β_M whose valuation is Π_M, and a fraction $\beta_L = 1 - \beta_H - \beta_M$ whose valuation is Π_L (with $\Pi_H > \Pi_M > \Pi_L > 0$). We call these individuals H-types, M-types, and L-types, respectively.[11] For simplicity, let us also fix a uniform noise distribution over $[0, \varepsilon]$ and a quadratic cost function $c(e) = \frac{1}{2}e^2$ throughout.

Non-discriminatory contests. Consider first a non-dscriminatory contest with m prizes, and suppose that every H-type exerts the same positive effort σ_H, a fraction $\lambda \in (0, 1)$ of M-types exerts positive effort σ_M, and all other individuals exert zero effort. As in the original model, there exist a uniform threshold τ such that only individuals with signals above τ win. Assuming that $\tau > \varepsilon$ (so a zero effort has no chance of winning), the capacity constraint (3) can be written as

$$\beta_H \left(1 - (\tau - \sigma_H)\right) + \lambda\beta_M \left(1 - F(\tau - \sigma_M)\right) \;=\; m. \tag{8}$$

Denote by $U_\theta(e_i|\tau) = (1 - F(\tau - e_i))\,\Pi_\theta - c(e_i)$ the expected payoff function for a θ-type ($\theta \in \{H, M, L\}$), given signal threshold τ. As before, $U_\theta(e_i|\tau)$ will have two local maxima, one at zero and one at $\sigma_\theta > 0$. The first-order condition for an interior

[11] The assumption that heterogeneity arises from different valuations is without loss of generality: By scaling the payoff functions, all individuals can have the same valuation Π but different effort costs, namely $\gamma_H c(\cdot)$, $\gamma_M c(\cdot)$, and $\gamma_L c(\cdot)$ (with $\gamma_H < \gamma_M < \gamma_L$).

payoff maximum for a θ-type is $f(\tau - \sigma_\theta)\Pi_\theta - c'(\sigma_\theta) = 0$, and using our functional form assumptions on f and c this implies $\sigma_\theta = \Pi_\theta/\varepsilon$. Substituting σ_θ into (8) and solving, we get τ (as a function of λ):

$$\tau \;=\; \varepsilon \cdot \frac{\beta_H \left[1 + \dfrac{\Pi_H}{\varepsilon^2}\right] + \lambda\beta_M \left[1 + \dfrac{\Pi_M}{\varepsilon^2}\right] - m}{\beta_H + \lambda\beta_M}. \tag{9}$$

Next, for a fraction $\lambda \in (0,1)$ of M-types to exert effort σ_M, M-type individuals must be indifferent between spending $\sigma_M = \Pi_M/\varepsilon$ and spending zero. Thus, we need

$$U_M(\sigma_M|\tau) \;=\; \left(1 - \frac{1}{\varepsilon}\left[\tau - \frac{\Pi_M}{\varepsilon}\right]\right)\Pi_M - \frac{1}{2}\left(\frac{\Pi_M}{\varepsilon}\right)^2 \;=\; 0. \tag{10}$$

Using (9) in (10) and performing some algebra, we can solve for the equilibrium fraction of M-types who exert positive effort:

$$\lambda(m) \;=\; \frac{m - \beta_H B}{\beta_M A}, \tag{11}$$

where $A = 1 - \Pi_M\left[1 + \Pi_M/(2\varepsilon^2)\right] + \Pi_M/\varepsilon^2$ and $B = 1 - \Pi_M\left[1 + \Pi_M/(2\varepsilon^2)\right] + \Pi_H/\varepsilon^2$. Finally, plugging (11) back into (9) we obtain the equilibrium cutoff signal above which an individual wins a prize. After simplifying, this cutoff can be written as

$$\tau^* \;=\; \frac{\Pi_M(2\varepsilon^2 + \Pi_M)}{2\varepsilon}. \tag{12}$$

Under some conditions on the parameters of the extended model, analogous to the previous condition $m < \overline{m}$, we have $\lambda(m) \in (0,1)$ and $\tau^* > \varepsilon$ as initially presumed.

By construction, in both contests the M-types are indifferent between exerting effort $\sigma_M = \Pi_M/\varepsilon$ and effort zero, as both yield a zero expected payoff. On the other hand, the expected payoff for H-types from effort $\sigma_H = \Pi_H/\varepsilon$ is positive:

$$\begin{aligned} U_H(\sigma_H|\tau^*) \;&=\; (1 - F(\tau^* - \sigma_H))\Pi_H - c(\sigma_H) \;>\; (1 - F(\tau^* - \sigma_M))\Pi_H - c(\sigma_M) \\ &> \; (1 - F(\tau^* - \sigma_M))\Pi_M - c(\sigma_M) \;=\; U_M(\sigma_M|\tau^*) \;=\; 0, \end{aligned}$$

where the first inequality follows from the first-order condition for H-types and the second from $\Pi_H > \Pi_M$. In a similar manner one can show that $U_L(\sigma_L|\tau^*) < 0$. Thus, it is strictly optimal for H-types to spend $\sigma_H > 0$, and strictly optimal for L-types to spend zero.

Discriminatory contests. Let us now turn to a discriminatory contest with group-specific award quotas m^1 and m^2. Applying (11) to each group separately, a fraction

$$\lambda^g(m^g) \;=\; \frac{m^g - \beta_H B}{\beta_M A} \tag{13}$$

of M-type individuals in group $g \in \{1,2\}$ spends effort σ_M. In the aggregate, then, a fraction $\alpha^1\lambda^1(m^1)+\alpha^2\lambda^2(m^2)$ of M-types spend effort σ_M in the discriminatory contest. Now recally that in the non-discriminatory contest with m prizes, this number was $\lambda(m)$; furthermore, it did not matter whether how many of the high-effort M-types belonged to group 1 and group 2. Thus, the equilibrium effort profile of the discriminatory contest is an equilibrium of the non-discriminatory contest as long as $\alpha^1\lambda^1(m^1)+\alpha^2\lambda^2(m^2) = \lambda(m)$. Using (11) and (13), we can write this equality as

$$\frac{1}{\beta_M A}\left(a^1 m^1 + \alpha^2 m^2 - \beta_H B\right) \;=\; \frac{1}{\beta_M A}\left(m - \beta_H B\right).$$

After cancelling common terms, this reduces to $\alpha^1 m^1 + \alpha^2 m^2 = m$, our original relation among discriminatory and non-discriminatory contests. By shifting efforts across M-types in both groups, the outcome of a non-discriminatory contest can hence be made to resemble that of a discriminatory contest with group quotas m^1 and m^2. In both contests individuals win if their signal is above τ^*, so that the fraction of winners in group $g = 1,2$ is the same in either case. It is therefore possible to extend our observational equivalence results to the heterogeneous case.[12]

6.2 Welfare considerations

Let us now turn to the second issue, ex-ante equality in payoffs across groups. Let C be a contest type and let $\mathbf{e}$ be an effort profile. For each $g \in \{1,2\}$ define

$$W^g(\mathbf{e}|C) \;=\; \frac{1}{\alpha^g}\int_{i\in I^g} U_i(\mathbf{e}|C)\,di$$

to be the average payoff of an individual in group g under effort profile $\mathbf{e}$, given a contest type C. In the equilibria characterized in Theorems 3–5, $W^1(\mathbf{e}|C) = W^2(\mathbf{e}|C)$. Despite this ex-ante equality in welfare in our model, we believe there are reasons why one might worry about discrimination and ex-post inequality.

[12] Doing so requires an additional restriction, however. Recall that, in the baseline model, the prize quotas had to be sufficiently small to generate asymmetric equilibria (i.e., $m, m^1, m^2 < \overline{m}$). In the extended model, such an upper bound alone is insufficient and an additional lower bound is needed. In particular, (11) and (13) imply that $\beta_H B \leq m^1, m^2, m \leq \beta_H B + \beta_M A$. The reason why both an upper and a lower bound are needed is that all H-types choose the same positive effort σ_H. If too few prizes were available in a contest this would not be an equilibrium. Instead, the H-types would choose asymmetric efforts and be indifferent, while both the M-types and the L-types would strictly prefer zero efforts.

Externalities. Most importantly, in reality there are likely to be positive externalities of both efforts (e.g., years of schooling) and prize wins (e.g., wages) within groups. A particularly compelling case can be made for families. If g denotes race, and interracial families are uncommon, then individuals belonging to the same family will also belong to the same group g. If we further think of efforts as parental education and prizes as parental income, then ex-post racial inequality implies that children in the disadvantaged group will have poorer and less educated parents on average than children in the advantaged group. To the extent that parental education and parental income matter for a child's development, the fact that parents may have been ex-ante indifferent between investing zero effort or a postive amount is irrelevant as far as their childrens' welfare is concerned. Similar externalities can arise in the form of neighborhood effects if residential neighborhoods are segregated by race, or in the form of peer effects if schools are segregated.

Such intra-group externalities can easily be incorporated into our model by changing individual payoffs from $U_i(\mathbf{e}|C)$ to

$$\hat{U}_i(\mathbf{e}|C) \;=\; U_i(\mathbf{e}|C) + \phi\big(\overline{e}^{g(i)}(\mathbf{e}|C), M^{g(i)}(\mathbf{e}|C)\big),$$

where $\overline{e}^g$ denotes the average effort and M^g the fraction of winners in group g, and ϕ is an increasing function. Clearly, individual incentives and decisions will not be affected by this change. However, the average welfare of group g now becomes

$$\hat{W}^g(\mathbf{e}|C) \;=\; \frac{1}{\alpha^g}\int_{i\in I^g}\hat{U}_i(\mathbf{e}|C)\,di \;=\; W^g(\mathbf{e}|C) + \phi\big(\overline{e}^g(\mathbf{e}|C), M^g(\mathbf{e}|C)\big),$$

and it is easy to see that, even if $W^g(\mathbf{e}|C)$ is the same for both groups, $\hat{W}^g(\mathbf{e}|C)$ will be larger in the group that generates more effort and more winners. In this case, it does not seem unreasonable for policy makers to take the effects of discrimination into account if they are concerned with the distribution of benefits across individuals.

Psychological effects. Even if outcomes are distributed equally across groups there may be reasons to be concerned with whether the institutions which generate these outcomes are discriminatory, or perceived as such. Consider, for example, a separate-but-equal contest $C = (D, m, m)$. This contest will always yield equal outcomes across groups in our model (and thus $W^1(\mathbf{e}|C) = W^2(\mathbf{e}|C)$), but it does so by distinguishing the groups. Making this distinction can itself be undesirable. For instance, in its landmark *Brown v. Board of Education* decision the U.S. Supreme Court notes:

> Does segregation of children in public schools solely on the basis of race, even though the physical facilities and other "tangible" factors may be equal, deprive the children of the minority group of equal educational opportunities? We believe that it does. [...] Segregation of white and colored children in public schools has

> a detrimental effect upon the colored children. The impact is greater when it has the sanction of the law; for the policy of separating the races is usually interpreted as denoting the inferiority of the negro group. A sense of inferiority affects the motivation of a child to learn. Segregation with the sanction of law, therefore, has a tendency to retard the educational and mental development of negro children and to deprive them of some of the benefits they would receive in a racially integrated school system. (United States Supreme Court, 1954.)

If these psychological factors play a role, then discrimination will affect individual welfare directly, even if it is of the separate-but-equal form. In addition, the mere perception of discrimination has also been shown to have adverse effects on individuals' mental health (Sellers and Shelton, 2003).

Like externalities, such effects can be incorporated into individual payoff functions easily. Let $\widetilde{C}_i = (\widetilde{\theta}_i, \widetilde{m}_i^1, \widetilde{m}_i^2)$ be a perception for individual i and set

$$\hat{U}_i(\mathbf{e}, \widetilde{C}_i|C) \;=\; U_i(\mathbf{e}|C) + \begin{cases} \phi(\widetilde{m}^{g(i)}) & \text{if } \widetilde{\theta}_i = ND, \\ 0 & \text{otherwise,} \end{cases}$$

where ϕ is a function representing possible detrimental effect for individual who perceives being discriminated against. If individuals in one group perceive the contest as discriminatory while individuals in the other groups do not, welfare will not be equal across groups even if prizes are equitably awarded.[13]

7 Conclusion

We developed a model of a contest with a continuum of prizes and players. The contest allowed for the possibility of discrimination against sub-groups of the population. We showed that discriminatory contest rules can be impossible to detect from observational data by both outside and inside observers.

Applying this contest model to labor markets, our results suggest that the presence of discrimination can be difficult to determine based on employment statistics typically available to government agencies or courts. For the same reason, individual labor market participants may rationally disagree about the presence or absence of discrimination in labor markets. Our model also suggests that competition and anti-discrimination policies may not eliminate outcome disparities in previously discriminatory situations, even under the stringent assumption that there are no economically relevant differences across population groups. These results are driven by the multiplicity of equilibria in non-discriminatory contests. In contrast to the statistical discrimination literature,

[13] For example, the "a sense of inferiority" experienced by an individual who believes to be engaged in a separate but equal contest can be modeled using this specification: Let ϕ be weakly increasing with a discontinuity at m, such that $\phi(\widetilde{m}^{g(i)}) < 0$ for $\widetilde{m}^{g(i)} \leq m$ and $\phi(\widetilde{m}^{g(i)}) \geq 0$ for $\widetilde{m}^{g(i)} > m$.

however, this multiplicity is not due to informational asymmetries. In particular, non-discriminatory contests can lead to seemingly discriminatory outcomes even if individual efforts, idiosyncratic random shocks, and final qualifications are perfectly observed.

Our work leaves a number of interesting questions for future research. On the theoretical side, it seems promising to investigate alternative forms of discrimination. For example, the contest could be such that one group competes for all prizes, while the other can compete for a subset of prizes only. Applied to the labor market, this situation would arise if, say, white applicants are considered for both managerial and manual labor positions, while black applicants are excluded from managerial jobs. Alternatively, the contest might be such that one group is "handicapped" in the sense that effort investments by members of this group are discounted by a certain factor relative to efforts from the other group. Second, one could explicitly introduce dynamics to the model, along with the possibility of changing perceptions over time. Such a model would provide a setting in which a rich set of policies can be studied. For example, the effects of temporary affirmative action on both labor market outcomes as well as perceptions of the labor market could be examined in such a dynamic model.

On the empirical side, our model remains to be tested against competing theories which can also explain unequal outcomes in labor markets and other areas. In the context of law enforcement, the literature has examined the question whether outcome inequalities across groups constitute evidence of racial prejudice (i.e., preference-based discrimination) instead of racial profiling (i.e., statistical discrimination). In particular, some authors have pointed to several difficulties in making this assessment using available outcome data (Knowles et al, 2001; Anwar and Fang, 2006). These difficulties occur because of variables that affect individual decisions but are not observed by the researcher. For example, arrest data usually do not contain all the information a police officer observes when making an arrest. As we have demonstrated in this paper, even if all individuals' characteristics are observed, color-conscious and color-blind contests can be distinguishable in their outcomes. Thus, it remains to be seen if, and to which extent, our theoretical model can be tested empirically.

Appendix

Proof of Proposition 1

Step 1. Inspection of the expected payoff function. First, notice that the probability of winning with effort $e_i \le \tau(m) - \varepsilon$ is zero, and the probability of winning with effort $e_i \ge \tau(m)-$ is one. Thus, the payoff $U(e_i|\tau(m))$ can be expressed as follows:

$$U(e_i|\tau(m)) = \begin{cases} -c(e_i) & \text{if } e_i < \tau(m) - \varepsilon, \\ (1 - F(\tau(m) - e_i))\Pi - c(e_i) & \text{if } e_i \in [\tau(m) - \varepsilon, \tau(m)], \\ \Pi - c(e_i) & \text{if } e_i > \tau(m). \end{cases} \tag{14}$$

Notice that $U(e_i|\tau(m))$ is continuous and strictly concave in e_i on each of these three parts separately, and has an interior local maximum at $\sigma(m) \in [\tau(m) - \varepsilon, \tau(m)]$. Thus, since c is strictly increasing, $U(e_i|\tau(m))$ has either exactly one local maximum at $\sigma(m)$ (this will be the case if $\tau(m) \le \varepsilon$, or exactly two local maxima at $\sigma(m)$ and zero (this will be the case if $\tau(m) > \varepsilon$). Furthermore, if there is a local maximum at zero, then it has a payoff level of $-c(0) = 0$.

Step 2. Monotonicity of payoffs in m. We will now focus on the local maximum at $\sigma(m)$. Let $U^*(m) = U_i((\sigma(m))_{j\in I})$ the payoff to every individual if every individual exerts effort $\sigma(m)$. Since the probability that an individual wins a prize, $(1 - F(\tau(m) - \sigma(m)))$, is then equal to m by construction, we can write

$$U^*(m) = m\Pi - c(\sigma(m)). \tag{15}$$

We show that U^* increases in m. By Assumption 1, $\sigma(m)$ is differentiable on $(0, 1)$, and thus U^* is differentiable with

$$\frac{d}{dm}U^*(m) = \Pi - c'(\sigma(m))\frac{d}{dm}\sigma(m). \tag{16}$$

This is positive if and only if

$$c'(\sigma(m))\frac{d}{dm}\sigma(m) < \Pi. \tag{17}$$

Differentiating both sides of (5) with respect to m (using the inverse function theorem for F^{-1}), we have

$$\begin{aligned} \frac{d}{dm}\sigma(m) &= -\frac{\Pi}{c''(\sigma(m))}\frac{f'(F^{-1}(1-m))}{f(F^{-1}(1-m))} \\ &= \frac{\Pi}{c''(\sigma(m))}\frac{f'(F^{-1}(1-m))}{c'(\sigma(m))/\Pi}. \end{aligned} \tag{18}$$

Using (18) in (17), dividing both sides by Π, substituting $F^{-1}(1-m) = \tau(m) - \sigma(m)$ and rearranging, the condition (17) becomes

$$f'(\tau(m) - \sigma(m)) > -\frac{1}{\Pi}c''(\sigma(m)). \tag{19}$$

By Assumption 1 (iv) this is satisfied; $U^*(m)$ thus strictly increases in m.

Step 3. Construction of $\overline{m}$. The value of $\overline{m}$ is now set so that $U^*(\overline{m}) = 0$. Define

$$\begin{aligned} \sigma(0) &= \lim_{m\to 0} \sigma(m) = [c']^{-1}(f(\varepsilon)\Pi) > 0, \\ \sigma(1) &= \lim_{m\to 1} \sigma(m) = [c']^{-1}(f(0)\Pi) > 0. \end{aligned}$$

Define $U^*(0)$ and $U^*(1)$ accordingly, using $\sigma(0)$ and $\sigma(1)$ in (15).[14] Since $U^*(m)$ is continuous and strictly increasing in m, for there to be $\overline{m} \in (0,1)$ such that $U^*(\overline{m}) = 0$, we need $U^*(0) < 0$ and $U^*(1) > 0$. The first condition is satisfied since $\sigma(0) > 0$ implies

$$U^*(0) = -c(\sigma(0)) < 0.$$

The second condition is satisfied if

$$U^*(1) = \Pi - c(\sigma(1)) > 0.$$

Since c is strictly convex we have

$$c(\sigma(1)) < \sigma(1)c'(\sigma(1)) = \sigma(1)c'([c']^{-1}(f(0)\Pi)) = \sigma(1)f(0)\Pi$$

and hence

$$U^*(1) > \Pi(1 - \sigma(1)f(0)).$$

Assumption 1 (iii) implies that $\sigma(1) = [c']^{-1}(f(0)\Pi) \leq 1/f(0)$; thus $U^*(1)$ is guaranteed to be positive. Therefore, there exists $\overline{m}$ such that $U^*(\overline{m}) = 0$, or $\overline{m}\Pi = c(\sigma(\overline{m}))$.

Step 4. Verification of equilibria. We now finish the proof. Recall that $U(e_i|\tau(m))$ has up to two local maxima: One at $\sigma(m)$, and another at zero if $\tau(m) > \varepsilon$ with associated payoff zero. Suppose first that $m \geq \overline{m}$. Then $U(\sigma(m)|\tau(m)) = U^*(\overline{m}) \geq 0$, so that deviating to zero will always result in a weakly lower payoff than exerting effort $\sigma(m)$. Hence, exerting effort $\sigma(m)$ is a best response for all individuals. This proves part (a) of the result.

[14] These are *not* payoffs in games where $m = 0$ or $m = 1$, but the limit of payoffs in games where $m \to 0$ or $m \to 1$.

Now suppose that $0 < m < \overline{m}$. In this case, $\sigma(m) > 0$ and $U(\sigma(m)|\tau(m)) = U^*(m) < 0$. By spending zero effort, an individual receives at least a zero payoff; thus $\sigma(m)$ is not a symmetric equilibrium effort anymore. To construct an asymmetric equilibrium, we claim first that $\tau(\overline{m}) > \varepsilon$. To see that this so, suppose to the contrary that $\tau(\overline{m}) \leq \varepsilon$, in which case $0 \in [\tau(m) - \varepsilon, \tau(m)]$. Recall that $U(e_i|\tau(\overline{m}))$ is continuous, strictly concave on $(\tau(m) - \varepsilon, \tau(m))$, and maximized on this interval at $\sigma(m)$ with a value of zero. Thus, it follows that $U(0|\tau(\overline{m})) < 0$. However, by spending zero effort a player is guaranteed at least a zero payoff, a contradiction. Therefore, $\tau(\overline{m}) > \varepsilon$.

Next, observe that $\tau(\overline{m}) > \varepsilon$ implies that a player who spends zero effort against this threshold has a zero chance of winning and therefore obtains a payoff of zero. Thus, if $\tau(\overline{m})$ is the minimum signal needed for winning, every player is indifferent between spending zero and spending $\sigma(\overline{m})$. Since these are the only two local maxima of $U(e_i|\tau(\overline{m}))$, all other effort levels will result in a negative payoff. If a measure $\lambda(m) = m/\overline{m}$ of individuals spends $\sigma(\overline{m})$ while the rest spends zero, then a measure $\overline{m}\lambda(m) = m$ of players will obtain signals $t_i \geq \tau(\overline{m})$, so that $\tau(\overline{m})$ is in fact the minimum winning signal, making the effort choices optimal for all players. This proves part (b) of the result.

Finally, to prove (c), suppose $m < \overline{m}$ and assume every individual spends $\sigma(\overline{m})$ with probability $\lambda(m)$ and zero with probability $1 - \lambda(m)$ (where $\lambda(m) = m/\overline{m}$ is defined as before). Because there is a continuum of individuals, exactly a fraction $\lambda(m)$ of individuals will spend $\sigma(\overline{m})$. As shown above, this means that any one individual is then indifferent between spending zero (and not winning) and spending $\sigma(\overline{m})$ and winning with probability $\overline{m}$ (with all other effort levels strictly worse). Thus it is a best response for this individual to randomize between $\sigma(\overline{m})$ and zero with probabilities $\lambda(m)$ and $1 - \lambda(m)$; making the profile a symmetric mixed strategy quilibrium. □

Proof of Theorem 3

Consider the contest $C_D = (D, m^1, m^2)$ and let $\mathbf{e}$ be an effort profile. By definition of a discriminatory contest, $M^1(\mathbf{e}|C_D) = m^1$ and $M^2(\mathbf{e}|C_D) = m^2$ for any $\mathbf{e}$, including any equilibrium profile $\mathbf{e}^*$. Since $\sigma(1) < 1/f(0)$ we can use Proposition 2 to fix such a profile $\mathbf{e}^*$, and because $m^1, m^2 \leq \overline{m}$ we use the profile described in Proposition 2 (b): A fraction $m^g/\overline{m}$ of group $g \in \{1, 2\}$ spends effort $\sigma(\overline{m})$ and wins if $\sigma(m) + \nu_i \geq \tau(\overline{m}) = T^g(\mathbf{e}^*|C_D)$, while the remaining population spends zero effort (and never wins).

The fraction of the total population spending effort $\sigma(\overline{m})$ in profile $\mathbf{e}^*$ is therefore $\alpha^1 m^1/\overline{m} + \alpha^2 m^2/\overline{m} = m/\overline{m}$. Since $m \leq m$, Proposition 1 (b) implies that this is also an equilibrium of C_{ND}; this proves (i). Proposition 1 (c) also states that a high-effort individual wins in C_{ND} if $\sigma(\overline{m}) + \nu_i \geq \tau(\overline{m})$. Since C_{ND} is non-discriminatory, this implies $T^1(\mathbf{e}^*|C_{ND}) = T^2(\mathbf{e}^*|C_{ND}) = \tau(\overline{m})$. Because this equals $T^1(\mathbf{e}^*|C_D)$ and $T^2(\mathbf{e}^*|C_D)$, we have proven (ii). Finally, in ε^* the fraction of high-effort individuals in group-g is $m^g/\overline{m}$. Each of these individuals wins if $\sigma_(\overline{m}) + \nu_i \geq \tau(\overline{m})$, which has probability $\overline{m}$. Thus, a fraction m^g of the individuals in group g obtains winning signals, so $M^g(\mathbf{e}^*|C_{ND}) = m^g = M^g(\mathbf{e}^*|C_D)$ for $g = 1, 2$. This proves (iii). $\square$

Proof of Theorem 4

Consider the contest C_{ND} with $m > \overline{m}$ prizes. Proposition 1 (a) implies that $\mathbf{e}^* = (\sigma(m))_{i \in I}$ is an equilibrium profile, with observed signal thresholds $T^1(\mathbf{e}^*|C_{ND}) = T^2(\mathbf{e}^*|C_{ND}) = \tau(m)$. Since the profile is symmetric, a fraction m of individuals in each group wins; hence $M^1(\mathbf{e}^*|C_{ND}) = M^2(\mathbf{e}^*|C_{ND}) = m$. Next, consider the separate-but-equal contest $C_D = (D, m, m)$. Proposition 2 (a) implies that $\mathbf{e}^*$ is an equilibrium in C_D as well (which proves (i)), with observed signal thresholds $T^1(\mathbf{e}^*|C_D) = T^2(\mathbf{e}^*|C_D) = \tau(m)$ for both groups (which proves (ii)). Furthermore, $M^1(\mathbf{e}^*|C_D) = M^2(\mathbf{e}^*|C_D) = m$ (this is true in $C_D = (D, m, m)$ for all effort profiles, not only $\mathbf{e}^*$), which proves (iii). $\square$

Proof of Theorem 5

Consider first the case $m \leq \overline{m}$. If the actual contest C is non-discriminatory, choose μ^1 and μ^2 to satisfy $\mu^1, \mu^2 \leq \overline{m}$ and $\alpha^1\mu^1 + \alpha^2\mu^2 = m$, where m is the measure of prizes available in C. If the actual contest C is discriminatory, choose $\mu^1 = m^1$ and $\mu^2 = m^2$, where m^1 and m^2 are the shares allocated to group 1 and group 2 in C. Now let $\mathbf{e}^*$ be as described in condition (i) of the theorem: A fraction $\mu^g/\overline{m}$ of individuals from group g spends effort $e_i^* = \sigma(\overline{m})$, while the rest spend $e_i^* = 0$. Further, let $\widetilde{\mathbf{C}}^*$ be as described in condition (ii) of the theorem: For every individual who perceives a discriminatory contest (if there are any), set $\widetilde{m}_i^{g*} = \mu^g$ for $g = 1, 2$.

Using Proposition 1 and 2, the effort profile is an equilibrium in every contest that is part of the perception profile $\widetilde{\mathbf{C}}^*$. Thus, e_i^* maximizes $U_i(e_i^*|\widetilde{C}_i^*)$ for every i, even if $\widetilde{C}_i^* \neq C$. This shows that $(\mathbf{e}^*, \widetilde{\mathbf{C}}^*)$ satisfies condition (i) in Definition 1. Note that $\mathbf{e}_i^*$ is also an equilibrium of the actual contest C. Theorem 3 then implies that $M^g(\mathbf{e}^*|C) = \mu^g$, and since $\widetilde{m}_i^{g*} = \mu^g$ if $\widetilde{\theta}_i^* = D$ by construction, condition (ii) in Definition 1 is met as well. Finally, Theorem 3 implies that $T^1(\mathbf{e}^*|C) = T^2(\mathbf{e}^*|C)$, so condition (iii) in Definition 1 is satisfied. This shows that $(\mathbf{e}^*, \widetilde{\mathbf{C}}^*)$ is a rational perceptions equilibrium of C.

Now consider $m > \overline{m}$ and a contests C that is either non-discriminatory (C_{ND}) or separate-but-equal ($C_D = (D, m, m)$). The theorem requires all individuals to spend $\varepsilon_i^* = \sigma(m)$, have perceptions $\widetilde{C}_i^* \in \{C_{ND}, C_D\}$. Since $\mathbf{e}^* = (\sigma(m))_{i \in I}$ is an equilibrium in both C_{ND} and C_D (Proposition 1 and 2), condition (i) of Definition 1 is satisfied. Theorem 4 implies $M^1(\mathbf{e}^*|C_D) = M^2(\mathbf{e}^*|C_D) = M^1(\mathbf{e}^*|C_{ND}) = M^2(\mathbf{e}^*|C_{ND}) = m$ and $T^1(\mathbf{e}^*|C_D) = T^2(\mathbf{e}^*|C_D) = T^1(\mathbf{e}^*|C_{ND}) = T^2(\mathbf{e}^*|C_{ND}) = \tau(m)$, so conditions (ii) and (iii) hold as well. $\square$

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