Early round upsets and championship blowouts*

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Abstract

In equilibrium play of a two-round tournament we find that underdogs exert more effort in the opening round while favorites save more effort for the final. Ability differences between players are therefore compressed in the opening round so upsets are more likely, and amplified in the final so blowouts are more likely. Measures that reduce the need to strategically allocate effort across games make for a more exciting final but a less exciting opening round. Consistent with the model, introduction of a one-day rest period between regional semi-final and final matches in the NCAA men's basketball tournament was found to increase the favorite's victory margin in the semi-finals by about five points. Non-sports applications of the model include the allocation of resources across primaries and general elections by candidates and the allocation of resources across a career ladder by managers.

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1 Introduction

Popular discussions of tournament play often celebrate the success of underdogs in early rounds and decry the prevalence of unexciting finals. Such a pattern might be just a statistical illusion. There are many early round matches and some upsets are inevitable. And there is only one final so it is unlikely to be the most exciting match of the tournament. In this paper we consider instead whether the pattern might have a real foundation in the strategic allocation of effort by tournament participants.

We analyze a two-round tournament in which a favorite and underdog play in each of the two semi-finals and then one player from each match advances to the final. Victory in a match is probabilistic in that the chance of winning is increasing in each player's relative quality and in his relative effort expenditure.[1] Each player has a fixed amount of effort to exert over the two rounds, unused effort is of no value, and the only payoff is from winning the tournament. To maximize the chance of winning, each player must balance out the benefits of expending more effort in the semi-final against the opportunity cost of having less effort available for the final.

Favorites and underdogs both have an incentive to conserve resources for the final, but the trade-offs they each face are different. A favorite plays a weak opponent in the semi-final, but is likely to face a tough opponent in the final, so it has a strong incentive to hold back. Conversely, an underdog already plays a tough opponent in the semi-final, so it has less incentive to conserve effort for the final.[2] Because of these different incentives, we find that in any symmetric Nash equilibrium underdogs exert more effort than favorites in the semi-final. The extra effort does not fully compensate for lower ability and underdogs are still likely to lose, but the chance of an upset rises. In the final round each player expends all its remaining resources so differences in abilities are no longer compressed by strategic considerations. Instead, an underdog who makes it to the final has fewer resources left to spend than the favorite, so differences in abilities are amplified and the chance of a blowout by the favorite rises.

If a favored player loses to an underdog in the early rounds it is often accused of "looking past" the underdog to its next match. Even if a player's strategy is optimal ex ante, it might turn out to be unsuccessful ex post, so such criticism is often unfair. Our results imply that it is rational for the favorite to hold back on resources in the semi-final even if it correctly anticipates that in equilibrium the underdog will be playing harder. Sometimes the strategy will backfire, but on average the favorite benefits from being in a better position for the final.

These results are derived using a standard contest model in which each player's probability of victory in a match is a strictly increasing function of his ability and effort.[3] Such models were first developed to analyze

[1]Being a better player can be interpreted to mean that the player uses resources more efficiently, or that the player has a larger resource endowment. Both interpretations are equivalent in our model, so we use the first one without loss of generality.

[2]Our model assumes that there is no "runner-up" prize for reaching the final but losing. Since the underdog has less chance of winning the final, such a prize gives the underdog even more incentive to spend resources on the opening round at the expense of being more likely to lose the final. For simplicity we do not include this effect in the formal model.

[3]A related literature, which has also been applied to a wide range of situations, considers all-pay auctions in which the highest bidder wins for certain (Baye, Kovenock, and de Vries, 1996).

rent-seeking (Tullock, 1967, 1980) and similar models are used in a wide variety of areas including patent races (Loury, 1979), election campaigns (Snyder, 1989), compensation schemes (Nalebuff and Stiglitz, 1983), career ladders (Lazear and Rosen, 1981), lobbying (Baye, Kovenock, and de Vries, 1993), and sports contests (Szymanski, 2003).[4] More specifically, we model the contest as a sequential elimination ladder tournament in which players first compete in separate groups and the winners then compete against each other (Rosen, 1986).

Our result that underdogs do indeed "try harder" than favorites in the semi-finals contrasts with that of Rosen (1986).[5] Because the favorite has a better chance of victory in the final, Rosen finds that winning a semi-final match is more valuable to the favorite so the favorite tries harder than the underdog. We reach the opposite conclusion because we assume that there is a fixed supply of effort to be used across rounds and unused effort in the tournament has no value. Since players lose nothing from trying hard other than having less effort for the final, and since favorites have more incentive to save effort for the final, underdogs try harder in the semi-finals. While not directly comparable, our results also differ in flavor from results on relative effort levels in single-round tournaments. For instance, Baik (2004) finds that favorites and underdogs exert the same effort, while Dixit (1987) finds that underdogs have an incentive to precommit to less effort and that favorites have an incentive to precommit to more effort.

The effect of asymmetric abilities in sequential elimination ladder tournaments is also considered by Groh, Moldovanu, Sela, and Sunde (2003) in an analysis of optimal seedings. The primary difference with our model is we assume there is a resource constraint for total effort in the tournament. The effect of such resource constraints is analyzed by Stein and Rapoport (2003). Their model differs in that they consider the case where players have identical abilities and they concentrate on tournament design issues. Amegashie (2004) also considers a tournament with budget constraints and asymmetric abilities, but rather than having a ladder structure the players all compete against each other in a single opening round match and then the best performers are selected to compete against each other in the final round.

If players have separate budgets for each round then the strategic factors we examine disappear. That is, all players will "give 100%" in each round so the probability of an upset in the opening round will not be increased by the favorite withholding effort. We exploit this implication to test our model empirically by looking at data from the NCAA men's basketball tournament. Before 1969, the regional semi-finals and regional finals were played back-to-back on two consecutive days so teams had little time to recover from the semi-final or prepare for the final. In 1969, the NCAA introduced a rest day between the matches, thereby reducing the need to allocate effort and preparation time strategically. Since favorites were more free to "play one game at a time" and focus on defeating their semi-final opponents, our model predicts that semi-final upsets were less likely starting in 1969 than in previous years. Consistent with the model's prediction we find that after introduction of the rest day the number of upsets fell and that the average victory margin for the favored team increased by about five points despite a long-term trend toward greater parity.

[4]For a survey see Nitzan (1994). Baye and Hoppe (2002) show the formal connection between many of these models.

[5]Rosen considers this question briefly in Section IV. The bulk of his paper examines other issues.

In addition to sports tournaments, our results apply to other situations with similar tournament structures, including multi-round hiring decisions, multi-division promotion ladders, and election campaigns with a primary and general election. For instance, if the favored candidate in a primary election conserves resources for the general election, the underdog candidate is more likely to win, but on average the favored candidate is still wise to hold back. Rules that restrict the amount of resources spent on the primaries are typically thought to favor underdogs because favorites are likely to have more resources. Our analysis suggests that, when resources endowments are the same, strategic allocation of resources across the primary and general elections helps weaker candidates, so capping resources in the primary can instead help the stronger candidate.[6]

2 The Model

Four players compete in a tournament with a semi-final and final round. Each player is endowed with 1 unit of effort to allocate across the two rounds. In each of two semi-finals two players compete by simultaneously choosing effort levels which influence the probability of winning the match.[7] The two semi-final winners advance to the final and compete for a prize of normalized value 1 by expending their remaining effort resources. There is no runner-up prize for a player who advances to the final but loses. The only cost of effort is the opportunity cost of having less effort available for the other round, and effort saved by a player who does not advance to the final has no value.[8]

In each given match (semi-final or final), the probability with which a player wins that match depends on his own effort level in the match, the effort level of the opposing player, and his relative ability compared to the opposing player. In particular, if a player expends effort $x \in [0, 1]$, the opposing player expends effort level $y \in [0, 1]$, and the relative ability of the first player is measured by $r > 0$, then the first player wins with probability

$$f(r, x, y) = \begin{cases} \frac{rx}{rx+y} & \text{if } x > 0 \text{ or } y > 0, \\ \frac{r}{r+1} & \text{if } x = y = 0. \end{cases} \tag{1}$$

This is an asymmetric version of the familiar Tullock success function.[9] Note that the marginal return to

[6]Resources can be though of quite generally. For instance, "going negative" is a costly strategy for the candidate making criticisms in that it loses voter goodwill. Underdogs might then be more inclined than favorites to use negative advertising in the primary.

[7]In sports contests the simultaneity assumption is most appropriate regarding the allocation of preparation time by players and coaches before the match. Regarding physical effort in the match itself, it can be adjusted based on circumstances. For instance, the coach can give more playing time to the best players if the team is behind. But the impact on the outcome is still probabilistic so the coach cannot be sure of how much playing time is appropriate, and the basic intuition of the model is unaffected.

[8]We therefore implicitly assume an extreme version of convex effort costs in which effort has zero cost up to the first unit, and infinite cost thereafter. With such a cost function it is clearly optimal to use the one unit of resources, but not more, so the model can be reduced to the allocation problem described in the text.

[9]This function is popular because of its simplicity and the fact that it typically yields pure strategy equilibria. When the

effort (as measured by the probability of victory) in a given match is positive and diminishing,

$$\begin{aligned}\frac{\partial}{\partial x}f(r,x,y) &= \frac{ry}{(rx+y)^2} > 0,\\ \frac{\partial^2}{(\partial x)^2}f(r,x,y) &= \frac{-2r^2y}{(rx+y)^3} < 0.\end{aligned} \tag{2}$$

We assume that there are only two types of players, "strong" favorites and "weak" underdogs. The difference between these types is that a favorite's ability is higher than that of an underdog. Specifically, we measure this ability difference by the parameter $g > 1$. If a favorite is matched against an underdog the favorite's relative ability is given by $r = g > 1$ and the underdog's relative ability by $r = 1/g < 1$. If either a favorite is matched against another favorite or an underdog against another underdog, the relative ability of either player is $r = 1$. We assume the semi-final pairings consist of one favorite and one underdog.

2.1 The players' allocation decisions

Let s_1 be the effort of the (strong) favorite in the first semi-final and w_1 be the effort of the (weak) underdog in that semi-final. The probability that the favorite wins the first semi-final is then $f(g, s_1, w_1)$ and the probability that the underdog wins is $f(1/g, w_1, s_1)$. Similarly in the second semi-final let s_2 be the effort of the favorite and w_2 be the effort of the underdog, so the probability that the favorite wins the second semi-final is $f(g, s_2, w_2)$ and the probability that the underdog wins is $f(1/g, w_2, s_2)$.

Since each player's total effort budget is 1, and unused effort has no value, if a player makes it to the final it will expend effort equal to 1 minus its effort in the semi-final. Consider the favorite in the first semi-final. It has a $f(g, s_1, w_1)$ chance of making it to the final. If it makes it to the final the chance of facing an equally strong opponent is $f(g, s_2, w_2)$, in which case the chance of victory is $f(1, 1-s_1, 1-s_2)$, and the chance of facing a weak opponent is $f(1/g, w_2, s_2)$, in which case the chance of victory is $f(g, 1-s_1, 1-w_2)$. Therefore for $i, j = 1, 2$ and $i \neq j$ the probability that favorite in the ith semi-final wins the tournament is

$$\pi_i^S(s_i, w_i, s_j, w_j) = \frac{gs_i}{gs_i+w_i}\left[\frac{gs_j}{gs_j+w_j}\frac{(1-s_i)}{(1-s_i)+(1-s_j)} + \frac{w_j}{gs_j+w_j}\frac{g\,(1-s_i)}{g\,(1-s_i)+(1-w_j)}\right] \tag{3}$$

and, similarly, the probability that the underdog in the ith semi-final wins the tournament is

$$\pi_i^W(s_i, w_i, s_j, w_j) = \frac{w_i}{gs_i+w_i}\left[\frac{gs_j}{gs_j+w_j}\frac{(1-w_i)}{g\,(1-s_j)+(1-w_i)} + \frac{w_j}{gs_j+w_j}\frac{(1-w_i)}{(1-w_i)+(1-w_j)}\right]. \tag{4}$$

Since there are no effort costs and since the prize is 1 from victory in the final and 0 from anything less, the expected payoff to each player is just the probability of winning the tournament.

probability of victory is very sensitive to relative effort only mixed strategy equilibria may exist (Baye, Kovenock, and de Vries, 1994).

2.2 Equilibrium

In equilibrium each player maximizes the probability of winning the tournament taking the allocation decisions by the other players as given, including the players in the other semi-final. As seen from the probability functions (3) and (4), each player faces a trade-off. Higher effort in the semi-final increases the chance of making it to the final, but at the opportunity cost of a lower chance of succeeding in the final. Because $f(g,x,y)$ is concave in x, there are diminishing returns from effort in both rounds. Therefore each player's probability of winning the tournament is a concave function of effort by the player: it is initially increasing when the marginal return to effort in the semi-final is high and the marginal cost of having less effort for the final is low, and then decreasing when the marginal return to effort in the semi-final is low and the marginal cost of having less effort for the final is high.

For $i,j=1,2$ and $i \neq j$ the first-order condition for maximization of (3) is

$$\begin{aligned} \frac{\partial \pi_i^S}{\partial s_i} \equiv & \frac{g w_i}{(g s_i + w_i)^2}\left[\frac{g s_j}{g s_j + w_j}\frac{(1-s_i)}{(1-s_i)+(1-s_j)} + \frac{w_j}{g s_j + w_j}\frac{g(1-s_i)}{g(1-s_i)+(1-w_j)}\right] - \\ & \frac{g s_i}{g s_i + w_i}\left[\frac{g s_j}{g s_j + w_j}\frac{(1-s_j)}{((1-s_i)+(1-s_j))^2} + \frac{w_j}{g s_j + w_j}\frac{g(1-w_j)}{(g(1-s_i)+(1-w_j))^2}\right] = 0, \end{aligned} \tag{5}$$

and similarly the first-order condition for maximization of (4) is

$$\begin{aligned} \frac{\partial \pi_i^S}{\partial w_i} \equiv & \frac{g s_i}{(g s_i + w_i)^2}\left[\frac{g s_j}{g s_j + w_j}\frac{(1-w_i)}{g(1-s_j)+(1-w_i)} + \frac{w_j}{g s_j + w_j}\frac{(1-w_i)}{(1-w_i)+(1-w_j)}\right] - \\ & \frac{w_i}{g s_i + w_i}\left[\frac{g s_j}{g s_j + w_j}\frac{g(1-s_j)}{(g(1-s_j)+(1-w_i))^2} + \frac{w_j}{g s_j + w_j}\frac{(1-w_j)}{((1-w_i)+(1-w_j))^2}\right] = 0. \end{aligned} \tag{6}$$

Since (3) and (4) are concave functions in their respective decision variables, the first-order conditions are also sufficient for a maximum. Throughout the rest of the paper, we only consider symmetric Nash equilibria, meaning that the favorite in one semi-final uses the same strategy as the favorite in the other semi-final, and likewise for the underdogs. Henceforth when we refer to equilibrium, we will mean a symmetric equilibrium in which $s_1 = s_2 = s$ and $w_1 = w_2 = w$. Invoking symmetry and making some minor simplifications, the two first order conditions for the favorites from (5) both reduce to

$$\frac{w}{gs+w}\left[s\frac{1}{2}+w\frac{(1-s)}{b}\right] = s\left[s\frac{1}{4(1-s)}+w\frac{(1-w)}{b^2}\right], \tag{7}$$

where $b = g(1-s)+(1-w)$. Similarly, the two first order conditions for the underdogs from (6) reduce to

$$\frac{gs}{gs+w}\left[gs\frac{(1-w)}{b}+w\frac{1}{2}\right] = w\left[gs\frac{g(1-s)}{b^2}+w\frac{1}{4(1-w)}\right]. \tag{8}$$

Note that effort levels at the corners cannot be part of any equilibrium. If, for instance, favorites spend zero effort in the first round, they are assured to lose the tournament unless underdogs also spend zero effort. But in that case, each team can discontinuously increase its chance of winning the tournament by allocating

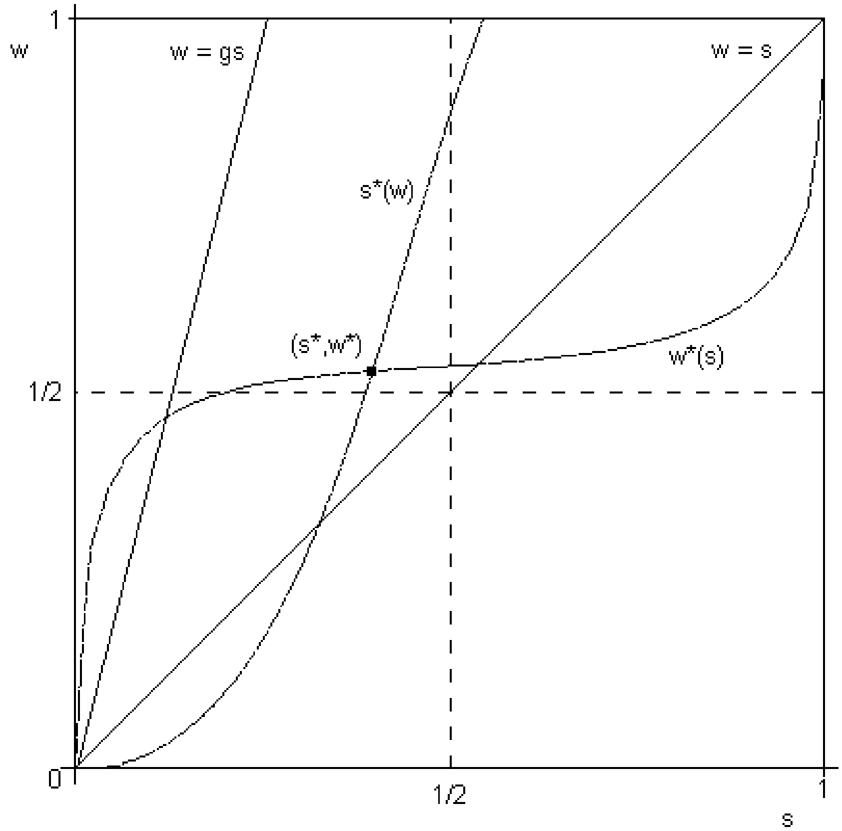

Figure 1: Reaction functions for symmetric equilibrium.

an infinitesimal amount of effort to the first round. A similar argument shows that allocating all available resources to the first round is not optimal either. Thus, to find equilibria of the model we may restrict our attention to effort levels that are strictly between zero and one.

For each $w \in (0,1)$, call $s^*(w)$ the value of s that solves (7), and likewise for each $s \in (0,1)$, call $w^*(s)$ the value of w that solves (8) (in the Appendix we show that these solutions are indeed well-defined). A symmetric Nash equilibrium in pure strategies is then a pair (s^*, w^*) that satisfies $s^* = s^*(w^*)$ and $w^* = w^*(s^*)$.

Figure 1 depicts representative reaction functions $s^*(w)$ and $w^*(s)$ generated by (7) and (8).[10] A symmetric equilibrium occurs at the point where they intersect. Note that this intersection is in the northwest half of the diagram where $s < w$, implying that underdogs exert more effort in the semi-final and that

[10]These functions already include the assumption that both favorites follow the same strategy and both underdogs follow the same strategy. Hence they limit our attention to symmetric Nash equilibria. The particular functions shown were generated by setting $g = 4$.

favorites have more effort remaining for the final, $1-s > 1-w$. Note also that the equilibrium lies in the region where $w < gs$, implying from (1) that $f(1/g, w^*, s^*) < \frac{1}{2}$. Therefore, although underdogs exert more effort than favorites in the semi-final, they do not exert so much more as to become the new favorite.[11] In fact, as stated in the following proposition, these properties always hold in our model.[12]

Proposition 1 *A symmetric pure strategy Nash equilibrium* (s^*, w^*) *exists. In any such equilibrium, the underdog exerts more effort than the favorite in the semi-final, but not enough to overcome the favorite's ability advantage,* $s^* < w^* < gs^*$.

The proof of Proposition 1 is in the Appendix. We first show that $s^*(w)$ and $w^*(s)$ each intersect the 45-degree line at exactly one point in $(0,1)^2$ and that $s^*(w)$ intersects it from below while $w^*(s)$ intersects it from above. As can be seen from examination of Figure 1, this implies that s^* and w^* intersect each other in the interior of the strategy space at least once so a symmetric Nash equilibrium always exists. It is then shown that s^* intersects at a lower value than w^* does and that $s^*(w)$ is strictly increasing in w. These restrictions imply that any such intersection is above the 45-degree line, so $s^* < w^*$. Finally, to show that $w^* < gs^*$ we use proof by contradiction based on direct manipulation of the equilibrium conditions.[13]

Why do underdogs "front-load" effort to early rounds, while favorites "back-load" effort to the finals? To gain some intuition for the result, consider the problem of a favorite. In the semi-final he is paired against an underdog. If he gets to the final there is a chance he will face an underdog again, but also a chance he will face the other favorite. Therefore the expected quality of a favorite's opponents increases as the tournament progresses. Conversely, an underdog is paired against a favorite in the semi-final. If he gets to the final, there is a chance that he will have to play a favorite again, but also a chance that he will play an underdog. Therefore the expected quality of an underdog's opponents decreases as the tournament progresses. Given these changes in opponent quality over the course of the tournament, an equal effort allocation for all players is clearly not an equilibrium. Favorites would have an incentive to shift some effort to the final where the marginal effect on the probability of winning is higher. Similarly, underdogs would have an incentive to shift some effort to the first round. Only if the underdogs exert more effort than favorites in the semi-final, and if the favorites exert more effort than underdogs in the final, can each player's marginal return from effort in the semi-final be equal to the opportunity cost of having less effort remaining for the final.

The strategic allocation of effort identified in Proposition 1 arises because the players have a single budget constraint for the whole tournament and can substitute their effort across rounds. To gain more insight into the model, it is worth comparing how outcomes differ when effort expenditures by the two players in a match are equal. This would result if players naively allocated half of their effort to each round. It would also result

[11]In the example of Figure 1 where $g = 4$, strategic allocation of effort raises the probability of an upset from $1/(1+g) = .2$ to $w^*/(gs^* + w^*) = .529/(4(.397)+.529) = .250$, which is still well below 1/2.

[12]Again we are following the literature in limiting attention to symmetric equilibria. There may also be asymmetric equilibria and we cannot be assured that they have the same properties as the symmetric equilibria we examine.

[13]Although we have not found any such situations, there may be multiple intersections, implying the existence of multiple equilibria. Any symmetric Nash equilibrium in pure strategies has the properties that we identify.

if the players had a separate effort budget in each round and could not substitute effort across rounds. For instance, if the interval between rounds was sufficient for the players to fully recover, then the players would have a full unit of effort to expend in each round.[14] Since the success function $f(r,x,y)$ is homogenous of degree zero in (x,y) the exact amount of effort in a match does not matter as long as it is the same for both players. Let the amount be $e \in [0,1]$.[15]

One question is whether strategic allocation of effort across rounds makes an upset in the semi-final more likely than if the players exerted equal effort. As would be expected, $w^* > s^*$ implies that this is true,

$$f(\frac{1}{g}, w^*, s^*) - f(\frac{1}{g}, e, e) = \frac{g\,(w^* - s^*)}{(gs^* + w^*)\,(g+1)} > 0. \tag{9}$$

Now consider how this higher probability of an upset in the semi-final affects the likely matchups in the final. The final is between a favorite and an underdog if either the favorite in the first semi-final wins and the underdog in the second semi-final wins, or vice-versa. Because strategic allocation of effort pushes the chance of an upset in each semi-final closer to 1/2 but not above it, the probability of a final between a favorite and an underdog is higher than in the equal effort case. Comparing,

$$2f(g, s^*, w^*)f(\frac{1}{g}, w^*, s^*) - 2f(g, e, e)f(\frac{1}{g}, e, e) = 2g\frac{(w^* - s^*)\left(g^2 s^* - w^*\right)}{(gs^* + w^*)^2\,(g+1)^2} > 0, \tag{10}$$

where the inequality follows from $w^* > s^*$, $gs^* > w^*$, and $g > 1$. Not only does strategic allocation of effort lead to more favorite-underdog matches in the final, but the underdog has less remaining resources, so the favorite is more likely to win these matches than in the case of equal effort expenditures. In particular,

$$f(g, 1 - s^*, 1 - w^*) - f(g, e, e) = \frac{g(w^* - s^*)}{(g(1 - s^*) + 1 - w^*)\,(g+1)} > 0, \tag{11}$$

where the inequality follows from $w^* > s^*$.

These results from inequalities (9) – (11) are stated formally in the following proposition.

Proposition 2 *Compared to the equal effort case, in any symmetric pure strategy Nash equilibrium: (i) the probability that an underdog defeats a favorite in the semi-final is higher; (ii) the probability that a favorite and an underdog meet in the final is higher; and (iii) the probability that a favorite-underdog final is won by the favorite is higher.*

Although the model does not directly predict the margin of victory, the margin is likely to be correlated with relative effort levels and the probability of victory. Therefore Propositions 1 and 2 support the basic intuition that the strategic allocation of resources across the tournament makes close matches in the semi-finals and a "blowout" in the championship more likely.

[14] That they would in fact use all of it follows from our simplifying assumption that unused effort has no value.

[15] For simplicity we assume that all effort levels in the tournament are the same, but the results hold as long as effort levels by the two players in each match are the same, even if effort varies across matches.

Now consider how the allocation of effort affects whether a favorite or underdog is most likely to win the whole tournament. From (3), with equal effort the probability that a given favorite wins the tournament is, for $i = 1, 2$,

$$\pi_i^S(e, e, e, e) = \frac{g}{g+1}\left(\frac{g}{g+1}\frac{1}{2} + \frac{1}{g+1}\frac{g}{g+1}\right). \tag{12}$$

With strategic allocation of effort the probability is, for $i = 1, 2$,

$$\pi_i^S(s^*, w^*, s^*, w^*) = \frac{gs^*}{gs^*+w^*}\left(\frac{gs^*}{gs^*+w^*}\frac{1}{2} + \frac{w^*}{gs^*+w^*}\frac{g(1-s^*)}{g(1-s^*)+(1-w^*)}\right). \tag{13}$$

The respective probabilities that one of the two favorites wins are therefore $2\pi_i^S(e, e, e, e)$ and $2\pi_i^S(s^*, w^*, s^*, w^*)$. Comparing (12) and (13), there are two effects from allocating effort across rounds. First, since

$$\frac{g}{g+1} > \frac{gs^*}{gs^* + w^*}$$

as implied by Proposition 2(i), favorites are less likely to advance to the final than with equal effort levels. Second, if they do make it to the final they have more resources left to compete. This second effect does not matter if two favorites meet since the probability of victory is 1/2 in either case, but if a favorite meets an underdog the probability of victory rises to

$$\frac{g(1-s^*)}{g(1-s^*) + (1-w^*)} > \frac{g}{g+1}$$

as shown in Proposition 2(iii). From direct comparison of (12) and (13) the relative strength of the two effects is not obvious. The following proposition, which is proven in the Appendix, uses indirect methods based on an underdog's probability of winning the tournament to show that the first effect always dominates the second effect. Even though allocation of effort across rounds makes it more likely that the favorite wins a match with an underdog, the probability that two underdogs reach the final increases enough that a favorite is less likely overall to win the tournament than in the case of equal effort expenditures.

Proposition 3 *Compared to the equal effort case, in any symmetric pure strategy Nash equilibrium the probability that a favorite wins the tournament is lower.*

3 Empirical Test

Because of their formalized structure and data availability, sporting contests are particularly amenable to empirical tests of strategic behavior. To test whether the strategic resource allocation effects derived in our model are present in actual contests, we use data from the NCAA men's basketball tournament. This sequential elimination tournament features invited U.S. college teams which first compete to be one of the "Final Four" regional champions and then compete for the national title. The tournament is attractive for our purposes because a structural change in the tournament's schedule allows us to contrast team behavior under two different resource constraints.[16] Before 1969, semi-final and final matches were played on consecutive days so coaches and players had little time to recover from the semi-finals and prepare for the finals. Starting in 1969, the NCAA introduced a rest day between the matches in order to "provide greater preparation time for the coaches involved" and "give more rest to the players".[17]

The rest day was introduced in both the regional championships and in the national championship. Because there are four regional championships every year, but only one national championship, there are four times as many observations on the regional level, so we test our model using regional data. While the entire tournament extends over more than just the two regional rounds, there is a multi-day (usually about one week) break between any preliminary matches and the regional semi-finals, and about a one week break between the regional finals and the national championships. Therefore, from the perspective of allocating resources across matches, the regional semi-finals and finals can be viewed in relative isolation from the rest of the tournament and our basic model of a two-round tournament can be applied.[18]

Our model predicts that the introduction of the rest day should have had differing effects on favorites and underdogs. Before 1969 when teams had to allocate their resources across the two rounds, favorites had a strong incentive to hold back on effort in the semi-final because they expected to play a better team in the final. Underdogs also had some incentive to conserve effort, but they were already playing a strong team in the semi-final so they had less incentive to hold back. Therefore, from Proposition 1, without the rest day we expect underdogs to allocate more effort to the semi-final than favorites do.[19] After introduction of the rest day, teams could recover from the semi-final before the final so the need to allocate resources across the tournament should have declined or disappeared. Therefore, as discussed in the previous section, teams should have come closer to fully exerting themselves in each match. Based on this result, Proposition 2(i) predicts there will be fewer upsets in the semi-finals after introduction of the rest day, Proposition 2(ii) predicts there will be fewer pairings in the final between the favorite from one semi-final and the underdog from the other, Proposition 2(iii) predicts such matches will be less likely to be won by the favorite, and

[16] This tournament is also attractive for testing the model because the duration of matches is not endogenous to player strategies except for rare overtime matches. In tournaments where the duration of matches is endogenous, e.g. the NBA tournament where a match is won after one team wins a majority of n games, or tennis tournaments where a match is won after one player wins a majority of n sets, a favorite that tries to save resources can instead find itself wasting resources on a longer match than necessary.

[17] These were the first two reasons cited by the "Report of the Executive Committee" in the NCAA's *Proceedings of the 62nd Annual Convention - 1968*. Better press coverage was the third reason.

[18] Since semi-final matches are played sequentially rather than simultaneously, players in the second match may have the opportunity to adjust their effort levels based on the outcome of the first match. However, such adjustments will be incomplete since it is too late for players and coaches to adjust their preparation time for the semi-final. For simplicity we abstract from this issue.

[19] Recall that we are assuming there is no cost to effort other than the opportunity cost of having less effort available for the next match. This simplifying assumption may not be appropriate for professional tournaments where players have been shown to vary their effort based on financial rewards (Ehrenberg and Bognanno, 1990). The idea that college basketball players in the tournament are willing to "go all out" is indirectly supported by McClure and Spector (1997) who find that higher financial rewards for colleges do not affect performance levels.

Proposition 3 predicts that the overall probability of a favorite winning the regional championship will higher.

Since all of the model's predictions follow from the main result of Proposition 1, we concentrate our formal analysis on measuring relative effort in the semi-final matches. The other predictions, which are more indirect and therefore more difficult to test, are briefly analyzed at the end of the empirical section.

3.1 Data and preliminary analysis

For each match in the tournament we observe the victor, the point scores of the two teams, and whether or not the game went into overtime.[20] To measure the quality of teams we use the final UPI poll which ranked the top 20 teams after the regular season and immediately before the start of the tournament.[21] Ideally we would like to observe each team's effort level and the resulting probability of winning, but clearly this is not possible. However, the relative scores of the two teams can be used as a noisy measure of relative effort.[22] To the extent that we can condition on the teams' relative quality, this measure will be more accurate.

To evaluate the impact of the rest day, we look at symmetric windows on either side of the change in 1969. The NCAA tournament began in 1939 but the modern system of four regional championships did not start until 1952.[23] Therefore the broadest possible window includes the 17 years 1952-1968 and the subsequent 17 years 1969-1985. We also consider a narrower 10-year window on either side of the change to reduce the impact of other unmeasured factors that might be changing concurrently. Every year there are four regional championship tournaments with two semi-final matches each. Therefore for the 34 years of data in the 17-year window there are a possible 272 matches in the sample, and for the 20 years of data in the 10-year window there are a possible 160 matches in the sample. Unranked teams, which are assigned the censored rank of 21, cannot be compared with other unranked teams so we discard matches in which both teams are unranked, leaving a sample of 249 matches for the 17-year window and 151 matches for the 10-year window.

As a first step to get some perspective on the data let us examine how average winning margins of favored teams have changed over time. Positive margins are assigned to victories by favored teams, negative margins are assigned to upsets, and zero margins are assigned to games that go into overtime. We compute the

[20]Tournament data is from the official NCAA site, http://www.ncaasports.com.

[21]The AP poll is the other major poll, but for the period 1963-1968 the AP only reported the top 10 teams. The NCAA did not start its own ranking of teams in the tournament until 1979. UPI rankings are from http://www.sportsstats.com/bball/rankings/national.rankings.by.year. Results are very similar if we use the AP top 10 data available from http://www.ncaasports.com. Rankings for teams in the 11-20 range are comparatively noisy so matches between teams in this range contribute little to our regression results.

[22]An alternative to the actual scores is Las Vegas point spreads. Assuming that gamblers anticipate the effects we consider, this ex ante measure should be less noisy, but we were unable to find point spread data for most of the sample period.

[23]Before 1951 there were only 8 teams in the in the tournament and only two regional championships. The number of participants rose to 16 teams in 1951 but the creation of four regional championships (and the concurrent adjustment to rest periods before the national championship) did not occur until the following year. Note that the number of participants increased further to 22 in 1953, and then ranged from 23 to 25 until 1975 when 32 teams participated. The number was again increased to 40 teams in 1979, to 48 teams in 1980, and finally to 64 teams in 1985.

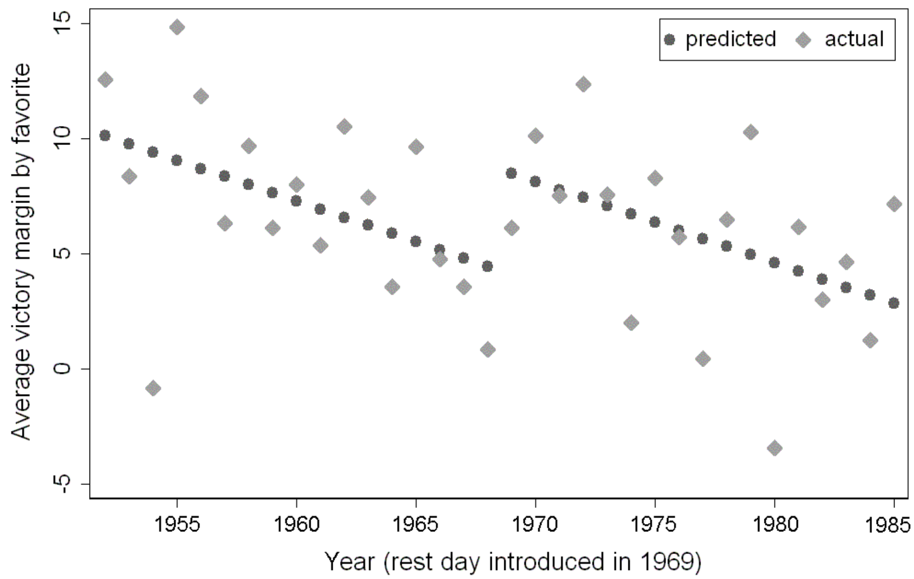

Figure 2: Average semi-finals victory margin with prediction from trend and rest day dummy variable

average margin for each year and regress the resulting sequence against a time trend and a rest day dummy variable that is set to zero until 1968 and to one after that. The result of this regression for the 17-year window is depicted in Figure 2. (Results for the 10-year window are similar.) Note that there is a long-term trend toward greater parity, presumably as athletic ability rises asymptotically toward physical maximums.[24] Consistent with Proposition 1, this trend appears to have been temporarily reversed in 1969 when the rest day was introduced. For the 17-year window, inclusion of the rest day dummy variable raises the adjusted R^2 from .146 to .218. For the 10-year window, where the long-run trend toward parity is less apparent, inclusion raises the adjusted R^2 from only -.030 to .265. However, while suggestive, these regressions are not conclusive because they are based on annual averages rather than on match-level data.[25]

[24]Such a trend would be captured in our model by a decrease in g. Greater parity is most apparent in individual sports such as running where world records have improved at a decreasing rate and victory margins have narrowed. In the NCAA tournament such a trend could also result from the increasing number of teams admitted to the tournament. Toward the end of the period so many teams were starting the tournament that surviving to the regional semi-finals (the "Sweet Sixteen") had become a strong signal of a team's ability, even if a team was poorly ranked before the tournament. However, the number of participants was nearly constant from 1953 to 1975 during which most of the fall in victory margins occurred. The issue of survival bias through the course of a tournament is analyzed in Abrevaya (2002).

[25]The coefficient for the dummy variable is 6.11 in the 10-year window regression and 4.33 in the 17-year window regression. The t-statistics are 2.36 and 1.69. The dummy variable coefficients for robust regressions are similar and the t-statistics are 2.14 and 2.03 respectively.

Table 1: Regression results for impact of rest day on semi-final victory margins

	10-year window (1959-1978)				Full 17-year window (1952-1985)			
	OLS		Robust		OLS		Robust	
	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
Fav. Score − Under. Score								
Constant	39.057	1.88	42.713	2.07	28.474	3.34	33.056	3.89
Year	-0.520	1.59	-0.591	1.82	-0.353	2.51	-0.434	3.10
Rest Day	5.897	1.56	7.544	2.01	4.376	1.60	6.095	2.24
Fav. Score − Under. Score								
Constant	34.817	1.72	42.163	2.13	24.357	2.91	29.120	3.50
Year	-0.525	1.65	-0.640	2.02	-0.360	2.63	-0.435	3.20
Under. Rank − Fav. Rank	0.459	2.95	0.424	2.74	0.455	3.87	0.427	3.67
Rest Day	6.126	1.66	7.775	2.13	4.487	1.68	5.485	2.21
$(\text{Fav. Score})^{\frac{1}{2}} - (\text{Under. Score})^{\frac{1}{2}}$								
Constant	2.388	1.99	2.584	2.16	1.672	3.32	1.928	3.84
Year	-0.032	1.70	-0.036	1.89	-0.021	2.48	-0.025	3.01
Rest Day	0.359	1.65	0.433	1.99	0.258	1.60	0.340	2.11
$(\text{Fav. Score})^{\frac{1}{2}} - (\text{Under. Score})^{\frac{1}{2}}$								
Constant	2.055	1.77	2.435	2.11	1.406	2.86	1.658	4.30
Year	-0.032	1.74	-0.038	2.08	-0.021	2.64	-0.025	3.16
$(\text{Under. Rank})^{\frac{1}{2}} - (\text{Fav. Rank})^{\frac{1}{2}}$	0.151	3.48	0.151	3.49	0.145	4.25	0.146	4.30
Rest Day	0.381	1.81	0.449	2.14	0.279	1.78	0.332	2.14

Year variable equals calendar year minus 1900. Rest Day dummy variable equals 1 starting in Year 69.

3.2 Estimation and results

To see whether the patterns suggested by Figure 2 are statistically significant, we use match-level data for the 10-year and 17-year windows before and after the change. Match-level data allows us to avoid statistical problems due to averaging and also allows us to condition on the rankings of the two teams so that the victory margin is a more accurate measure of relative efforts. We perform standard OLS regressions and also robust regressions that iteratively reweight the observations to reduce the impact of large outliers.[26] In our case such outliers could arise from the stochastic relationship between victory margins and relative effort or from the imperfect (and censored) measurement of team ability.[27]

Looking at Table 1, we first regress the victory margin of the favorite against the year trend and the rest day dummy. This regression is the match-level equivalent of the regression shown in Figure 2. Second, we

[26] We use the *rreg* command from STATA which uses a mixture of Huber weights and bi-weights to reweight the regression.

[27] For each specification the robust regression procedure assigns the smallest weight to #1 UCLA's 49-point victory over unranked Wyoming in 1967. The different regressions predict only about a 9-point victory, but undefeated UCLA (the eventual tournament champion) was an unusually strong #1 team, Wyoming with a 15-12 record was an unusually weak unranked team, and UCLA, which had not made it to the tournament the previous year after winning the tournament two years in a row, might have been less averse to running up the score than the typical favorite.

perform the same regression except we condition on the difference in rankings between the underdog and the favorite where better teams have lower ranks. Third, we regress the difference in the square roots of the scores against the year trend and the rest day dummy. Fourth, we again condition on the difference in rankings between the underdog and the favorite, but we use square roots of the rankings.

The regression results tend to support the basic trend in Figure 2 of decreasing margins over time with a jump in 1969 when the rest day is introduced. In particular, when we condition on the rankings the rest day dummy is significant at the 10% level in all of the OLS regressions and at the 5% level in all of the robust regressions.[28] Coefficient signs for the other variables are also as predicted and significant, with more significant results for the full 17-year window.[29] Using square roots tends to increase the overall predictive power of the regressions, but has little impact on the significance of the rest day dummy.[30] Adjusted R^2s (available only for the OLS regressions) are small as is typical in cross-sectional data, reaching a high of .101 for the 17-year window regression with square root victory margins and square root rankings.

Regarding alternative measures of effort besides victory margins, one attractive measure is minutes played by the best players, but scorekeepers did not record this statistic regularly until late in the sample period. A related measure, for which data is available for all matches in the 10-year window around 1969, is percentage of total points scored by the starting players. Our model predicts that before 1969 favorites should have relied on their starting players less than underdogs did so as to save their energies for the final, and that this difference should have narrowed after introduction of the rest day in 1969. This measure of effort turns out to be somewhat noisy, presumably because the number of players with strong offensive talents varies substantially across teams, but the overall trend is consistent with the model. Before 1969, starting players for favored teams accounted for about 5% less of their team's total points than did starting players for underdog teams, and this difference completely disappeared after 1969. Regressing this difference in share of points by starting players against a time trend and the rest day dummy, the rest day dummy is always positive as predicted and consistent with a 5%-10% shift, but the coefficient is typically not significant.[31]

3.3 Other predictions

Now consider briefly the other predictions of the model. Proposition 2(i) predicts a decrease in the number of semi-final upsets following the change in 1969. The binary outcome of an upset or not is a noisier measure of relative effort than is the victory margin, but the numbers are consistent with the theory. For instance,

[28] If one-sided tests are used the rest day dummy is also significant at the 5% level in the OLS regressions. The (unreported) results for median regression, another method for handling outliers, are of similar significance as the robust regression results.

[29] Similar results also hold for the unbalanced window 1952–2000.

[30] Results are also very similar for different ways of conditioning on relative strength, such as using logs or taking the square root of the difference in ranks rather than the difference in the square root of ranks.

[31] The results improve if we restrict the sample to matches where the stronger teams are in different semi-finals. Using annual average data, the dummy variable is then significant at the 5% level in the robust regression. Using match-level data and performing the equivalent regressions as in Table 1, the rest day dummy is significant at the 10% level in the OLS regressions but is still not significant in the robust regressions.

in the five years 1964–1968 before the change there were 13 upsets in the semi-finals, while in the five years 1969–1973 after the change there were only 8 upsets. Looking at a narrower window around the change, there were 8 upsets in 1967 and 1968, and only 2 upsets in 1969 and 1970. However, with wider time windows, the long-run trend toward greater parity washes out the resource allocation effects observed around 1969. For instance, in the full 17 years 1952–1968 there were 28 upsets, while in the subsequent 17 years 1969–1985 there were 39 upsets.

Proposition 2(ii) predicts that the number of finals featuring the favored team from one semi-final and the underdog from the other semi-final should decrease after introduction of the rest day. It turns out that there is little difference in the number of favorite-underdog pairings in the finals before and after 1969. For instance, in the five years before the change there were 7 such pairings and in the five years after the change there were 8 such pairings. The number of pairings is even noisier than the total number of upsets because in many cases there are upsets in both semi-finals, and such cases are more likely when upsets are more likely. Therefore we simply do not have a large enough sample to accurately test the prediction. A similar problem occurs when we try to test whether favorites are more likely to win final matches against underdogs before introduction of the rest day – the implication of Proposition 2(iii). Overall, only about one fourth of all games result in upsets, and these include cases where there are upsets in both semi-finals. Furthermore, the model assumes that the stronger teams are in different semi-finals, which is not always the case. This does not present a problem for the basic intuition that the stronger team in each semi-final has more incentive to conserve resources,[32] but it can change the prediction regarding the finals. Therefore, to test Proposition 2(iii) we need to limit the sample to cases where there is an upset in only one semi-final and where the two strongest teams are in different semi-finals. Doing so reduces the number of usable data points by so much that an accurate test becomes impossible. For instance, in the ten years before the change, only 10 out of 40 matches fit our criteria, and only 12 do so in the ten years after the change.

Proposition 3 predicts that a favored team is more likely to win the tournament after introduction of the rest day. This prediction, too, is difficult to test because there are only four regional championships each year and favored teams are still very likely to win, with or without the rest day. Nevertheless, the numbers are consistent with our theory in that in the five years before the change, 15 of the 20 regional championships were won by one of the two best teams from the semi-finals, and in the five years after the change this share increased to 18 out of 20.[33]

[32] For instance, if a strong team and a very strong team meet in one semi-final and a weak team and a very weak team meet in the other semi-final, then the very strong team still has more incentive to hold back than the strong team, and the weak team still has more incentive to hold back than the very weak team. This pattern holds in numerical simulations based on generalizations of equations (5)–(6).

[33] If we only consider matches where the favorites are in different semi-finals, 10 of 13 regional championships are won by favorites in the five years before the change, and 15 of 15 regional championships are won by favorites in the five years after the change.

4 Conclusion

This paper develops and tests a model of a two-round sequential elimination tournament among resource-constrained players of varying ability. We take the design of the tournament as given, but clearly these results have implications for tournament design (Lazear and Rosen, 1981). For sports tournaments, the main concern is probably the excitement value of the matches as measured by intensity of the play and uncertainty over the likely victor (Chan, Courty, and Li, 2003). This paper implies that increasing excitement at one level of the tournament involves a trade-off in reducing excitement at the other level. For election campaigns, a key concern for each party is ensuring that the strongest candidate makes it to the general election (Klumpp and Polborn, 2003). This paper suggests that spending constraints and other restrictions at the primary level can reduce the problem of a weak candidate upsetting a better candidate who saves resources for the general election. For career ladders, a main concern is ensuring that the best manager makes it to the top. This paper indicates that strategic allocation of effort, e.g. managers delaying having children so as to be less encumbered for competition early in their careers, increases the chance of a less talented manager prevailing. Under such circumstances performance measures alone may not be the best selection criteria.

Appendix

In this Appendix we prove the propositions that are stated in the main text. To this end, we first establish the following intermediate result:

Lemma 1 *For every $w \in (0,1)$, there exists a unique $s^*(w) \in (0,1)$ that solves* (7)*, and for every $s \in (0,1)$, there exists a unique $w^*(s) \in (0,1)$ that solves* (8)*. Furthermore, s^* and w^* are continuous functions on $(0,1)$, and s^* increases in w.*

For equation (7) denote the left-hand side as $F_L^S(s,w)$ and the right-hand side as $F_R^S(s,w)$. Similarly for equation (8) denote the left-hand side as $F_L^W(s,w)$ and the right-hand side as $F_R^W(s,w)$. The proof is organized in two main steps, one for s^* and one for w^*. Each step is divided into three substeps: (a) existence, (b) uniqueness, and (c) continuity. For s^* there is an additional substep (d) in which we establish monotonicity.

Step 1. s^* exists and is unique and continuous in w.

Step 1a.
Observe that, for fixed $w \in (0,1)$, F_R^S strictly increases in s, $F_R^S(0,w) = 0$, and $F_R^S(1,w) = \infty$. Since $F_L^S(0,w)$ and $F_L^S(1,w)$ are finite positive numbers, there exists $s^* \in (0,1)$ such that $F_L^S(s^*,w) = F_R^S(s^*,w)$.

Step 1b.
To show that s^* is unique for given w, we now show that F_L^S decreases in s. Differentiating with respect to

s and simplifying, we obtain

$$\frac{\partial F_L^S(s,w)}{\partial s} = -\frac{1}{2}w^2\frac{-1+4w-3w^2-2gsw+g^2(1+s^2-2s)+2gs}{(gs+w)^2b^2},$$

which is non-positive if and only if the term in the numerator,

$$\big[-1+4w-3w^2-2gsw\big]+\big[2gs+g^2(1+s^2-2s)\big]\equiv L, \tag{14}$$

is non-negative. Our approach is to minimize (14). Consider the value for w first. The second term in brackets in (14) is independent of w, while the first term is minimized if $w\in\{0,1\}$. If $w=0$, then $L=-1+[g^2(1+s^2-2s)+2gs]$, which is minimized if $s=(g-1)/g$, so that $L=2(g-1)>0$. If $w=1$, then $L=-2gs+[g^2(1+s^2-2s)+2gs]=g^2(1+s^2-2s)$, which is minimized if $s=1$, so that $L=0$. Hence $L\geq 0$ and F_L^S decreases in s, and a unique s^* exists such that $F_L^S(s^*,w)=F_R^S(s^*,w)$.

Step 1c.
Since F_L^S and F_R^S are continuous in w for $w\in(0,1)$, s^* is continuous on $(0,1)$.

Step 1d.
It can be shown that $F_L^S(s,w)=0$ if and only if

$$h(s,w)=\frac{s}{2(1-s)}\left(\frac{w(1-s)}{gs+w}-\frac{s}{2}\right)+\frac{w}{g(1-s)+1-w}\left(1-s-\frac{s(1-w)}{g(1-s)+1-w}\right)=0. \tag{15}$$

It is easy to see that $h(s,w)$ increases in w. We further know from Step 1b that for each w there is a unique $s^*(w)$ that solves (15). Since $h(0,w)\geq 0$, $h(s,w)$ must be downward sloping in s at $s=s^*(w)$. From the implicit function theorem, it follows that $s^*(w)$ increases in w.

Step 2. w^* exists and is unique and continuous in s.

Step 2a.
Observe that, for fixed $s\in(0,1)$, F_R^W strictly increases in w, $F_R^W(s,0)=0$, and $F_R^W(s,1)=\infty$. Since $F_L^W(s,0)$ and $F_L^W(s,1)$ are finite positive numbers, there exists $w^*\in(0,1)$ such that $F_L^W(s,w^*)=F_R^W(s,w^*)$.

Step 2b.
Establishing uniqueness of w^* is more complicated than for s^*, since F_L^W may be increasing in w. We show the following: If F_L^W is increasing in w on a subset of $(0,1)$, then it is concave on $(0,1)$. Noting that $F_L^W(s,0)>0=F_R^W(s,0)$, concavity implies that the two curves intersect exactly once. After some algebra, the first two derivatives of F_L^W can be written as follows:

$$\begin{aligned}\frac{\partial}{\partial w}F_L^W(s,w) &= \frac{1}{2}\frac{g^2s^2}{(gs+w)^2}\left[1-2\frac{L_1(w)}{b^2}\right],\\ \frac{\partial^2}{\partial w^2}F_L^W(s,w) &= -\frac{g^2s^2}{(gs+w)^3}\left[1-2\frac{L_2(w)}{b^3}\right],\end{aligned}$$

where

$$\begin{aligned} L_1(w) &= (1-s)(g+g^2s)+(1-w)^2, \\ L_2(w) &= g^3s(2s^2-3s+1)+g^2(1-s)(1-3sw)+g(1-s)(2-3w)+(1-w)^3. \end{aligned}$$

Now let $M_1(w) = 2L_1(w) - b^2$ and $M_2(w) = 2L_2(w) - b^3$. These functions are continuous in $w \in (0,1)$. Observe that $M_1(w) < (>)\, 0$ iff $F_L^W(s,w)$ is increasing (decreasing) at w, and $M_2(w) < (>)\, 0$ iff $F_L^W(s,w)$ is concave (convex) at w. In the following, it will be convenient to extend F_L^W, L_1, L_2, M_1 and M_2 to all $w \in \mathbb{R}$, and assume that $s \in [0,1]$.

Let us first examine the slope of F_L^W. Notice that M_1 is quadratic in w with a positive coefficient. This implies that F_L^W will either be always decreasing in w on $\mathbb{R}$, or have one local maximum and one local minimum in w on $\mathbb{R}$. If this is the case, then the solutions for $M_1(w) = 0$ are given by

$$\underline{w} = 1-g(1-s)-\sqrt{A}, \;\; \overline{w} = 1-g(1-s)+\sqrt{A},$$

where $A = g^2(4s^2-6s+2) - 2g(1-s)$ and $\underline{w}$ corresponds to the local minimum of F_B^L and $\overline{w}$ to the local maximum. We now verify that $\underline{w} < 0$ or $\underline{w} = 1$. For $\underline{w}$ to be a real number, $A \geq 0$, which implies $s \geq 1$ or $s \leq \frac{1}{2}(1-\frac{1}{g})$. In the first case, only $s = 1$ is relevant, and $\underline{w} = 1$. In the second case, note that $1-g(1-s) < 0$ for all $s < 1 - \frac{1}{g}$. Thus, if $s < \frac{1}{2}(1-\frac{1}{g})$, $w_1 < 0$. For the slope of F_L^W, this leaves us with three possible cases:

(1) $\underline{w} \leq \overline{w} \leq 0$ or $\underline{w} = 1 \leq \overline{w} \;\; \Rightarrow \;\; F_B^L$ is decreasing on $[0,1]$,

(2) $\underline{w} < 0 < 1 \leq \overline{w} \;\; \Rightarrow \;\; F_L^W$ is increasing on $[0,1]$,

(3) $\underline{w} < 0 < \overline{w} < 1 \;\; \Rightarrow \;\; F_L^W$ is first increasing and then decreasing on $[0,1]$.

In case (1), Step 2b is complete. For the other two cases, we need to check the curvature of F_L^W; in particular we want to show that F_L^W is concave. If it is, then Step 2b is complete. It can be shown that the following relationship between M_1 and M_2 holds:

$$\frac{\partial M_2(w)}{\partial w} = -3M_1(w).$$

In cases (2) and (3) above, $\underline{w} < 0$ and $\overline{w} > 0$. Since $M_1(\overline{w}) = 0$, so F_L^W and M_2 have local maxima at $\overline{w}$. Furthermore, $M_2(\overline{w}) \leq 0$, for otherwise F_L^W would be convex at a local maximum, which is impossible. Since $\overline{w}$ is the only extremum of M_2 on $[0,1]$, it must be that $M_2(w) \leq 0$ for $w \in [0,1]$. Therefore, F_L^W is concave at $w \in [0,1] \supset (0,1)$.

Step 2c.
Finally, since F_L^W and F_R^W are continuous in s for $s \in (0,1)$, w^* is continuous on $(0,1)$. ∎

4.1 Proof of Proposition 1

Part 1: Existence. Any interior intersection of s^* and w^* from Lemma 1 is a symmetric Nash equilibrium. We now examine the points at which s^* and w^* intersect the 45-degree line in (s,w)-space. Substituting $w=s$ into (7), we get

$$-s\frac{g^2s+(4g+11)s+2g-6}{4(1-s)(g+1)^2}=0.$$

There is one non-zero solution:

$$\psi^S=2\frac{g+3}{g^2+4g+11}.$$

Thus, $s^*(\psi^S)=\psi^S$, and s^* intersects the 45-degree line at $w=\psi^S$. Similarly, from (8) we get

$$\psi^W=2\frac{g(3g+1)}{11g^2+4g+1}.$$

Thus, $w^*(\psi^W)=\psi^W$, and w^* intersects the 45-degree line at $s=\psi^W$.

The numbers ψ^S and ψ^W are unique, well-defined, and positive. We now show that $\psi^S<1$ and $\psi^W<1$; this ensures that w^* and s^* intersect the 45-degree line exactly once in $(0,1)$. For ψ^S, we have

$$\psi^S=2\frac{g+3}{g^2+4g+11}=2\frac{g+3}{(g+3)(g+1)+8}<2\frac{g+3}{(g+3)(g+1)}=\frac{2}{g+1}\leq 1.$$

For ψ^W, we have

$$\psi^W=2\frac{g(3g+1)}{11g^2+4g+1}=2\frac{g(3g+1)}{(3g+1)(\frac{11}{3}g+\frac{1}{9})+\frac{8}{9}}<2\frac{g(3g+1)}{(3g+1)(\frac{11}{3}g+\frac{1}{9})}=\frac{g}{\frac{11}{3}g+\frac{1}{9}}<\frac{2}{11/3}=\frac{6}{11}<1.$$

Next, recall that by Lemma 1, s^* and w^* are continuous functions on $(0,1)$ and have values in $(0,1)$. Any intersection is a symmetric Nash equilibrium. So to prove existence, we need to show that they intersect in the interior. To do so, we first verify that $\lim_{w\to1}s^*(w)\in(0,1)$. For s^*, multiply (7) by $b^2(1-s)(gs+w)$ to get

$$w(1-s)\left[\frac{1}{2}sb^2+wb(1-s)\right]=s(gs+w)\left[\frac{1}{4}sb^2+w(1-w)(1-s)\right]. \tag{16}$$

The curve defined by (16) coincides with s^* for $w\in(0,1)$, but extends continuously to $[0,1]$. Since neither $(s=1,w=1)$ nor $(s=0,w=1)$ satisfy (16), we conclude that $0<\lim_{w\to1}s^*(w)<1$. The same steps applied to (8) show that $\lim_{s\to1}w^*(s)\in(0,1)$. Next, since s^* crosses the 45-degree line exactly once in $(0,1)$, the fact that $\lim_{w\to1}s^*(w)\in(0,1)$ implies that for $w<(>)\psi^S$, $s^*(w)<(>)w$. Likewise, since w^* crosses the 45-degree line exactly once in $(0,1)$, the fact that $\lim_{s\to1}w^*(s)\in(0,1)$ implies that for $s<(>)\psi^W$, $w^*(s)>(<)s$. It is then impossible for s^* and w^* not to intersect each other in $(0,1)^2$. Hence, a symmetric pure strategy equilibrium exists.

Part 2: $s<w$. When $g=1$, $\psi^S=\psi^W=1/2$. When $g>1$, we claim that $\psi^S<\psi^W$. To see this, simply observe that

$$\frac{d\psi^S}{dg}=-2\frac{g^2+6g+1}{(g^2+4g+11)^2}<0$$

and

$$\frac{d\psi^W}{dg} = 2\frac{g^2+6g+1}{(11g^2+4g+1)^2} > 0.$$

Since both s^* and w^* are continuous functions on $(0,1), \psi^S < \psi^W$ implies that s^* and w^* intersect at a point above the 45-degree line, i.e. where $w > s$. Furthermore, since s^* increases, there cannot be an intersection of s^* and w^* below the 45-degree line, so in every equilibrium $s^* < w^*$.

Part 3: $w^* < gs^*$. Dividing (7) by (8), we get

$$\frac{\frac{w}{gs+w}\left(\frac{1}{2}s+w\frac{1-s}{b}\right)}{\frac{gs}{gs+w}\left(\frac{1}{2}w+gs\frac{1-w}{b}\right)} = \frac{s\left(s\frac{1}{4(1-s)}+w\frac{1-w}{b^2}\right)}{w\left(w\frac{1}{4(1-w)}+g^2s\frac{1-s}{b^2}\right)}.$$

Rearranging and simplifying yields

$$\underbrace{\frac{w^2(1-s)}{gs^2(1-w)}}_{A_1} \times \underbrace{\frac{g^2s^2(1-w)4(1-s)\left[b+2(1-s)\frac{w}{s}\right]+\left[swb^3+2w^2(1-s)b^2\right]}{w^2(1-s)4(1-w)\left[b+2(1-w)\frac{gs}{w}\right]+\left[swb^3+2gs^2(1-w)b^2\right]}}_{A_2} = 1. \tag{17}$$

Condition (17) is necessary (but not sufficient) for equilibrium. Now suppose, contrary to the proposition, that $w \geq gs$. It follows that $gs/w \leq 1$, $gs^2 \leq w^2$, $1–w < 1–s$, and $w/s > 1$, implying

$$A_2 \geq \frac{g^2s^2(1-w)4(1-s)\left[b+2(1-s)\right]+\left[swb^3+2w^2(1-s)b^2\right]}{w^2(1-s)4(1-s)\left[b+2(1-s)\right]+\left[swb^3+2w^2(1-s)b^2\right]}. \tag{18}$$

Note that $\frac{\rho_1}{\rho_2} \leq 1$ and $c > 0$ implies $\frac{\rho_1+c}{\rho_2+c} \geq \frac{\rho_1}{\rho_2}$. This fact and $1-w < 1-s$ implies

$$A_2 \geq \frac{g^2s^2(1-w)4(1-s)\left[b+2(1-s)\right]}{w^2(1-s)4(1-s)\left[b+2(1-s)\right]} = \frac{g^2s^2(1-w)}{w^2(1-s)} = g\frac{1}{A_1},$$

and therefore $A_1A_2 \geq A_1 g\frac{1}{A_1} = g > 1$ so condition (17) cannot hold. Therefore (s,w) with $w \geq gs$ cannot be an equilibrium.

4.2 Proof of Proposition 3

Note that $\pi_1^S + \pi_2^S + \pi_1^W + \pi_2^W = 1$. Therefore to show that $2\pi_i^S(s^*,w^*,s^*,w^*) < 2\pi_i^S(e,e,e,e)$ for $i = 1,2$ it is equivalent to show that $2\pi_i^W(s^*,w^*,s^*,w^*) > 2\pi_i^W(e,e,e,e)$. To prove this we will show that $\pi_i^W(s^*,w^*,s^*,w^*) \geq \pi_i^W(s^*,s^*,s^*,w^*) > \pi_i^W(s^*,s^*,s^*,s^*) = \pi_i^W(e,e,e,e)$. First note that $\pi_i^W(s^*,w^*,s^*,w^*)$ is the underdog's equilibrium probability of winning the tournament while $\pi_i^W(s^*,s^*,s^*,w^*)$ is the underdog's probability of winning the tournament when the underdog chooses effort s^* instead of the equilibrium effort w^*. By the definition of a Nash equilibrium, $\pi_i^W(s^*,w^*,s^*,w^*) \geq \pi_i^W(s^*,s^*,s^*,w^*)$. Now comparing $\pi_i^W(s^*,s^*,s^*,w^*)$ with $\pi_i^W(s^*,s^*,s^*,s^*)$,

$$\pi_i^W(s^*,s^*,s^*,w^*) = \frac{1}{g+1}\left[\frac{gs^*}{gs^*+w^*}\frac{1}{g+1}+\frac{w^*}{gs^*+w^*}\frac{(1-s^*)}{(1-s^*)+(1-w^*)}\right]$$

and

$$\pi_i^W(s^*, s^*, s^*, s^*) = \frac{1}{g+1}\left[\frac{g}{g+1}\frac{1}{g+1} + \frac{1}{g+1}\frac{1}{2}\right] \tag{19}$$

so

$$\pi_i^W(s^*, s^*, s^*, w^*) - \pi_i^W(s^*, s^*, s^*, s^*) = \frac{1}{g+1}\frac{[w^*(1+3g) + g(g-1)(2-s^*)](w^*-s^*)}{2(w^*+gs^*)(2-s^*-w^*)(g+1)^2} > 0,$$

where the inequality follows from $w^* > s^*$ and $g > 1$. The final equality that $\pi_i^W(s^*, s^*, s^*, s^*) = \pi_i^W(e, e, e, e)$ follows from (12) and (19).

5 References

1. Abrevaya, Jason (2002), "Ladder tournaments and underdogs: lessons from professional bowling," *Journal of Economic Behavior & Organization* 47, 87–101.

2. Amegashie, J. Atsu (2004), "Burning out in sequential elimination contests," working paper.

3. Baik, Kyung Hwan (2004), "Two-player asymmetric contests with ratio-form contest success functions," *Economic Inquiry* 42, 679–689.

4. Baye, Michael R. and Heidrun C. Hoppe (2003), "The strategic equivalence of rent-seeking, innovation, and patent-race games," *Games and Economic Behavior* 44, 217–226.

5. Baye, Michael R., Dan Kovenock, and Casper de Vries (1994), "The solution to the Tullock rent-seeking game when R>2: mixed-strategy equilibria and mean dissipation rates," *Public Choice* 81, 363–380.

6. Baye, Michael R., Dan Kovenock, and Casper de Vries (1993), "Rigging the lobbying process: an application of the all-pay auction," *American Economic Review* 83, 289–294.

7. Baye, Michael R., Dan Kovenock, and Casper de Vries (1996), "The all-pay auction with complete information," *Economic Theory* 8, 291–305.

8. Chan, William, Pascal Courty, and Hao Li (2003), "Suspense," working paper.

9. Dixit, Avinash (1987), "Strategic behavior in contests," *American Economic Review* 77, 891–898.

10. Ehrenberg, Ronald G. and Michael L. Bognanno (1990), "Do tournaments have incentive effects?" *Journal of Political Economy* 98, 1307–1324.

11. Groh, Christian, Benny Moldovanu, Aner Sela, and Uwe Sunde (2003), "Optimal seedings in elimination tournaments," working paper.

12. Klumpp, Tilman and Mattias K. Polborn (2003), "Primaries and the New Hampshire effect," working paper.

13. Lazear, Edward P. and Sherwin Rosen (1981), "Rank-order tournaments as optimum labor contracts," *Journal of Political Economy* 89, 841–864.

14. Loury, Glenn C. (1979), "Market structure and innovation," *Quarterly Journal of Economics* 93, 395-410.

15. McClure, James E. and Lee C. Spector (1997), "Tournament performance and 'agency problems': an empirical investigation of 'March Madness'," *Journal of Economics and Finance* 21, 61–68.

16. Nalebuff, Barry and Joseph Stiglitz (1983), "Prizes and incentives: toward a general theory of compensation and competition," *Bell Journal of Economics* 14, 21–43.

17. Nitzan, Shmuel (1994), "Modelling rent-seeking contests," *European Journal of Political Economy* 10, 41–60.

18. Rosen, Sherwin (1986), "Prizes and incentives in elimination tournaments," *American Economic Review* 76, 701–715.

19. Snyder, James M. (1989), "Election goals and the allocation of campaign resources," *Econometrica* 57, 637–660.

20. Stein, William E. and Amnon Rapoport (2003), "Symmetric multistage contests with budget constraints," working paper.

21. Szymanski, Stefan (2003), "The economic design of sporting contests," *Journal of Economic Literature* 41, 1137–1187.

22. Tullock, Gordon (1967), "The welfare costs of tariffs, monopolies and theft," *Western Economic Journal* 5, 224–232.

23. Tullock, Gordon (1980), "Efficient rent-seeking," in J.M. Buchanan, R. Tollison, and G. Tullock (eds.), *Toward a Theory of the Rent-Seeking Society*, Texas A&M Press, 267–292.