SUM RATE MAXIMIZATION FOR MULTI-PAIR TWO-WAY RELAYING WITH SINGLE-ANTENNA AMPLIFY AND FORWARD RELAYS

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ABSTRACT

We consider a multi-pair two-way relay network with multiple single antenna amplify-and-forward relays. The sum rate maximization problem subject to a total transmit power constraint is studied for such network. The optimization problem is non-convex. First, we show that the problem is a monotonic optimization problem and propose a polyblock approximation algorithm for obtaining the global optimum. However, this algorithm is only suitable for benchmarking because of its high computational complexity. After observing that the necessary optimality condition for our problem is similar to that of the generalized eigenvalue problem, we propose to use the generalized power iterative algorithm which can approach the global optimum recursively. Finally, we propose the total signal-to-interference-plus-noise ratio (SINR) eigen-beamformer which is a closed-form suboptimal solution that reduces the computational complexity significantly. Simulation results show that the proposed algorithms outperform the existing scheme. Moreover, the total SINR eigen-beamformer almost achieves the performance of the optimal solution.

Index Terms— Two-way relaying, amplify and forward, sum rate maximization, monotonic optimization.

1. INTRODUCTION

Relay networks are important for future mobile networks since they can improve the network performance by extending the coverage and increasing the network capacity. In contrast to one-way relaying, two-way relaying techniques can compensate the spectral efficiency loss due to the half-duplex constraint of the relay and, therefore, use the radio resources in a more efficient manner. The optimal beamforming design for the sum rate maximization in two-way amplify and forward (AF) relay networks with one pair of users has been studied in [1] and [2]. Only a few references deal with multi-pair two-way AF relay networks, which include adaptive power allocation [3] and distributed beamforming [4]. Moreover, the optimum beamforming design for maximizing the sum rate of this system has not been studied prior to our work.

In this paper, we consider the problem of maximizing the sum rate of the multi-pair two-way AF relaying network with a total power budget. This optimization problem is non-convex. Since the framework of monotonic optimization is applicable to our task, we first propose a global optimization approach which is based on the polyblock approximation algorithm. However, because of its high computational complexity, this algorithm is only suitable for benchmarking. Afterwards, observing that the optimality conditions of our problem have a similar structure as the generalized eigenvalue problem, we apply a recursive algebraic solution, i.e., the generalized power iterative algorithm [5] which is also globally optimal if there is a dominant eigenvalue. To reduce the computational complexity, we propose the total signal-to-interference-plus-noise ratio (SINR) eigen-beamformer which is a closed-form suboptimal solution. Simulation results show that all the proposed algorithms outperform the existing scheme in [4]. Moreover, the total SINR eigen-beamformer is very close to the optimal solution when there is a small number of relays and it suffers only a small loss in the high SNR regime when many relays exist.

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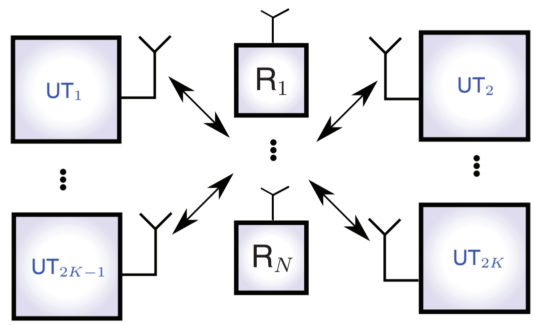

Fig. 1. Multi-pair two-way relaying with multiple single-antenna amplify and forward relays.

Notation: Uppercase and lower case bold letters denote matrices and vectors, respectively. The expectation operator, trace of a matrix, transpose, conjugate, and Hermitian transpose are denoted by $\mathbb{E}\{\cdot\}$, $\mathrm{Tr}\{\cdot\}$, $\{\cdot\}^{\mathrm{T}}$, $\{\cdot\}^*$, and $\{\cdot\}^{\mathrm{H}}$, respectively. The $m \times m$ identity matrix is $\boldsymbol{I}_m$. The Euclidean norm of a vector is denoted by $\|\cdot\|$ and $\succeq$ is the generalized inequality. The Hadamard (element-wise) product is denoted by $\odot$ and $\mathrm{diag}\{\boldsymbol{v}\}$ creates a diagonal matrix by aligning the elements of the vector $\boldsymbol{v}$ onto its diagonal entries.

2. SYSTEM MODEL

The scenario under investigation is shown in Fig. 1. K pairs of single-antenna users would like to communicate with each other via the help of N single-antenna relays. We assume perfect synchronization and the channel is frequency flat and quasistatic block fading. The vector channel from the $(2k-1)$th user (on the left-hand side of Fig. 1) to the relays is denoted as $\boldsymbol{f}_{2k-1} = [f_{2k-1,1}, f_{2k-1,2}, \ldots, f_{2k-1,N}]^{\mathrm{T}} \in \mathbb{C}^N$, while the channel from the $2k$th user (on the right-hand side of Fig. 1) to the relay is denoted as $\boldsymbol{g}_{2k} = [g_{2k,1}, g_{2k,2}, \ldots, g_{2k,N}]^{\mathrm{T}} \in \mathbb{C}^N$, for $k \in \{1, 2, \cdots, K\}$. For notational simplicity, we assume that the channels are *reciprocal* [1]. The transmission takes two time slots. In the first time slot, the signal received at all relays can be combined in a vector as

$$\boldsymbol{r} = \sum_{k=1}^{K} (\boldsymbol{f}_{2k-1} s_{2k-1} + \boldsymbol{g}_{2k} s_{2k}) + \boldsymbol{n}_{\mathrm{R}} \in \mathbb{C}^N \quad (1)$$

where s_{2k-1} and s_{2k} are i.i.d. symbols with zero mean and unit power. The vector $\boldsymbol{n}_{\mathrm{R}}$ denotes the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise and $\mathbb{E}\{\boldsymbol{n}_{\mathrm{R}} \boldsymbol{n}_{\mathrm{R}}^{\mathrm{H}}\} = \sigma_{\mathrm{R}}^2 \boldsymbol{I}_N$.

Afterwards, the AF relays broadcast the weighted signal as

$$\bar{\boldsymbol{r}} = \boldsymbol{W} \cdot \boldsymbol{r} \quad (2)$$

where $\boldsymbol{W} = \mathrm{diag}\{\boldsymbol{w}^*\}$ and $\boldsymbol{w} = [w_1, w_2, \ldots, w_N]^{\mathrm{T}}$ is the vector which consists of the N complex weights of all the relays.

In the second time slot, the received signal at the $(2k-1)$th user (on the left-hand side of Fig. 1) is expressed as [4]

$$\begin{aligned} y_{2k-1} \;=\; & \underbrace{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{F}_{2k-1} \boldsymbol{g}_{2k} s_{2k}}_{\text{desired signal}} + \underbrace{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{F}_{2k-1} \boldsymbol{f}_{2k-1} s_{2k-1}}_{\text{self-interference}} \\ + \; & \underbrace{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{F}_{2k-1} \sum_{\substack{\ell \neq k \\ \ell=1}}^{K} (\boldsymbol{f}_{2\ell-1} s_{2\ell-1} + \boldsymbol{g}_{2\ell} s_{2\ell})}_{\text{inter-pair interference}} \\ + \; & \underbrace{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{F}_{2k-1} \boldsymbol{n}_{\mathrm{R}} + n_{2k-1}}_{\text{effective noise}} \end{aligned} \quad (3)$$

where $\boldsymbol{F}_{2k-1} = \mathrm{diag}\{\boldsymbol{f}_{2k-1}\}$ and n_{2k-1} is the ZMCSCG noise with variance σ_{2k-1}^2. Similar expressions can be obtained for the $2k$th user.

Assume that perfect channel knowledge can be obtained such that the self-interference terms can be canceled. Let P_{R} be the total transmit power consumed by the relays in the network. Our goal is to find the weight vector $\boldsymbol{w}$ such that the sum rate of the system is maximized subject to the sum power constraint. Note that this power constraint is similar to [1], besides for the fact that we do not investigate the adaptation of the transmit powers of each user but consider them to be fixed.

3. SUM RATE MAXIMIZATION

The optimization problem can be formulated as

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \tfrac{1}{2} \sum_{m=1}^{2K} \log_2(1 + \mathrm{SINR}_m) \\ \text{subject to} \quad & \mathbb{E}\{\|\bar{\boldsymbol{r}}\|^2\} \leq P_{\mathrm{R}}, \end{aligned} \quad (4)$$

where the factor $1/2$ is due to the two channel uses (half duplex). When $m = 2k-1$, from the expression (3), the SINR of the mth user can be calculated as

$$\mathrm{SINR}_{2k-1} = \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{B}_{2k-1} \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} (\boldsymbol{D}_{2k-1} + \boldsymbol{E}_{2k-1}) \boldsymbol{w} + \sigma_{2k-1}^2} \quad (5)$$

where $\boldsymbol{D}_{2k-1} = \sum_{\substack{\ell \neq k \\ \ell=1}} (\tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{o})} \tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{o})\mathrm{H}} + \tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{e})} \tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{e})\mathrm{H}})$ and $\boldsymbol{B}_{2k-1} = \boldsymbol{h}_{2k-1} \boldsymbol{h}_{2k-1}^{\mathrm{H}}$ are $N \times N$ positive semidefinite Hermitian matrices. Matrices $\boldsymbol{D}_{2k-1}$ and $\boldsymbol{B}_{2k-1}$ are related to the interference power and the desired signal power, respectively, ($\boldsymbol{h}_{2k-1} = \boldsymbol{f}_{2k-1} \odot \boldsymbol{g}_{2k}$, $\tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{o})} = \boldsymbol{f}_{2k-1} \odot \boldsymbol{f}_{2\ell-1}$ and $\tilde{\boldsymbol{h}}_{2k-1,\ell}^{(\mathrm{e})} = \boldsymbol{f}_{2k-1} \odot \boldsymbol{g}_{2\ell}$). The term which is related to the forwarded noise from the relay is denoted by an $N \times N$ full rank diagonal matrix $\boldsymbol{E}_{2k-1} = \sigma_{\mathrm{R}}^2 \boldsymbol{F}_{2k-1} \boldsymbol{F}_{2k-1}^{\mathrm{H}}$. Similar SINR expression can be obtained when $m = 2k$. Furthermore, the total transmit power is given by $\mathbb{E}\{\|\bar{\boldsymbol{r}}\|^2\} = \boldsymbol{w}^{\mathrm{H}} \boldsymbol{\Gamma} \boldsymbol{w}$ with

$$\boldsymbol{\Gamma} = \sum_{k=1}^{K} (\boldsymbol{F}_{2k-1} \boldsymbol{F}_{2k-1}^{\mathrm{H}} + \boldsymbol{G}_{2k} \boldsymbol{G}_{2k}^{\mathrm{H}}) + \sigma_{\mathrm{R}}^2 \boldsymbol{I}_N. \quad (6)$$

To simplify the optimization problem we note that the inequality constraint in (4) has to be satisfied with equality at optimality. Otherwise, the optimal $\boldsymbol{w}$ can be scaled up to satisfy the constraint with equality while increasing the objective function, which contradicts the optimality. Inserting the constraint into the objective function in (4), the original problem can be reformulated as an *unconstrained* optimization problem

$$\max_{\boldsymbol{w}} \prod_{m=1}^{2K} \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w}} \quad (7)$$

where $\boldsymbol{C}_m = \boldsymbol{D}_m + \boldsymbol{E}_m + \frac{\sigma_m^2}{P_{\mathrm{R}}} \boldsymbol{\Gamma}$ and $\boldsymbol{A}_m = \boldsymbol{B}_m + \boldsymbol{C}_m$ are positive definite. Problem (7) is equivalent to (4) since the objective function is homogeneous and any scaling in $\boldsymbol{w}$ does not change the optimality. Nevertheless, if $\bar{\boldsymbol{w}}$ is the solution to (7), it should be scaled to fullfill the power constraint, i.e., the optimal solution to (4) is given by

$$\boldsymbol{w} = \sqrt{\frac{P_{\mathrm{R}}}{\bar{\boldsymbol{w}}^{\mathrm{H}} \boldsymbol{\Gamma} \bar{\boldsymbol{w}}}} \bar{\boldsymbol{w}}. \quad (8)$$

Problem (7) is non-convex and in general NP-hard.

3.1. Generalized Polyblock Algorithm

Monotonic optimization (see [6], [7]) deals with the maximization or minimization of an increasing function over an intersection of normal and reverse normal sets. The polyblock approximation approach is a unified algorithm to find the global optimum of the monotonic optimization problem. Prior work that used this approach in the area of wireless communications can be found in [8], [9]. We show that the problem (7) is a monotonic optimization problem and then propose a version of the polyblock algorithm to solve it.

Proposition 1. *Problem (7) is a monotonic optimization problem.*

Proof. Problem (7) is equivalent to the following problem

$$\max_{\boldsymbol{y}} \{\Phi(\boldsymbol{y}) | \boldsymbol{y} \in \mathbb{D}\} \quad (9)$$

where $\Phi(\boldsymbol{y}) = \prod_{m=1}^{2K} y_m$ and $\mathbb{D} = \mathbb{G} \cap \mathbb{L}$. The sets $\mathbb{G} = \{\boldsymbol{y} \in \mathbb{R}_+^{2K} | y_m \leq \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w}}, \boldsymbol{w} \in \mathbb{C}^N\}$ and $\mathbb{L} = \{\boldsymbol{y} \in \mathbb{R}_+^{2K} | y_m \geq \min_{\boldsymbol{w}} \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w}}\}$ are normal set and reverse normal set, respectively. Function $\Phi(\boldsymbol{y})$ is an increasing function since $\Phi(\bar{\boldsymbol{y}}) \geq \Phi(\tilde{\boldsymbol{y}})$ for $\bar{\boldsymbol{y}} \succeq \tilde{\boldsymbol{y}}$. Then the proof of the equivalence follows similar steps as in [7]. Thus, problem (7) is a monotonic optimization problem. The definitions of increasing function, normal set, and reverse normal set are the same as in [7]. □

A polyblock $\mathbb{P}$ with vertex set $\mathbb{T} \subset \mathbb{R}_+^{2K}$ is defined as the finite union of all the boxes $[\mathbf{0}, \boldsymbol{z}]$, $\boldsymbol{z} \in \mathbb{T}$. It is dominated by its proper vertices. A vertex $\boldsymbol{z}$ is proper if there is no $\bar{\boldsymbol{z}} \neq \boldsymbol{z}$ and $\bar{\boldsymbol{z}} \succeq \boldsymbol{z}$ for $\bar{\boldsymbol{z}} \in \mathbb{T}$.

According to Proposition 2 in [7], the global maximum of the problem (9), if exists, is attained on $\partial^+\mathbb{D}$, i.e., the upper boundary of $\mathbb{D}$. The main idea of the polyblock approximation algorithm for solving (9) is to approximate $\partial^+\mathbb{D}$ by polyblocks, i.e., construct a nested sequence of polyblocks which approximate $\mathbb{D}$ from above, that is,

$$\mathbb{P}_1 \supset \mathbb{P}_2 \supset \cdots \supset \mathbb{D} \text{ s.t. } \max_{\boldsymbol{y} \in \mathbb{P}_k} \Phi(\boldsymbol{y}) \to \max_{\boldsymbol{y} \in \mathbb{D}} \Phi(\boldsymbol{y}) \qquad (10)$$

when $k \to \infty$ and $\boldsymbol{y}_k \succeq \boldsymbol{y}_\ell$ for all $\ell \geq k$.

Now we outline how to construct the subset $\mathbb{P}_k$ in our case, which is clearly the critical step of a polyblock approximation. Let $\mathbb{T}_k$ be the proper vertex set of $\mathbb{P}_k$ and define the maximizer at iteration k as

$$\bar{\boldsymbol{y}}_k \in \arg\max_{\bar{\boldsymbol{y}}} \{\Phi(\bar{\boldsymbol{y}}) | \bar{\boldsymbol{y}} \in \mathbb{T}_k\}. \qquad (11)$$

Compute the unique intersection point of $\partial^+\mathbb{D}$ and $\bar{\boldsymbol{y}}_k$ as $\hat{\boldsymbol{y}}_k = \alpha_k \bar{\boldsymbol{y}}_k$ with $\alpha_k \in [0, 1]$. Then the proper vertex set $\mathbb{T}_{k+1}$ of $\mathbb{P}_{k+1}$ in step $k+1$ is the set obtained by substituting $\bar{\boldsymbol{y}}_k$ in $\mathbb{T}_k$ with the new vertices $\{\bar{\boldsymbol{y}}_k^1, \cdots, \bar{\boldsymbol{y}}_k^{2K}\}$ defined by

$$\bar{\boldsymbol{y}}_k^m = \bar{\boldsymbol{y}}_k - (\bar{y}_{k,m} - \hat{y}_{k,m})\boldsymbol{e}_m, \; m = 1, \cdots, 2K \qquad (12)$$

and removing all the improper vertices as well as the vertices not belonging to $\mathbb{L}$. The scalar $\bar{y}_{k,m}$ is the mth element of $\bar{\boldsymbol{y}}_k$ and $\boldsymbol{e}_m \in \mathbb{R}_+^{2K}$ is the mth unit vector. The factor α_k is calculated as [7]

$$\alpha_k = \max_{\boldsymbol{w}} \min_m \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w}}{\bar{y}_{k,m} \boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w}}. \qquad (13)$$

Although (13) is non-convex, it is an easier sub-problem which can be solved approximately ($\eta-$optimality) using the algorithm in [10]. Finally, the proposed (ϵ, η)-optimal solution using the polyblock algorithm is described in Table 1. The proof of the global convergence follows similar steps as in [7].

Table 1. (ϵ, η)-optimal polyblock algorithm for solving (7)

Initialization step: set initial vertex set $\mathbb{T}_0 = \{\boldsymbol{b}\}$,[1] maximum iteration number $N_{\max}$, and the threshold values ϵ, η.
Main step:
1: **for** $k = 1$ to $N_{\max}$ **do**
2: Solve (11) and (13) finding $\bar{\boldsymbol{y}}_k$ and $\eta-$optimal α_k.
3: Construct a smaller polyblock $\mathbb{P}_k$ using $\bar{\boldsymbol{y}}_k$ and α_k.
4: **if** $\max_m\{(\bar{y}_{k,m} - \hat{y}_{k,m})/\bar{y}_{k,m}\} \leq \epsilon$ **then**
5: **break**
6: **end if**
7: **end for**

3.2. Extended GPI Algorithm

The problem (7) can also be solved using the the general power iterative (GPI) algorithm which is introduced in [5]. However, the condition for applying GPI is not explicitly given in [5] and it is not trivial.

Let us briefly review the GPI method in [5]. According to the optimality condition, all the local maximizers for the problem (7) should satisfy

$$\left.\frac{\partial \lambda(\boldsymbol{w})}{\partial \boldsymbol{w}}\right|_{\boldsymbol{w}=\bar{\boldsymbol{w}}} = 0 \qquad (14)$$

[1]Here $\boldsymbol{b} \in \mathbb{R}_+^{2K}$ satisfies $b_m = \max_{\boldsymbol{w}} \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w}}$, $m = 1, \cdots, 2K$.

where $\lambda(\boldsymbol{w}) = \prod_{m=1}^{2K} \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{A}_m \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{C}_m \boldsymbol{w}}$. After differentiation and some algebraic manipulation, the condition in (14) can be converted into

$$\boldsymbol{V}(\bar{\boldsymbol{w}})\bar{\boldsymbol{w}} = \lambda(\bar{\boldsymbol{w}})\boldsymbol{Q}(\bar{\boldsymbol{w}})\bar{\boldsymbol{w}} \qquad (15)$$

where $\boldsymbol{V}(\bar{\boldsymbol{w}}) = \sum_{m=1}^{2K}(\prod_{i \neq m} \bar{\boldsymbol{w}}^{\mathrm{H}} \boldsymbol{A}_i \bar{\boldsymbol{w}})\boldsymbol{A}_m$ and $\boldsymbol{Q}(\bar{\boldsymbol{w}}) = \sum_{m=1}^{2K}(\prod_{i \neq m} \bar{\boldsymbol{w}}^{\mathrm{H}} \boldsymbol{C}_i \bar{\boldsymbol{w}})\boldsymbol{C}_m$. Equation (15) is a generalized eigenvalue problem and $\lambda(\bar{\boldsymbol{w}})$ can be thought as the generalized eigenvalue of matrices $\boldsymbol{V}(\bar{\boldsymbol{w}})$ and $\boldsymbol{Q}(\bar{\boldsymbol{w}})$. Thus, the maximum generalized eigenvalue $\lambda_{\max}(\bar{\boldsymbol{w}})$ is the maximum of the problem (7). Since both matrices are functions of $\bar{\boldsymbol{w}}$, a closed-form solution is not possible. Therefore, the authors in [5] apply the recursive power method of [11] to obtain the solution. It is also numerically shown that the GPI algorithm converges in 30 iterations. However, this is not true in general. In [11], it is shown that the power method converges only if the largest eigenvalue is dominant and the convergence speed depends on the ratio between the largest and the second largest eigenvalues. Although we can only demonstrate this via numerical simulations, we claim that GPI should have similar features as the original power method. Thus, the following conjecture is given.

Conjecture 1. *The GPI algorithm converges if there is a dominant eigenvalue. The convergence behavior depends on the dispersion of the eigenvalue profiles of the matrices of $\boldsymbol{A}_m$ and $\boldsymbol{C}_m$.*

Nevertheless, the GPI algorithm can be applied to our scenario especially when $\boldsymbol{D}_m$ is rank deficient ($N > 2(K-1)$ since $\mathrm{rank}\{\boldsymbol{D}_m\} = \min\{N, 2K-2\}$), i.e., there will be a dominant eigenvalue when SNR $\to \infty$. Moreover, it converges faster in the high SNR regime with a given error tolerance factor. For a detailed implementation one can be referred to [5].

3.3. Total SINR Eigen-Beamformer

Although the polyblock algorithm and the GPI algorithm solve the problem (7) in an optimal way, they require many iterations. In this section, we propose a closed-form suboptimal design. This closed-form solution is based on the observation that for our scenario nulling the inter-pair interferences by forcing every interference term to zero is equivalent to nulling the sum of the inter-pair interferences. That is, if the sum of the interference powers $\boldsymbol{w}^{\mathrm{H}}(\sum_{m=1}^{2K} \boldsymbol{D}_m)\boldsymbol{w} = \sum_{m=1}^{2K}(\boldsymbol{w}^{\mathrm{H}} \boldsymbol{D}_m \boldsymbol{w}) = 0$, it is clear that $\boldsymbol{w}^{\mathrm{H}} \boldsymbol{D}_m \boldsymbol{w} = 0$, for all m since $\boldsymbol{D}_m \succeq 0$.

Let us define $\boldsymbol{S}_{\mathrm{tot}} = \sum_{m=1}^{2K} \boldsymbol{B}_m$ and $\boldsymbol{U}_{\mathrm{tot}} = \sum_{m=1}^{2K} \boldsymbol{C}_m$. Thus, $\boldsymbol{w}^{\mathrm{H}} \boldsymbol{S}_{\mathrm{tot}} \boldsymbol{w}$ and $\boldsymbol{w}^{\mathrm{H}} \boldsymbol{U}_{\mathrm{tot}} \boldsymbol{w}$ are the sum of the signal power and the sum of the interference plus noise power of all the users, respectively. Then the proposed total SINR eigen-beamformer solves the following problem

$$\max_{\boldsymbol{w}} \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{S}_{\mathrm{tot}} \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{U}_{\mathrm{tot}} \boldsymbol{w}}. \qquad (16)$$

It is obvious that the maximum value of (16) is the maximum generalized eigenvalue $\lambda_{\max}\{\boldsymbol{S}_{\mathrm{tot}}, \boldsymbol{U}_{\mathrm{tot}}\}$ and the optimal $\boldsymbol{w}$ is the dominant eigenvector of the matrix $\boldsymbol{U}_{\mathrm{tot}}^{-1} \boldsymbol{S}_{\mathrm{tot}}$ ($\boldsymbol{U}_{\mathrm{tot}}$ is always invertible due to the noise term). In the end, a scaling has to be performed as in (8).

Remark 1. *Although all the proposed algorithms do not have any requirements on N, to cancel the interference completely $N > 2K(K-1)$ is required since the rank of the sum of the interference terms is equal to $\mathrm{rank}\{\sum_{m=1}^{2K} \boldsymbol{D}_m\} = 2K(K-1)$ [4]. If $N \leq 2K(K-1)$, the results will be unfair for some users since they will suffer from extremely lower throughputs compared to the other users.*

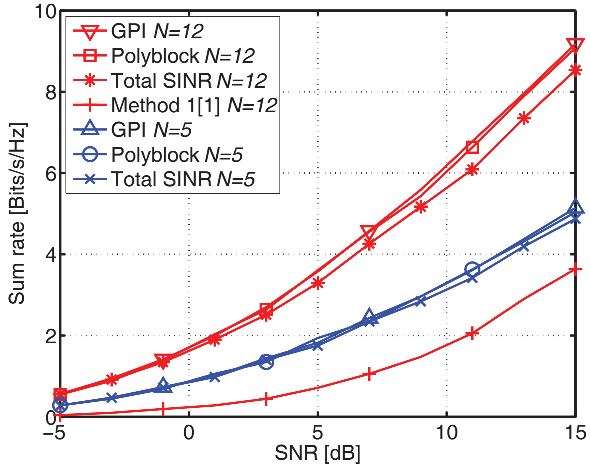

Fig. 2. Sum rate comparison.

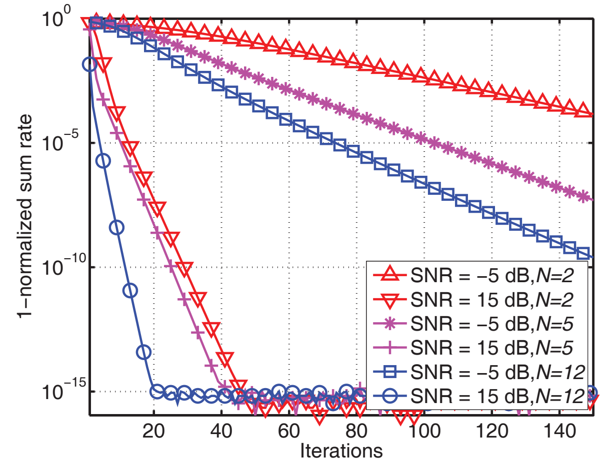

Fig. 3. Convergence property of GPI under different N and SNRs.

4. SIMULATION RESULTS

In this section, the performance of the proposed algorithms is evaluated via Monte-Carlo simulations. The simulated flat fading channels are spatially uncorrelated Rayleigh fading channels. The total relay power P_{R} is fixed to unity. The noise variances at all nodes are the same, i.e., $\sigma_{\mathrm{R}}^2 = \sigma_m^2$ and thus SNR $= 1/\sigma_m^2$. There are $K = 2$ pairs of users in the network. All the simulation results are obtained by averaging over 100 channel realizations. "Polyblock", "GPI", "Total SINR", and "Method 1" denote the algorithms in Sections 3.1, 3.2, 3.3, and [4], respectively. For the polyblock algorithm, $\epsilon = 10^{-1}$ and $\eta = 10^{-6}$.

Fig. 2 shows the comparison of different algorithms with $N = 5$ relays and $N = 12$ relays in the network. "Method 1" is available only for the case $N = 12$ since it requires that $N \geq 2K^2 + K$. It is obvious that "Polyblock", "GPI" and "Total SINR" outperform "Method 1". One possible reason is that in "Method 1" a part of the transmit power is used to force the self-interference power to a certain level. The polyblock algorithm performs slightly worse than the GPI algorithm. This is due to the (ϵ, η)-optimality. Moreover, the total SINR eigen-beamformer performs almost the same as the optimal solution with a small number of relays ($N = 5$) and suffers only a small loss when many relays ($N = 12$) exist.

Fig. 3 demonstrates the convergence property of the GPI algorithm under different N and SNRs. As we discussed in Section 3.2, the convergence speed increases when the number of relays increases in the network or the SNR is high.

5. CONCLUSION

In this paper, we have investigated the sum rate maximization problem in two-way AF relaying networks. Given a total network power constraint, the optimization problem fits into the monotonic optimization framework and thus can be solved using the generalized polyblock approximation algorithm. Since the optimality condition yields a generalized eigenvalue problem, we propose to apply the GPI algorithm which can also approach the global optimum since there is a dominant eigenvalue. To reduce the computational complexity, we propose the total SINR eigen-beamformer which maximizes the total SINR of the network. The total SINR eigen-beamformer only suffers a little loss compared to the two optimum solutions. All the proposed algorithms outperform the recently proposed algorithm in [4].

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