Communication in Financial Markets with Several Informed Traders*

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August 2006

Abstract

This paper investigates the incentives for informed traders in financial markets to reveal their information truthfully to the public. In the model, a subset of traders receive noisy signals about the value of a risky asset. The signals are composed of a directional component ("high" vs. "low") as well as a precision component that represents the quality of the directional component. Between trading periods, the informed agents make public announcements to the uninformed traders. With a sufficiently large number of informed traders, an equilibrium exists in which the directional components are credibly revealed, but not the precision components. Even though the informed traders retain some of their private information, the post-communication estimate of the asset value converges in probability to the full-information estimate as the number of informed traders increases.

Keywords: Cheap Talk, Communication, Informational Smallness, Insider Trading, Multiple Experts, Market Manipulation.

JEL code: G14, D84.

*The paper is based on a chapter of my Ph.D. thesis at the University of Western Ontario and was circulated previously under the title "Public Communication Devices in Financial Markets." I thank my dissertation committee Arthur Robson, Hari Govindan, and Al Slivinski for their guidance and support. I also thank Murali Agastya, Roland Benabou, Philippe Grégoire, Rick Harbaugh, Mike Peters, an anonymous referee and an associate editor, and seminar participants at various universities and conferences at which this paper was presented. All remaining errors are the responsibility of the author.

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1 Introduction

Privately informed participants in security markets often have the possibility to communicate their information to others. For example, corporate insiders can leak knowledge of events affecting their company to newspapers. In financial newsletters, or on financial television shows or websites, experts voice their opinions on company stocks. There are good reasons to question the credibility of such reports. Consider, for instance, an agent who is privately informed of a favorable event concerning a certain financial asset. The agent can profit from his privileged information by buying the asset and sell it once the news becomes public—a behavior that is called informed speculation. However, if the agent can communicate information to other traders who trust his report, he can use the following scheme to further increase his profits: First, the agent sells the asset. Then, contrary to what he knows, the agent claims to have unfavorable information. This will temporarily depress the market price, allowing the agent to close the short position with a profit. He then uses the proceeds to buy the asset and waits for the good news to become public. At this time the asset price appreciates, and the agent can profit a second time by selling the asset—such behavior is called (information-based) market manipulation. Of course, if the uninformed market participants are strategically aware, they will see through this scheme, and the agent's report cannot contain useful information in any equilibrium of this market game.

This paper shows that a third strategy can arise in certain cases: Informed speculation coupled with the communication of true information. We propose a simple model of communication in financial markets, based on the well-known model of Benabou and Laroque (1992). The environment features a risky asset of unknown value, to be revealed at a future date, and cash. Some traders receive independent signals that are imperfectly correlated with the asset's value. There are two trading dates, between which the informed agents make public announcements. After the second date, the true value of the asset is revealed. Although each agent is quantitatively negligible, informed traders do have some informational influence: They can strategically exploit the impact of their announcements on other agents' expectations and hence on the market price.

The asset can have two possible final values, zero and one. An informed trader's knowledge consists of a directional component and a quality component. The directional component is called the trader's *signal* and is either zero or one. The quality component is called the trader's *signal precision* and represents the probability with which the signal equals the true asset value. Both the directional and the quality component are private information of an informed trader. Equivalently, the signal and its precision can be composed into a single scalar variable which represents an informed trader's belief regarding the asset's value, and which ranges from "very optimistic" to "very pessimistic." In between these extreme points, moderately optimistic or pessimistic signal are possible.

Our main result states that a *direction revealing equilibrium* exists in the model if the number of informed traders is sufficiently large. In this equilibrium, the informed traders truthfully report their signal, but reports concerning the precision of the signals are not credible. To see why such an equilibrium exists, it is instructive to look at a simpler case for a moment, where the precision of each informed trader's signal is fixed and commonly known. We speak of *binary signals* in this case, and for a single informed trader the binary signals case corresponds precisely to the model of Benabou and Laroque (1992). Note that any agent who has bought the asset in period 1 clearly wants to make others believe that it is going to be of high value, and the opposite holds for an agent who has sold the asset. Thus, for truthful reporting of the signals it is necessary that traders with high signals buy the asset in the first period, and traders with low signals sell. This trading behavior becomes optimal once there are sufficiently many informed traders. The reason is that if every informed trader's signal is known, the true asset value can be predicted with high accuracy. Thus, if there are many informed traders who all report their signals truthfully, the second period price (call it P_2) will likely be close to the true asset value. Because of this, almost all profits a trader can make accrue between the first and the second trading period. Now suppose that a trader receives a high signal. He must believe that, with probability larger than half, P_2 is likely going to be close to one; at the same time the difference between this price and the true value, to be revealed after the second period, is likely to be small. The key observation is that this holds regardless of what message the trader sends, provided that the number of informed agents is high enough, as each trader becomes informationally small and no single message will, on average, influence P_2 significantly. Thus, the best the agent can do is to buy the asset and reveal his information in order to push the post-communication price in the direction that supports his initial trade. A similar argument can be made for an informed trader who has a low signal and sells the asset. With binary signals, therefore, a *fully revealing* equilibrium exists if the number of informed traders is sufficiently large.

When agents are privately informed about their signal's precision, a trader who bought the asset with a moderately optimistic signal would obviously have an incentive to report that his signal was extremely optimistic (that is, his precision was very high), as such a report would maximize P_2, if believed. Hence there is little hope of obtaining credible information about the precision of a trader's signal through communication. We show, however, that the signal itself can still be credibly communicated, giving rise to a direction revealing equilibrium. Even though in this case the informed traders retain some of their informational advantage, we show that the post-communication estimate of the asset's value converges in probability to the full information estimate as the number of informed traders increases. Thus, asymptotically the communication possibilities of the game aggregate all private information into the market price. We further show that, if N is large, the incentives to report the directional component truthfully are strict. Hence, even if there was a small cost of communication and traders had the option of not making an announcement, the equilibrium would remain intact.

The remainder of the paper is organized as follows. A discussion of some related papers is given in Section 2 below. The model is contained in Section 3, where we describe the trading environment, information structure, and communication possibilities. In Section 4 we state the main result, namely that (for the general case) a direction revealing equilibrium exists and asymptotically aggregates all information. The slightly stronger result for the binary signals case is then derived as a corollary. In Section 5 we prove the main result. Section 6 concludes with a discussion of some extensions of the model.

2 Related Literature

The literature that is related to this paper is vast; containing theoretical as well as empirical studies of insider trading, market manipulation, and communication games. We will not provide an exhaustive review of these papers here, and focus on only a few papers that are most directly related to this one.

The paper's basic model is borrowed from a seminal paper on communication in financial markets by Benabou and Laroque (1992). The conclusion of Benabou and Laroque (1992) is that, unless the game is repeated, the single opportunistic informed trader's report cannot be credible. For the public to believe the trader's report, there either has to be a significant chance that the trader is intrinsically honest and always reports the truth (the opportunistic type can then hide behind the honest type), or the game must be repeated so that the informed agent can build a reputation of being honest. Our model is the same as theirs, except that we allow for multiple informed traders and consider the more general case where traders possess private precision information.

A recent paper by Van Bommel (2003) considers disclosure of private information in a dynamic model of a financial market. In disclosure games the informed agent's decision is whether and how much of his private information to communicate, but where lying (i.e. communicating false information) is not possible. He shows that the informed agent finds it optimal to disclose privileged information in an effort to induce volatility on which he then trades. With positive probability (due to market noise) the price overshoots following the disclosure, allowing the sender to profit twice. Van Bommel then considers "opportunistic" agents who are just like the traders in our model in that they can make false statements. Similar to Benabou and Laroque (1992), unless the game is repeated, no endogenous incentives for the opportunistic type to reveal his information can exist.

Grégoire (2004) examines information disclosure in a multi-asset financial market with one informed trader per asset (an insider). The insiders are quantitatively large and compete for market depth by revealing private information relating to an insider's company, as this attracts liquidity to the market for their asset. Thus, although disclosure reduces the informational advantage of insiders, it can be beneficial because it also reduces the implicit transaction costs that arise from limited marked depth.

Finally, the paper is related to a growing literature on the aggregation of expert opinions, containing papers by Austen-Smith (1993), Wolinsky (2002), and Gerardi et al. (2005). These papers demonstrate how the multiplicity of experts (who do not necessarily possess the same information) can be exploited to construct elicitation schemes that would not be feasible with only a single expert. However, the models examined in these papers are much different from the trading game we analyze here, and will therefore not be discussed.

3 The Model

3.1 The market

A risky asset and a riskless numeraire good (cash) are traded on two days, $t = 1, 2$. The risky asset is of uncertain value $v \in \{0, 1\}$; the prior probability of either value is $\frac{1}{2}$. In each period $t \in \{1, 2\}$ the risky asset is traded at the price that clears the market, denoted by P_t. After period 2 the value v is revealed and the asset pays either one or zero units of cash. We will call this the consummation period.

Three types of agents participate in the market: Informed traders, uninformed traders (also called the *public*), and noise traders. The set of informed traders is $\mathcal{N} = \{1, \ldots, N\}$ and has measure zero. The set of uninformed traders is a continuum of measure 1. The set of noise traders is a continuum of measure $\alpha > 0$.[1]

Initially each informed and uninformed trader is endowed with a single unit of the riskless asset and zero units of the risky asset. Trade in the risky asset is purely speculative, and short sales are allowed under the restriction that a trader's wealth must always be non-negative. Informed traders are risk neutral and maximize their expected final wealth. Uninformed traders are risk averse and maximize the expected utility of their final wealth. The utility function of uninformed traders is denoted u and assumed increasing, concave, and twice continuously differentiable. Furthermore, u exhibits non-increasing absolute risk aversion: $-u''(w)/u'(w)$ is non-increasing in w.

To model noise, we assume that the aggregate demand function for the risky asset by the noise traders at times $t \in \{1, 2\}$ is given by $\alpha z(P_t, \epsilon^t)$, where $\alpha > 0$ is the measure of noise traders in the market, z is a continuous function that is strictly decreasing in P_t and strictly increasing in ϵ^t and satisfies $z(1/2, 0) = 0$. The ϵ^t are i.i.d. random variables drawn from a symmetric and atomless distribution on $[-\tau, \tau]$ ($\tau > 0$). We further assume that $z(1, \tau) < 0$ and $z(0, -\tau) > 0$. This means that at extremely high prices noise traders sell the asset for all realizations of ϵ^t, and vice versa for extremely low prices.

3.2 Information structure

Prior to the first trading day each informed trader $i \in \mathcal{N}$ privately observes two variables: $p_i \in [\underline{p}, \overline{p}] \subset (1/2, 1)$, which we call the trader's *signal precision*, and $s_i \in \{0, 1\}$, which we call the trader's *signal*. The p_i are i.i.d. random draws from support $[\underline{p}, \overline{p}]$, and we denote by $\hat{p} = E[p_i]$ the average signal precision. The s_i are then drawn such that $\Pr[s_i = v] = p_i$. Conditional on v, the s_i are therefore independent. Thus, each informed trader receives a private signal that is imperfectly correlated with the true state v, and is privately informed as to how precise this signal is. The prior distribution of v, as well as the distribution of the p_i, are common knowledge. Denote by $s = (s_1, \ldots, s_N)$ the vector of all signals, and by $p = (p_1, \ldots, p_N)$ the vector of all precision values.

A special case arises when $\underline{p} = \overline{p} \in (1/2, 1)$ so that the precision of each trader's signal is common knowledge. Trader i's private information then essentially collapses to $s_i \in \{0, 1\}$, and we refer to this case as the *binary signals case.* When $\underline{p} < \overline{p}$, we speak of the *continuous signals case.*

We denote $\Omega \equiv \{0, 1\} \times [\underline{p}, \overline{p}]$, the space of possible realizations of (s_i, p_i). Given a particular value of (s_i, p_i), trader i believes that

$$\sigma_i \equiv \text{Prob}[v = 1 | s_i, p_i] = s_i p_i + (1 - s_i)(1 - p_i) \in [0, 1]. \tag{1}$$

[1] The assumption that there are finitely many informed traders, but a continuum of uninformed and noise traders, may appear unnatural to some readers. It is made because it simplifies the analysis considerably. In particular, it allows us to focus on the informed traders impact on prices through their announcements only, and to neglect any effect their trade quantities may have.

Note that the scalar σ_i contains the same information as (s_i, p_i), and it will sometimes be convenient to denote the private information of an informed trader by σ_i instead of (s_i, p_i). Also, $\text{Prob}[s_i|p_i, v = 1] = \sigma_i$ and $\text{Prob}[s_i|p_i, v = 0] = 1 - \sigma_i$. Thus the expected value of the asset conditional on (s, p) is given by

$$\begin{aligned} \theta_F(s,p) \equiv \text{Prob}[v = 1|s,p] &= \frac{\text{Prob}[s|p, v = 1]}{\text{Prob}[s|p, v = 1] + \text{Prob}[s|p, v = 0]} \\ &= \frac{\prod_{i=1}^{N} \sigma_i}{\prod_{i=1}^{N} \sigma_i + \prod_{i=1}^{N} (1 - \sigma_i)} \end{aligned} \tag{2}$$

(recall that $v = 1$ and $v = 0$ are equally likely ex-ante.) We call $\theta_F(s, p)$ the *full-information estimate* of v.

3.3 Communication

Between the first and the second period, the informed traders can make public announcements concerning their private information. Each informed trader i sends a costless message $(m_i, q_i) \in \Omega$, and all informed and uninformed traders observe these messages. Denote by $m = (m_1, \ldots, m_N)$ the vector of all messages concerning the signals, and by $q = (q_1, \ldots, q_N)$ the vector of all messages concerning the precision of the traders' signals.

At the communication stage, informed traders i's information set is the tuple

$$(s_i, p_i, x_i^1, P_1) \in \Omega \times \mathbb{R} \times [0, 1].$$

A pure communication strategy for i is then a function that assigns to each possible information set a message $(m_i, q_i) \in \Omega$. A mixed communication strategy assigns to each information set a probability distribution over possible messages. A communication strategy is said to be *informative* or *fully revealing* if (s_i, p_i) can be inferred with certainty from the report (m_i, q_i) generated by the strategy. A strategy is *direction revealing* if s_i can be inferred with certainty from the report (m_i, q_i) and nothing is learned about p_i. For example, the pure strategy to report

$$(m_i, q_i) = (s_i, p_i) \;\; \forall s_i, p_i, x_i^1, P_1$$

is an informative strategy. Similarly, if i draws q_i uniformly from $[\underline{p}, \overline{p}]$ and then reports

$$(m_i, q_i) = (s_i, q_i) \;\; \forall s_i, p_i, x_i^1, P_1,$$

he follows a direction revealing strategy. From now on and without loss of generality, these strategies are meant when speaking of informative, or direction revealing, strategies.

3.4 Equilibrium

After observing the reports (m, q), the uninformed traders update their belief to a revised estimate, call it $\theta(m, q) \equiv \text{Prob}[v = 1|m, q] \in [0, 1]$. An equilibrium in this model is a collection of communication strategies and a public belief function $\theta : \Omega^N \to [0, 1]$, such that $\theta(m, q)$ is calculated via Bayes' rule using the vector of messages (m, q) and the communication strategies used by the informed traders, and trader i's communication strategy (together with his trade quantities) maximizes i's expected wealth for each $i \in \mathcal{N}$. An *informative* (or *fully revealing*) *equilibrium*, if it exists, is an equilibrium in which every informed trader uses an informative strategy. In an informative equilibrium, the public's post-communication belief $\theta(m, q)$ equals the full-information estimate $\theta_F(s, p)$. A *direction revealing equilibrium* is an equilibrium in which every informed trader uses a direction revealing strategy. Our results will concern precisely these two types of equilibria.

We conclude this section with two remarks concerning our equilibrium definition. First, the equilibrium concept employed is essentially that of sequential equilibrium, but utilizes the fact that the scalar $\theta(m, q)$ is a sufficient statistic for beliefs about each informed trader's individual information. Notice that with the fully revealing and direction revealing strategies described above there are no unsent messages; thus no specification of out-of-equilibrium beliefs is necessary. There are of course other, equivalent, strategies that do not send all possible messages. For example, the report $(m_i, q_i) = (s_i, \overline{p})$ is also direction revealing but only the highest precision value, $\overline{p}$, is reported. In this case, we would have to specify what an uninformed trader believes if, out of equilibrium, some report (m_i, q_i) with $q_i < \overline{p}$ was made. As is common in cheap-talk and signaling games, the game's structure does not induce any constraints on consistent out-of-equilibrium beliefs. Therefore one can always posit beliefs to construct the original equilibrium outcomes that obtain for the case of strategies with no unsent messages. Second, since the informed traders are of measure zero the first-period price P_1 will not be affected by their information and hence be independent of the informed trader's knowledge. However, in general (m, q) can depend on P_1. If this was the case in equilibrium, P_1 would have to be included as an argument in θ as well. Neither in a fully revealing equilibrium nor in a direction revealing equilibrium will this be necessary, however.

4 The Main Result

An informed trader can profit from speculative trading in three ways. He can trade from $t = 1$ to $t = 2$ (*pre-announcement speculation*), from $t = 2$ to the final consummation period (*post-announcement speculation*), or both. Benabou and Laroque (1992) demonstrate that, for the binary signals case, even if informed traders are restricted to post-announcement speculation, an informative equilibrium does not exist for $N = 1$. The results of this paper concerns what happens in the presence of several informed traders, each of whom is free to combine the two types of speculation. We establish two results which state that equilibria with better informational properties can be obtained if the number of informed traders is sufficiently large. The main result is the following:

Proposition 1. *There exists a constant $\overline{\alpha} > 0$ such that for all $0 < \alpha \leq \overline{\alpha}$, the following holds: If N is sufficiently large, a direction-revealing equilibrium exists. In this equilibrium, an informed trader buys the asset in period 1 and reports $m_i = 1$ when he has a high signal ($s_i = 1$), and sells it and reports $m_i = 0$ when he has a low signal ($s_i = 0$). Furthermore, as $N \to \infty$ the post-communication belief $\theta(m, q)$ converges in probability to the full-information estimate $\theta_F(s, p)$. That is, for all $\epsilon > 0$ and $\delta > 0$ there exists N^* such that*

$$Prob[\,\theta_F(s,p) - \epsilon < \theta(m,q) < \theta_F(s,p) + \epsilon\,] \;>\; 1 - \delta \;\;\forall N > N^*.$$

When all informed traders have the same, commonly known signal precision, the signals s_i, $i = 1, \ldots, N$ constitute the only private information held by informed traders. The direction revealing equilibrium of Proposition 1 thus becomes a fully revealing equilibrium. Since the binary information structure is employed in Benabou and Laroque (1992) for the case $N = 1$, we state this observation as a corollary to Proposition 1 above.

Corollary 1. *Consider the binary signals case. There exists a constant $\overline{\alpha} > 0$ such that for all $0 < \alpha \leq \overline{\alpha}$, the following holds: If N is sufficiently large, an informative (i.e. fully revealing) equilibrium exists. In this equilibrium, an informed trader buys the asset in period 1 and reports $m_i = 1$ when he has a high signal ($s_i = 1$), and sells it and reports $m_i = 0$ when he has a low signal ($s_i = 0$). Furthermore, the post-communication belief $\theta(m, q)$ equals the full-information estimate $\theta_F(s, p)$.*

5 Proof of Proposition 1

In this section we prove Proposition 1; Corollary 1 follows immediately from the Proposition and is not proven separately. To establish the existence of a direction revealing equilibrium, our approach is to look at the decision problem of a single informed trader, say i, and postulate direction revealing strategies and appropriate beliefs for everybody else. That is, we assume that every trader $j \neq i$ reports his signal s_i truthfully ($m_i = s_i$), and all uninformed traders believe that every informed trader reports his signal truthfully. Similarly, we assume that every trader $j \neq i$ follows a completely uninformative communication strategy with respect to q_i (for example, they randomize uniformly over possible reports $q_i \in [\underline{p}, \overline{p}]$), and all uninformed traders believe that the reports q_i contain no information about p_i. This hypothesis gives rise to a certain process by which the second-period market price P_2 is formed in response to the traders' messages. It then needs to be checked if—under this price process—the optimal communication strategy for i is in fact direction revealing if N is large enough. Finally, the convergence part of Proposition 1 is proven in the Appendix, as are several intermediate results.

5.1 Price formation

We begin with a preliminary result that characterizes the market prices P_1 and P_2. Note that these prices will not depend on the informed traders' demand quantities, as they are of measure zero. They will, however, depend on the demand by the uninformed and noise traders. The uninformed traders' demand further depends on their beliefs. In period 1, the uninformed traders simply believe that $\text{Prob}[v = 1] = 1/2$. In period 2, they will have updated this belief to, say, $\theta = \theta(m, q) = \text{Prob}[v = 1|m, q]$. The following Lemma characterizes P_1 and P_2 and their dependence on the uninformed traders' beliefs.

Lemma 1. *Let $\theta \in [0, 1]$ denote the publics's belief in period 2. If α is sufficiently small, unique market clearing price $P_1 \in [0, 1]$ and $P_2 \in [0, 1]$ exist. P_1 and P_2 are random variables with the following properties. Given P_1, P_2 is continuous and first-order stochastically increasing in θ. Given a fixed $\alpha > 0$, P_2 is bounded away from 0 and 1. Further, the supports of P_1 and P_2 are continuous in α, and if $\alpha = 0$ then $P_1 = 1/2$ and $P_2 = \theta$.*

Conditional on the public's belief, the market clearing prices are random variables due to noise. We let $M(\cdot|\theta)$ denote the c.d.f. of P_2, conditional on θ and P_1 (the dependence on P_1 has been suppressed in the notation). Thus, given P_1, we have $M(P_2|\theta) \leq M(P_2|\theta')$ if $\theta > \theta'$, with strict inequality for some P_2.

5.2 Belief formation

We now describe the formation of beliefs in detail. Look at period 2 first. Each informed trader has transmitted a message (m_i, q_i), and the informed as well as uninformed traders have observed the vector of messages (m, q). In equilibrium, the uninformed traders also know the communication strategies used by the informed. This allows them to update their beliefs θ accordingly. In a direction revealing equilibrium the public believes that $s = m$ and ignores q, and because all informed traders are ex-ante symmetric, the Bayesian post-communication belief will simply be a function of the number of high reports in m. Let this number be k, and for notational convenience write write $\theta(k)$ instead of $\theta(m, q)$. The public's belief, given a message vector m with k high reports, is then

$$\theta(k) = \text{Prob}[v = 1|k] = \frac{\text{Prob}[k|v = 1]}{\text{Prob}[k|v = 1] + \text{Prob}[k|v = 0]}. \tag{3}$$

Sometimes we will fix a trader i and let k_i be the number of high reports made by traders $j \neq i$. In this case, if $m_i \in \{0, 1\}$ is trader i's report, we use notation $\theta(k_{-i}, m_i)$ to denote the public's post-communication belief $\theta(k_{-i} + m_i)$. Given some scalar $\mu \in [0, 1]$, the binomial probability of k successes in N Bernoulli trials with success rate μ is given by

$$\psi(\mu, N, k) = \binom{N}{k} \mu^k (1 - \mu)^{N-k}. \tag{4}$$

If all traders' signals were known to be of precision μ, then the probability terms in (3) would be of the binomial form given in (4), that is

$$\theta(k) = \frac{\psi(\mu, N, k)}{\psi(\mu, N, k) + \psi(1 - \mu, N, k)}. \tag{5}$$

Unless we have binary signals, however, the public does not know how precise a trader's signal is; one therefore has to take the expectation of (5) with respect to all possible precision values. As the following Lemma establishes, this expectation can be obtained by replacing each trader's signal precision p_i with its expectation $\hat{p}$:

Lemma 2. *In any direction-revealing equilibrium, the public's belief following an announcement with k high reports is given by*

$$\theta(k) = \frac{\psi(\hat{p}, N, k)}{\psi(\hat{p}, N, k) + \psi(1 - \hat{p}, N, k)}. \tag{6}$$

Now turn to period 1. There, uninformed traders believe that $\text{Prob}[v = 1] = 1/2$. Each informed trader knows his own private signal s_i as well as p_i, or equivalently σ_i, and uses this information to update his belief about the other traders' signals. Recall that

k_{-i} denotes the number of high signals by traders $j \neq i$. Then, from the perspective of a trader with private information (s_i, p_i), k_{-i} is distributed according to

$$\begin{aligned} f(k_{-i}|s_i, p_i) &\equiv \text{Prob}[k_{-i}|s_i, p_i] \\ &= \sigma_i \psi(\hat{p}, N-1, k_{-i}) + (1-\sigma_i)\psi(1-\hat{p}, N-1, k_{-i}). \end{aligned} \tag{7}$$

Coupled with his own report m_i, trader i can use these probabilities in period 1 to form his own beliefs about the public's beliefs at t_2. For example, if i plans on reporting a high signal ($m_i = 1$), the probability that the public's post-communication belief is $\theta(k)$ is given by $f(k-1|s_i, p_i)$.

5.3 The informed trader's problem

Because informed traders are risk neutral and quantitatively negligible, we can rule out "mixed portfolios" that contain both a long and a short position of the risky asset. In each period, traders use their entire wealth to either buy the asset, or as collateral for a short sale. We will thus compare the expected values of the two extreme portfolios that can be chosen in period 1; one containing $1/P_1$ shares (a *long* portfolio), the other containing $P_1/(1-P_1)$ units of cash and $-1/(1-P_1)$ shares (a *short* portfolio). The initial position a trader enters in the first period will accordingly be denoted by L (for long) or S (for short).

Consider first period 2. An informed agent has inherited the portfolio $\rho \in \{L, S\}$ he chose in the previous period and now faces the second period price P_2. The market value of his portfolio at price P_2 is the agent's *interim wealth*. Furthermore, i knows m_{-i}, the reports made by the other informed traders, as well as his own signal s_i and his signal's precision p_i. A sufficient statistic for m_{-i} is k_{-i}, the number of high signals reported by the other traders. Trader i revises his expectation of v to

$$\begin{aligned} \theta_i(k_{-i}, s_i, p_i) &= E[v|k_{-i}, s_i, p_i] \\ &= \frac{\sigma_i \psi(\hat{p}, N-1, k_{-i})}{\sigma_i \psi(\hat{p}, N-1, k_{-i}) + (1-\sigma_i)\psi(1-\hat{p}, N-1, k_{-i})}, \end{aligned} \tag{8}$$

where σ_i is defined in (1), and the expected precision $\hat{p}$ was used for the same reason by trader i as it is by the public when forming belief $\theta(k)$. Note that even if $m_i = s_i$ (i.e. i reports his signal truthfully), due to i's private precision information $\theta_i(k_{-i}, s_i, p_i)$ is typically not the the same as $\theta(k)$, the public's belief.

Consider first the case $\rho = L$. The trader's interim wealth is then P_2/P_1. The agent's optimal strategy is now either to hold his inherited long position until the asset value is revealed, or to close the long position and enter a short position containing $-P_2/[P_1(1-P_2)]$ units of the asset. We denote these strategies by H (hold) and R (reverse), respectively. The expected value of i's final wealth is then given by one of the following:

$$\begin{aligned} w^{L,H}(P_2, k_{-i}, s_i, p_i) &= \frac{\theta_i(k_{-i}, s_i, p_i)}{P_1}, \\ w^{L,R}(P_2, k_{-i}, s_i, p_i) &= \frac{P_2}{P_1} \cdot \frac{1-\theta_i(k_{-i}, s_i, p_i)}{1-P_2}. \end{aligned}$$

Similarly, if the trader went short initially, his interim wealth is $(1-P_2)/(1-P_1)$, and the expected value of his final wealth is one of

$$\begin{aligned} w^{S,H}(P_2,k_{-i},s_i,p_i) &= \frac{1-\theta_i(k_{-i},s_i,p_i)}{1-P_1}, \\ w^{S,R}(P_2,k_{-i},s_i,p_i) &= \frac{1-P_2}{1-P_1}\cdot\frac{\theta_i(k_{-i},s_i,p_i)}{P_2}. \end{aligned}$$

For each P_2, k_{-i}, and (s_i,p_i), trader i chooses the second period trade H or R that maximizes his expected final wealth. Thus, assuming optimal behavior, the value of the inherited portfolio ρ in the second period is

$$w^\rho(P_2,k_{-i},s_i,p_i) = \max\left\{ w^{\rho,H}(P_2,k_{-i},s_i,p_i)\,,\, w^{\rho,R}(P_2,k_{-i},s_i,p_i) \right\}. \tag{9}$$

Let us now go back to the first period, in which the informed trader must decide on $\rho \in \{L,S\}$ and $m_i \in \{0,1\}$. Agent i knows his private signal s_i as well as its precision p_i, but the realizations of P_2 and k_{-i} are unknown. He forms expectations over these variables as follows: Given s_i and p_i, k_{-i} is distributed according to $f(\cdot|s_i,p_i)$. Given k_{-i} and m_i, the public's belief in $t=2$ is given by $\theta(k_{-i},m_i)\equiv\theta(k_{-i}+m_i)$. Thus P_2 is distributed according to $M(\cdot|\theta(k_{-i},m_i))$. The expectation of (9) can therefore be written as

$$\begin{aligned} w^\rho(s_i,p_i,m_i) &\equiv E[w^\rho(P_2,k_{-i},s_i,p_i)|s_i,p_i,m_i] \\ &= \sum_{k_{-i}=0}^{N-1} f(k_{-i}|s_i,p_i)h^\rho(k_{-i},s_i,p_i,m_i), \end{aligned} \tag{10}$$

where

$$h^\rho(k_{-i},s_i,p_i,m_i) = \int_0^1 w^\rho(P_2,k_{-i},s_i,p_i)dM(P_2|\theta(k_{-i},m_i)) \tag{11}$$

is the expected value of portfolio ρ, given s_i, p_i, m_i, and k_{-i}.

The trader's problem in the first period is thus to maximize (10) with respect to $\rho \in \{L,S\}$ and $m_i \in \{0,1\}$. To solve this problem, two results need to be established. The first one says that the optimal announcement m_i is determined by the choice of ρ, not by the trader's signal. More precisely, it is optimal for an informed trader to set $m_i=1$ if he has entered a long position, and $m_i=0$ if he has entered a short position at t_1:

Lemma 3. *If all $j\neq i$ use a direction revealing strategy, and the public believes that the reports m_i are truthful, then $w^L(s_i,p_i,1)\geq w^L(s_i,p_i,0)$ and $w^S(s_i,p_i,0)\geq w^S(s_i,p_i,1)$ for all $(s_i,p_i)\in\Omega$.*

Thus, if i sets $m_i=1$ if and only if he has entered a long position in period 1, his message reveals his position to the public. Thus, an informative equilibrium exists if a trader with a high signal buys the asset in period 1, and a trader with a low signal sells it—just like a conventional speculator does.[2]

The second result says that, if there is some positive measure of noise traders, an informed trader's announcement does not affect the value of his portfolio significantly if N is large:

[2]It should be noted that the incentives to reveal one's position to the public are strict when N is

Lemma 4. *Suppose all $j \neq i$ use a direction revealing strategy, and the public believes that the reports m_i are truthful. If $\alpha > 0$, then for all $(s_i, p_i) \in \Omega$ and $\rho \in \{L, S\}$, $|w^\rho(s_i, p_i, 1) - w^\rho(s_i, p_i, 0)| \to 0$ as $N \to \infty$.*

5.4 Existence of a direction revealing equilibrium

We are now prepared to prove the existence part of Proposition 1. Suppose that $s_i = 1$. By Lemma 3, a direction revealing equilibrium exists if $w^L(1, p_i, 1) > w^S(1, p_i, 0)$ and $w^S(0, p_i, 0) > w^L(0, p_i, 1)$. The first inequality will be shown below, and the same argument can be repeated to show the second inequality. This proves that a direction revealing equilibrium exists in which the risky asset is traded according to the informed traders' private signals in the first period. These signals are then reported truthfully between periods 1 and 2 in order to "push" the second-period price P_2 in a direction that supports the initial trades.

To show that $w^L(1, p_i, 1) > w^S(1, p_i, 0)$ if N is large, assume for a moment the limiting case $\alpha = 0$, so that $P_1 = 1/2$ and $P_2 = \theta(k)$. Suppose an informed trader can trade a non-zero quantity at these prices. (Of course, if there is no noise, a zero quantity will be traded at these prices. But $\alpha = 0$ is only an intermediate assumption and will be relaxed in due course.) To derive a lower bound for $w^L(1, p_i, 1)$, note that a trader can always hold the long position and not trade in period 2. In this case, his final wealth will be $2p_i$. Thus, assuming optimal trading behavior in period 2, we have

$$w^L(1, p_i, 1) \geq 2p_i. \tag{12}$$

Consider next the value $w^S(1, p_i, 1)$. We have

$$h^S(k_{-i}, 1, p_i, 1) = \max\left\{2(1 - \theta_i(k_{-i}, 1, p_i)), 2\theta_i(k_{-i}, 1, p_i)\frac{1 - \theta(k_{-i}, 1)}{\theta(k_{-i}, 1)}\right\}. \tag{13}$$

We now derive an upper bound for $h^S(k_{-i}, 1, p_i, 1)$. After some algebraic manipulations and using (5) and (8), we can write (13) as

$$h^S(k_{-i}, 1, p_i, 1) = 2\frac{\psi(1 - \hat{p}, N - 1, k_{-i})}{\sigma_i \psi(\hat{p}, N - 1, k_{-i}) + (1 - \sigma_i)\psi(1 - \hat{p}, N - 1, k_{-i})} \max\left\{1 - \sigma_i, \frac{1 - \hat{p}}{\hat{p}}\sigma_i\right\}.$$

Therefore, to compute an upper bound for

$$w^S(1, p_i, 1) = \sum_{k_{-i}=0}^{N-1} f(k_{-i}|1, p_i) h^S(k_{-i}, 1, p_i, 1)$$

large. To see this, consider an informed trader with private signal $s_i = 1$, and suppose $m_i = 1$. If N is sufficiently large, it is possible that even for large p_i we have $\theta_i(k_{-i}, 1, p_i) < 1/2$. In this case and if $\epsilon^2 > 0$, the second-period price satisfies $P_2 > \theta_i(k_{-i}, 1, p_i)$, and i reverts his long position. Thus, with positive probability i benefits strictly from sending message $m_i = 1$, as it increases P_2 and thus his interim wealth to be used to fund the short sale. If $P_2 \leq \theta_i(k_{-i}, 1, p_i)$, i simply holds his long position in period 2 and is thus indifferent between the messages he can send. Thus, on expectation i strictly prefers to reveal his long position to the public.

we have to consider two cases. First, suppose $1-\sigma_i \geq \frac{1-\hat{p}}{\hat{p}}\sigma_i$. Then

$$w^S(1,p_i,1) = \sum_{k_{-i}=0}^{N-1} 2(1-\sigma_i)\psi(1-\hat{p},N-1,k_{-i}) = 2(1-\sigma_i) = 2(1-p_i). \quad (14)$$

Next, suppose $1-\sigma_i < \frac{1-\hat{p}}{\hat{p}}\sigma_i$. Then

$$w^S(1,p_i,1) = \sum_{k_{-i}=0}^{N-1} 2\frac{1-\hat{p}}{\hat{p}}\sigma_i\psi(1-\hat{p},N-1,k_{-i}) = 2\frac{1-\hat{p}}{\hat{p}}\sigma_i = 2\frac{1-\hat{p}}{\hat{p}}p_i. \quad (15)$$

Since $\underline{p} > \frac{1}{2}$, for each realization of $p_i \in [\underline{p},\overline{p}]$ (12) exceeds (14) and (15) by a constant C that is bounded away from zero:

$$\begin{aligned} w^L(1,p_i,1) - w^S(1,p_i,1) &\geq 2p_i - \max\{2(1-p_i), 2\frac{1-\hat{p}}{\hat{p}}p_i\} \\ &\geq C \equiv \min\{2(2\underline{p}-1), 2\underline{p}(1-\frac{1-\hat{p}}{\hat{p}})\} > 0. \end{aligned}$$

C is further independent of N. Now increase the level of noise slightly to some $\alpha > 0$. By continuity of prices in α, $w^S(1,p_i,1) - w^L(1,p_i,1)$ is still bounded away from zero by a constant that is independent of N. If N is now chosen large enough, Lemma 4 implies $w^L(1,p_i,1) > w^S(1,p_i,0) \approx w^S(1,p_i,1)$, as needed to be shown.

6 Discussion

For a direction revealing equilibrium to exist, a sufficiently large number of informed traders is needed. The effect of increasing N in the model is that each agent becomes informationally small, reducing the impact of any single trader's announcement on beliefs and prices: If one is informed of $N-1$ signals, and N is large, v can be predicted with high accuracy. This implies that between periods 1 and 2, the asset prices moves from $P_1 \approx 1/2$ to $P_2 \approx 0$ or $P_2 \approx 1$, and an additional N^{th} signal would have only little impact on this price movement. Our result hence relies on the interplay between an agent's informational advantage due to his private signal in period 1, and his "informational smallness" at the communication stage, formally stated in Lemma 4.

We conclude with a brief discussion of several further aspects of the model and our results. These concern the existence of other equilibria, alternative assumptions on the timing in our model, and alternative specifications of the information structure.

Other equilibria. It is easy to see that for continuous signals no fully revealing equilibria can exist. If they did, then $\theta(m,q) = \theta_F(s,p)$. Notice that the full-information estimate in (2) increases in each σ_i, and P_2 increases in the public's belief. Hence a trader who bought the asset in period 1 and is hence interested in seeing as high a second-period price as possible can push P_2 up by reporting the highest possible signal precision ($q_i = \overline{p}$). He will do so always, and thus no report concerning p_i can be credible. Thus, the direction revealing equilibrium is the most informative equilibrium in the continuous signals case.[3] The model

[3]Of course, there exist 2^N-1 equivalent equilibria that can be obtained by permuting the message sets.

possesses many other (less informative) equilibria as well: A babbling equilibrium, in which no information is revealed, always exists of course. It is also possible for asymmetric equilibria to exist in which some informed traders use direction revealing strategies and others use uninformative strategies, or to find symmetric equilibria that are direction revealing after some realizations of P_1 and uninformative after others.

Timing of announcements and trades. Interestingly, the direction revealing equilibrium, if it exists, is robust with respect to extensions of the model in which traders can choose the timing of their trades and their announcements. Consider, for example, a model with $T > 2$ periods, where after period T the value v is revealed, the asset can be traded in any of the T periods, and communication takes place between the trading periods. One can show that a sequential equilibrium exists in which the vector of signals s is reported truthfully at the earliest possible date (i.e. between periods 1 and 2) and the market price from period 2 onward contains that information. This equilibrium, however, requires that the informed traders remain quantitatively negligible and act as a price taker. If the quantities traded by the informed participants exerted price pressure, then these traders would likely split their trades into smaller blocks and trade them over time. The incentives to reveal information immediately after the first trading round would then be weakened. However, it is not obvious what the equilibria of such a model might be.

Alternative information structures. Finally, note that even though we have assumed continuous signals, the set of possible valuations of the risky asset was still binary, i.e. $v \in \{0, 1\}$. Alternatively, one can depart from the binary case by assuming that $v \in \{v_1, \ldots, v_L\} \subset [0, 1]$ with ex-ante expected value $E[v] = 1/2$. Assume that each informed trader draws a signal $s_i \in V$ such that $\text{Prob}[s_i = v] = p > 1/L$ and $\text{Prob}[s_i = v'] = (1-p)/(L-1)\ \forall v' \neq v$. In this case, the precision of each trader's signal is p and commonly known, but signals are not binary because the asset value is not binary. With this information structure and if N is large enough, the model has an equilibrium that is direction revealing in the sense that an agent's report still communicates the direction of his initial trade (i.e. long vs. short). However, the information aggregation properties of this equilibrium are much weaker than in the model of Section 3. To see this, note that the direction of an informed agent's trade is informative only as to whether $E[v|s_i] > 1/2$ or not. Observing the direction of the agents' initial trades is thus informative only in so far as $v < / > \frac{1}{2}$ can be learned with arbitrarily high probability, but not more. There will hence be some residual uncertainty in the post-communication belief. This residual uncertainty concerns the precise value of the asset and does not vanish as $N \to \infty$.

Appendix

Proof of Lemma 1

The market clearing price P_2.

Suppose an uninformed enters period 2 with wealth $w > 0$ and belief $\theta \in [0, 1]$. Let y^2 denote the demand quantity of an uninformed trader. The trader's final wealth will be $\overline{w} = w + (1 - P_2)y^2$ if $v = 1$, and $\underline{w} = w - P_2 y^2$ if $v = 0$. The expected utility in period 2 can thus be written as

$$v(y^2, P_2, \theta) = \theta u(\overline{w}) + (1-\theta)u(\underline{w}),$$

which is maximized by choice of a demand $y^2 \in [-\frac{w}{1-P_2}, \frac{w}{P_2}]$. Since u is continuous and strictly concave, there is a unique maximizer, which we also denote by y^2, and which in continuous in P_2.

We will now show that y^2 strictly decreases in P_2 and (weakly) increases in θ. In case of a corner solution, either $y^2 = -\frac{w}{1-P_2}$ or $y^2 = \frac{w}{P_2}$, all of which are strictly decreasing P_2 and independent of θ. Now focus on the case of an interior maximum. The first order condition can be stated as

$$\frac{u'(\overline{w})}{u'(\underline{w})} = \frac{1-\theta}{\theta}\frac{P_2}{1-P_2}.$$

Thus y^2 has the sign of $\theta - P_2$. By the implicit function theorem, y^2 is continuously differentiable in both P_2 and θ. To see how y^2 varies with P_2, note first that $\partial y^2/\partial P_2$ has the sign of

$$\begin{aligned}\frac{\partial^2 v(y^2, P_2, \theta)}{\partial y^2 \partial P_2} &= -\theta u'(\overline{w}) - (1-\theta)u'(\underline{w}) \\ &\quad + y^2[-\theta(1-P_2)u''(\overline{w}) + (1-\theta)P_2 u''(\underline{w})]. \qquad (16)\end{aligned}$$

The first two terms in (16) are strictly negative. To show that the third term is non-positive, assume first that $P_2 \leq \theta$ (and therefore $y^2 \geq 0$) and write the expression in brackets as

$$-\theta(1-P_2)u'(\overline{w})\frac{u''(\overline{w})}{u'(\overline{w})} + (1-\theta)P_2 u'(\underline{w})\frac{u''(\underline{w})}{u'(\underline{w})}.$$

Because u has non-increasing absolute risk aversion, this cannot be greater than

$$\begin{aligned}&-\theta(1-P_2)u'(\overline{w})\frac{u''(\underline{w})}{u'(\underline{w})} + (1-\theta)P_2 u'(\underline{w})\frac{u''(\underline{w})}{u'(\underline{w})} \\ &= \frac{u''(\underline{w})}{u'(\underline{w})}[-\theta(1-P_2)u'(\overline{w}) + (1-\theta)P_2 u'(\underline{w})] = 0\end{aligned}$$

by the first-order condition. An analogous argument applies for $P_2 \leq \theta$ and $y^2 \leq 0$, so that y^2 strictly decreases in P_2. Similarly, $\partial y^2/\partial\theta$ has the sign of

$$\frac{\partial^2 v(y^2, P_2, \theta)}{\partial y^2 \partial \theta} = (1-P_2)u'(\overline{w}) + P_2 u'(\underline{w}) > 0,$$

so that y^2 increases in θ.

Let $y^2(P_2, \theta)$ be the uninformed traders' demand as a function of P_2 and θ. The demand by noise traders in period 2 is $\alpha z(P_2, \epsilon^2)$. Since there is a zero measure of informed traders, market clearing requires that

$$\xi^2(P_2, \theta, \epsilon^2) \equiv y^2(\theta, P_2) + \alpha z(P_2, \epsilon^2) = 0. \qquad (17)$$

Note that the excess demand function in (17) is continuous in α, continuous and strictly increasing in θ, and continuous and strictly decreasing in P_2. Our assumption that

$z(1,\tau) < 0$ and $z(0,-\tau) > 0$ guarantees that for every $\epsilon^2 \in [-\tau,\tau]$ and every $\theta \in [0,1]$ there is a unique $P_2^* \in [0,1]$ such that $\xi^2(P_2^*,\theta,\epsilon^2) = 0$. By continuity of ξ^2 in θ, P_2^* is continuous in θ. Thus, by continuity of (17) in α, the support of P_2^* is continuous in α and if $\alpha = 0$ then $P_2^* = \theta$. An upper bound for P_2 is given by the implicit relationship

$$\xi^2(P_2,1,\tau) = y^2(1,P_2) + \alpha z(P_2,\tau) = 0 \tag{18}$$

(i.e. both ϵ^2 and θ are at their highest possible values). Since z^2 and y^2 are continuous and strictly decreasing in P_2, $y^2(1,1) = 0$, and $z(1,\tau) < 0$, the value P_2 which solves (18) must be strictly less than one. The same argument can be made to find a lower bound on P_2 which exceeds zero. Finally, ϵ^2 and θ independent and ξ^2 strictly increasing in θ implies P_2^* is increasing in θ in the sense of first-order stochastic dominance.

The market clearing price P_1.

In period 1, an uninformed trader maximizes with respect to his demand $y^1 \in [-\frac{1}{1-P_1},\frac{1}{P_1}]$ the expectation of the indirect utility which he receives in period 2 at wealth level $1 + y^1(P_2 - P_1)$. This expectation is taken jointly over θ and P_2, and is continuous in α by the previous step; hence the uninformed traders' demand $y^1(P_1)$ is continuous in α. The noise traders' demand in period 1 is $\alpha z(P_1,\epsilon^1)$. Thus market clearing in period 1 requires

$$\xi^1(P_1,\epsilon^1) \equiv y^1(P_1) + \alpha z(P_1,\epsilon^1) = 0. \tag{19}$$

Now consider the case $\alpha = 0$. Since the quantity traded in period 2 must then be then zero, the uninformed trader's problem in period 1 is the same as the problem analyzed above, with $\theta = 1/2$, $w = 1$, and $P_2 = P_1$. Thus, $\xi^1(P_1,\epsilon^1)$ slopes downward strictly and is zero at $P_1^* = \frac{1}{2}$. By continuity, then, the same holds for small enough but positive α. Thus a unique market clearing price P_1^* exists, has support that is continuous in α, and if $\alpha = 0$ then $P_1^* = 1/2$. □

Proof of Lemma 2

In general, post-communication beliefs are given by the expression in (5). To prove the Lemma we must therefore show that

$$\text{Prob}[k|v=1] = \psi(\hat{p},N,k) \quad \text{and} \quad \text{Prob}[k|v=0] = \psi(1-\hat{p},N,k).$$

Given k and N, define $m_k^N = (1,\ldots,1,0,\ldots,0)$; that is the first k entries are equal to one and the remaining $N-k$ entries are equal to zero. Given this vector, let $m_k^N(1)$ denote its first entry. Assume that each one of the entries in m is an independent Bernoulli trial with success rate p_i, and p_i itself is a random variable with expected value $\hat{p}$. Let $P(N,k)$ be the probability of realization m_k^N. Since there are $\binom{N}{k}$ vectors of length N with k one-entries and $N-k$ zero-entries, all of which must have the same probability as m_k^N, we have $\text{Prob}[k|v=1] = \binom{N}{k}P(N,k)$. The probability $P(N,k)$ can be written as

$$P(N,k) = \text{Prob}[m_k^N(1) = 1]P(N-1,k-1) = \hat{p}P(N-1,k-1).$$

$P(N-1,k-1)$ itself can be written as $\hat{p}P(N-2,k-2)$, and continuing in this fashion we have

$$P(N,k) = \hat{p}P(N-1,k-1) = \hat{p}^2 P(N-2,k-2) = \ldots = \hat{p}^k P(N-k,0).$$

Similarly, the probability $P(N-k,0)$ can be written as

$$P(N-k,0) = \text{Prob}[m_0^{N-k}(1)=0]P(N-k-1,0) = (1-\hat{p})P(N-k-1,0) = \ldots = (1-\hat{p})^{N-k},$$

and therefore

$$\text{Prob}[k|v=1] = \binom{N}{k} P(N,k) = \binom{N}{k} \hat{p}^k (1-\hat{p})^{N-k} = \psi(\hat{p},N,k).$$

□

Proof of Lemma 3

The argument is made for the case $\rho = L$. For this case, rewrite the term inside the integral in (11) as

$$g(P_2,k_{-i},s_i,p_i) \equiv \max\left\{ \tfrac{\theta_i(k_{-i},s_i,p_i)}{P_1}, \tfrac{P_2}{P_1}\tfrac{1-\theta_i(k_{-i},s_i,p_i)}{1-P_2} \right\}. \tag{20}$$

The second term on the right-hand side of (20) is strictly increasing in P_2, while the first term is independent of P_2. Thus, $g(P_2,k,\sigma_i)$ is increasing in P_2. Next, note that $\theta(k+1) > \theta(k)$ for all $0 \leq k < N-1$. Therefore $M(P_2|\theta(k+1)) \leq M(P_2|\theta(k))$ by Lemma 1, and we have

$$\begin{aligned} h^L(k_{-i},s_i,p_i,1) &= \int_0^1 g(P_2,k_{-i},s_i,p_i)dM(P_2|\theta(k+1)) \\ &\geq \int_0^1 g(P_2,k_{-i},s_i,p_i)dM(P_2|\theta(k)) = h^L(k_{-i},s_i,p_i,0) \end{aligned} \tag{21}$$

for all k. Hence,

$$\sum_{k_{-i}=0}^{N-1} f(k_{-i}|s_i,p_i)h^L(k_{-i},s_i,p_i,1) \geq \sum_{k_{-i}=0}^{N-1} f(k_{-i}|s_i,p_i)h^L(k_{-i},s_i,p_i,0),$$

so that $m_i = 1$ is optimal if $\rho = L$. A similar argument can be made to show that $m_i = 0$ is optimal if $\rho = S$. □

Proof of Lemma 4

Fix $\mu \in (1/2,1)$ and $\phi \in (0,1)$. Define

$$f(k) = \phi\psi(\mu,N,k) + (1-\phi)\psi(1-\mu,N,k)$$

and

$$\theta(k) = \frac{\phi\psi(\mu,N,k)}{\phi\psi(\mu,N,k) + (1-\phi)\psi(1-\mu,N,k)},$$

and let $h : [0,1] \to \mathbb{R}$ be some bounded and continuous function. We first show that

$$\sum_{k=0}^{N-1} f(k)\left[h(\theta(k+1)) - h(N, \theta(k))\right] \quad \to \quad 0$$

as $N \to \infty$. To see this, rewrite $\theta(k)$ as

$$\theta(k) = \left[1 + \frac{1-\phi}{\phi}\left(\frac{\mu}{1-\mu}\right)^{N-2k}\right]^{-1}.$$

One can see that $\theta(k)$ goes to 0 (1) as $N-2k$ goes to $+\infty$ ($-\infty$). Thus, $\theta(k+1)-\theta(k) \to 0$ as $|N-2k| \to \infty$. Since h is continuous, $h(\theta(k+1))-h(\theta(k)) \to 0$ as $|N-2k| \to \infty$. Since f is a weighted sum of two binomial probability distributions with success rates $\mu > 1/2$ and $1-\mu < 1/2$, $\sum_{|N-2k|<d} f(k) \to 0$ as $N \to \infty$ for every $d > 0$. Together with the fact that $\sum_{k=0}^{N-1} f(k) \to 1$, this implies that

$$\sum_{k=0}^{N-1} f(k)\left(h(\theta(k+1)) - h(\theta(k))\right) \quad \to \quad 0$$

as $N \to \infty$.

To complete the proof of the Lemma, note that the term inside the integral in (11) is a continuous and bounded function of P_2 (as P_2 is bounded by Lemma 1). Since $M(\cdot|\theta)$ is continuous in θ by Lemma 1, (11) implies that $h^\rho(k_{-i}, s_i, p_i, m_i)$ is a bounded and continuous function of $\theta(k_{-i} + m_i)$. Therefore

$$w^\rho(s_i, p_i, 1) - w^\rho(s_i, p_i, 0) = \sum_{k_{-i}=0}^{N-1} f(k_{-i}|s_i, p_i)\left[h^\rho(k_{-i}, s_i, p_i, 1) - h^\rho(k_{-i}, s_i, p_i, 0)\right] \quad \to \quad 0$$

as $N \to \infty$. □

Proof of the convergence result in Proposition 1

Note that we can express $\theta(k)$ as

$$\theta(k) = E[\theta_F(s,p)|k(s) = k],$$

and taking expectations with respect to k we obtain

$$E[\theta(k)] = E[E[\theta_F(s,p)|k(s) = k]] = E[\theta_F(s,p)].$$

Now suppose $v = 1$. By Lemma 2 $\text{Prob}[k|v=1] = \psi(\hat{p}, N, k)$; thus we have

$$E[\theta(k)|v = 1] = \sum_{k=0}^{N} \text{Prob}[k|v=1]\theta(k) = \sum_{k=0}^{N} \psi(\hat{p}, N, k)\theta(k).$$

Since $\hat{p} > 1/2$, $\theta(k) \to 1$ as $2k - N \to \infty$ (as shown in the proof of Lemma 4). Furthermore, $\sum_{2k-N<d} \psi(\hat{p}, N, k) \to 0$ as $N \to \infty$ for every $d > 0$. Taken together, these two facts imply that

$$E[\theta(k)|v = 1] = E[\theta_F(s, p)|v = 1] = \sum_{k=0}^{N} \psi(\hat{p}, N, k)\theta(k) \to 1 \tag{22}$$

as $N \to \infty$. Because $\theta_F(s, p) \leq 1$ and $\theta(k) \leq 1$, (22) can only hold if $\theta(k)$ converges in probability to $\theta_F(s, p)$, conditional on $v = 1$. For $v = 0$, one can similarly show that

$$E[\theta(k)|v = 0] = E[\theta_F(s, p)|v = 0] = \sum_{k=0}^{N} \psi(1 - \hat{p}, N, k)\theta(k) \to 0, \tag{23}$$

and since $\theta_F(s, p) \geq 0$ and $\theta(k) \geq 0$, (23) implies that $\theta(k)$ converges in probability to $\theta_F(s, p)$, conditional on $v = 0$. The two cases are exhaustive and the result follows. □

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