TWO-WAY RELAY BEAMFORMING DESIGN: PROPORTIONAL FAIR AND MAX-MIN RATE FAIR APPROACHES USING POTDC

Arash Khabbazibasmenj[†] *and* *Sergiy A. Vorobyov*[†,*]

[†]Dept. of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada
[*] School of Electrical Engineering, Aalto University, Espoo, Finland
Emails: {khabbazi,svorobyo}@ualberta.ca

ABSTRACT

The challenge in designing relay beamforming in two-way relaying systems is the non-convex nature of the corresponding optimization problem. In this work, we concentrate on the mathematical issues of such design for the cases when the max-min rate and proportional fairness are used as the design criteria. We show that the corresponding optimization problems belong to the class of difference-of-convex functions (DC) programming problems. Due to the specific structure of the corresponding DC problems, they can be efficiently addressed by using the polynomial-time DC (POTDC) algorithm which guarantees to find the Karush-Kuhn-Tucker (KKT) optimal point in polynomial-time. We have also shown earlier that the question of global optimality of the POTDC algorithm boils down to a simple numerical convexity check for a certain one-dimensional optimal value function.

***Index Terms*—** Difference-of-convex functions optimization, Max-min rate fairness, Proportional fairness, Two-way relaying.

1. INTRODUCTION

Two-way relaying (TWR) is a certain realization of the network coding [1] in which both terminals transmit their signals to the relay simultaneously through a multiple access channel (MAC) [2]. After receiving the transmitted signals corrupted by the additive noise, relay processes the mixture and then broadcasts it to the terminals. The most common relaying protocols are amplify-and-forward (AF) [3] and decode-and-forward (DF) [4]. In this work, it is assumed that the relay uses the AF relaying protocol which is more practical compared to other protocols in terms of the processing delay and processing energy consumption.

One fundamental problem associated with TWR systems is the relay beamforming design based on the available channel state information (CSI) [5]–[12]. It is usually designed so that a specific performance criterion is optimized under constraints on the available resources and/or quality of service (QoS) requirements. The optimization criterion for most of the relay beamforming methods is the sum-rate [5]-[10].

The importance of the user fairness in asymmetric TWR systems has been recently demonstrated in [2], [11] and [12]. The authors of [2] study the optimal power allocation problem for single antenna users and single antenna relay where the sum-rate is maximized under the fairness constraint. Relay beamforming and optimal power allocation for a pair of single antenna users and several single antenna relays based on max-min signal-to-noise ratio (data rate) has been also considered in [11] and [12].

The main difficultly of the relay beamforming design in TWR system is the non-convex nature of the corresponding optimization problem. In this work, we study the relay beamforming for two single antenna users and one AF multi-antenna (multiple-input multiple-output (MIMO)) relay when the max-min rate and proportional fairness are used as the design criteria. It is shown that the corresponding optimization problems can be recast as difference-of-convex functions (DC) programming problems. Although DC problems do not generally have polynomial-time solution, we handle the corresponding optimization problems using the so-called polynomial-time DC (POTDC) algorithm, which guarantees to find the Karush-Kuhn-Tucker (KKT) optimal point in polynomial time. Moreover, the global optimality can be checked by a simple numerical test.

2. SYSTEM MODEL

Consider two single antenna terminals that communicate via a MIMO AF relay equipped with M_R antennas through frequency-flat quasi-static block fading channels. Every data transmission between the terminals takes place in two phases. In the first phase, both terminals transmit their signals to the relay simultaneously. Then the received signal at the relay, which is a combination of both transmitted signals, can be expressed as

$$\mathbf{r} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{n}_R \tag{1}$$

where $\mathbf{h}_i = [h_{i,1}, \ldots, h_{i,M_R}]^T \in \mathbb{C}^{M_R}$ denotes the channel vector between terminal i and the relay, x_i is the transmitted symbol from terminal i, $\mathbf{n}_R \in \mathbb{C}^{M_R}$ is the additive

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noise at the relay, and $(\cdot)^T$ stands for the transpose. Let $P_{T,i} \triangleq \mathbb{E}\{|x_i|^2\}$ and $\mathbf{R}_{N,R} \triangleq \mathbb{E}\{\mathbf{n}_R\mathbf{n}_R^H\}$ denote the average transmit power of terminal i and noise covariance matrix at the relay, respectively, where $\mathbb{E}\{\cdot\}$ and $(\cdot)^H$ stand for the mathematical expectation and the Hermitian transpose, respectively.

In the second phase, relay amplifies the received signal through multiplying it by the relay beamforming matrix $\mathbf{G} \in \mathbb{C}^{M_R \times M_R}$ as $\bar{\mathbf{r}} = \mathbf{G}\mathbf{r}$, and then retransmits it to the terminals. The signals received by the terminals can be expressed as

$$y_1 = h_{1,1}^{(e)} x_1 + h_{1,2}^{(e)} x_2 + \tilde{n}_1 \quad (2)$$

$$y_2 = h_{2,2}^{(e)} x_2 + h_{2,1}^{(e)} x_1 + \tilde{n}_2 \quad (3)$$

where $h_{i,j}^{(e)} = \mathbf{h}_i^T \mathbf{G} \mathbf{h}_j$ denotes the effective channel from terminal j to terminal i for $i,j = 1,2$ and $\tilde{n}_i = \mathbf{h}_i^T \mathbf{G} \mathbf{n}_R + n_i$ is the effective noise at terminal i which contains the terminal's own noise and the noise forwarded by the relay. The self-interference can be subtracted by each terminal since its own transmitted signal is known. After canceling the self-interference, the TWR system is decoupled into two parallel single-user single-input single-output (SISO) systems. Then the rate of terminal i can be expressed as $r_i = (1/2) \cdot \ln\left(1 + P_{R,i}/\tilde{P}_{N,i}\right)$, where $P_{R,1} \triangleq \mathbb{E}\left\{\left|h_{1,2}^{(e)} x_2\right|^2\right\}$, $P_{R,2} \triangleq \mathbb{E}\left\{\left|h_{2,1}^{(e)} x_1\right|^2\right\}$, and $\tilde{P}_{N,i} \triangleq \mathbb{E}\left\{|\tilde{n}_i|^2\right\}$ are the powers of the desired signals and the effective noise term at terminal i. Note that the factor $1/2$ results from the two time slots needed for the bidirectional transmission.

Using the vectorization of the relay beamforming matrix, i.e., introducing $\mathbf{g} = \text{vec}\{\mathbf{G}\}$, where $\text{vec}\{\cdot\}$ denotes the vectorization operator, the powers of the desired signals at the terminals, the relay transmission power and the power of effective noise term at the terminals can be equivalently expressed as the following quadratic forms of $\mathbf{g}$ [9]:

$$P_{R,1} = \mathbf{g}^H \mathbf{K}_{2,1} \mathbf{g} P_{T,2}, \quad P_{R,2} = \mathbf{g}^H \mathbf{K}_{1,2} \mathbf{g} P_{T,1} \quad (4)$$

$$\mathbb{E}\{\|\bar{\mathbf{r}}\|_2^2\} = \mathbf{g}^H \mathbf{Q} \mathbf{g}, \qquad \tilde{P}_{N,i} = \mathbf{g}^H \mathbf{J}_i \mathbf{g} + P_{N,i} \quad (5)$$

where $\mathbf{K}_{i,j} \triangleq \left[\left(\mathbf{h}_i \mathbf{h}_i^H\right) \otimes \left(\mathbf{h}_j \mathbf{h}_j^H\right)\right]^T$, $\mathbf{Q} \triangleq \mathbf{R}_R^T \otimes \mathbf{I}_{M_R}$, $\mathbf{J}_i \triangleq \left[\mathbf{R}_{N,R} \otimes \left(\mathbf{h}_i \mathbf{h}_i^H\right)\right]^T$, $\otimes$ denotes the Kronecker product, $\mathbf{I}_{M_R}$ is the identity matrix, and $\mathbf{R}_R \triangleq \mathbb{E}\{\mathbf{r}\mathbf{r}^H\}$ is the covariance matrix of the received signal at the multi-antenna relay, which can be derived as

$$\mathbf{R}_R = \mathbf{h}_1 \mathbf{h}_1^H P_{T,1} + \mathbf{h}_2 \mathbf{h}_2^H P_{T,2} + \mathbf{R}_{N,R}. \quad (6)$$

3. MAIN RESULTS

Proportional Fairness : Proportional fairness has been initially introduced and applied in game theory [13]. In application to the resource allocation problem, it is known to provide a good trade-off between the maximum sum-rate and the user fairness [14]. It is also well-known that a proportionally fair resource allocation/beamformer maximizes the sum of the logarithmic average sum-rate [15]. The relay beamforming problem based on the proportional fairness criterion and subject to the total power constraint of the relay can be expressed as the following optimization problem:

$$\mathbf{g}_{\text{opt}} = \underset{\mathbf{g}^H \mathbf{Q} \mathbf{g} \leq P_{T,R}}{\arg\max} \; \frac{1}{4} \ln\Big(1 + \frac{P_{R,1}}{\tilde{P}_{N,1}}\Big) \ln\Big(1 + \frac{P_{R,2}}{\tilde{P}_{N,2}}\Big) \quad (7)$$

where $P_{T,R}$ denotes the total transmit power of the relay and the rate is measured in nats per second. Using the fact that the received signal-to-noise ratio (SNR) of both terminals is an increasing function of the norm of $\mathbf{g}$ in a fixed direction of $\mathbf{g}$ [9], the inequality constraint of the optimization problem (7) must be satisfied with equality at optimality. Based on the latter fact and also replacing $P_{N,i}$ in the definition of $\tilde{P}_{N,i}$ (5) by $(P_{N,i}/P_{T,R}) \cdot \mathbf{g}^H \mathbf{Q} \mathbf{g}$, the optimization problem (7) can be expressed as the following homogeneous problem

$$\mathbf{g}_{\text{opt}} = \arg\max_{\mathbf{g}} \ln\Big(\frac{\mathbf{g}^H \mathbf{A}_1 \mathbf{g}}{\mathbf{g}^H \mathbf{B}_1 \mathbf{g}}\Big) \ln\Big(\frac{\mathbf{g}^H \mathbf{A}_2 \mathbf{g}}{\mathbf{g}^H \mathbf{B}_2 \mathbf{g}}\Big) \quad (8)$$

where $\mathbf{B}_i \triangleq \mathbf{J}_i + \frac{P_{N,i}}{P_{T,R}} \mathbf{Q}$, $\mathbf{A}_1 \triangleq \mathbf{K}_{2,1} P_{T,2} + \mathbf{B}_1$, and $\mathbf{A}_2 \triangleq \mathbf{K}_{1,2} P_{T,1} + \mathbf{B}_2$. We recall that a function is referred to as $0-$homogenous or shortly as homogenous if the value of the function at any arbitrary point is independent of any positive scaling [16]. Note that since the objective function in (8) is homogeneous, the equality constraint can be dropped. Moreover, since logarithm is a strictly increasing function and the objective function of the optimization problem (7) or, equivalently, (8) is positive, by taking logarithm of (8), the problem can be equivalently recast as

$$\mathbf{g}_{\text{opt}} = \arg\max_{\mathbf{g}} \ln\bigg(\ln\Big(\frac{\mathbf{g}^H \mathbf{A}_1 \mathbf{g}}{\mathbf{g}^H \mathbf{B}_1 \mathbf{g}}\Big)\bigg) + \ln\bigg(\ln\Big(\frac{\mathbf{g}^H \mathbf{A}_2 \mathbf{g}}{\mathbf{g}^H \mathbf{B}_2 \mathbf{g}}\Big)\bigg). \quad (9)$$

Using the fact that the problem (9) is homogeneous and defining additional variables α and β, (9) can be equivalently rewritten as

$$\max_{\mathbf{g},\alpha,\beta} \; \ln\Big(\ln(\mathbf{g}^H \mathbf{A}_1 \mathbf{g})\Big) + \ln\Big(\ln(\alpha) - \ln(\beta)\Big)$$
$$\text{s.t.} \quad \mathbf{g}^H \mathbf{B}_1 \mathbf{g} = 1, \; \mathbf{g}^H \mathbf{A}_2 \mathbf{g} = \alpha, \; \mathbf{g}^H \mathbf{B}_2 \mathbf{g} = \beta. \quad (10)$$

Defining the matrix $\mathbf{X} \triangleq \mathbf{g}\mathbf{g}^H$ and using the semidefinite relaxation (SDR), i.e., dropping the rank one constraint, the problem (10) can be further expressed as

$$\max_{\mathbf{X},\alpha,\beta} \ln\bigg(\ln\Big(\text{tr}(\mathbf{A}_1 \mathbf{X})\Big)\bigg) + \ln\Big(\ln(\alpha) - \ln(\beta)\Big)$$
$$\text{s.t.} \quad \text{tr}(\mathbf{B}_1 \mathbf{X}) = 1, \; \text{tr}(\mathbf{A}_2 \mathbf{X}) = \alpha, \text{tr}(\mathbf{B}_2 \mathbf{X}) = \beta. \quad (11)$$

The optimal solution of the problem (10) can be easily extracted from the optimal solution of the problem (11) using rank reduction techniques (see [9] and [17]).

Defining the additional variable γ, the problem (11) can be finally recast as

$$\max_{\mathbf{X},\alpha,\beta,\gamma} \ln\Big(\ln\big(\mathrm{tr}(\mathbf{A}_1\mathbf{X})\big)\Big) + \ln\big(\ln(\alpha) - \gamma\big)$$
$$\text{s.t.} \quad \mathrm{tr}(\mathbf{B}_1\mathbf{X}) = 1, \qquad \mathrm{tr}(\mathbf{A}_2\mathbf{X}) = \alpha$$
$$\mathrm{tr}(\mathbf{B}_2\mathbf{X}) = \beta, \qquad \ln(\beta) \leq \gamma. \tag{12}$$

This is a DC programming problem and it has similar mathematical structure to the problem that we addressed in [9]. It can be easily handled using the POTDC algorithm. Indeed, the objective function of (12) is concave and all the constraints except the last one are convex where the last one is a DC constraint. The POTDC algorithm finds the KKT optimal point for such type of DC programming problems [9]. Moreover, the global optimality of the POTDC algorithm reduces to the question of convexity of a certain one-dimensional optimal value function (see [18] for details). Such convexity can be easily checked numerically by using the convexity on lines property of the convex functions. The corresponding numerical test then can be viewed as a simple global optimality test.

Max-Min Rate Fairness: The max-min rate fair resource allocation/beamformer aims at maximizing the minimum received rate for each terminal subject to the total power constraint at the relay. The corresponding optimization problem can be expressed as

$$\mathbf{g}_{\mathrm{opt}} = \arg\max_{\mathbf{g}^H\mathbf{Q}\mathbf{g}\leq P_{T,R}} \min\left\{\frac{1}{2}\ln\Big(1+\frac{P_{R,1}}{\tilde{P}_{N,1}}\Big), \frac{1}{2}\ln\Big(1+\frac{P_{R,2}}{\tilde{P}_{N,2}}\Big)\right\}. \tag{13}$$

Similar to the proportional fairness beamforming problem (7), this problem can be equivalently expressed as the following homogeneous problem

$$\mathbf{g}_{\mathrm{opt}} = \arg\max_{\mathbf{g}} \min\left\{\ln\Big(\frac{\mathbf{g}^H\mathbf{A}_1\mathbf{g}}{\mathbf{g}^H\mathbf{B}_1\mathbf{g}}\Big), \ln\Big(\frac{\mathbf{g}^H\mathbf{A}_2\mathbf{g}}{\mathbf{g}^H\mathbf{B}_2\mathbf{g}}\Big)\right\}. \tag{14}$$

Defining the additional variables α and β and using the fact that the problem (14) is homogeneous, (14) can be equivalently recast as

$$\max_{\alpha,\beta} \max_{\mathbf{g}} \min\left\{\ln\big(\mathbf{g}^H\mathbf{A}_1\mathbf{g}\big), \ln(\alpha) - \ln(\beta)\right\}$$
$$\text{s.t.} \quad \mathbf{g}^H\mathbf{B}_1\mathbf{g} = 1, \mathbf{g}^H\mathbf{A}_2\mathbf{g} = \alpha, \mathbf{g}^H\mathbf{B}_2\mathbf{g} = \beta. \tag{15}$$

Exchanging the order of maximum and minimum in the objective of (15) can simplify this problem significantly so that the POTDC algorithm can then be directly applied to it. The following lemma considers the possibility of such exchange in the order of maximum and minimum.

Lemma 1: For fixed values of α and β, the following optimization problems have the same optimal values, i.e., $p_1 = p_2$,

$$p_1 \triangleq \max_{\mathbf{g}} \min\left\{ \ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}), \ln(\alpha) - \ln(\beta) \right\}$$
$$\text{s.t.} \quad \mathbf{g}^H\mathbf{B}_1\mathbf{g} = 1, \mathbf{g}^H\mathbf{A}_2\mathbf{g} = \alpha, \mathbf{g}^H\mathbf{B}_2\mathbf{g} = \beta \tag{16}$$

and

$$p_2 \triangleq \min\left\{ \max_{\mathbf{g}} \ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}), \ln(\alpha) - \ln(\beta) \right\}$$
$$\text{s.t.} \quad \mathbf{g}^H\mathbf{B}_1\mathbf{g} = 1, \quad \mathbf{g}^H\mathbf{A}_2\mathbf{g} = \alpha$$
$$\mathbf{g}^H\mathbf{B}_2\mathbf{g} = \beta. \tag{17}$$

Proof. We define first the feasible set of the optimization problems (16) and (17) as

$$\mathcal{S} = \{\mathbf{g} \mid \mathbf{g}^H\mathbf{B}_1\mathbf{g} = 1,\ \mathbf{g}^H\mathbf{A}_2\mathbf{g} = \alpha,\ \mathbf{g}^H\mathbf{B}_2\mathbf{g} = \beta\}.$$

Two different cases are possible. If for every $\mathbf{g} \in \mathcal{S}$, $\ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}) \leq \ln(\alpha) - \ln(\beta)$, then it can be easily verified that

$$p_1 = \max_{\mathbf{g}\in\mathcal{S}} \ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}). \tag{18}$$

Furthermore, since for $\mathbf{g} \in \mathcal{S}$, $\ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}) \leq \ln(\alpha) - \ln(\beta)$, it is also true that $\max_{\mathbf{g}\in\mathcal{S}} \ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}) \leq \ln(\alpha) - \ln(\beta)$ and therefore

$$p_2 = \max_{\mathbf{g}\in\mathcal{S}} \ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}). \tag{19}$$

Hence, trivially $p_1 = p_2$.

In the other case, let D denote the set of all vectors $\mathbf{g} \in \mathcal{S}$ such that $\ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}) > \ln(\alpha) - \ln(\beta)$ and let $\tilde{D}$ denote its complement. Considering the inner minimization problem of the problem (16), it can be simply concluded that p_1 is the maximum of the following function over $\mathbf{g} \in \mathcal{S}$

$$k(\mathbf{g}) \triangleq \begin{cases} \ln(\alpha) - \ln(\beta), & \mathbf{g} \in D \\ \ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}), & \mathbf{g} \in \tilde{D}. \end{cases} \tag{20}$$

Since for $\mathbf{g} \in \tilde{D}$, $k(\mathbf{g}) = \ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}) \leq \ln(\alpha) - \ln(\beta)$, it is resulted that $p_1 = \ln(\alpha) - \ln(\beta)$.

Moreover, since for $\mathbf{g} \in D$, $\ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}) > \ln(\alpha) - \ln(\beta)$, it is also true that $\max_{\mathbf{g}\in\mathcal{S}} \ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}) > \ln(\alpha) - \ln(\beta)$. Therefore $p_2 = \ln(\alpha) - \ln(\beta)$ that completes the proof. □

Using Lemma 1, the optimization problem (15) can be equivalently rewritten as

$$\max_{\alpha,\beta} \min\left\{ \max_{\mathbf{g}} \ln(\mathbf{g}^H\mathbf{A}_1\mathbf{g}), \ln(\alpha) - \ln(\beta) \right\}$$
$$\text{s.t.} \quad \mathbf{g}^H\mathbf{B}_1\mathbf{g} = 1, \quad \mathbf{g}^H\mathbf{A}_2\mathbf{g} = \alpha$$
$$\mathbf{g}^H\mathbf{B}_2\mathbf{g} = \beta. \tag{21}$$

In terms of the matrix $\mathbf{X} \triangleq \mathbf{g}\mathbf{g}^H$ and using SDR, the problem (21) can be further expressed as

$$\max_{\alpha,\beta} \min\left\{ \max_{\mathbf{X}} \ln(\mathrm{tr}(\mathbf{A}_1\mathbf{X})), \ln(\alpha) - \ln(\beta) \right\}$$
$$\text{s.t.} \quad \mathrm{tr}(\mathbf{B}_1\mathbf{X}) = 1, \quad \mathrm{tr}(\mathbf{A}_2\mathbf{X}) = \alpha$$
$$\mathrm{tr}(\mathbf{B}_2\mathbf{X}) = \beta \tag{22}$$

where the optimal solution of the problem (21) can be extracted form the optimal solution of the problem (22) as it

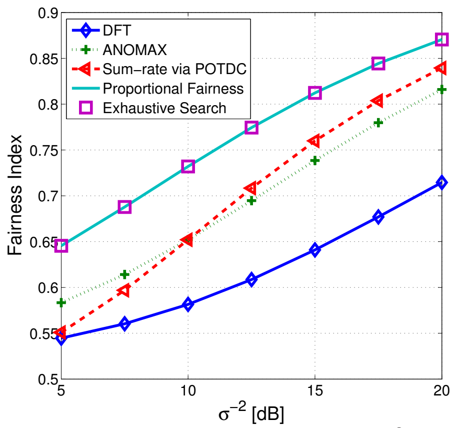

Fig. 1. Fairness index versus σ^{-2}.

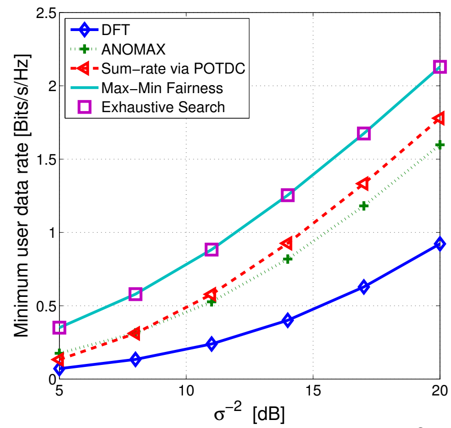

Fig. 2. Minimum data rate versus σ^{-2}.

has been shown in [9]. Therefore, these two problems are equivalent. Eventually, defining the additional variable t, the problem (22) can be recast as

$$\begin{aligned} \max_{\alpha,\beta,\mathbf{X},t} \quad & t \\ \text{s.t.} \quad & \text{tr}(\mathbf{B}_1\mathbf{X}) = 1, \qquad \text{tr}(\mathbf{A}_2\mathbf{X}) = \alpha \\ & \text{tr}(\mathbf{B}_2\mathbf{X}) = \beta, \qquad \ln(\,\text{tr}(\mathbf{A}_1\mathbf{X})\,) \geq t \\ & \ln(\alpha) - \ln(\beta) \geq t. \end{aligned} \tag{23}$$

The objective function is concave and all the constraints of the problem (23) except the last constraint are convex. The last constraint is the DC constraint. Thus, the problem (23) can be addressed using the POTDC algorithm [9] as well.

4. SIMULATION RESULTS

Two single antenna users are communicating through an AF MIMO relay equipped with $M_R = 3$ antennas. The transmit power of the users and the total transmit power of the relay are all equal to 1. All the channels are modeled as Rayleigh fading with variances equal to 4. The noise power for all the antenna elements is assumed to be the same, and it is denoted as σ^2 except the second user whose noise power is 20 dB larger than for the rest of the noises. The difference in noise power is used for modeling the asymmetric environmental conditions for the users. In order to generate each point in the simulations, 50 independent simulation runs are used.

We compare the proposed proportional fairness and the max-min fairness beamforming methods with other relay beamforming methods in terms of the fairness index defined as $(r_1^2 + r_2^2)/(2 \cdot (r_1 + r_2)^2)$ [19], [20] and the minimum user data rate, respectively. The methods used for performance comparison are the sum-rate maximization based relay beamforming of [9], the algebraic norm-maximizing (ANOMAX) transmit strategy of [21], and the discrete Fourier transform (DFT) beamforming. The latter is used as a bench mark which does not use any CSI [21]. Furthermore, the proposed relay beamforming methods are compared with the global optima of the optimization problems (7) and (13) found using exhaustive search. Fig. 1 shows the fairness index for the proportional fairness beamforming method in comparison to that of the other methods tested versus σ^{-2}. Fig. 2 shows the minimum data rate of the max-min fairness beamformer in comparison to that of the other aforementioned methods also versus σ^{-2}. From these figures, it can be seen that the proposed methods outperform the other state-of-art relay beamforming methods in the scenario with the noise power asymmetry at the terminals. Moreover, the POTDC algorithm is able to find global optima of the corresponding optimization problems.

5. CONCLUSION

The relay beamforming design problem for AF TWR based on the max-min rate and proportional fairness criteria has been studied. It is shown that these design problems can be recast as DC programming problems which can be efficiently addressed using the POTDC algorithm. The POTDC algorithm is guaranteed to find the KKT optimal point. Moreover, its global optimality in each specific case can be easily checked by the means of a simple numerical global optimality test that aims at ensuring that a certain one-dimensional optimal value function is convex.

6. RELATION TO PRIOR WORK

We have recently shown in [9] and [10] that the relay beamforming design problem in AF TWR systems based on the sum-rate maximization criterion belongs to the class of DC programming problems. Although, DC problems are NP-hard in general, we have developed the so-called POTDC algorithm for addressing such problems. This work is an extension of the aforementioned contributions for the cases when the max-min rate and proportional fairness are used as the design criteria. The importance of the fairness in TWR systems has been recently highlighted in [2], [11] and [12].

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