POLYNOMIAL-TIME DC (POTDC) FOR SUM-RATE MAXIMIZATION IN TWO-WAY AF MIMO RELAYING

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ABSTRACT

The problem of sum-rate maximization in two-way amplify-and-forward (AF) multiple-input multiple-output (MIMO) relaying is considered. Mathematically, this problem is equivalent to the constrained maximization of the product of quadratic ratios that is a non-convex problem. Such problems appear also in many other applications. This problem can be further relaxed into a difference-of-convex functions (DC) programming problem, which is typically solved using the branch-and-bound method without polynomial-time complexity guarantees. We, however, develop a polynomial-time convex optimization-based algorithm for solving the corresponding DC programming problem named polynomial-time DC (POTDC). POTDC is based on a specific parameterization of the problem, semi-definite programming (SDP) relaxation, linearization, and iterations over a single parameter. The complexity of the problem solved at each iteration of the algorithm is equivalent to that of the SDP problem. The effectiveness of the proposed POTDC method for the sum-rate maximization in two-way AF MIMO relay systems is shown.

***Index Terms*—** Two-way relaying, sum-rate, difference-of-convex functions, semi-definite relaxation.

1. INTRODUCTION

Two-way relaying is able to overcome the rate loss associated with the conventional one-way relaying. The design of rate-optimal strategies in two-way relaying systems is one of the most important problems in the area [1]. Although the rate-optimal strategy for two-way relaying is in general unknown, the capacity region for the case of amplify-and-forward (AF) multiple-input multiple-output (MIMO) relaying system with two single-antenna terminals has been discussed in [2].

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In this paper, we consider a two-way relaying system with two single-antenna terminals and one AF MIMO relay and aim at finding the relay transmit strategy (amplification matrix) to maximize the sum-rate of both terminals. Mathematically, the corresponding problem of finding the optimal relay amplification matrix is equivalent to maximizing the product of quadratic ratios under the quadratic power constraint on the available power at the relay. Using a certain parameterization of the problem, rewriting it as a difference-of-convex functions (DC) programming problem, and further using semi-definite programming (SDP) relaxation together with linearization and an iterative search over a single parameter, a polynomial-time algorithm for solving this problem is developed. The algorithm is called polynomial-time DC (POTDC) and its complexity is equivalent to the complexity of the SDP problem, which has to be solved at each iteration. The effectiveness of the proposed POTDC method for sum-rate maximization in two-way AF MIMO relay systems is demonstrated via simulations.

2. SYSTEM MODEL AND PROBLEM FORMULATION

In a two-way relaying system, two single-antenna terminals (users) communicate via an AF relay equipped with M_R antennas. The channels between the terminals and the relay are modeled as frequency-flat quasi-static block fading and every data transmission occurs in two phases. In the first phase, both single antenna terminals transmit their signals to the relay. The received signal at the relay is then given as

$$\mathbf{r} = \mathbf{h}_1^{(f)} x_1 + \mathbf{h}_2^{(f)} x_2 + \mathbf{n}_R \qquad (1)$$

where $\mathbf{h}_i^{(f)} = [h_{i,1}, \ldots, h_{i,M_R}]^T \in \mathbb{C}^{M_R}$ is the (forward) channel vector between terminal i and the relay, x_i is the transmitted symbol from terminal i, $\mathbf{n}_R \in \mathbb{C}^{M_R}$ is the additive noise at the relay, and $(\cdot)^T$ stands for the transpose. The average transmit power of terminal i equals $P_{T,i} = \mathbb{E}\{|x_i|^2\}$ and the noise covariance matrix at the relay is $\mathbf{R}_{N,R} = \mathbb{E}\{\mathbf{n}_R \mathbf{n}_R^H\}$ where $\mathbb{E}\{\cdot\}$ and $(\cdot)^H$ denote the mathematical expectation and the Hermitian transpose, respectively.

In the second phase, after amplifying the received signal using a relay amplification matrix, the relay retransmits the signal to the terminals. The transmitted signal from the relay is $\bar{\mathbf{r}} = \mathbf{G}\mathbf{r}$ where $\mathbf{G} \in \mathbb{C}^{M_R \times M_R}$ is the relay amplification matrix. The transmit power of the relay is $\mathbb{E}\{\|\bar{\mathbf{r}}\|_2^2\} = \mathbf{g}^H \mathbf{Q} \mathbf{g}$, where $\mathbf{Q} = \left(\mathbf{R}_R^T \otimes \mathbf{I}_{M_R}\right)$, $\mathbf{g} = \mathrm{vec}\{\mathbf{G}\}$, $\|\cdot\|_2$ and $\otimes$ denote the Euclidian norm of a vector and the Kronecker product, respectively, $\mathrm{vec}(\cdot)$ is a vectorization operation of a matrix, and $\mathbf{R}_R = \mathbb{E}\{\mathbf{r}\mathbf{r}^H\}$ is the covariance matrix of $\mathbf{r}$ given by

$$\mathbf{R}_R = \mathbf{h}_1^{(f)} \left(\mathbf{h}_1^{(f)}\right)^H P_{T,1} + \mathbf{h}_2^{(f)} \left(\mathbf{h}_2^{(f)}\right)^H P_{T,2} + \mathbf{R}_{N,R}. \tag{2}$$

The signals received by the terminals via the (backward) channels $(\mathbf{h}_1^{(b)})^T$ and $(\mathbf{h}_2^{(b)})^T$ can be expressed as

$$y_1 = h_{1,1}^{(e)} x_1 + h_{1,2}^{(e)} x_2 + \tilde{n}_1 \tag{3}$$

$$y_2 = h_{2,2}^{(e)} x_2 + h_{2,1}^{(e)} x_1 + \tilde{n}_2 \tag{4}$$

where $h_{i,j}^{(e)} = \left(\mathbf{h}_i^{(b)}\right)^T \mathbf{G} \mathbf{h}_j^{(f)}$ denotes the effective channel between terminal i and terminal j for $i, j = 1, 2$ and $\tilde{n}_i = \left(\mathbf{h}_i^{(b)}\right)^T \mathbf{G} \mathbf{n}_R + n_i$ is the effective noise at terminal i which contains the terminal's own noise and the noise forwarded by the relay. The self-interference (the first terms of (3) and (4)) can be subtracted by the terminal since its own transmitted signal is known. Moreover, the required channel knowledge for this step can be obtained, for example, via the least squares (LS) compound channel estimator of [3]. After the cancellation of the self-interference, the two-way relaying system is decoupled into two parallel single-user single-input single-output (SISO) systems. Then the rate of terminal i can be expressed as $r_i = (1/2) \cdot \mathrm{ld}\left(1 + P_{R,i}/\tilde{P}_{N,i}\right)$, where $\mathrm{ld}(\cdot)$ stands for the logarithm in base 2, $P_{R,1} = \mathbb{E}\left\{\left|h_{1,2}^{(e)} x_2\right|^2\right\}$, $P_{R,2} = \mathbb{E}\left\{\left|h_{2,1}^{(e)} x_1\right|^2\right\}$, and $\tilde{P}_{N,i} = \mathbb{E}\left\{|\tilde{n}_i|^2\right\}$ for $i = 1, 2$ are the powers of the desired signal and the effective noise term at terminal i. Note that the factor $1/2$ results from the two time slots needed for the bidirectional transmission. The sum-rate maximization problem is then formulated as finding the relay amplification matrix $\mathbf{G}$ which maximizes the sum-rate $r_1 + r_2$ subject to the total power constraint at the relay. Mathematically, it can be written as [4]

$$\mathbf{g}_{\mathrm{opt}} = \underset{\mathbf{g} | \mathbf{g}^H \cdot \mathbf{Q} \cdot \mathbf{g} \leq P_{T,R}}{\arg\max} \frac{1}{2} \mathrm{ld}\left[\left(1 + \frac{P_{R,1}}{\tilde{P}_{N,1}}\right)\left(1 + \frac{P_{R,2}}{\tilde{P}_{N,2}}\right)\right] \tag{5}$$

Using the vectorization of the relay amplification matrix and introducing $\mathbf{g} = \mathrm{vec}\{\mathbf{G}\}$, the powers of the desired signal and the effective noise term at terminals can be conveniently expressed as quadratic form with respect to $\mathbf{g}$ as

$$P_{R,1} = \mathbf{g}^H \mathbf{K}_{2,1} \mathbf{g} P_{T,2} \tag{6}$$

$$P_{R,2} = \mathbf{g}^H \mathbf{K}_{1,2} \mathbf{g} P_{T,1} \tag{7}$$

$$\tilde{P}_{N,i} = \mathbf{g}^H \mathbf{J}_i \mathbf{g} + P_{N,i}, \; i = 1, 2 \tag{8}$$

where the matrices $\mathbf{K}_{2,1}$, $\mathbf{K}_{1,2}$, and $\mathbf{J}_i$, $i = 1, 2$ are defined as

$$\mathbf{K}_{2,1} = \left[\left(\mathbf{h}_2^{(f)} \left(\mathbf{h}_2^{(f)}\right)^H\right) \otimes \left(\mathbf{h}_1^{(b)} \left(\mathbf{h}_1^{(b)}\right)^H\right)\right]^T \tag{9}$$

$$\mathbf{K}_{1,2} = \left[\left(\mathbf{h}_1^{(f)} \left(\mathbf{h}_1^{(f)}\right)^H\right) \otimes \left(\mathbf{h}_2^{(b)} \left(\mathbf{h}_2^{(b)}\right)^H\right)\right]^T \tag{10}$$

$$\mathbf{J}_i = \left[\mathbf{R}_{N,R} \otimes \left(\mathbf{h}_i^{(b)} \left(\mathbf{h}_i^{(b)}\right)^H\right)\right]^T. \tag{11}$$

3. POTDC FOR SUM-RATE MAXIMIZATION IN TWO-WAY AF MIMO RELAYING

Since the cost function of the problem (5) increases monotonically with the norm of $\mathbf{g}$, the inequality constraint in (5) (total relay power constraint) has to be active at the optimal point (the details will be available in [5]). Using this fact, the sum-rate maximization problem can be simplified as

$$\mathbf{g}_{\mathrm{opt}} = \underset{\mathbf{g} | \mathbf{g}^H \mathbf{Q} \mathbf{g} = P_{T,R}}{\arg\max} \frac{\mathbf{g}^H \mathbf{A}_1 \mathbf{g}}{\mathbf{g}^H \mathbf{B}_1 \mathbf{g}} \cdot \frac{\mathbf{g}^H \mathbf{A}_2 \mathbf{g}}{\mathbf{g}^H \mathbf{B}_2 \mathbf{g}} \tag{12}$$

where $\mathbf{B}_i$ and $\mathbf{A}_i, i = 1, 2$ are given by

$$\mathbf{B}_i = \mathbf{J}_i + \frac{P_{N,i}}{P_{T,R}} \mathbf{Q}, \quad i = 1, 2 \tag{13}$$

$$\mathbf{A}_1 = \mathbf{K}_{2,1} \cdot P_{T,2} + \mathbf{B}_1, \;\; \mathbf{A}_2 = \mathbf{K}_{1,2} \cdot P_{T,1} + \mathbf{B}_2. \tag{14}$$

It is easy to verify that the matrices $\mathbf{A}_i$, $i = 1, 2$ and $\mathbf{B}_i$, $i = 1, 2$ are positive definite and therefore the optimization problem (12) is the maximization of the product of two Rayleigh quotients.

Since the problem (12) is homogeneous, its single equality constraint can be dropped and the term $\mathbf{g}^H \mathbf{B}_1 \mathbf{g}$ can be equated to one at the optimal point. Then, introducing new variables α and β, (12) can be recast as

$$\begin{aligned} \max_{\mathbf{g}, \alpha, \beta} \quad & \mathbf{g}^H \mathbf{A}_1 \mathbf{x} \cdot \frac{\alpha}{\beta} \\ \text{s.t.} \quad & \mathbf{g}^H \mathbf{B}_1 \mathbf{g} = 1 \\ & \mathbf{g}^H \mathbf{A}_2 \mathbf{g} = \alpha \\ & \mathbf{g}^H \mathbf{B}_2 \mathbf{g} = \beta \end{aligned} \tag{15}$$

Using the fact that the quadratic form $\mathbf{g}^H \mathbf{B}_1 \mathbf{g}$ is set to one, it can be easily checked that $\alpha \in [\lambda_\alpha, \gamma_\alpha]$ (here γ_α and λ_α denote the largest and smallest eigenvalues of the matrix $\mathbf{B}_1^{-1} \mathbf{A}_2$, respectively) and $\beta \in [\lambda_\beta, \gamma_\beta]$ (here γ_β and

λ_β denote the largest and smallest eigenvalues of the matrix $\mathbf{B}_1^{-1}\mathbf{B}_2$, respectively).

By introducing the matrix $\mathbf{X} \triangleq \mathbf{g}\mathbf{g}^H$ and using the fact that for any arbitrary matrix $\mathbf{Z}$, the equation $\mathbf{g}^H\mathbf{Z}\mathbf{g} = \mathrm{tr}(\mathbf{Z} \cdot \mathbf{g}\mathbf{g}^H)$ holds, the optimization problem (15) can be equivalently expressed as

$$\begin{aligned} \max_{\mathbf{X},\alpha,\beta} \quad & \mathrm{tr}(\mathbf{A}_1\mathbf{X}) \cdot \frac{\alpha}{\beta} \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{B}_1\mathbf{X}) = 1 \\ & \mathrm{tr}(\mathbf{A}_2\mathbf{X}) = \alpha \\ & \mathrm{tr}(\mathbf{B}_2\mathbf{X}) = \beta \\ & \mathrm{rank}(\mathbf{X}) = 1, \quad \mathbf{X} \succeq \mathbf{0}. \end{aligned} \tag{16}$$

Let $\mathbf{X}_{\alpha,\beta}$ denote the optimal solution of (16) with respect to $\mathbf{X}$ for some fixed values of the variables α and β and without considering the rank-one constraint. Applying the rank reduction technique [7], it is possible to construct another optimal rank-one solution based on $\mathbf{X}_{\alpha,\beta}$ if the number of constraints does not exceed three, as it is in our problem. The later fact implies that for fixed α and β, the optimal value of the objective function of (16) with respect to $\mathbf{X}$ is the same with or without considering the rank-one constraint. Thus, we can drop the rank one constraint in (16), solve the so-obtained relaxed problem, and then construct an optimal rank-one solution once the optimal $\mathbf{X}_{\mathrm{opt}}$, α_{opt}, and β_{opt} are obtained.

Dropping the rank-one constraint and also taking logarithm of the objective function, results in the following optimization problem

$$\begin{aligned} \max_{\mathbf{X},\alpha,\beta} \quad & \log(\mathrm{tr}(\mathbf{A}_1\mathbf{X})) + \log(\alpha) - \log(\beta) \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{B}_1\mathbf{X}) = 1 \\ & \mathrm{tr}(\mathbf{A}_2\mathbf{X}) = \alpha \\ & \mathrm{tr}(\mathbf{B}_2\mathbf{X}) = \beta, \quad \mathbf{X} \succeq \mathbf{0} \end{aligned} \tag{17}$$

which is a DC programming problem. The available algorithms in the literature for solving DC programming problems are based on the so-called branch-and-bound method. Therefore, although the relaxed problem boils down to the known family of DC problems, there exists still no solution for such DC problems with guaranteed polynomial time complexity. In what follows, we develop a new method for solving (17) efficiently in polynomial time. To fulfil this goal, we introduce a new additional variable t and rewrite (17) equivalently as

$$\begin{aligned} \max_{\mathbf{X},\alpha,\beta,\mathbf{t}} \quad & \log(\mathrm{tr}(\mathbf{A}_1\mathbf{X})) + \log(\alpha) - t \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{B}_1\mathbf{X}) = 1 \\ & \mathrm{tr}(\mathbf{A}_2\mathbf{X}) = \alpha \\ & \mathrm{tr}(\mathbf{B}_2\mathbf{X}) = \beta \\ & \mathbf{X} \succeq \mathbf{0}, \quad \log(\beta) \leq t. \end{aligned} \tag{18}$$

The objective function of the optimization problem (18) is concave and all the constraints, except the constraint $\log(\beta) \leq t$, are convex. Using a similar idea as in [6], we develop an iterative method for solving (17) by linearizing the non-convex term $\log(\beta)$ in a single non-convex constraint $\log(\beta) \leq t$ around a suitably selected point in each iteration. Specifically, the linearizing point in each iteration is selected so that the iterative algorithm moves closer to the optimal point in every iteration. In the first iteration, we start with an arbitrary point selected in the interval $[\lambda_\beta, \gamma_\beta]$ and denote it as β_c. Then the non-convex function $\log(\beta)$ is replaced by its linear approximation around this point β_c, that is,

$$\log(\beta) \approx \log(\beta_c) + \frac{1}{\beta_c}(\beta - \beta_c). \tag{19}$$

It results in the following convex optimization problem

$$\begin{aligned} \max_{\mathbf{X},\alpha,\beta,\mathbf{t}} \quad & \log(\mathrm{tr}(\mathbf{A}_1\mathbf{X})) + \log(\alpha) - t \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{B}_1\mathbf{X}) = 1 \\ & \mathrm{tr}(\mathbf{A}_2\mathbf{X}) = \alpha \\ & \mathrm{tr}(\mathbf{B}_2\mathbf{X}) = \beta, \quad \mathbf{X} \succeq \mathbf{0} \\ & \log(\beta_c) + \frac{1}{\beta_c}(\beta - \beta_c) \leq t. \end{aligned} \tag{20}$$

The problem (20) is convex and can be efficiently solved using the interior-point-based numerical methods. Once the optimal solution of this problem in the first iteration, denoted as $\mathbf{X}_{\mathrm{opt}}^{(1)}$, $\alpha_{\mathrm{opt}}^{(1)}$, $\beta_{\mathrm{opt}}^{(1)}$ and $t_{\mathrm{opt}}^{(1)}$ is found, the POTDC algorithm proceeds to the second iteration by replacing the function $\log(\beta)$ by its linear approximation around $\beta_{\mathrm{opt}}^{(1)}$ found from the previous (first) iteration. The resulting optimization problem, in the second iteration, has the same structure as the problem (20) in which β_c has to be replaced by $\beta_{\mathrm{opt}}^{(1)}$. This process continues and every iteration is obtained by replacing $\log(\beta)$ at the iteration k by its linearization of type (19) around $\beta_{\mathrm{opt}}^{(k-1)}$ found in the iteration $k-1$. The iterations terminate when the difference between the optimal values of two consequent iterations is less than a desired threshold. It is guaranteed that such stopping criteria will be satisfied due to the following result.

Result 1: The optimal values of the optimization problem (20) obtained over all iterations are non-decreasing.

Proof. Considering the linearized problem (20) in the iteration $k+1$, it is easy to verify that $\mathbf{X}_{\mathrm{opt}}^{(k)}$, $\alpha_{\mathrm{opt}}^{(k)}$, $\beta_{\mathrm{opt}}^{(k)}$ and $t_{\mathrm{opt}}^{(k)}$ give a feasible point for this problem. Therefore, it can be concluded that the optimal value at the iteration $k+1$ must be greater than or equal to the optimal value in the iteration k which completes the proof. □

The POTDC algorithm has a polynomial-time complexity, since it requires to solve a finite number of convex problems. As soon as the solution of (18) is found, the solution of (15), which is equivalent to the solution of the sum-rate maximization problem (12), can be found using one of the existing methods for extracting a rank one solution. Solutions obtained over all iterations of the POTDC algorithm including the final solution are regular points of the problem (18), i.e., at these points, the gradients of the equality and active inequality constraints are linearly independent. Moreover, the final solution satisfies the Karush-Kuhn-Tucker (KKT) conditions that follows from a similar fact proved in [8]. Therefore, the optimal solution found using the above described algorithm is guaranteed to be at least a local optimal point of the problem. The global optimality is further investigated in [5].

4. SIMULATION RESULTS

In our simulation scenario, the communication between two single antenna users is supported by an AF MIMO relay equipped with $M_R = 4$ antennas. The total transmit powers of the terminals, i.e., $P_{T,1}$ and $P_{T,2}$, as well as the total transmit power of the MIMO relay $P_{T,R}$ are all assumed to be equal to 1. The noise powers of the relay antennas and the users are assumed to be equal to σ^2 where σ^{-2} denotes the signal-to-noise ratio (SNR). The channels between the terminals and the relay are assumed to be uncorrelated Rayleigh fading channels and it is also assumed that reciprocity holds. The variance of the channel coefficients between the first terminal and the relay antennas and between the second terminal and the relay antennas are assumed to be equal to 1 and 0.1, respectively. To obtain each simulated point, 100 independent simulation runs are used.

The POTDC algorithm is compared in terms of the sum-rate with 2-D semi-algebraic solution via generalized eigenvectors (RAGES) of [4], 1-D RAGES of [4], the algebraic norm-maximizing (ANOMAX) transmit strategy of [9], and the discrete Fourier transform (DFT) method when the relay precoding matrix is a scaled DFT matrix, i.e., no channel knowledge is used. The DFT method serves as a benchmark for evaluating the gain archived by using channel knowledge.

Fig. 1 shows the sum-rate achieved by the aforementioned methods versus the SNR. It can be observed that the sum-rate results corresponding to the POTDC method, 2-D RAGES and 1-D RAGES coincide. As it was mentioned earlier, it is analytically guaranteed that the POTDC method converges to at least a local maximum of the sum-rate maximization problem. Moreover, our extensive simulation results confirm that the new proposed method converges to the global maximum of the problem in all simulation runs. It can also be observed that the 2-D RAGES and 1-D RAGES are optimal as well. The ANOMAX and DFT methods, however, do not achieve the maximum sum-rate, and the loss in the sum-rate related to the DFT method is quite significant.

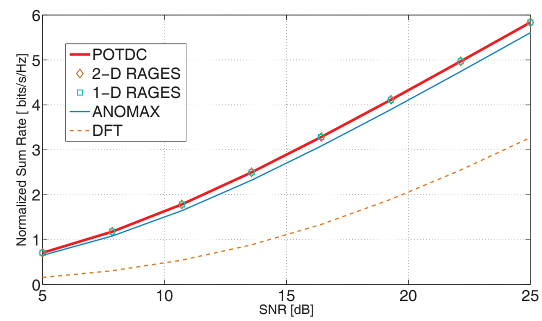

Fig. 1. Sum-rate versus SNR for $M_R = 4$ antennas.

5. CONCLUSIONS

We have shown that the sum-rate maximization problem in two-way AF MIMO relaying belongs to the class of DC programming problems. Although DC programming problems are typically solved using the branch-and-bound method, this method does not have any polynomial time guarantees for its worst-case complexity. Therefore, we have developed the POTDC algorithm for finding the global maximum of the aforementioned problem with polynomial time worst-case complexity. The effectiveness of the proposed POTDC method is demonstrated via simulations.

6. REFERENCES

[1] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimization linear non-regenerative multicarrier MIMO relay communication systems," *IEEE Trans. Signal Processing*, vol. 57, pp. 4837–4852, Dec. 2009.

[2] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE Journal on Selected Areas of Communications*, vol. 27, no. 5, pp. 699–712, June 2009.

[3] F. Roemer and M. Haardt, "Tensor-based channel estimation (TENCE) and iterative refinements for two-way relaying with multiple antennas and spatial reuse," *IEEE Trans. Signal Processing*, vol. 58, no. 11, pp. 5720–5735, Nov. 2010.

[4] F. Roemer and M. Haardt, "Sum-rate maximization in two-way relaying systems with MIMO amplify and forward relays via generalized eigenvectors," in *Proc. 18-th European Sig. Proc. Conf.*, Aalborg, Denmark, Aug. 2010.

[5] A. Khabbazibasmenj, F. Roemer, S. A. Vorobyov, and M. Haardt, "Sum-Rate Maximization in Two-Way AF MIMO Relaying: Polynomial Time Solutions to a Class of DC Programming Problems," submitted to *IEEE Trans. Signal Processing.*

[6] A. Khabbazibasmenj and S. A. Vorobyov "A computationally efficient robust adaptive beamforming for general-rank signal model with positive semi-definite constraint," in *Proc. Inter. Workshop Comp. Advances in Multi-Sensor Adaptive Processing*, San Juan, Puerto Rico, Dec. 2011, pp. 185-188.

[7] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Trans. Signal Processing*, vol. 58, pp. 664–678, Feb. 2010.

[8] A. Beck, A. Ben-Tal and L. Tetruashvili, "A sequential parametric convex approximation method with applications to nonconvex truss topology design problems," *Journal of Global Optimization*, vol. 47, pp. 29–51, Jan. 2010.

[9] F. Roemer and M. Haardt, "Algebraic norm-maximizing (ANOMAX) transmit strategy for two-way relaying with MIMO amplify and forward relays," *IEEE Sig. Proc. Letters*, vol. 16, pp. 909-912, Oct. 2009.