Birefringence

• Many crystalline substances have optical properties that are not the same in all directions within any given sample – these materials are *optically anisotropic.*

• In order to see how this might arise, and some of the consequences, consider light that is incident on the following anisotropic system:

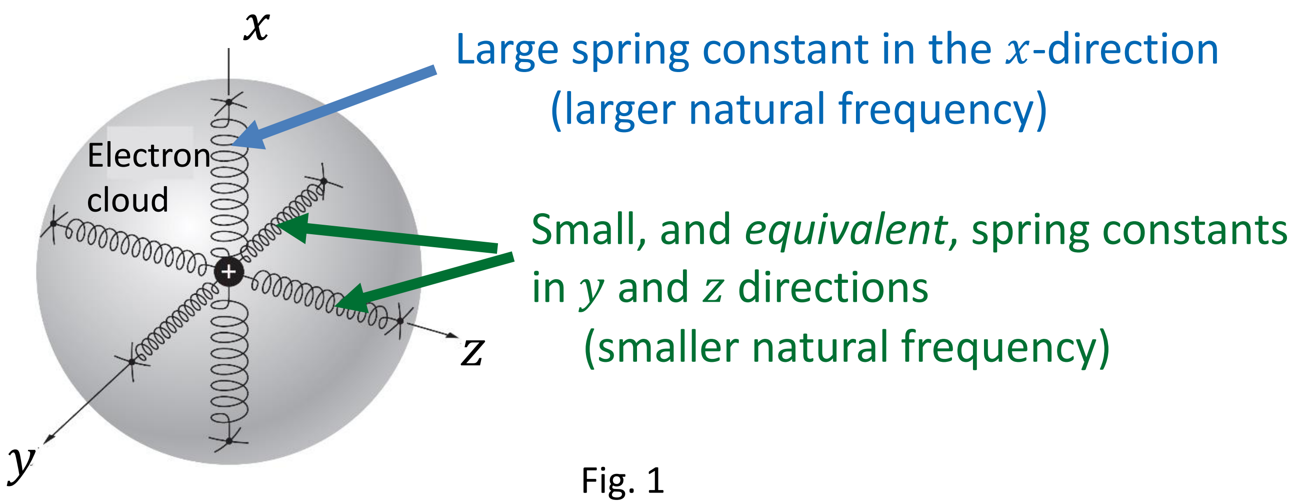


Fig. 1

[Note: The following discussion makes qualitative use of our previous discussion on dispersion $n(\omega)$ in Sec. 3.]

• If the light's $\vec{\mathbf{E}}$ oscillates parallel to the stiff springs, i.e., in a direction of strong binding (x-axis in Fig. 1), the electron's natural frequency ("$\omega_0 = \sqrt{k_s/m}$" ; k_s = spring constant) would be high.

• If instead, the light's $\vec{\mathbf{E}}$ oscillates parallel to the soft springs, i.e., in a direction of weaker binding (y-axis, or z-axis, or in the yz plane), the electron's natural frequency would be smaller.

Consequence:

$n(\omega)$ can be different for different oscillation directions of $\vec{\mathbf{E}}$.

For the hypothetical material displayed in Fig. 1, the behavior of $n(\omega)$ might look qualitatively like:

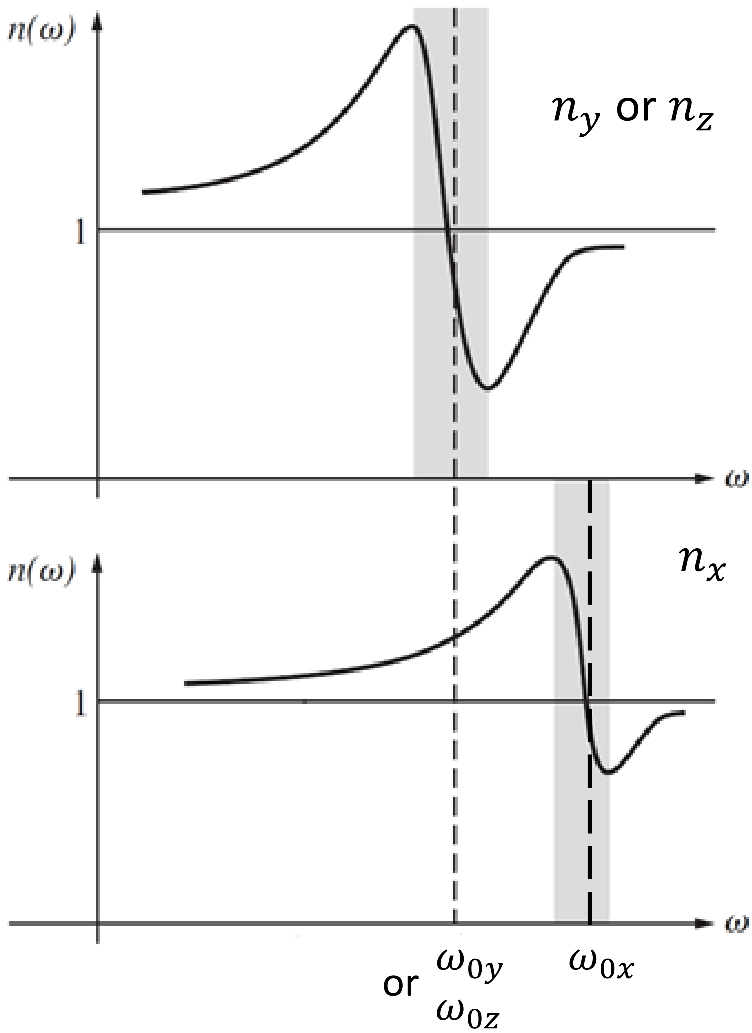


Fig. 2

- Depending on the direction of oscillation of $\vec{\mathbf{E}}$ (the polarization of the light), the incident light with a particular value of ω might "experience" n_x, or n_y $(= n_z)$, or some "combination" of both n_x and n_y $(= n_z)$.

Birefringence (con't)

- **Birefringence** is the optical property of a material having an index of refraction that depends on the light's polarization. A material possessing birefringence is said to be **birefringent.** A common manifestation of birefringence is the existence of *double refraction*, i.e., formation of a double image, due to the existence of the different refractive indices.

Fig. 3

<u>Note</u>: Often the natural frequencies (e.g. $\omega_{0x}, \omega_{0y}, \omega_{0z}$ in Fig. 2) of actual birefringent crystals are significantly above the visible range. Hence, in the visible range, two different indices of refraction are apparent, but absorption effects do not need to be considered. e.g. Recall from our discussion of $n(\omega)$ in Sec. 3 that absorption is only strong near the natural frequencies, such as the shaded regions of Fig. 2.

Optic axis (OA):

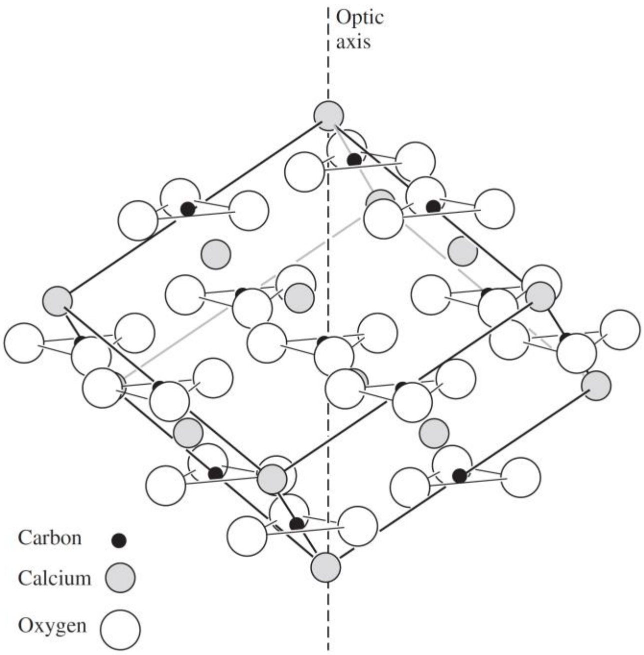
- A ***direction*** (not actually an "axis") in the birefringent crystal along which a ray of transmitted light experiences no birefringence, i.e., ray "experiences" only one refractive index, irrespective of the orientation of its $\vec{\mathbf{E}}$.

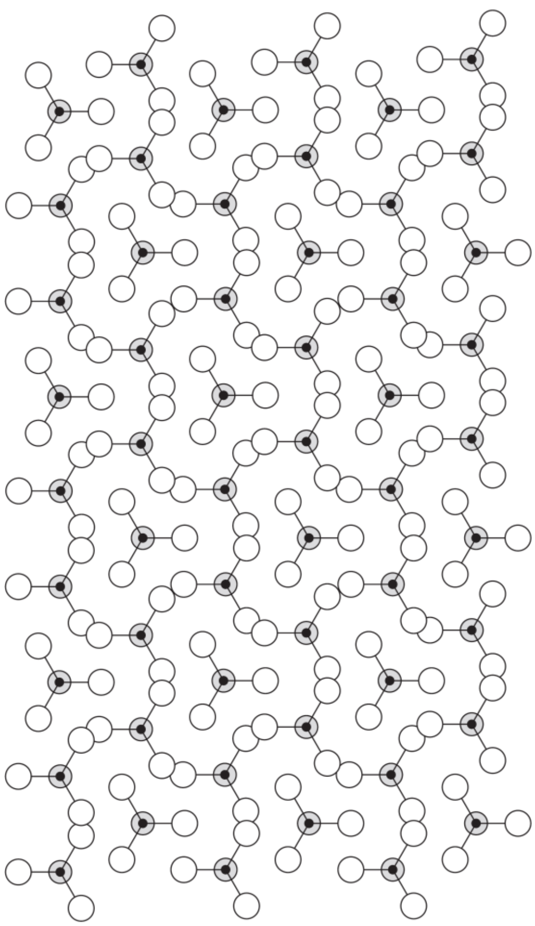
(A beam sent along the OA will not undergo double refraction.)

(Q) For the hypothetical birefringent material displayed in Fig. 1, which axis (e.g. x, y, or z) defines the direction of the optic axis?

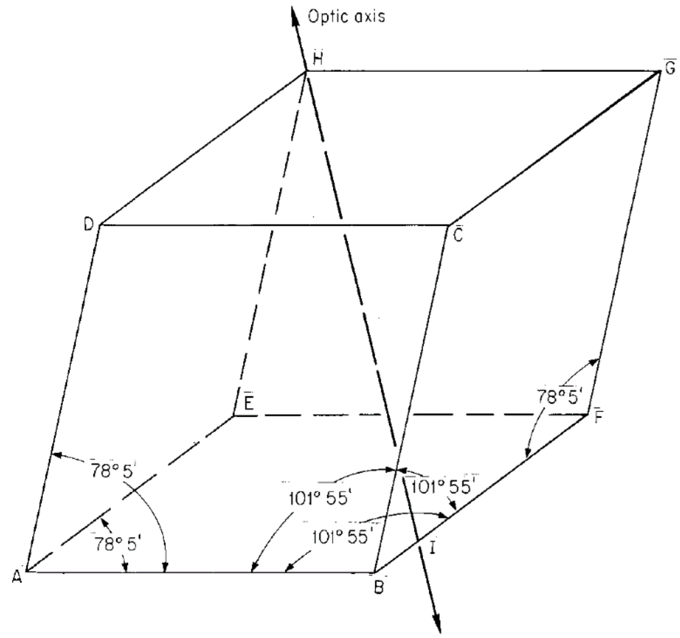
Birefringence (con't)

Calcite ($CaCO_3$) is a well-known birefringent material.





Atomic arrangement for calcite looking down the optic axis



Schematic representation of a rhombohedral calcite crystal showing the angles between faces . The optic axis passes through corner *H* and point *I* on side *BF* .

Fig. 4

Birefringence (con't)

Below is an example of a particular situation regarding a calcite crystal: (from Hecht)

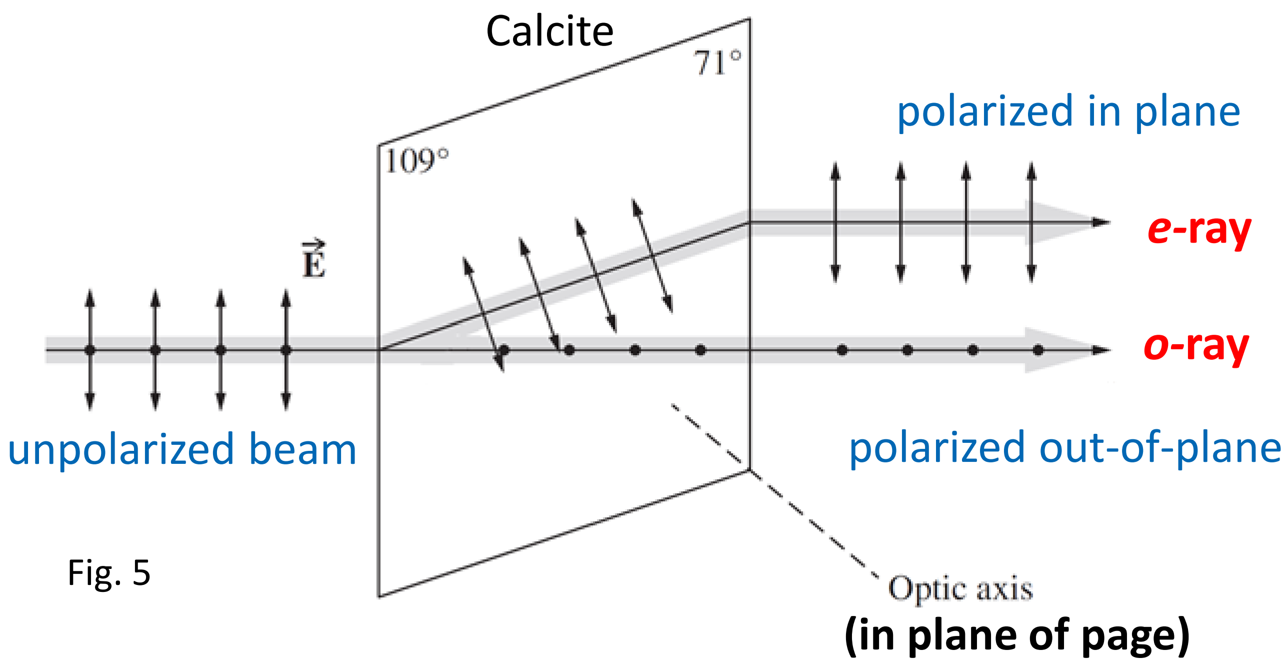


Fig. 5

• Incident unpolarized beam is split into two beams in the calcite, i.e., double refraction.

• Recall that according to Snell's Law, light incident normal to the surface should be transmitted without deviation (i.e., $\theta_t = \theta_i = 0^0$).

⇒ The **ordinary rays**, or ***o*-rays**, obey Snell's Law. Also, notice the ray emerge linearly polarized out-of-plane.

⇒ The **extraordinary rays**, or ***e*-rays**, do *not* obey Snell's Law. Although $\theta_i = 0^0$ on the left surface, $\theta_t \neq 0^0$. Also, notice the rays emerge linearly polarized in-plane.

(An upcoming handout presents a simplified model, due to Huygen, for the above behavior.)

Birefringence (con't)

Experimentally:

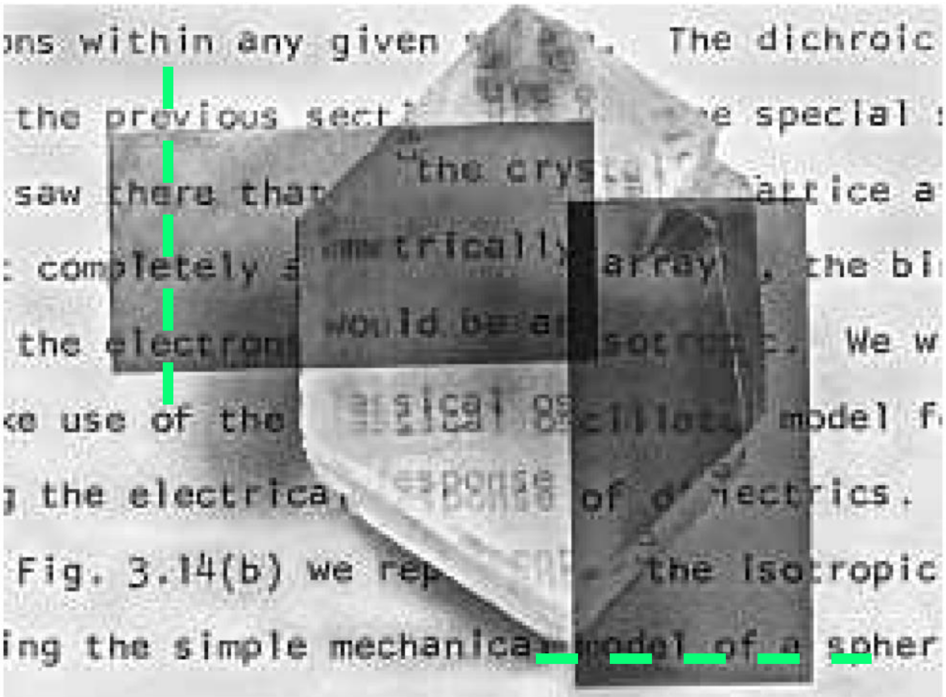


Fig. 6

- The transmission axes of the two polarizers are parallel to their short edges (indicated by the dashed green lines).

- Where the image is doubled, the lower, un-deflected one is the *ordinary* image. It is polarized "horizontally" in the photograph.

- Where the image is doubled, the upper, deflected one is the *extraordinary* image. It is polarized "vertically" in the photograph.

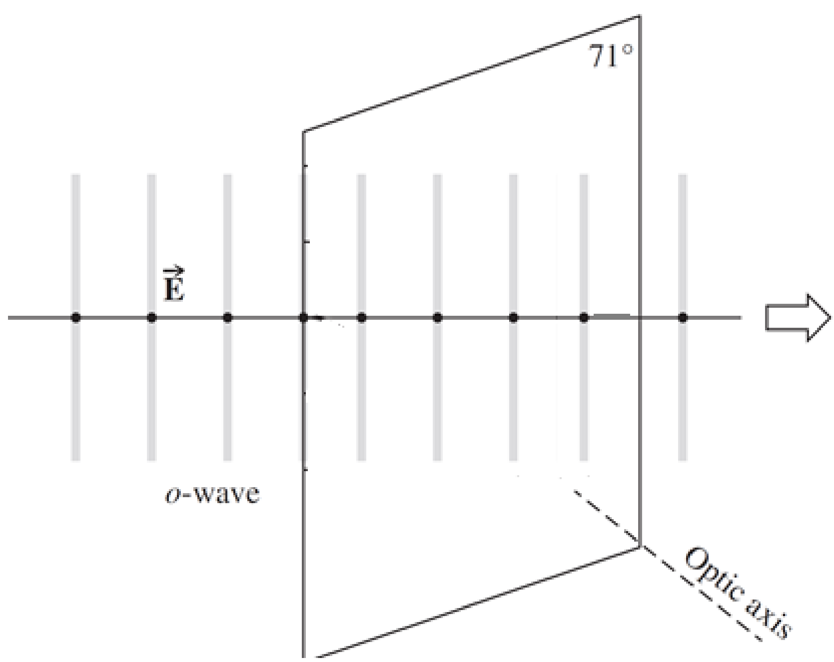
Note the emerging P-states (linearly polarized) are orthogonal.

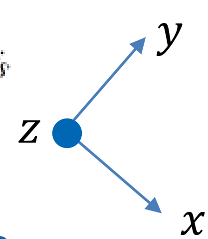
Birefringence (con't)

- In general, the behavior of a beam incident on the birefringent material will depend on the orientation of its $\vec{\mathbf{E}}$ with respect to the optic axis (OA).

(i) If $\vec{\mathbf{E}}$ is fully ***perpendicular*** to the OA, the ray will "see" one refractive index, i.e., the one that is ⊥ to the OA. Instead of labelling it as $n_\perp$, this refractive index is conventionally labelled as n_o. **The ray is an "*o*-ray", and obeys Snell's Law with $\boldsymbol{n = n_o}$.** In the material, the o-ray will have a speed of $v_\perp = c/n_o$.

e.g. The o-ray in Fig. 4. It is associated with the electric field component of the incident unpolarized light which is oscillating perpendicular to the plane of the page (represented by the solid black dots).



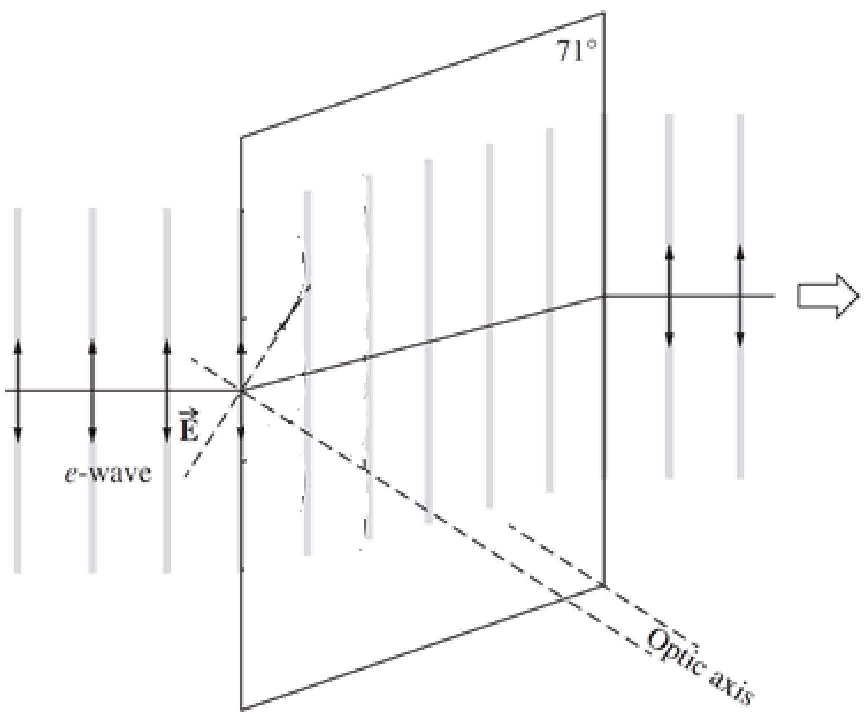


$\vec{\mathbf{E}}$ is oscillating along z-axis, always perpendicular to the optic axis, and sees $n_z = n_o$

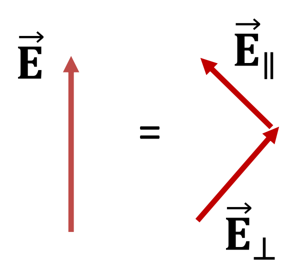
Birefringence (con't)

(ii) If $\vec{\mathbf{E}}$ has components **parallel and perpendicular** to the OA, it will "see" ***both*** n_o and n_e. The label n_e is conventionally used to label the refractive index ∥ to the OA. The ray is an "*e*-ray", and will *not* obey Snell's Law. (The component ∥ to the OA propagates with a velocity determined by n_e, and the component ⊥ to the OA propagates with velocity determined by n_o.) The resulting behavior of the ray can be "complicated" and we will **not** be discussing it mathematically.

e.g. The e-ray in Fig. 4. It is associated with the electric field component of the incident un-polarized light which is oscillating *in the plane* of the page.



Notice the incident $\vec{\mathbf{E}}$ in the figure can be decomposed into components ∥ and ⊥ to the OA.

e.g. 

(iii) If $\vec{\mathbf{E}}$ is **fully parallel** to the OA, it will "see" one refractive index n_e (the refractive index ∥ to the OA). **The ray is still considered to an *e*-ray**, but in this special case, **it will still obey Snell's Law with $\boldsymbol{n = n_e}$**. In the material, the e-ray will have a speed of $v_{\parallel} = c/n_e$.

e.g. Not relevant in Fig. 4, but will be important for later discussions.

Qualitative Example:

- Suppose a light beam is incident on a birefringent crystal along its optic axis.

$\Rightarrow$ Within the crystal, all possible orientations of the ray's $\vec{\mathbf{E}}$ are $\perp$ to the OA.

$\Rightarrow$ The ray will always "see" $n_\perp$ (the refractive index perpendicular to the OA), which in the literature is labelled as n_o. It is an o-ray, and Snell's Law is obeyed with the value of n_o for the refractive index.

$\Rightarrow$ Since Snell's Law is obeyed, there is no double refraction, i.e., no splitting of the beam.

$\Rightarrow$ The speed of this ray that is propagating along the OA is $\mathrm{v} = c/n_o$

Birefringence (con't)

Refractive Indices of Some Uniaxial Birefringent Crystals (λ_0 = 589.3 nm)

Crystal	n_o	n_e	type
Calcite	1.6584	1.4864	negative
Ice	1.309	1.313	positive
KDP	1.51	1.47	negative
Lithium niobate	2.30	2.21	negative
Quartz	1.5443	1.5534	positive
Rutile (TiO_2)	2.616	2.903	positive
Sodium nitrate	1.5854	1.3369	negative
Tourmaline	1.669	1.638	negative

Uniaxial = only one optic axis.

Birefringence = $\Delta n = n_e - n_o$

$\Rightarrow$ if birefringence <0, then material is *negative uniaxial*
(e.g. calcite, $\Delta n = -0.172$)

$$n_e < n_o\ ;\ v_\parallel > v_\perp$$

$\Rightarrow$ if birefringence >0, then material is *positive uniaxial*

(e.g. ice, quartz, rutile)

$$n_e > n_o\ ; v_\perp > v_\parallel$$

Extra Info: **Biaxial** birefringent crystals have two OAs and three refractive indices.