Matching Markets and Social Networks

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Abstract

We consider a spatial two-sided matching market with a *network friction*, where exchange between any pair of individuals requires that the individuals know each other. Such relationships are costly and must be formed before individuals learn their availability for trade. Our theoretical results characterize the basic geometry of small stable networks. We then use simulation techniques to examine the structure and size of larger stable networks. We show that regular networks (i.e., networks in which individuals know those within a given distance from themselves) are not necessarily stable. If they are, these networks grow in size as uncertainty increases and/or network costs decrease. Furthermore, an imbalanced market where (on expectation) one side is rationed by the other tends to decrease network size.

Keywords: Social and economic networks, network formation, matching markets, spatial differentiation.

JEL codes: C71, C78, D20, D85.

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1 Introduction

In many market environments, individuals who know each other are more likely to trade than anonymous agents. This can be for several reasons. First, by establishing a social relationship with other individuals, agents can acquire relevant information about the good or service to be exchanged (an obvious case in point is marriage). Second, social relationships may reduce transaction costs, for instance by substituting trust for monitoring. Third, prevailing cultural norms may require that individuals become acquainted with one another personally before engaging in commercial transactions. These personal relationships constitute a social network, which facilitates exchange in non-anonymous market environments.

The present paper investigates the role of social networks in matching markets. We take as the underlying economic environment a two-sided matching market with non-transferable utility and horizontal heterogeneity. Potential buyers and sellers of a differentiated good are located along a line, with the surplus from a buyer-seller match decreasing in the distance between the agents. Thus, every agent prefers to be in a close match rather than a distant match.[1] We introduce uncertainty by assuming that, a priori, it is not known which potential sellers and buyers will be on the market. As an example, consider the matching of workers (sellers of labor) and firms (buyers of labor): A worker may only know that (with some probability) he becomes unemployed in the future, in which case he would enter the supply side of the labor market. Similarly, a firm may only know that (with some probability) it will have a job vacancy in the future, in which case it would enter the demand side of the labor market. Before the agents learn whether or not they are on the market, they engage in a cooperative game of network formation. We assume that links to other agents are costly, but also necessary for future exchange. Once it is revealed which agents are on the market, a stable matching of agents on the market is found *subject to the constraint that only agents linked in the social network can be paired.* We call this constraint a network friction.[2]

The main questions we ask are: What is the value of a network in the matching environment; who knows whom in equilibrium; and how does the structure and size of the social network depend on the parameters of the underlying matching market? We demonstrate how the network payoff function can be derived from the matching market model and use this payoff to define pairwise stable networks.[3] Under certain restrictions, the network

[1]The study of two-sided matching problems was pioneered by Gale and Shapley (1962), Shapley and Shubik (1972), and Becker (1973); for a survey of the classic results in two-sided matching, see Roth and Sotomayor (1990). The particular setup used here, a two-sided market with horizontal heterogeneity, is studied in Eeckhout (2000), Clark (2003, 2006), and Klumpp (2009).

[2]Pissarides (2000) defines a friction as "anything that interferes with the smooth and instantaneous exchange of goods and services," with the implication that "individuals are prepared to spend time and other resources on exchange." The most common way to introduce frictions to matching markets is by assuming that agents meet potential match partners randomly over time, and then decide whether to stay with their partner or wait to draw another partner at random. This is called a *search friction.*

[3]Pairwise stability, introduced in Jackson and Wolinsky (1996), means that no individual prefers to sever

payoff will be simple enough to allow us to answer the next two questions analytically. In particular, we derive conditions for the existence of a pairwise stable network in which each individual knows her k nearest neighbors, for $k \leq 3$. This will be called a regular network of size k. We further show that k depends non-monotonically on the parameters of the model: If the probability that an agent has a positive supply or demand of the good increases, network size grows at first but then shrinks. We then use simulations to compute the pairwise stable networks for a wider set of parameters. We show that in general, regular pairwise stable networks need not exist. When they do exist, our numerical results confirm that, ceteris paribus, network size is non-monotonic in the probability of needing a trading partner, and is maximized when each buyer and seller has probability $1/2$ of being on the market.

These results highlight the fact that social networks serve an insurance function in our model. Links to other agents are necessary for an individual to be paired when the individual needs a partner, which a priori is uncertain. Reaching out to a large set of other agents, therefore, acts as insurance against future needs, whose value is largest when uncertainty is high. In our model, this is the case when the probabilities that an individual will be on the market are in an intermediate range. In our simulations we also show that an imbalanced market—in which the expected number of agents on one side is larger than the expected number of agents on the other side—tends to decrease the size of social networks. The reason is that, even though social networks facilitate exchange, they also facilitate competition over match partners. In a state in which one side of the market is rationed by the other on expectation, competition over match partners will be fierce, thereby diminishing the value a link creates for the individuals involved.[4]

The importance of social networks for economic outcomes has long been recognized. Early studies by Myers and Shultz (1951), Rees and Shultz (1970), and Granovetter (1973), for example, found that over half of the workers surveyed had obtained jobs through previously known social contacts. These early works have since spawned a large and growing literature that investigates the role of networks in labor markets and other environments theoretically, as well as the incentives for individuals to establish their social ties.[5] An important class of models commonly used to study emergent networks is based on Jackson and Wolinsky's (1996) *connections model*.[6] In the basic connections model, it is assumed

a link to another, and no pair of unlinked individuals mutually prefers to establish a link.

[4]This result suggests that, contrary to popular opinion, the value of social networks for finding employment in a recession (i.e., if there are more job seekers than job openings on the other) may be less than in a balanced labor market.

[5]For example, Montgomery (1991) examines the decision of employers to use their workers' social contacts for hiring referrals. A series of recent papers by Calvó-Armengol and Jackson (2004, 2007) and Calvó-Armengol and Zenou (2004) examine dynamic models in which information about job opening travels through the workers' network.

[6]Other models of strategic network formation include contributions by Boorman (1975), Myerson (1977), and Dutta and Mutuswami (1997), and Bloch and Jackson (2007).

that every connection between two individuals provides a certain benefit to the individuals which is discounted by the number of steps involved in the connection. That is, a friend of a friend is assumed less valuable than a friend. Numerous extensions of the basic model have been proposed: For example, the *coauthors model* (also Jackson and Wolinsky, 1996) allows for a negative value of indirect connections, Johnson and Gilles (2000) introduce a spatial topography to the set of agents, and Carayol and Roux (2009) introduce dynamic link formation to the connections model.[7]

We borrow from Jackson and Wolinsky (1996) the solution concept of pairwise stability, and from Johnson and Gilles (2000) the notion of spatial differentiation among the individuals in the network. However, there is an important difference between the connections model and the framework presented in this paper. In our model, the value of a network depends on how well it facilitates exchange in the underlying matching market. No such market exists in the connections model, and one may think of its payoff function as a reduced form for the payoff generated by the network in *some* underlying economic environment. One contribution of our paper is to develop a structural approach to examine the formation of networks in one particular such environment —a two-sided matching market, which we model explicitly. In this setting, we demonstrate how a simple non-anonymity requirement for trade gives rise a social network whose payoff function cannot be adequately described in terms of the reduced-form payoff functions commonly assumed in the connections model. For example, the value of some indirect connections will be negative, while the value of others will be positive. To understand this property, consider a marriage market. A man (say i) is always hurt when a woman he knows (say j) becomes acquainted with another man (say i'), as i is now in direct competition with i' for j's hand. On the other hand, i benefits if i' befriends another woman j', since this reduces the amount of competition i can expect from i' over woman j (as i' could now also marry j'). Our model is a step toward understanding the role of social networks in such environments.

A model with a similar motivation to ours is developed by Galeotti and Merlino (2010). Their model shares with ours the assumption of uncertainty about which jobs will become vacant, and which workers will become unemployed. Workers invest in networking before these uncertainties are resolved. Galeotti and Merlino (2010) show that, similar to our results, networking activities are most intense for intermediate degrees of uncertainty. However, their model does not include an explicit description of a two-sided matching market, and the agents are not differentiated in the same spatial sense as is assumed here.

Our paper is also related to previous work on buyer-seller networks. Kranton and Minehart (2000, 2001) and Blume et al. (2009) examine settings in which buyers must know potential sellers before they can buy a good from them. Kranton and Minehart (2000) and Blume et al. (2009) examine equilibrium prices and allocations in a model where the network as well as the buyers' valuations for the good are given. This perspective

[7] For a detailed review of the literature on social and economic networks, including the connections model, see Jackson (2008).

corresponds to the final stage of our model, at which the network is established and the uncertainty about agents' trading needs has been lifted. In Kranton and Minehart (2001), prior uncertainty about the buyers' valuations as well as a network formation game are introduced. This perspective corresponds to the entire sequence of stages in our model. Despite these similarities, there are several key differences between our model and theirs. First, there is no sense in which buyers or sellers can be considered "close" or "distant," which is a central aspect both of our model and of its results. Second, utility is assumed transferable; in particular, the buyers pay a price for a seller's good that is determined in an auction. The terms of trade in the market are thus endogenous, while we take them as exogenous.[8] Finally, Kranton and Minehart (2001) consider a non-cooperative game of link formation, while we use a coalitional specification.

The remainder of the paper is organized as follows. Section 2 contains our model of a matching market with network frictions. In Section 3, we explore the relationship between the uncertainty in the matching market on the one hand, and the expected benefit of social networks on the other. The next two sections, Sections 4 and 5, treat the network as endogenous: In Section 4 we model the network formation stage as a cooperative game and characterize equilibrium networks under certain simplifying assumptions. In Section 5 we perform numerical simulations for less restrictive assumptions; these extend and complement the result of Section 4. Section 6 concludes.

2 The Model

In this section, we describe our model of a matching market with network frictions. The agents belong to two groups which we call sellers and buyers. This terminology does not preclude other interpretations of two-sided matching environments, such as men and women in a marriage market, or workers and jobs in a job-worker assignment problem.

2.1 The matching market

Let $\mathcal{C}_S = \mathcal{C}_B = \{1, 2, 3, \ldots, N\}$. Each $i \in \mathcal{C}_S$ is the location of a seller, and each $j \in \mathcal{C}_B$ is the location of a buyer. Given a seller-buyer pair $(i, j) \in \mathcal{C}_S \times \mathcal{C}_B$, define their ***distance*** as follows:

$$|i, j| \;\equiv\; \min\{\, |i - j| \,,\, |i + N - j| \,\}.$$

Thus, one can imagine the locations of the sellers and buyers as a set of points arranged on a circle, and $|i, j|$ represents the shortest distance on that circle from i to j. In general, we

[8] Our assumption of non-transferable utility lends itself to a more tractable characterization of stable assignments in the matching market. Klumpp (2009) shows that in the two-sided matching market we consider in this paper, this tractability would be lost if utility was transferable. Moreover, the set of possible side payments that support a stable assignment would not be unique. To avoid these complications, we here focus on the non-transferable utility case.

will assume all addition and subtraction operations on $\mathcal{C}_S$ and $\mathcal{C}_B$ are modulo N.[9]

Each seller supplies either zero or one units of an indivisible good, and each buyer demands either zero or one units of that good. We let $x_i \in \{0, 1\}$ denote the quantity supplied by seller i, and $y_j \in \{0, 1\}$ the quantity demanded by buyer j. The vectors $x = (x_1, \ldots, x_N)$ and $y = (y_1, \ldots, y_N)$ will be called the ***supply configuration*** and ***demand configuration***, respectively. We assume that these are random variables. Specifically, we assume that each x_i equals one with with probability p and zero with probability $1 - p$. Likewise, each y_j equals one with probability p and zero with probability $1 - p$. All x_i and y_j are drawn independently. We sometimes refer to $x_i = 1$ as a positive supply shock, and to $y_j = 1$ as a positive demand shock.

If $x_i = y_i = 1$, then the pair seller-buyer (i, j) have coincidence of wants, and we denote by

$$D(x, y) = \{(i, j) \in \mathcal{C}_S \times \mathcal{C}_B : x_i = y_j = 1\}$$

the set of all seller-buyer pairs that satisfy this condition. If $(i, j) \in D(x, y)$ do trade, the exchange generates value $u(|i, j|)$ for both seller i and buyer j. The utility function u is positive and strictly decreasing (this represents horizontal differentiation among the agents). Utility is non-transferable; that is, no side-payments are allowed. Finally, an agent who does not trade with another agent obtains a zero payoff.

2.2 Networks

Now assume that in order to trade with one another, the agents in the seller-buyer pair (i, j) must also know each other. That is, they must have established a social relationship prior to trading. We call this requirement a ***network friction***.

We model the social relations among the players as a bi-partite network whose vertices are the sellers on one side, and the buyers on the other. Formally, a ***link*** $(i, j) \in \mathcal{C}_S \times \mathcal{C}_B$ between seller i and buyer j represents a social relationship among i and j. A ***network*** is then a collection of links $G \subseteq \mathcal{C}_S \times \mathcal{C}_B$. In Section 4 we will introduce a cooperative game of network formation through which the sellers and buyers in the matching markets establish their links. Until then, we take the network $G \subseteq \mathcal{C}_S \times \mathcal{C}_B$ as given.

Given a supply-demand configuration (x, y) and a network G, the set of matches that can form in the matching market with the network friction is

$$D(x, y|G) \equiv D(x, y) \cap G.$$

The intersection of $D(x, y)$ and G represents the fact that a feasible trade must now satisfy *two* conditions: The previous requirement that agents have coincidence of wants ($x_i = y_j = 1$), and the new requirement that i and j be connected in the network G. Note that a frictionless market is subsumed as a special case in our setup: If $G = \mathcal{C}_S \times \mathcal{C}_B$ (i.e., G is the ***complete network***) then $D(x, y|G) = D(x, y)$ for all x and y.

[9]For example, the location three spots to the left of location 1 is $N - 2$.

2.3 Assignments

An ***assignment***, or matching, specifies who trades with whom. Formally, an assignment is a set $M \subseteq D(x, y|G)$ such that

(a) each seller is matched with at most one buyer: $(i, j) \in M$ and $j \neq j'$ implies $(i, j') \notin M$;

(b) each buyer is matched with at most one seller: $(i, j) \in M$ and $i \neq i'$ implies $(i', j) \notin M$.

For fixed x and y, we denote by $v_i^S(M)$ the surplus that seller $i \in \mathcal{C}_S$ receives in assignment M,

$$v_i^S(M) = \begin{cases} u(|i,j|) & \text{if } \exists j \in \mathcal{C}_B \text{ s.t. } (i,j) \in M, \\ 0 & \text{otherwise,} \end{cases}$$

and by $v_j^B(M)$ the surplus that buyer $j \in \mathcal{C}_B$ receives,

$$v_j^B(M) = \begin{cases} u(|i,j|) & \text{if } \exists i \in \mathcal{C}_S \text{ s.t. } (i,j) \in M, \\ 0 & \text{otherwise.} \end{cases}$$

A stable assignment (or equilibrium assignment) is defined as follows:

Definition 1. Given a network G and a supply-demand configuration (x, y), an assignment $M \subseteq D(x, y|G)$ is ***stable*** if there does not exist $(i, j) \in D(x, y|G) \backslash M$ such that $u(|i, j|) > v_i^S(M)$ and $u(|i, j|) > v_j^B(M)$.

That is, there does not exist a seller-buyer pair in $D(x, y|G)$ such that both agents prefer being matched with one another over being matched with their assigned partner, or being unmatched. Note that, since u is positive, all sellers and buyers prefer being matched over being unmatched.[10]

2.4 Market clearing on a network: The inside-out algorithm

Stable assignments, as defined in Definition 1, have a simple recursive structure. Note that, if $(i, j) \in D(x, y|G)$, then i and j have coincidence of want and are linked in G; thus they can trade with each other and receive a positive surplus. Since all agents prefer shorter matches to longer matches, both i and j may decline to transact with one another if a closer match partner is available who agrees to enter into a match. For this to be the case, this third agent must not have an even closer potential match partner available herself. For instance, buyer j would decline to obtain the good from seller i if there exists another seller k such that $(k, j) \in D(x, y|G)$, $|j, k| < |i, k|$, *and* if there does not exist a buyer l such that $(k, l) \in D(x, y|G)$, $|k, l| < |k, j|$, *and* l does not herself decline the transaction with k.

[10] If this was not the case, we would have had to include the requirement that all agents obtain non-negative surpluses.

These considerations show that a stable assignment $M \subseteq D(x, y|G)$ can be built iteratively, using the following "inside-out" market clearing process.[11] Let

$$D^0 = D(x, y|G) \tag{1}$$

be the set of all potential matches. If $(i, j) \in D^0$ and $i = j$, then buyer i and seller j are located just across from each other, and neither of them could obtain a higher utility matching with any other agent. All such matches are therefore part of the stable assignment. Thus, we set

$$M^1 = \{ (i, i) \in D^0 \}.$$

The matches remaining are those that do not involve an agent who is already in a match in M_0. Let this set be

$$D^1 = D^0 \setminus \{ (i, j) : \exists k \text{ s.t. } (i, k) \in M^1 \text{ or } (k, j) \in M^1 \}.$$

The next matches to form are those for which $|i, j| = 1$, as these generate the next highest utility. There is now a need for tie-breaking. For example, if $y_{-1} = x_0 = y_1 = 1$ (all other x_i and y_j zero) there will be two possible matches, $(0, 1)$ and $(0, -1)$, but only one can ultimately be in the assignment. We assume that from all possible matches $(i, j) \in D^1$ with $|i, j| = 1$, priority is given to the "left-to-right" matches $(i, i+1)$ before any remaining "right-to-left" matches $(i, i-1)$ are cleared. Thus, first define

$$\begin{aligned} M^2 &= \{ (i, i+1) \in D^1 \}, \\ D^2 &= D^1 \setminus \{ (i, j) : \exists k \text{ s.t. } (i, k) \in M^2 \text{ or } (k, j) \in M^2 \}, \end{aligned}$$

and then

$$\begin{aligned} M^3 &= \{ (i, i-1) \in D^2 \}, \\ D^3 &= D^2 \setminus \{ (i, j) : \exists k \text{ s.t. } (i, k) \in M^3 \text{ or } (k, j) \in M^3 \}. \end{aligned}$$

Then proceed to the next-longest matches:

$$\begin{aligned} M^4 &= \{ (i, i+2) \in D^3 \}, \\ D^4 &= D^3 \setminus \{ (i, j) : \exists k \text{ s.t. } (i, k) \in M^4 \text{ or } (k, j) \in M^4 \}, \\ M^5 &= \{ (i, i-2) \in D^4 \}, \\ D^5 &= D^4 \setminus \{ (i, j) : \exists k \text{ s.t. } (i, k) \in M^5 \text{ or } (k, j) \in M^5 \}. \end{aligned}$$

Proceed in this fashion for all subsequent matches. The overall assignment resulting from this algorithm,

$$M(x, y|G) \equiv \bigcup_{\sigma=1,2,3,\ldots} M^\sigma,$$

is stable per Definition 1. Note that this is typically not the only stable assignment, as we could have broken ties in a different way. However, $M(x, y|G)$ is the unique assignment under the assumed tie-breaking rule.

[11] This algorithm is described (for the frictionless case) in detail in Klumpp (2009), and for similar environments also in Alcalde (1995), Eeckhout (2000), and Clark (2003).

3 The Value of a Network

The aim of this section is to address the question of how valuable the social network G is to every seller $i \in \mathcal{C}_S$ and buyer $j \in \mathcal{C}_B$. Note that, unlike the connections model (Jackson and Wolinsky, 1996; Johnson and Gilles, 2000), we do not assume that there is a given value of connections in the network. Instead, the value of the network to the individuals must be derived based on how well it facilitates trade in the underlying matching market. Below, we show how this can be done. We also provide a series of examples to demonstrate the complex nature of network values in our setup. The networks discussed in Section 3.3 will then be used for our results in Section 4 and Section 5.

3.1 Utilization probabilities

To start with, recall that given a network G and a supply-demand configuration (x, y), it is straightforward to compute a stable assignment $M(x, y|G)$ by applying the inside-out algorithm of Section 2.4 to the set of potential matches $D(x, y|G)$. Note, however, that the supply and demand configurations x and y are random variables. This implies that the stable matching $M(x, y|G)$ itself is random. Thus, from an "interim" perspective where G is known but x and y are not, there is a probability that trade will occur along each link $(i, j) \in G$. This probability is called the ***utilization probability*** of link $(i, j) \in G$ and denoted by

$$\varphi(i, j|G) \;=\; Pr\big[\,(i, j) \in M(x, y|G)\,\big]. \tag{2}$$

Because $M(x, y|G)$ is unique (owing to our tie-breaking assumption), (2) is well-defined.

The expected benefit of a link $(i, j) \in G$ for both seller i and buyer j is then given by the product $\varphi(i, j|G) \cdot u(|i, j|)$. That is, the probability that the match (i, j) happens, multiplied by the value of the match if it happens. For a given seller i or buyer j, the value generated by network G can now be computed by adding the expected benefits of all links belonging to this agent:

$$U_i^S(G) \equiv \sum_{j:(i,j)\in G} \varphi(i, j|G)\, u(|i, j|), \tag{3}$$

$$U_j^B(G) \equiv \sum_{i:(i,j)\in G} \varphi(i, j|G)\, u(|i, j|). \tag{4}$$

As can be seen from (3)–(4), the crucial determinants of the expected network benefits for an agent are the link utilization probabilities $\varphi(i, j|G)$. These, in turn, depend on the probability p that any given seller or buyer is on the market. The following examples demonstrate how the utilization probabilities of network links can be found.

3.2 Example 1: A simple network

Consider the network $G = \{(2,1), (2,2), (2,3), (3,3)\}$, shown in Figure 1. We are interested in the probability that seller 2 utilizes each of her three links $(2,1)$, $(2,2)$, $(2,3)$.

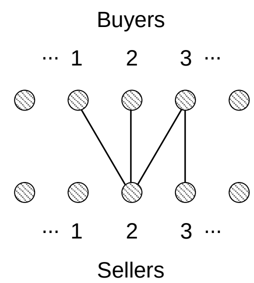

Figure 1: A network containing four links

Let us first consider $\varphi(2,2|G)$. Clearly, if $x_2 = y_2 = 1$ then the seller-buyer pair $(2,2)$ will trade, and this event has probability p^2. The demand and supply shocks of other agents do not matter for this match, as seller 2 and buyer 2 are mutually most-preferred match partners. Thus, the probability that seller 2 and buyer 2 trade in G is $\varphi(2,2|G) = p^2$ (see Figure 2).[12]

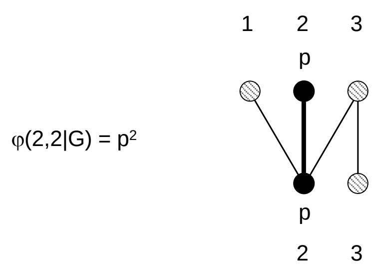

Figure 2: Computation of $\varphi(2,2|G)$

Next, let us consider $\varphi(2,3|G)$. For the pair $(2,3)$ to trade we need $x_2 = y_3 = 1$; this has probability p^2. At the same time, seller 2 must not trade with buyer 2 (to whom she is linked and whom she would prefer). For this to be the case, we need $y_2 = 0$; this event has probability $1 - p$. Similarly, buyer 3 must not trade with seller 3 (to whom she is linked and whom she would prefer), and for this to be the case we need $x_3 = 0$; this event also has probability $1 - p$. Now, seller 2 is also linked to buyer 1, but the value of y_1 does not matter for the match $(2,3)$, given our tie-breaking rule: Given the same distance, a left-to-right match is cleared before a right-to-left match. Thus, the probability that seller 2 and buyer 3 trade in G is $\varphi(2,3|G) = p^2(1-p)^2$ (see Figure 3).

[12]In all figures in this section, a black dot indicates a positive demand or supply quantity, a white dot indicates a zero quantity, and a gray dot indicates either a positive or a zero quantity.

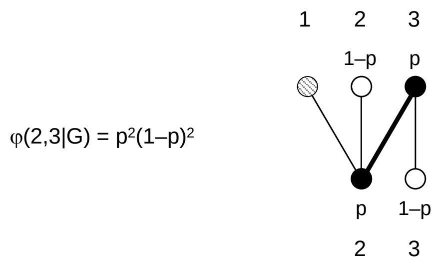

Figure 3: Computation of $\varphi(2,3|G)$

Finally, consider $\varphi(2,1|G)$. For the pair $(2,1)$ to trade we need $x_2 = y_1 = 1$; this has probability p^2. For the same reason as in the previous step, seller 2 must not trade with buyer 2, so we need $y_2 = 0$, which has probability $1-p$. Seller 2 must also not trade with buyer 3, as a potential match $(2,3)$ would beat the match $(2,1)$ by our tie-breaking assumption. For this to be the case we need either $y_3 = 0$ (probability $1-p$), or $x_3 = y_3 = 1$ (probability p^2). In the latter case, seller 2 would like to trade with buyer 3, but buyer 3 would refuse the trade in favor of the closer seller 3. Thus, the probability that seller 2 and buyer 1 trade in G is $\varphi(2,1|G) = p^2(1-p)(1-p+p^2)$ (see Figure 4).

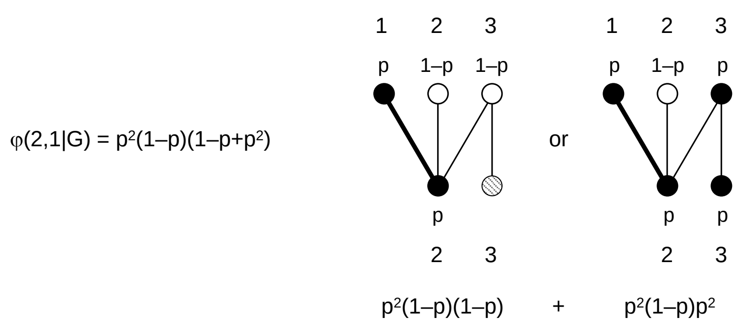

Figure 4: Computation of $\varphi(2,1|G)$

The expected benefit of the network $G = \{(2,1),(2,2),(2,3),(3,3)\}$ for seller 2 is therefore given by

$$\begin{aligned} U_2^S(G) &= \varphi(2,2|G)\cdot u(0) + \varphi(2,3|G)\cdot u(1) + \varphi(2,1|G)\cdot u(1) \\ &= p^2\cdot u(0) + p^2(1-p)^2\cdot u(1) + p^2(1-p)(1-p+p^2)\cdot u(1) \\ &= p^2\Big[u(0) + (1-p)(2-2p+p^2)u(1)\Big]. \end{aligned}$$

3.3 Example 2: Regular networks

We now introduce a class of networks we call ***regular networks***. In a regular network of size $k \geq 0$, denoted $G(k)$, each agent is linked to her "k nearest neighbors." Because agents prefer shorter matches over longer ones, regular networks are natural candidates for networks which emerge as "equilibrium networks" in our model (a network formation stage will be formally introduced in the next section).

In keeping with the order in which matches are resolved in the inside-out market clearing algorithm, the meaning of "k nearest neighbors" is the following:

– If k is zero, $G(k)$ is the empty network. That is, $G(0) = \emptyset$.

– If k is odd, every seller is linked to the one buyer who resides at the same location, as well as the $(k-1)/2$ nearest buyers on both her left and right side. Similarly, every buyer is linked to the seller who resides at the same location, as well as to the $(k-1)/2$ nearest sellers on both her left and right side.

– If k is even (and positive), every seller is linked to the buyer who resides at the same location, as well as the $(k-2)/2$ nearest buyers on both her left and the $k/2$ nearest buyers to her right side (and similarly for every buyer).

In a regular network, links of the form (i, i) are called 1st-order links. Links of the form $(i, i+1)$ are 2nd-order links, $(i, i-1)$ are 3rd-order links, $(i, i+2)$ are 4th-order links, $(i, i-2)$ are 5th-order links, and so on. Thus, the regular network $G(k)$ contains all lth-order links for $l = 1, \ldots, k$. Figure 5 depicts the first six regular networks.

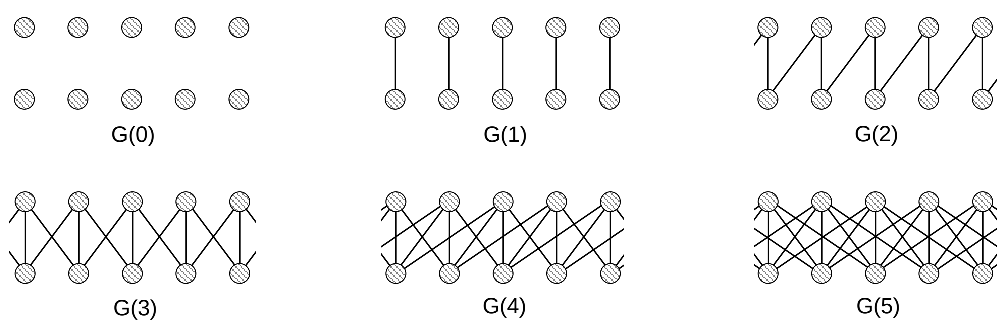

Figure 5: The regular networks $G(0), \ldots, G(5)$

For regular networks $G(k)$, it is possible to derive the utilization probabilities $\varphi(i, j|G(k))$ analytically, provided k is not too large. This can be done in the same way as in the previous examples. For $G(0)$, since it is the empty network, we have

$$\varphi(i, j|G(0)) \;=\; 0 \quad \text{for all } i, j. \tag{5}$$

In $G(1)$ on the other hand, each agent is linked to exactly one other agent. Thus, seller i and buyer i trade if and only if $x_i = y_i = 1$, which has probability p^2. It follows that

$$\varphi(i, j|G(1)) \;=\; \begin{cases} p^2 & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases} \tag{6}$$

The regular network $G(2)$ contains 1st-order and 2nd-order links. The utilization probability of a 1st-order link in $G(2)$ is still p^2. Now consider the 2nd-order links, between sellers i and buyers $i+1$. For the pair $(i, i+1)$ to trade, we need $x_i = y_{i+1} = 1$, which has probability p^2. In addition, we need $y_i = 0$ (otherwise, seller i would trade with buyer i) and $x_{i+1} = 0$ (otherwise buyer $i+1$ would trade with seller $i+1$); each of these events has probability $1-p$. Thus, the utilization probabilities for the links in $G(2)$ are given by

$$\varphi(i,j|G(2)) = \begin{cases} p^2 & \text{if } i = j, \\ p^2(1-p)^2 & \text{if } j = i+1, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Next, for $G(3)$ we get one additional term:

$$\varphi(i,j|G(3)) = \begin{cases} p^2 & \text{if } i = j, \\ p^2(1-p)^2 & \text{if } j = i+1, \\ p^2(1-p)^2(1-p+p^2)^2 & \text{if } j = i-1, \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

To see why $\varphi(i, i-1|G(3)) = p^2(1-p)^2(1-p+p^2)^2$, consider what must happen for seller i and buyer $i-1$ to trade: As before, we need $x_i = y_{i-1} = 1$, which has probability p^2. Also as before, we need to ensure that seller i does not trade with buyer i, and that buyer $i-1$ does not trade with seller $i-1$. This will be the case if and only if $x_{i-1} = y_i = 0$, which has probability $(1-p)^2$. What is new for 3rd-order links is that we must now ensure that seller i does not trade with buyer $i+1$. This will be the case either if $y_{i+1} = 0$ (buyer $i+1$ is not on the market), or if $y_{i+1} = x_{i+1} = 1$ (buyer $i+1$ is on the market but trades with seller $i+1$). This gives us the factor $1-p+p^2$. Similarly, we must ensure that buyer $i-1$ does not trade with seller $i-2$. This will be the case either if $x_{i-2} = 0$ or if $x_{i-2} = y_{i-2} = 1$, yielding another factor $1-p+p^2$.

On could, in principle, carry on in the same manner for successively larger networks. For example, with $G(4)$ we must include the utilization probabilities of 4th-order links:

$$\varphi(i,j|G(4)) = \begin{cases} p^2 & \text{if } i = j, \\ p^2(1-p)^2 & \text{if } j = i+1, \\ p^2(1-p)^2(1-p+p^2)^2 & \text{if } j = i-1, \\ p^2(1-p)^2(1-2p+2p^2)(1-p+p^2(1+(1-p)^2))^2 & \text{if } j = i+2, \\ 0 & \text{otherwise,} \end{cases} \tag{9}$$

(The analytical derivation of (9) is in the Appendix.) Since matches are cleared "from the inside out" (i.e., closer matches have priority over more distant ones), the utilization probability of an lth-order link will be the same in $G(k)$ and in $G(k')$ if $k, k' \geq l$, and equal to zero if $k < l$. However, the utilization probabilities of the higher-order links become increasingly unwieldy. Also note that, going from the third to the fourth line in (9), the utilization probability of an lth-order link in $G(k)$ cannot generally be expressed as the corresponding probability of the $(l-1)$st-order link times a new term (this simple pattern is valid only up to 3rd-order links).

In any event, the value of the regular network $G(k)$ is the same for every seller and every buyer and can be found by summing the expected benefits of lth-order links for $k = 1, \ldots, k$. Thus, we get the following network values:

$$\begin{aligned}
U_i^S(G(0)) &= U_j^B(G(0)) &=& \; 0, \\
U_i^S(G(1)) &= U_j^B(G(1)) &=& \; p^2 u(0), \\
U_i^S(G(2)) &= U_j^B(G(2)) &=& \; p^2 \Big[u(0) + (1-p)^2 u(1) \Big], \\
U_i^S(G(3)) &= U_j^B(G(3)) &=& \; p^2 \Big[u(0) + (1-p)^2 \big[1 + (1-p+p^2)^2 \big] u(1) \Big],
\end{aligned}$$

and so on.

3.4 External effects and the value of indirect connections

In our model, a link between two agents enables this pair of agents to trade, but an agent cannot trade with another agent to whom he is only indirectly linked. For example, consider seller i who knows buyer j, who in turn knows seller i', who in turn knows buyer j'. In this case, we say that seller i has an ***indirect connection*** to buyer j'. The indirect connection from i to j' does not allow i and j' to trade—for this, the network would have to contain a direct link (i, j'). However, the fact that trade can only take place over direct connections does not imply that agents derive a zero benefit from indirect connections. More generally, the value generated by a network for an individual depends not only on the links this individual has to others, but on *all* links in the network, including links which connect two entirely different agents.

The reason for this dependence is that links in our network generate externalities on the utilization probability of other links. To see this, define the ***external value*** of the link $(i', j') \in G$ to seller i as

$$z_i^S(i', j'|G) \equiv U_i^S(G) - U_i^S(G - (i', j')).$$

That is, $z_i^S(i', j'|G)$ is the difference between the overall value that i derives from network G and the value i derives from the network that arises after (i', j') is removed from G. The external value of an indirect link to buyer $j \neq j'$ can be defined similarly as $z_j^B(i', j'|G) \equiv U_j^B(G) - U_j^B(G - (i', j'))$. We will show that $z_i^S(i', j'|G)$ is not zero, and that it can be positive *or* negative (the same is true for z_j^B). The possibility of negative link externalities shows that the role of social networks in matching markets cannot generally be viewed as a special case of the connections model.

Example 3: A negative externality. As an example of the first possibility, consider the network $G = \{(1, 2), (2, 2)\}$, depicted on the left side of Figure 6.

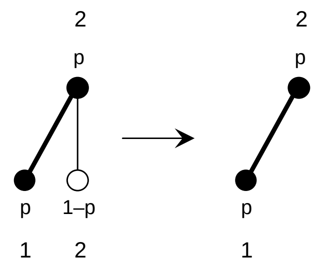

Figure 6: Removing link $(2, 2)$ increases the probability that link $(1, 2)$ is utilized

Since seller 1 is directly linked only to buyer 2, we have $U_i^S(G) = \varphi(1, 2|G)u(1)$. For seller 1 and buyer 2 to trade, we must have $x_1 = y_2 = 1$, as well as $x_2 = 0$ (otherwise buyer 2 would prefer to trade with seller 2 instead of seller 1). Thus, $\varphi(1, 2|G) = p^2(1-p)$. Deleting link $(2, 2)$ from G eliminates the requirement that $x_2 = 0$: As seller 2 and buyer 2 are no longer linked they cannot trade, regardless of the value of x_2 (see the right side of Figure 6). Thus $\varphi(1, 2|G - (2, 2)) = p^2$, and we have

$$z_i^S(2, 2|G(2)) = \varphi(1, 2|G)u(1) - \varphi(1, 2|G - (2, 2))u(1) = -p^3u(1) < 0.$$

Notice that seller 1 values the link between seller 2 and buyer 2 (and thus the indirect connection between himself and seller 2) negatively because this link puts seller 1 in direct competition with seller 2 over buyer 2. Removing link $(2, 2)$ hence benefits seller 1.

Example 4: A positive externality. To see the opposite effect, consider the network $G = \{(1, 1), (1, 2), (4, 2)\}$ depicted on the left side of Figure 3.4.

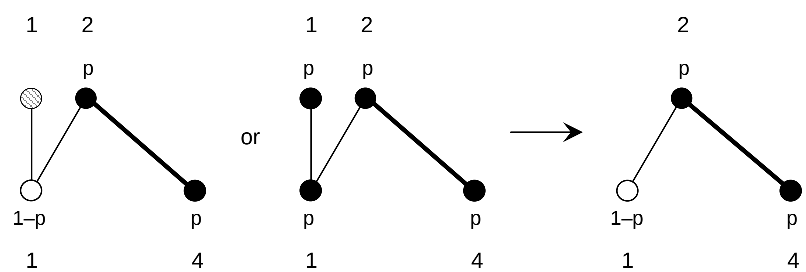

Figure 7: Removing link $(1, 1)$ decreases the probability that link $(4, 2)$ is utilized

Since seller 4 is directly linked only to buyer 2, we have $U_4^S(G) = \varphi(4, 2|G)u(2)$. For seller 4 and buyer 2 to trade, we must have $x_4 = y_2 = 1$; this has probability p^2. At the same time, buyer 2 must not trade with seller 1 (who would be preferred over seller 3 due to 1's location). This requires that either $x_1 = 0$ (probability $1 - p$), or $x_1 = y_1 = 1$ (probability p^2). Thus, $\varphi(4, 2|G) = p^2(1 - p + p^2)$. If we remove the link $(1, 1)$ from G, we eliminate the last possibility, so that $\varphi(4, 2|G - (1, 1) = p^2(1 - p)$ (see the right side of Figure 3.4). Thus, the value of link $(1, 1)$ in G to seller 4 is

$$z_4^S(1, 1|G) = \varphi(4, 2|G)u(2) - \varphi(4, 2|G - (1, 1))u(2) = p^4u(2) > 0.$$

Seller 4 values the link between seller 1 and buyer 1 (and thus the indirect connection between himself and buyer 1) positively because this link reduces the competition seller 4 faces from seller 1 over buyer 2. Removing link $(1,1)$ hence hurts seller 4.

These examples demonstrate that, even though we only assume a very simple non-anonymity condition which requires direct links between individuals for trade, the indirect connections of the resulting social network do affect the value the network generates for the agents. More precisely, indirect connections affect the value of direct links, as indirect connections can both strengthen and weaken the competition over match partners, thereby generating either negative or positive externalities for other agents.

4 Network Formation: Theory

The main focus of this and the next section is on the networks that emerge endogenously, given the matching environment described in the previous section.

To this end, let us assume that the network G is formed in a cooperative game *before* the supply and demand configuration (x, y) is known. Specifically, assume that for every link $(i, j) \in G$, both the seller i and the buyer j incur a networking cost $c > 0$. This cost can be interpreted as the time spent by i and j in cultivating their relationship, and must be paid regardless of whether or not i and j eventually trade with each other.

4.1 Pairwise stable networks

Given the networking cost c, the *net-of-cost* value generated by network G for seller i is given by

$$V_i^S(G) \;\equiv\; U_i^S(G) - \#\{j : (i,j) \in G\} \cdot c = \sum_{j:(i,j)\in G} \Big[\varphi(i,j|G)u(|i,j|) - c\Big].$$

That is, $V_i^S(G)$ is the expected benefit of the network G for seller i minus the network cost for i. Similarly, the net value of network G for buyer j is

$$V_j^B(G) \;\equiv\; U_j^B(G) - \#\{i : (i,j) \in G\} \cdot c = \sum_{i:(i,j)\in G} \Big[\varphi(i,j|G)u(|i,j|) - c\Big].$$

Using V_i^S and V_j^B as payoff functions, we can now define an equilibrium network by employing the pairwise stability concept of Jackson and Wolinsky (1996):

Definition 2. A network $G \subseteq \mathcal{C}_S \times \mathcal{C}_B$ is ***pairwise stable*** if the following holds for each $(i, j) \in \mathcal{C}_S \times \mathcal{C}_B$:

(a) $(i, j) \in G$ implies $V_i^S(G) \geq V_i^S(G - (i, j))$ and $V_j^B(G) \geq V_j^B(G - (i, j))$,

(b) $(i, j) \notin G$ implies $V_i^S(G \cup (i, j)) \leq V_i^S(G)$ or $V_j^B(G \cup (i, j)) \leq V_j^B(G)$.

Pairwise stability is a relatively weak definition of network stability, imposing only two requirements: Neither party in a link has a strict preference for severing the link (condition (a)), and no two unlinked agents strictly prefer forming a link (condition (b)).[13] Note that the first condition can equivalently be expressed as the requirement that the external value of every link G be at least c for the two individuals connected by the link:

$$z_i^S(i,j|G),\ z_j^B(i,j|G) \geq c \quad \forall (i,j) \in G.$$

Similarly, the second condition says that the external value of every link not in G would be at most c (for the two individuals involved) if it was added to G:

$$z_i^S(i,j|G \cup (i,j)),\ z_j^B(i,j|G \cup (i,j)) \leq c \quad \forall (i,j) \notin G.$$

In order to determine whether a network G is pairwise stable, one needs to know the utilization probabilities $\varphi(i,j|G)$ for all its links $(i,j) \in G$, as well as the utilization probabilities of links in the network that result when links are added to, or subtracted from, G. In principle, these can be found in the same way as demonstrated in Section 3. However, as is apparent from the examples presented there, doing so may not be practically feasible except for simple cases. Below, one such simple case will be discussed.

4.2 Stability of small regular networks

Given the fact that the utility from a match decreases with the distance between the two match partners, it seems intuitive that an agent would want to form close links first and then add more distant links until the cost of doing so outweighs the benefit. This reasoning suggests that regular networks are natural candidates for pairwise stable networks.

To determine analytically whether regular networks are stable, we need to make an additional assumption: For the time being, assume that $u(0) = 1$, $u(1) = \delta < 1$, and $u(d) = 0$ for $d > 1$. This implies that a seller i wants to form links to at most three buyers j: The buyer directly across from the seller ($j = i$), as well as the buyers one step to the seller's left or right ($j = i \pm 1$). This simplifying assumption allows us to restrict our attention to the regular networks $G(0), \ldots, G(3)$, for which we already know the relevant utilization probabilities (see Example 2 of Section 3). One of these networks will be a pairwise stable network, as the following result states (the proof is in the Appendix):

[13]The reason why pairwise stability is a weak equilibrium concept is that it precludes, among other things, deviations that result from combining the moves (a) and (b) of Definition 2. For example, a network may be pairwise stable even if there are two unlinked agents who both prefer to cut one link to another agent and replace it with a link to one another. Thus, to verify pairwise stability of a candidate network, it needs to be compared to only a small number of alternative networks. Pairwise stability can therefore be implemented in numerical simulations with relative ease, which is the main reason why we adopt it here.

Proposition 1. *Suppose that $u(0) = 1$, $u(1) = \delta < 1$, and $u(d) = 0$ for $d \geq 2$. There exists a regular, pairwise stable network $G(k)$, where*

$$k = \begin{cases} 0 & \text{if} \quad c \geq p^2, \\ 1 & \text{if} \quad p^2 \geq c \geq p^2(1-p)^2\delta, \\ 2 & \text{if} \quad p^2(1-p)^2\delta \geq c \geq p^2(1-p)^2(1-p+p^2)^2\delta, \\ 3 & \text{if} \quad p^2(1-p)^2(1-p+p^2)^2\delta \geq c. \end{cases}$$

We remark here that, in general, the result in Proposition 1 does *not* always carry over to larger regular networks. That is, if we assumed that u stayed sufficiently large long enough for more links to be added, then agents would not necessarily want to add successively longer links to their networks. Thus, contrary to what one might perhaps expect, decreasing c does not necessarily create a series of regular networks of increasing size, as it does in the simple case of Proposition 1. For regular networks to be stable in general, one must not only assume that u decreases but that it decreases at a sufficiently fast rate. This effect will be demonstrated, through simulations, in Section 5.

4.3 Comparative statics

For $\delta = 0.75$, Figure 8 shows the stable networks identified in Proposition 1 graphically. (There are typically more stable networks than the ones characterized in Proposition 1; however, these will not be regular.)

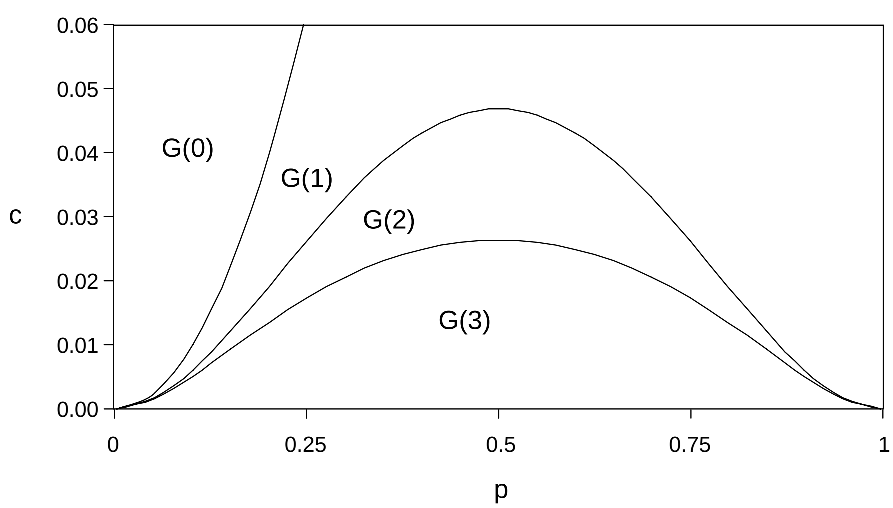

Figure 8: Pairwise stable networks when $u(0) = 1$, $u(1) = \delta = 0.75$, $u(d) = 0$ $(d > 1)$

As expected, network size increases as the networking cost c falls. It is more interesting, however, to examine how network size adjusts in response to a change in the parameter p. Even though it only applies to certain utility functions u, the plot in Figure 8 reveals an important aspect of networks in matching markets: The size of equilibrium networks is non-monotonic in p. As p goes from zero to one the following happens (provided c is small enough): At first, the only stable network is the empty one, as it is too unlikely that agents ever need a partner for the cost c of a single link to be incurred. Then, as p increases, the stable network grows first to $G(1)$, then to $G(2)$, and then to $G(3)$, which will be reached before p reaches $1/2$. As p increases further, however, the network shrinks.

There are two reasons for this. First, while it is true that there is now a larger chance that an agent will have a trading need, any seller will also be more likely to meet a buyer's needs (and vice versa), which means that there is less reason for any agent to maintain a large network for "insurance reasons." Second, even conditional on an agent's acquaintances being unable to fulfill this individual's trading needs, the chance that an added link would remedy this situation becomes smaller, as a more distant agent will be more likely to have a closer match partner available herself. In the end, when $p = 1$, there is indeed no reason for any individual to be linked to anyone other than the individual directly across from her, as this agent can, with probability one, fulfill the individual's trading needs.[14] Thus, networks are large when the agents' uncertainty about their future needs is large, and this is the case when $p = 1/2$.[15]

5 Network Formation: Numerical Analysis

In this section, we explore the structure and size of stable networks numerically. We compute utilization probabilities $\varphi(i, j|G)$ for networks other than the ones considered previously, through simulation methods. We then use these probabilities to check under which conditions the regular network structure revealed by Proposition 1 carries over to larger networks and more general utility functions u. We also consider the networks which would arise under asymmetric probabilities for the supply and demand shocks.

5.1 Monte-Carlo simulation of utilization probabilities

Recall from Definition 2 that, in order to check whether a given network G is pairwise stable, one needs to compute the value of G for every seller and buyer, as well as the values of networks that result from adding a link to G or deleting a link from G. In order to do so, one requires the utilization probabilities of the links in these networks. As we have shown,

[14] Observe that, as $p \to 1$, network size decreases but does not become zero: Even if $p = 1$, it is necessary to know at least one other agent in order to trade. Thus, the graph in Figure 8 is asymmetric in that the line dividing the empty network $G(0)$ from the network $G(1)$ only appears on the left side.

[15] The insurance function of networks is also demonstrated in Bajeux-Besnainoua, Joshi, and Vonortas (2010). There, a link between two firms gives each firm the option to invest in joint project at some future date. The value of a link is hence an option value, which depends on the probability that the link will be utilized given an underlying stochastic process for the profitability of the project. In our model, a link between two agents gives the two players jointly the option to trade with one another. The value of a link is, in a sense, an option value, as it depends on the probability that the link will be utilized given the underlying uncertainty of being on the matching market.

even if we restrict G to be in the class of regular networks the analytical derivation of these probabilities becomes exceedingly cumbersome for networks other than very small ones.

The aim of this section, therefore, is to test the pairwise stability of larger regular networks through numerical methods. We simulated the market clearing process described in Section 2.4 for the following networks:

1. The "candidate" network $G(k)$, for $k = 1, \ldots 10$.
2. The network that arises from adding a single link (i, j) to $G(k)$, up to 20th-order links. (Due to computational constraints, we were unable to check for longer links. At the end of Section 5.2 we argue below that this is not a severe limitation.)
3. The network that arises from deleting a single link (i, j) from $G(k)$.

The specifics of our computations are as follows. We set $N = 50$ and considered $p \in \{0.05, 0.10, 0.15, \ldots, 0.95\}$. For each p-value, we drew $500,000$ supply-demand configurations (x, y) to which the "inside-out" market clearing algorithm was subsequently applied, given the network in question. By counting how often each link was utilized, we obtained numerical estimates of the link utilization probabilities. The results of these simulations are presented in Figures 9 and 10, for $p = 0.5$.[16]

Adding links to networks. Figure 9 shows the utilization probabilities of links in regular networks, as well as links added to regular networks. The figure is plotted for the case $p = 0.5$. The links are numbered $1, \ldots, 15$, by the order in which they are resolved in the inside-out algorithm.[17] For example, the column in cell $\langle G(10), 8\rangle$ depicts the probability that an 8th-order (i.e., $(i, i+4)$) is utilized in network $G(10)$, while the column in cell $\langle G(10), 12\rangle$ depicts the probability with which a 12th order link (i.e., $(i, i+6)$) would be used if a single such link were added to the network $G(10)$.

Deleting links from networks. Figure 10 shows the utilization probabilities of all links belonging to an agent who has deleted one single link from the regular network $G(10)$ (while all other agents maintain their links). Again, this is plotted for the case $p = 0.5$, and the links are numbered by the order in which they are resolved in the inside-out algorithm. For example, the column in cell $\langle 8, 6\rangle$ ("existing link 8," "deleted link 6") depicts the probability that an 8th-order link is utilized by an agent who has deleted his 6th-order link from $G(10)$. Note that we only need to perform this exercise for $G(10)$, the largest regular network we consider: Since matches are cleared "from the inside out," the corresponding probabilities for $G(k)$ $(k < 10)$ must the same as those displayed in the figure. (For example, the column in cell $\langle 8, 6\rangle$ also depicts the probability that an 8th-order link is utilized by an agent who has deleted his 6th-order link from $G(9)$.)

[16]The complete set of simulation results is available, in tabular form, from the authors.

[17]The figure only displays links up to order 15, but we computed up to order 20.

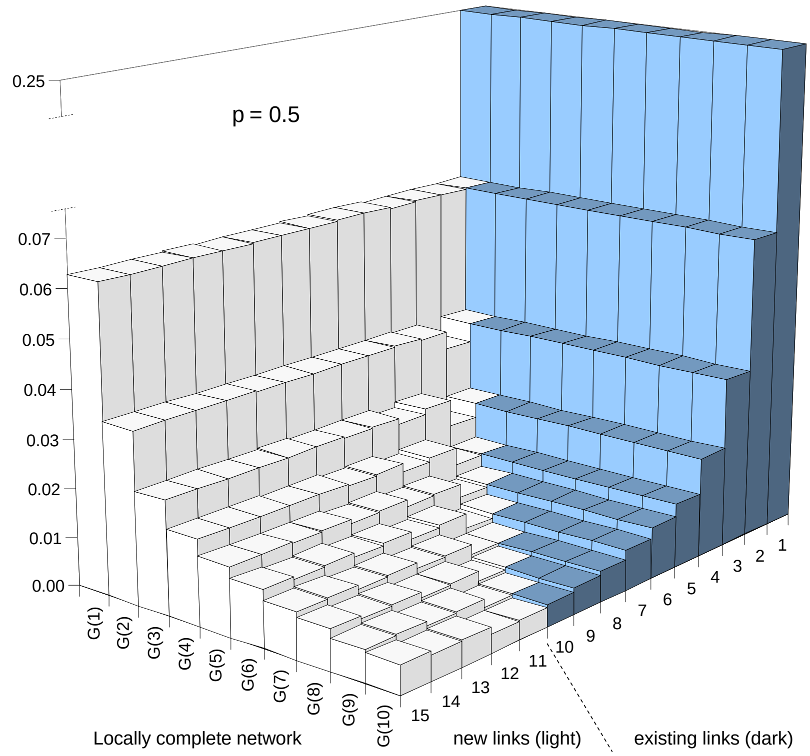

Figure 9: Simulated utilization probabilities: Link addition ($p = 0.5$)

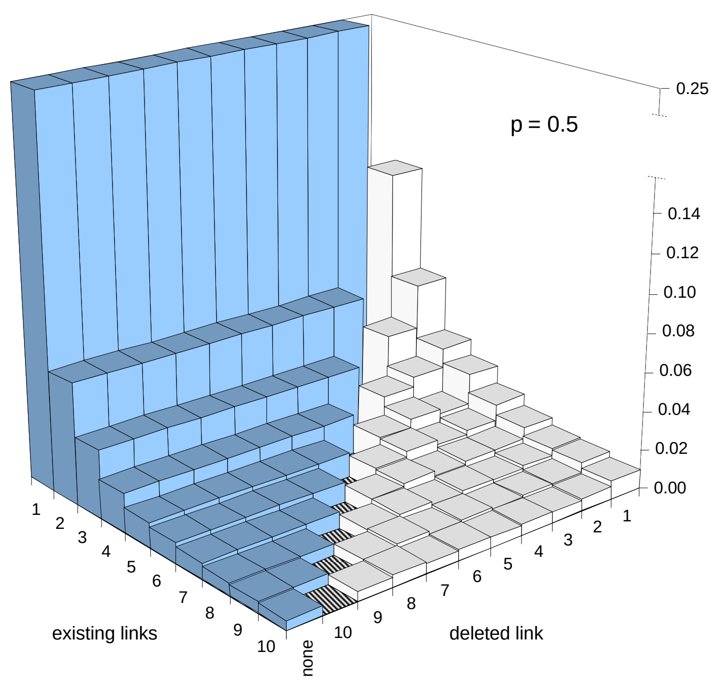

Figure 10: Simulated utilization probabilities: Link deletion ($p = 0.5$)

5.2 Testing for pairwise stability

Let us now fix a parametric class for the match utility function, $u(d) = \delta^d$ for $\delta < 1$. Given values for the parameters p, c, and δ, and using our simulated probabilities, it is easy to verify whether or not a given network $G(k)$ is pairwise stable. It turns out that, in general, we cannot expect regular networks to be stable, as the following example illustrates:

Example 5. Let $c = 0.006$, $p = 0.5$, and $\delta = 0.95$. Table 1 shows the values for seller i of every regular network from $G(0)$ to $G(10)$, as well as the best possible link addition and the best possible link deletion for this individual (i.e., the maximum value generated by deleting some existing link, and the maximum value generated by adding some new link).[18] As can be seen from the values in the table, no regular network is pairwise stable: For networks $G(7)$ and smaller there will always exist a link that should be added to the network; and for networks $G(8)$ and larger there will always exist a link that should be deleted. Thus, a pairwise stable, regular network does not exist in this case (we do not know which network G is pairwise stable in this case, or if one exists.)

Delete		$V_i^S(G)$	$\boldsymbol{k}$	$V_i^S(G(k))$	**Add**		$V_i^S(G)$
			0	.00000	#1	(i,i)	.24400
			1	.24400	#2	$(i,i{+}1)$	.29738
			2	.29738	#3	$(i,i{-}1)$	.32477
			3	.32477	#6	$(i,i{+}3)$	.33902
			4	.33761	#7	$(i,i{-}3)$	.34575
			5	.34346	#10	$(i,i{+}5)$	.34800
			6	.34648	#11	$(i,i{-}5)$	.34905
			7	.34753	#12	$(i,i{+}6)$	.34827
#8	$(i,i{+}4)$	.34753	8	.34660			
#9	$(i,i{-}4)$	.34660	9	.34503			
#10	$(i,i{+}5)$	.34503	10	.34256			

Table 1: No $G(k)$ is pairwise stable ($p{=}0.5$, $c{=}0.006$, $\delta{=}0.95$)

Example 5 is somewhat surprising: The fact that the utility of matches decreases in the distance between match partners suggests that agents would indeed want to build networks by adding successively longer links—generating a regular network of growing size in the process, until the added benefit of an additional link falls below the cost c. However, this intuition is incorrect, for the following reason. Recall that the expected benefit of a link (i,j) added to a network G is not $u(|i,j|)$, but $\varphi(i,j|G \cup (i,j))u(|i,j|)$. Even though

[18]Note that the symmetry of the regular network $G(k)$ across agents implies that $V_i^S(G(k)) = V_j^B(G(k))$ and $V_i^S(G(k) \cup (i,j)) = V_j^B(G(k) \cup (i,j))$. Thus, if seller i benefits from adding a link to buyer j, so does buyer j; it is therefore sufficient to only check if one of these agents wants to add a link to test for pairwise stability. For general networks G, of course, this is not the case.

$u(|i,j|)$ is decreasing in the distance $|i,j|$, the utilization probability $\varphi(i,j|G \cup (i,j))$ of an added link (i,j) may be increasing in $|i,j|$, at least locally.[19] Thus, given $G(k)$ and $(i,j),(i,j') \notin G(k)$ with $|i,j| > |i,j'|$, it is possible that $\varphi(i,j)|G(k) \cup (i,j))u(|i,j|) > \varphi(i,j')|G(k) \cup (i,j'))u(|i,j'|)$. This, in turn, implies that agents may want to create networks with "holes" instead of regular networks. For example, as can be seen in Table 1 the best addition to $G(6)$ is not the 7th-order link $(i,i-3)$ but instead the 11th-order link $(i,i-5)$

To counteract this phenomenon, we must assume that the u-function is decreasing at a sufficiently fast rate for the expected link benefit $\varphi(i,j|G(k) \cup (i,j))u(|i,j|)$ to be decreasing in $|i,j|$. If this is satisfied, the most beneficial links to add to $G(k)$ are the ones of the order $k+1$. In our simulations, we found $\delta \leq 0.8$ to be sufficient for this purpose for all p. (Similarly, one must assure that the best link to delete from $G(k)$ is a kth-order link, and for this we found $\delta \leq 0.94$ to be sufficient.) A regular network can then be constructed by successively adding longer links, until the expected benefit of a new link falls below its cost c. At this point, we will have reached a regular, pairwise stable network. The following example demonstrates this process:

Example 6. As in the previous example, let $c = 0.006$ and $p = 0.5$. This time, however, use $\delta = 0.75$. Table 2 shows the regular network $G(5)$ to be pairwise stable: For every $k < 5$ the individuals want to add the $(k+1)$st-order links to $G(k)$, and for every $k > 5$ the individuals want to delete their kth-order links. For $G(5)$, on the other hand, even the best deletion and the best addition will result in values that are less than $V_i^S(G(5))$; therefore $G(5)$ is pairwise stable.

Delete		$V_i^S(G)$	k	$V_i^S(G(k))$	**Add**		$V_i^S(G)$
			0	.00000	#1	(i,i)	.24400
			1	.24400	#2	$(i,i+1)$	.28510
			2	.28510	#3	$(i,i-1)$	.30547
			3	.30547	#4	$(i,i+2)$	.31106
			4	.31106	#5	$(i,i-2)$	.31245
#5	$(i,i-2)$	(.31106)	**5**	**.31245**	#6	$(i,i+3)$	(.31088)
#6	$(i,i+3)$	.31245	6	.31088			
#7	$(i,i-3)$	.31088	7	.30835			
#8	$(i,i+4)$	.30835	8	.30432			
#9	$(i,i-4)$	.30432	9	.30004			
#10	$(i,i+5)$	.30004	10	.29512			

Table 2: $G(5)$ is pairwise stable ($p=0.5$, $c=0.006$, $\delta=0.75$)

[19] This can be seen by inspecting Figure 9. For example, the gray columns representing the utilization probabilities for links added to $G(7)$ clearly have increasing parts when going from link #8 to link #15.

Let us now briefly revisit the issue that we do not consider links longer than 20th-order links in our simulations. It is conceivable that some network $G(k)$ (with $k \leq 10$) is not stable because agents want to add, say, a 21st-order link but not any shorter links of order $11, \ldots, 20$. Our verification procedure would then miss this possibility. In this regard, a low δ-value in the utility function u means that the expected benefit of higher-order links will decrease quickly. Take $\delta = 0.75$, for example, and suppose that we have verified that the agents do not wish to add links of order 11 through 20 to $G(10)$. Then for them wanting to add a 21st-order link would require the utilization probability of this link to be more than four times as large as the utilization probability of the last link in $G(10)$, in order to override the decrease in u. This appears very unlikely to be the case, as the variation in the utilization probabilities for added links is less than 85% of the smallest probability in each one of our simulations.

5.3 Results

We assume that the match surplus function is $u(d) = \delta^d$, with $\delta = 0.75$. This value is below the threshold beyond which stable regular networks may fail to exist. Figure 11 shows the size of regular, stable networks as a function of both the networking cost c and the probability p.

As one would expect, network size increases as c decreases. Going from top to bottom in the graph, each line represents the jump from network $G(k)$ to $G(k+1)$. Furthermore, for a given c, network size initially increases as p increases, is the largest when $p = 0.5$, and then decreases as p increases further. This is consistent with our theoretical analysis in Section 4. To understand this effect, notice that a link to other agents can be regarded as (imperfect) insurance against the risk of future trading needs. Large networks are costlier, but also provide better insurance. As p increases, the risk of being in need of a future match partner increases. At the same time, each individual is more likely to be available for trade; thus, the network grows in size. However, there is a countervailing effect: Consider seller i contemplating the creation of a link to a distant buyer j. If p increases, it becomes more likely that some buyer closer to i is available to trade with i, so linking to the more distant buyer j is less beneficial on expectation. Second, *even if no closeby buyers are available*, buyer j is now less likely to be available for i, as he himself is more likely to be in a match with a seller who is closer to j than i is. This increased within-network competition over match partners eventually overrides the “insurance benefits” of large networks. In the extreme, if $p = 1$, a single link from seller i to buyer $j = i$ provides perfect insurance for seller i (and vice versa), while any additional links would be of no value to either i or j. Thus, for very large values of p, the stable network is $G(1)$.[20]

We now turn to an extension of our model not considered so far. Assume that each

[20]It should also be noted that changes in networking costs seem to play a larger role in determining network size than do changes in p. This is evidenced by the relative “flatness” of the curves in the middle part of the p-range. For example, for $c = 0.005$, the stable network is $G(5)$ for all $p \in [0.2, 0.8]$.

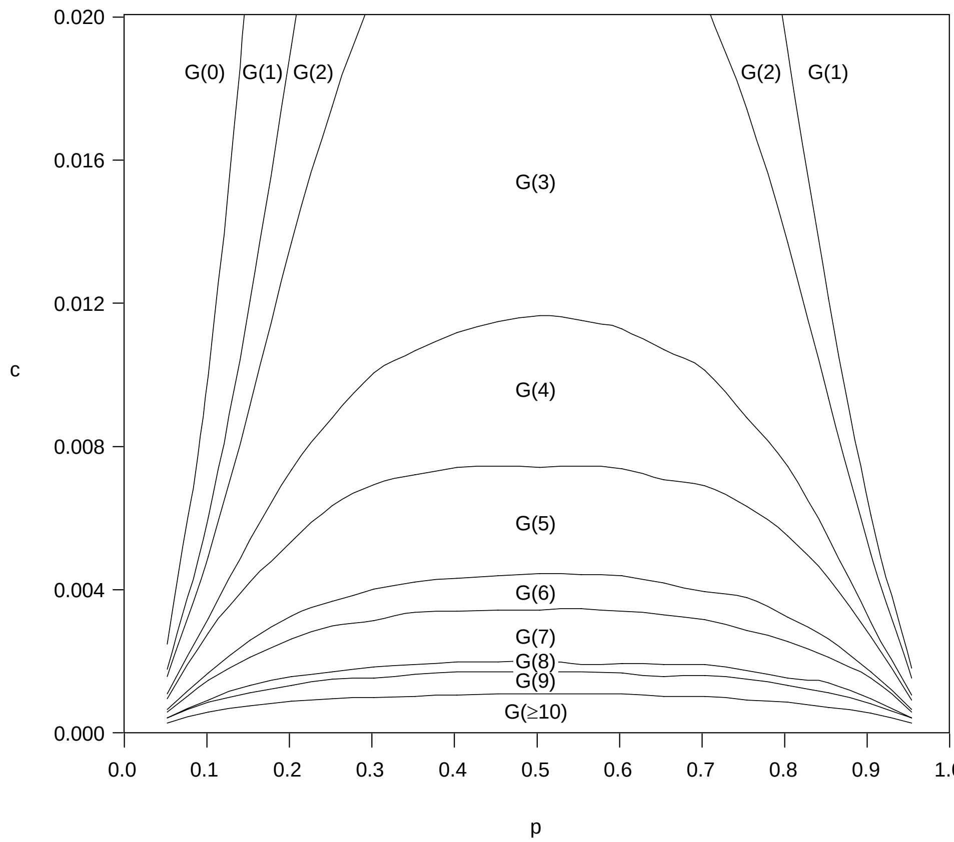

Figure 11: Pairwise stable networks $G(0), \ldots, G(10)$ ($\delta = 0.75$)

seller is on the market with probability p and each buyer is on the market with probability q (which is not necessarily the same as p). For example, suppose the underlying matching environment is a job market, with networking being a tool for workers (sellers) to find jobs, and for firms (buyers) to get to know potential hires. An imbalance between p and q, such that $p > q$, then represents a recessionary situation in the sense that the number of job seekers on expectation exceeds the number of job openings. Similarly, if $q > p$, we could interpret this as a labor shortage. As a response to such imbalances, should we expect an increased or a decreased use of job networking? To answer this question, we simulated the utilization probabilities and network values for all (p, q)-combinations with $p, q \in \{0.05, 0.10, \ldots, 0.95\}$ (if $p = q$ then this corresponds to the symmetric probability model considered so far).

Figure 12 shows the stable networks as a function of the networking cost c and the sellers' probability p, with the corresponding probability for the buyers being constant at $q = 0.25$. The match surplus function is the same as before. Unlike the symmetric case, network size is no longer maximized at $p = 0.5$. Instead, given a fixed value for c, networks tend to be largest when p is slightly larger than 0.25 but less than 0.5. Starting at the vertical line where $p = q = 0.25$, an increase in p increases the network size by at most one

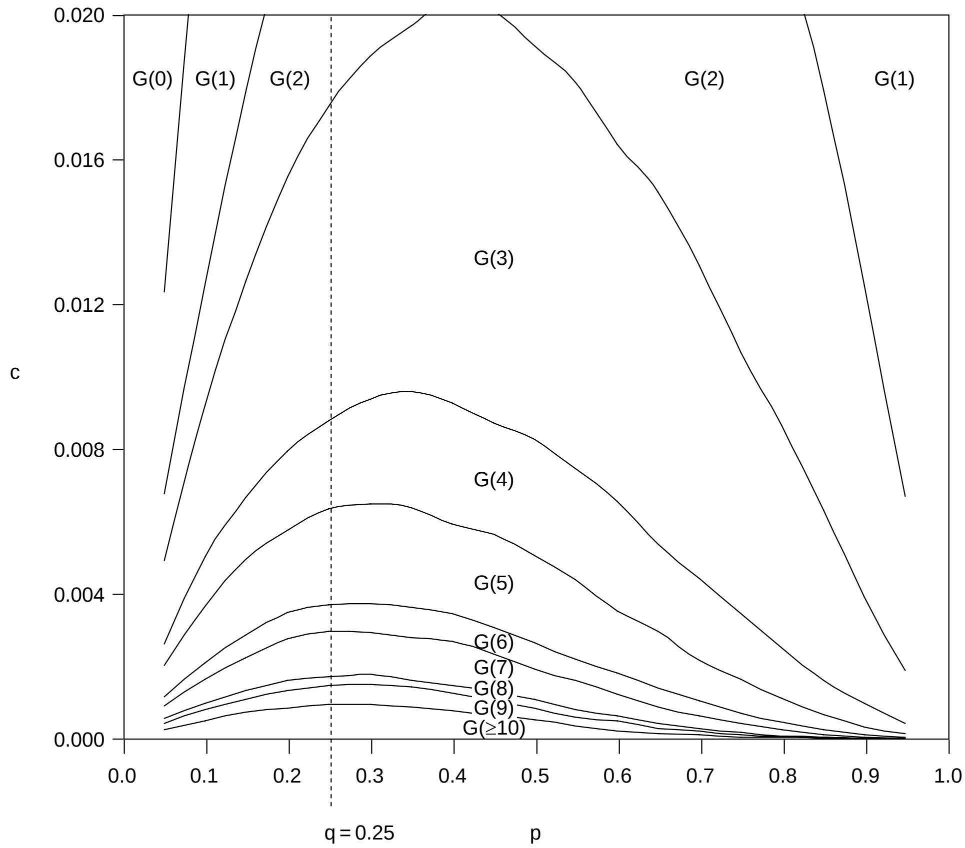

Figure 12: Pairwise stable networks with asymmetric shocks ($q = 0.5$, $\delta = 0.75$)

link. Furthermore, for small values of c the largest networks tend to arise when $p \approx q$.

Figure 13 provides an alternative perspective. There, we hold the per-link cost fixed at $c = 0.006$, but let both p and q vary between 0.05 and 0.95. The (p, q)-space is partitioned into regions by the size of the stable network that arises. The largest equilibrium network is $G(5)$. The region in which $G(5)$ is stable is a tilted ovoid shape, oriented along the 45°-line where supply and demand are balanced on expectation (i.e., $p = q$). Starting at any point on the 45°-line and moving away from the line either horizontally or vertically decreases network size.

6 Conclusion

Using a two-sided matching market as our starting point, we examined the social networks that arise if agents must know each other in order to trade. We assumed that the individuals are spatially separated and prefer closeby match partners over those located further away. In this setting we examined under which circumstances regular equilibrium networks exist, in which individuals know their k nearest neighbors. We further examined the effect of

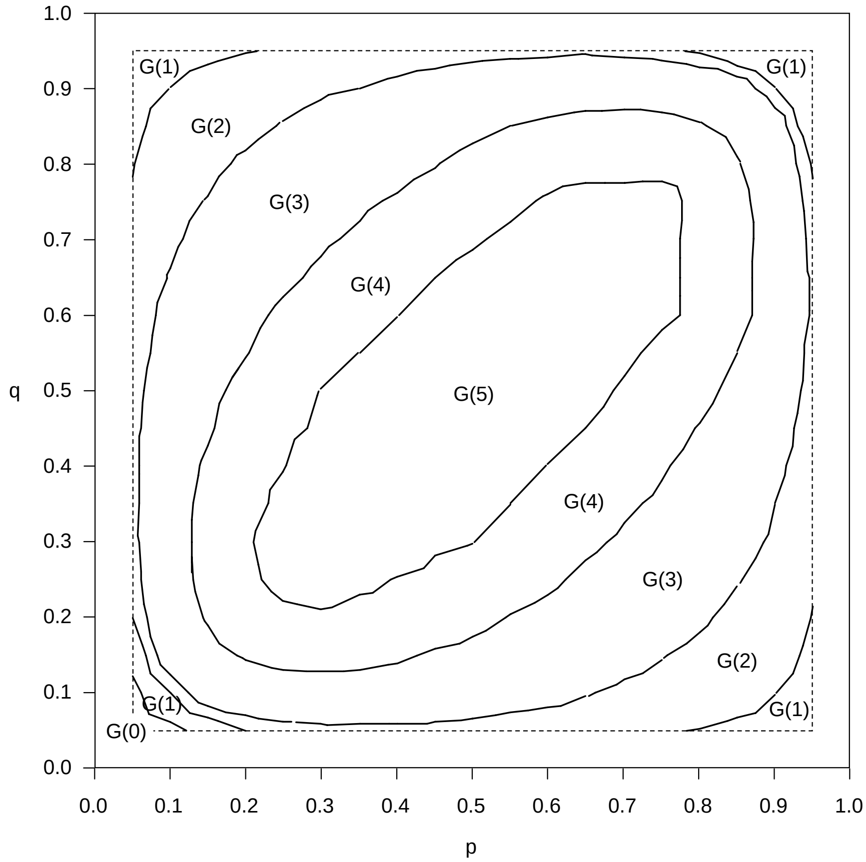

Figure 13: Pairwise stable networks with asymmetric shocks ($c = 0.006$, $\delta = 0.75$)

uncertainty on the size of these networks. Our theoretical and numerical results suggest two things. First, networking is most intense in balanced markets where supply and demand shocks happen to individual agents with roughly the same probability. Within the set of balanced markets, the largest networks arise when $p = q = 0.5$. Second, an unbalanced market tends to not increase networks, unless the imbalance is very small.[21]

There are several interesting questions which we do not address in this paper. It seems promising to investigate the structure and size of efficient networks, as well as the equilib-

[21]This result contradicts some popular advice that in a recession, when there are more job seekers than openings, job seekers should rely more heavily on networking to find employment. For example, a March 24, 2009 article on CNN.com reports that as of February 2009, the number of registered users on the job networking website LinkedIn.com had more than doubled over the previous year, and suggests that the 2008/2009 recession was responsible for this growth. (See `http://money.cnn.com/2009/03/24/technology/hempel_linkedin.fortune/index.htm`.) However, these registrations were presumably made *after* the users had learned of their unemployment. In the context of our model, on the other hand, networking is an ex-ante activity which individuals undertake to insure against the risk of becoming unemployed. Everything else equal, if this risk increases then networks will typically decrease as the benefits of links are being "competed away."

rium networks that arise when no stable, regular networks exist. Furthermore, one could examine the effects of a network friction in other matching environments, such as one-sided markets, or markets that exhibit vertical instead of horizontal differentiation or which allow for transferable utility. These questions are left for future research.

Appendix

Derivation of equation (9)

The utilization probabilities for 1st-order, 2nd-order, and 3rd-order links are derived in the text already. To see why the utilization probability of a 4th-order link is $\varphi(i, i+2|G(4)) = p^2(1-p)^2(1-2p+2p^2)(1-p+p^2(1+(1-p)^2))^2$, consider what must happen for seller i and buyer $i-1$ to trade:

- First, $x_i = y_{i+2} = 1$, which has probability p^2.
- Second, we need to ensure that seller i does not trade with buyer i, and that buyer $i+2$ would trade with seller $i+2$. This will be the case if and only if $x_{i+2} = y_i = 0$, which has probability $(1-p)^2$.
- Third, we must ensure that seller i does not trade with buyer $i+1$, and that buyer $i+2$ does not trade with seller $i+1$. This will be the case if and only if one of the following holds: $x_{i+1} = y_{i+1} = 0$ (neither seller $i+1$ nor buyer $i+1$ are on the market), or $x_{i+1} = y_{i+1} = 1$ (both seller $i+1$ and buyer $i+1$ are on the market and trade with each other). This gives us the factor $(1-p)^2 + p^2 = 1 - 2p + 2p^2$.
- Fourth, we must ensure that seller i does not trade with buyer $i-1$. This will be the case if and only if one of the following holds: $y_{i-1} = 0$ (buyer $i-1$ is not on the market), or $y_{i-1} = x_{i-1} = 1$ (buyer $i-1$ is on the market but trades with seller $i-1$), or $y_{i-1} = x_{i-2} = 1$ and $x_{i-1} = y_{i-2} = 0$ (buyer $i-1$ is on the market and trades with seller $i-2$, who accepts because buyer $i-2$ is not on the market). This gives us the factor $(1-p) + p^2 + p^2(1-p)^2 = 1 - p + p^2(1+(1-p)^2)$. Similarly, we must ensure that buyer $i+2$ does not trade with seller $i+3$. This will be the case if and only if one of the following holds: $x_{i+3} = 0$ (seller $i+3$ is not on the market), or $x_{i+3} = y_{i+3} = 1$ (seller $i+3$ is on the market but trades with buyer $i+3$), or $x_{i+3} = y_{i+4} = 1$ and $x_{i+4} = y_{i+3} = 0$ (seller $i+3$ is on the market and trades with buyer $i+4$, who accepts because seller $i+4$ is not on the market). This gives us another factor $(1-p) + p^2 + p^2(1-p)^2 = 1 - p + p^2(1+(1-p)^2)$.

Proof of Proposition 1

Throughout the proof, we will employ the utilization probabilities (5)–(8), derived in Section 3. In addition to these probabilities, it will be necessary to derive the utilization probabilities for links in networks which result from adding links to regular networks, or deleting links from them. These can be found by taking the same steps as in the examples of Section 3; that is, by determining the supply-demand configurations which will lead to trade over a link and adding their probabilities. In all but a few cases this will be straightforward and not be reiterated here; the rest is explained in the footnotes.

Stability of $G(0)$**.** Note that for every network G and every link $(i,j) \in G$, $\varphi(i,j|G) \leq p^2$ (because for i and j to trade, it is necessary but not sufficient that $x_i = y_i = 1$). This implies that if $p^2 \cdot u(0) = p^2 \leq c$, the expected benefit of a link does not strictly exceed its cost. Thus, if $c \geq p^2$, the empty network $G(0)$ is pairwise stable.

Stability of $G(1)$**.** Now assume $c \leq p^2$ and consider the regular network $G(1)$. The expected net value of $G(1)$ for each agent is

$$V_i^S(G(1)) = V_j^B(G(1)) = \varphi(i,i|G(1)) \cdot 1 - c = p^2 \cdot 1 - c > 0.$$

If a buyer or seller cut the single link she has in $G(1)$, she would receive an expected payoff of zero. Thus, no agent wants to cut her link from $G(1)$.

Consider next the possibility of adding a link between two agents. Since $u(2) < c$, the only possible additions are those from some seller i to buyer $i+1$ or $i-1$. Consider first the network $G' = G(1) \cup (i, i+1)$, for some seller i. It can be verified that $\varphi(i,i|G') = p^2$ and $\varphi(i,i+1|G') = p^2(1-p)^2$. The expected net payoffs to seller i and buyer $i+1$ in G' are this

$$\begin{aligned} V_i^S(G') = V_{i+1}^B(G') &= \varphi(i,i|G') \cdot 1 + \varphi(i,i+1|G') \cdot \delta - 2c \\ &= p^2 + p^2(1-p)^2\delta - 2c. \end{aligned}$$

Now consider the network $G'' = G(1) \cup (i, i-1)$. It can be verified that $\varphi(i,i|G'') = p^2$ and $\varphi(i,i-1|G'') = p^2(1-p)^2$, and thus

$$V_i^S(G'') = V_{i-1}^B(G'') = p^2 + p^2(1-p)^2\delta - 2c.$$

Seller i and buyer $i+1$ (resp. $i-1$) hence prefer the network $G(1)$ to G' (resp. G'') if $c \geq p^2(1-p)^2\delta$. Thus, as long as $p^2 \geq c \geq p^2(1-p)^2\delta$, the network $G(1)$ is pairwise stable.

Stability of $G(2)$**.** Now assume $c \leq p^2(1-p)^2\delta$ and consider the regular network $G(1)$. The expected net value of $G(2)$ for each agent is

$$V_i^S(G(2)) = V_j^B(G(2)) = p^2 + p^2(1-p)^2\delta - 2c.$$

Consider first the possibility of seller i cutting her link to buyer $i+1$. In this case $\varphi(i,i|G(2) - (i,i+1)) = p^2$, and seller i's expected payoff becomes

$$V_i^S(G(2) - (i,i+1)) = p^2 - c,$$

which is less than $p^2 + p^2(1-p)^2\delta - 2c$ (since $c \leq p^2(1-p)^2\delta$ is assumed). On the other hand, if seller i cuts her link to buyer i we have $\varphi(i, i+1|G(2) - (i,i)) = p^2(1-p)$.[22] Thus, seller i's expected payoff becomes

$$V_i^S(G(2) - (i,i)) = p^2(1-p)\delta - c.$$

Clearly $V_i^S(G(2)-(i,i)) < w_i^S(G(2)-(i,i+1))$, and since seller i does not want to sever the link to buyer $i+1$ she also does not want to sever the link to buyer i. Symmetric arguments apply to the case where a buyer is cutting one of her links to a seller. It follows that no agent wants to cut a link from $G(2)$.

Consider next the possibility of adding a link between two agents. Since $u(2) < c$, the only possible addition is a link from some seller i to buyer $i-1$. Thus, consider the network $G' = G(2) \cup (i, i-1)$, for some seller i. It can be verified that $\varphi(i,i|G') = p^2$, $\varphi(i, i+1|G') = p^2(1-p)^2$ and $\varphi(i, i-1|G') = p^2(1-p)^2(1-p+p^2)^2$. The expected net payoff to seller i and buyer $i-1$ are

$$\begin{aligned} V_i^S(G') = V_{i-1}^B(G') &= \varphi(i,i|G') \cdot 1 + \varphi(i,i+1|G') \cdot \delta + \varphi(i,i-1|G') - 3c \\ &= p^2 + p^2(1-p)^2\delta + p^2(1-p)^2(1-p+p^2)^2\delta - 3c. \end{aligned}$$

Seller i and buyer $i-1$ hence prefer the network $G(2)$ to G' if $c \geq p^2(1-p)^2(1-p+p^2)^2\delta$. Thus, as long as $p^2(1-p)^2(1-p+p^2)^2\delta \geq c \geq p^2(1-p)^2\delta$, the network $G(2)$ is pairwise stable.

Stability of $G(3)$**.** Now assume $c \leq p^2(1-p)^2(1-p+p^2)^2\delta$ and consider the regular network $G(3)$. The expected net value of $G(3)$ for each agent is

$$V_i^S(G(3)) = V_j^B(G(3)) = p^2 + p^2(1-p)^2\delta + p^2(1-p)^2(1-p+p^2)^2\delta - 3c.$$

Given that $u(d) < c$ for $d > 1$, clearly no additional links will be worthwhile, so we only need to consider the possibility of cutting links. First, suppose seller i cuts her link to buyer $i-1$. In this case $\varphi(i,i|G(3) - (i,i-1)) = p^2$ and $\varphi(i,i+1|G(3) - (i,i-1) = p^2(1-p)^2$, and seller i's expected payoff becomes

$$V_i^S(G(3) - (i,i-1)) = p^2 + p^2(1-p)^2\delta - 2c,$$

which is less than $p^2 + p^2(1-p)^2\delta + p^2(1-p)^2(1-p+p^2)^2\delta - 3c$ (since $c \leq p^2(1-p)^2(1-p+p^2)^2\delta$ is assumed). Second, suppose seller i cuts her link to buyer $i+1$. In this case $\varphi(i,i|G(3) - (i,i+1)) = p^2$ and $\varphi(i,i-1|G(3) - (i,i+1)) = p^2(1-p)^2(1-p+p^2)$,[23] and seller i's expected payoff becomes

$$V_i^S(G(3) - (i,i+1)) = p^2 + p^2(1-p)^2(1-p+p^2)\delta - 2c.$$

[22] For seller i to trade with buyer $i+1$, we need $x_i = y_{i+1} = 1$ (probability p^2) and $x_{i+1} = 0$ (probability $1-p$).

[23] For seller i to trade with buyer $i-1$, we need $x_i = y_{i-1} = 1$ (p^2), $x_{i-1} = y_i = 0$ (probability $(1-p)^2$), and either $x_{i-2} = 0$ or $x_{i-2} = y_{i-2} = 1$ (probability $1-p+p^2$).

Note that $V_i^S(G(2)-(i,i+1)) < V_i^S(G(2)-(i,i-1))$, and since seller i does not want to sever the link to buyer $i-1$, she also does not want to sever the link to buyer $i+1$. Third, suppose seller i cuts her link to buyer i. In this case $\varphi(i,i+1|G(3)-(i,i)) = p^2(1-p)$ and $\varphi(i,i-1|G(3)-(i,i)) = p^2(1-p)(1-p+p^2)^2$,[24] and seller i's expected payoff becomes

$$V_i^S(G(3)-(i,i)) = p^2(1-p)\delta + p^2(1-p)(1-p+p^2)^2\delta - 2c.$$

We now show that $V_i^S(G(2)-(i,i)) < V_i^S(G(2)-(i,i-1))$. After some algebraic manipulations, this inequality can be written as

$$\frac{1}{(1-p)\delta} + 1 - p > 1 + (1-p+p^2)^2,$$

and since $\delta < 1$ it is sufficient to show

$$\frac{1}{(1-p)} + 1 - p > 1 + (1-p+p^2)^2.$$

After further algebra, this can be expressed as $(1+p)/(1-p^4) > 1$, which is satisfied for all $p \in (0,1)$. Thus, since seller i does not want to sever the link to buyer $i-1$ she also does not want to sever the link to buyer i. It follows that for $c \leq p^2(1-p)^2(1-p+p^2)^2\delta$, the network $G(3)$ is pariwise stable. □

[24]For seller i to trade with buyer $i+1$, we need $x_i = y_{i+1} = 1$ (probability p^2) and $x_{i+1} = 0$ (probability $1-p$). For seller i to trade with buyer $i-1$, we need $x_i = y_{i-1} = 1$ (probability p^2) and $x_{i-1} = 0$ (probability $1-p$). In addition, either $x_{i-2} = 0$ or $x_{i-2} = y_{i-2} = 1$ (probability $1-p+p^2$), and similarly either $y_{i+1} = 0$ or $x_{i+1} = y_{i+1} = 1$ (probability $1-p+p^2$).

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