TRANSMIT BEAMSPACE DESIGN FOR DIRECTION FINDING IN COLOCATED MIMO RADAR WITH ARBITRARY RECEIVE ARRAY

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ABSTRACT

The transmit beamspace design problem for colocated multiple-input multiple-output (MIMO) radar is considered. We show that the MIMO radar transmit beampattern can be designed so that it is as close as possible to the desired one, the power is uniformly distributed across the transmit antennas, and most significantly, the rotational invariance property at the receive array with arbitrary geometry is satisfied. The latter enables a straightforward application of search-free direction of arrival estimation techniques such as ESPRIT in the unconventional case with the receive array of arbitrary geometry. The transmit beamspace design problem is cast as an optimization problem which is non-convex in general, but can be solved efficiently using the semi-definite programming relaxation technique.

***Index Terms*—** Colocated MIMO radar, direction of arrival (DOA) estimation, transmit beamspace design

1. INTRODUCTION

Multiple-input multiple-output (MIMO) radar is a new emerging technique which offers significant performance improvements compared to its traditional counterpart, that is, single-input multiple-output radar [1]. The performance improvements can be attributed to the fact that the transmit signals as well as transmit beamforming techniques in MIMO radar can be chosen/designed quiet freely [1], [2]. For example, the additional degrees of freedom provided by MIMO radar can be used to obtain the transmit beampattern which is as close as possible to a desired beampattern [1], [3]. The increased aperture of the virtual array in MIMO radar is also instrumental for improving the direction of arrival (DOA) estimation performance using, for example, the search-free estimation of signal parameters via rotational invariance techniques (ESPRIT) [4]. However, the known search-free DOA estimation techniques in application to MIMO radar use the conventional idea of the virtual/receive array partitioning [4]. Thus, for achieving the rotational invariance property (RIP) between different subarrays, either the transmit or receive arrays should be a uniform linear array (ULA). It has been shown in [5], [6], however, that the RIP for MIMO radar can be also achieved for transmit and receive arrays of arbitrary geometry in unconventional way through an appropriate design of the transmit beamspace matrix. Moreover, in addition to the desired RIP, one obtains a further DOA estimation performance improvement due to the transmit energy focusing. The problem of designing such transmit beamspace matrices which satisfy a number of practical constraints is an open problem of significant interest.

In this paper, we consider the transmit beamspace design problem for MIMO radar. The practical requirements for such design are that the transmit beampattern must be as close as possible to the desired beampattern and the transmit power is uniform across the transmit antennas [3]. However, we additionally require that the RIP is satisfied for a receive array of arbitrary geometry. An important special case of two orthonormal waveforms and ULA at the transmitter is considered and new insights to the problem of transmit beamspace design are provided. The corresponding problem is cast as a non-convex quadratically constrained quadratic programming (QCQP) problem which is NP-hard in general, but can be efficiently solved using the semi-definite programming (SDP) relaxation technique [7]– [9].

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2. SYSTEM MODEL

Consider a mono-static radar system with a transmit array being an M-antenna ULA and a receive array being an N-antenna array with arbitrary geometry. Let $\mathbf{a}(\theta)$ and $\mathbf{b}(\theta)$ denote the steering vectors of the transmit and receive arrays, respectively. The transmit energy focusing can be achieved using an $M \times K$ transmit beamspace matrix $\mathbf{W}$ where $K \leq M$ is the number of orthonormal basis waveforms [5]. Then the $M \times 1$ vector of transmitted signals in baseband can be expressed as

$$\mathbf{s}(t) = \mathbf{W}\boldsymbol{\phi}(t), \quad 0 \leq t \leq T \tag{1}$$

where $\boldsymbol{\phi}(t) = [\phi_1(t), \ldots, \phi_K(t)]$ is the set of orthonormal basis waveforms such that $\int_0^T \boldsymbol{\phi}(t)\boldsymbol{\phi}^H(t) = \mathbf{I}_K$, $(\cdot)^H$ stands for the Hermitian transpose, $\mathbf{I}_K$ is the identity matrix of size $K \times K$, and T is the radar pulse width.

Using (1), the array transmit beampattern can be written as

$$p(\theta) = \left\|\mathbf{W}^H \mathbf{d}(\theta)\right\|^2 \quad (2)$$

where $\|\cdot\|$ denotes the Euclidian norm of a vector, $\mathbf{d}(\theta) = \mathbf{a}^*(\theta)$, and $(\cdot)^*$ stands for the conjugation.

Assuming that L targets are present, the $N \times 1$ receive array observation vector can be written as [5]

$$\mathbf{x}(t,\tau) = \sum_{l=1}^{L} \beta_l(\tau)\mathbf{b}(\theta_l)\left(\mathbf{W}^H \mathbf{d}(\theta_l)\right)^H \boldsymbol{\phi}(t) + \mathbf{z}(t,\tau) \quad (3)$$

where t and τ are the fast and slow time indexes, respectively, $\beta_l(\theta_l)$ is the reflection coefficient of the target located at the angle θ_l with variance σ_β^2, and $\mathbf{z}(t,\tau)$ is the $N \times 1$ vector of zero-mean white Gaussian noise. By matched filtering $\mathbf{x}(t,\tau)$ to each of the orthonormal basis waveforms $\phi_k(t), k = 1,\ldots,K$, the $N \times 1$ virtual data vectors can be obtained as

$$\begin{aligned}\mathbf{y}_k(\tau) &= \int_{\mathrm{T}} \mathbf{x}(t,\tau)\phi_k^*(t)dt \\ &= \sum_{l=1}^{L} \beta_l(\tau)e^{j\psi_k(\theta_l)}\left|\mathbf{w}_k^H \mathbf{d}(\theta_l)\right|\mathbf{b}(\theta_l) + \mathbf{z}_k(\tau) \quad (4)\end{aligned}$$

where $\mathbf{w}_k$ is the kth column of $\mathbf{W}$, $\psi_k(\theta)$ is the phase of the inner product $\mathbf{d}^H(\theta)\mathbf{w}_k$, and $\mathbf{z}_k(\tau) \triangleq \int_{\mathrm{T}} \mathbf{z}(t,\tau)\phi_k^*(t)dt$ is the $N \times 1$ noise term whose covariance is $\sigma_z^2 \mathbf{I}_N$. Note that $\mathbf{z}_k(\tau)$ and $\mathbf{z}_{k'}(\tau)$ $(k \neq k')$ are independent due to the orthogonality between $\phi_k(t)$ and $\phi_{k'}(t)$.

It is worth noting that if $\mathbf{w}_k$ $(k = 1,\ldots,K)$ are designed such that the equality $|\mathbf{w}_k^H \mathbf{d}(\theta)| = |\mathbf{w}_{k'}^H \mathbf{d}(\theta)|$ is satisfied, then the RIP between $\mathbf{y}_k$ and $\mathbf{y}_{k'}$ holds, i.e., the signal component of $\mathbf{y}_k$ associated with the lth target is the same as the corresponding signal component of $\mathbf{y}_{k'}$ up to a phase rotation that is given by $\psi_k(\theta_l) - \psi_{k'}(\theta_l)$.

3. TRANSMIT BEAMSPACE DESIGN

3.1. Main Idea and Problem Formulation

Main Idea: Focusing the transmission power on a desired sector, i.e., the sector where the targets are located, results in significant improvement in the DOA estimation performance [5], [6]. Moreover, such desired properties as the RIP can be achieved in MIMO radar with transmit beamspace preprocessing in unconventional way by appropriate design of the transmit beamspace matrix $\mathbf{W}$. Although it has been shown in [5], [6] that one can focus the transmission power in the desired sector and simultaneously obtain K data sets with RIP, which allows to apply the search-free direction finding techniques such as ESPRIT and PARAFAC, the design of such transmit beamspace matrices, which satisfy a number of practical constraints, is an open problem of significant interest.

We aim at designing $\mathbf{W}$ such that the transmit beampattern (2) is as close as possible to the desired beampattern, the RIP holds, i.e., $|\mathbf{w}_1^H \mathbf{d}(\theta)| = \ldots = |\mathbf{w}_K^H \mathbf{d}(\theta)|$, and the power distribution among transmit antenna elements is uniform. First and last requirements are the practical requirements recognized in, for example, [3], while the second requirement enables applying search free DOA estimation techniques as noticed above.

To develop a simple practical DOA estimation algorithm based on transmit beamspace preprocessing, we consider in this paper the special case of two orthonormal waveforms. Thus, the dimension of $\mathbf{W}$ is $M \times 2$. Then under the aforementioned assumption of ULA at the MIMO radar transmitter, the RIP can be satisfied by the matrix $\mathbf{W} = [\mathbf{w}, \tilde{\mathbf{w}}^*]$ where $\tilde{\mathbf{w}}$ is the flipped version of vector $\mathbf{w}$, i.e., $\tilde{\mathbf{w}}(i) = \mathbf{w}(M - i + 1)$, $i = 1,\ldots,M$. Indeed, in this case, $|\mathbf{w}^H \mathbf{d}(\theta)| = |(\tilde{\mathbf{w}}^*)^H \mathbf{d}(\theta)|$ and the RIP is satisfied.

Problem Formulation: Using the minimum error criteria [3], the beampattern design problem which satisfies the aforementioned requirements can be formulated as the following optimization problem

$$\begin{aligned}&\min_{\mathbf{w}} \max_{\theta_q} \left|P_d(\theta_q) - \|[\mathbf{w}\,\tilde{\mathbf{w}}^*]^H \mathbf{d}(\theta_q)\|^2\right| \\ &\text{s.t. } |\mathbf{w}(i)|^2 + |\tilde{\mathbf{w}}(i)|^2 = \frac{E}{M}, \quad i = 1,\ldots,M \quad (5)\end{aligned}$$

where $P_d(\theta)$ is the desired beampattern, E is the total transmit power, and $\{\theta_q : q = 1,\ldots,Q\}$ is a uniform grid that approximates the interval $[-\pi/2, \pi/2]$ into Q number of directions. The constraint used in (5) ensures that the transmit power distribution across the antenna elements is uniform.

3.2. Solution

Assuming that the number of the transmit antenna elements M is even[1], the optimization problem (5) can be equivalently rewritten as

$$\begin{aligned}&\min_{\mathbf{w},z} z \\ &\text{s.t. } \frac{P_d(\theta_q)}{2} - |\mathbf{w}^H \mathbf{d}(\theta_q)|^2 \leq z,\ q = 1,\ldots,Q \\ &\quad\ \ \frac{P_d(\theta_q)}{2} - |\mathbf{w}^H \mathbf{d}(\theta_q)|^2 \geq -z,\ q = 1,\ldots,Q \\ &\quad\ \ |\mathbf{w}(i)|^2 + |\mathbf{w}(M-i+1)|^2 = \frac{E}{M},\ i = 1,\ldots,\frac{M}{2}. \quad (6)\end{aligned}$$

It is worth noting that the problem (6) has significantly larger number of degrees of freedom than the beamforming problem for the phased-array case when the magnitudes of $\mathbf{w}(i)$, $i = 1,\ldots,M$ are fixed. The problem (6) belongs to the class of non-convex QCQP problems which are NP-hard in

[1] The case of odd M can be considered in completely similar way.

general. However, a well developed SDP relaxation technique can be used to solve it [7]– [9]. Indeed, using the facts that $|\mathbf{w}^H\mathbf{d}(\theta_q)|^2 = Tr(\mathbf{d}(\theta_q)\mathbf{d}^H(\theta_q)\mathbf{w}\mathbf{w}^H)$ and $|\mathbf{w}(i)|^2 + |\mathbf{w}(M-i+1)|^2 = Tr(\mathbf{w}\mathbf{w}^H\mathbf{A}_i), i = 1,\ldots,M/2$, where $Tr(\cdot)$ stands for the trace and $\mathbf{A}_i$ is an $M \times M$ matrix such that $\mathbf{A}_i(i,i) = \mathbf{A}_i(M-(i-1), M-(i-1)) = 1$ and the rest of the elements are equal to zero, the problem (6) can be cast as

$$\begin{aligned} &\min_{\mathbf{w},z} z \\ \text{s.t. } &\frac{P_d(\theta_q)}{2} - Tr(\mathbf{d}(\theta_q)\mathbf{d}^H(\theta_q)\mathbf{w}\mathbf{w}^H) \leq z,\ q = 1,\ldots,Q \\ &\frac{P_d(\theta_q)}{2} - Tr(\mathbf{d}(\theta_q)\mathbf{d}^H(\theta_q)\mathbf{w}\mathbf{w}^H) \geq -z,\ q = 1,\ldots,Q \\ &Tr(\mathbf{w}\mathbf{w}^H\mathbf{A}_i) = \frac{E}{M},\ i = 1,\ldots,\frac{M}{2} \end{aligned} \quad (7)$$

Introducing the new variable $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$, the problem (7) can be equivalently written as

$$\begin{aligned} &\min_{\mathbf{X},z} z \\ \text{s.t. } &\frac{P_d(\theta_q)}{2} - Tr(\mathbf{d}(\theta_q)\mathbf{d}^H(\theta_q)\mathbf{X}) \leq z,\ q = 1,\ldots,Q \\ &\frac{P_d(\theta_q)}{2} - Tr(\mathbf{d}(\theta_q)\mathbf{d}^H(\theta_q)\mathbf{X}) \geq -z,\ q = 1,\ldots,Q \\ &Tr(\mathbf{X}\mathbf{A}_i) = \frac{E}{M},\ i = 1,\ldots,\frac{M}{2};\ rank(\mathbf{X}) = 1 \end{aligned} \quad (8)$$

where $\mathbf{X}$ is the Hermitian matrix and $rank(\cdot)$ denotes the rank of a matrix. Note that the last two constraints in (8) imply that the matrix $\mathbf{X}$ is positive semi-definite. The problem (8) is non-convex with respect to $\mathbf{X}$ because the last constraint is not convex. However, by means of the SDP relaxation technique, this constraint can be replaced by another constraint which implies that the matrix $\mathbf{X}$ is positive semi-definite, that is, $\mathbf{X} \succeq 0$. The resulting problem is the relaxed version of (8) and it is a convex SDP problem which can be efficiently solved using, for example, interior-point methods. When the relaxed problem is solved, extraction of the solution of the original problem is typically done via the so-called *randomization* techniques.

Let $\mathbf{X}_{opt}$ denote the optimal solution of the relaxed problem. If the rank of $\mathbf{X}_{opt}$ is one, the optimal solution of the original problem (6) can be obtained by simply finding the principal eigenvector of $\mathbf{X}_{opt}$. However, if the rank of the matrix $\mathbf{X}_{opt}$ is higher than one, the randomization approach can be used. Various randomization techniques have been developed and are generally based on generating a set of candidate vectors and then choosing the candidate which gives the minimum of the objective function of the original problem. Our randomization procedure can be described as follows. Let $\mathbf{X}_{opt} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$ denote the eigen-decomposition of $\mathbf{X}_{opt}$. The candidate vector k can be chosen as $\mathbf{w}_{can,k} = \mathbf{U}\mathbf{\Sigma}^{1/2}\mathbf{v}_k$ where $\mathbf{v}_k$ is random vector whose elements are random variables uniformly distributed on the unit circle in the complex plane. Candidate vectors are not always feasible and should be mapped to a nearby feasible point. This mapping is problem dependent [9]. In our case, if the condition $|\mathbf{w}_{can,k}(i)|^2 + |\mathbf{w}_{can,k}(M-i+1)|^2 = E/M$ does not hold, we can map this vector to a nearby feasible point by scaling $\mathbf{w}_{can,k}(i)$ and $\mathbf{w}_{can,k}(M-i+1)$ to satisfy this constraint. Among the candidate vectors we then choose the one which gives the minimum objective function, i.e., the one with minimum $\max_{\theta_q} \left| P_d(\theta_q)/2 - |\mathbf{w}^H_{can,k}\mathbf{d}(\theta_q)|^2 \right|$.

4. SIMULATION RESULTS

Consider a MIMO radar which consists of a ULA of $M = 10$ omni-directional transmit antennas spaced half a wavelength apart from each other and an arbitrary array of $N = 10$ receive antennas whose locations are drawn uniformly within the interval [0, 4.5] measured in wavelength. Temporally and spatially independent white Gaussian noise with zero mean and unit variance is assumed. Two targets are located in the directions $-5°$ and $5°$. The total transmit energy $E = 1$ is taken, the total number of 50 snapshots are used to compute the sample covariance matrix for all methods tested, and the RMSEs for all methods are computed based on 1000 independent runs. The desired beampattern is assumed to be

$$p(\theta) = \begin{cases} M, & \text{if } -10° \leq \theta \leq 10° \\ 0, & \text{otherwise} \end{cases}. \quad (9)$$

For the method of [4], the following baseband signals are used

$$\phi_m(t) = \sqrt{\frac{1}{T}} e^{j2\pi\frac{m}{T}t}, \quad m = 1,\ldots,M \quad (10)$$

while for the proposed transmit beamspace method, the first two waveforms of (10) are used. ESPRIT based DOA estimation is performed using (i) the proposed method with the transmit beamspace matrix $\mathbf{W}_0$ obtained by solving the problem (5) (referred to as 'new ESPRIT – Equal gain'); (ii) the proposed method with the transmit beamspace matrix $\mathbf{W}_0\mathbf{U}_{2\times 2}$ (referred to as 'new ESPRIT – Not equal gain'), where $\mathbf{U}_{2\times 2}$ is a unitary matrix defined as

$$\mathbf{U} = \begin{bmatrix} 0.6925 + j0.3994 & 0.4903 + j0.3468 \\ -0.4755 + j0.3669 & 0.6753 - j0.4279 \end{bmatrix} \quad (11)$$

and (iii) the method of [4] (referred to as 'ESPRIT of [4]'). Note that $\mathbf{W}_0$ and $\mathbf{W}_0\mathbf{U}$ have the same transmit beampattern and as a result the same transmit power on the desired sector, however, compared to the former, the latter one does not satisfy the RIP precisely. In the ESPRIT method of [4], the $M \times N$ virtual steering vector is partitioned into two subarrays of size $(M-1) \times N$ such that the first subarray contains the first $(M-1) \times N$ elements while the second subarray contains the last $(M-1)N$ elements. In two other proposed

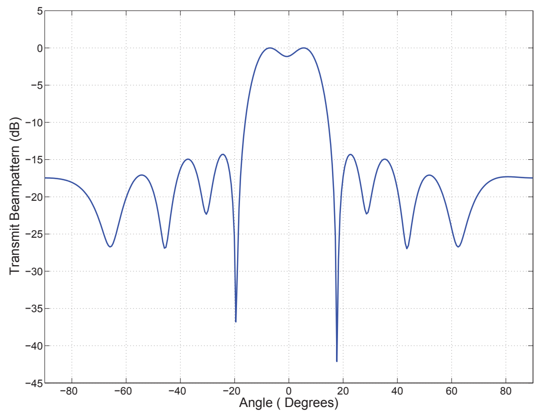

Fig. 1. Transmit Beampattern versus angle θ.

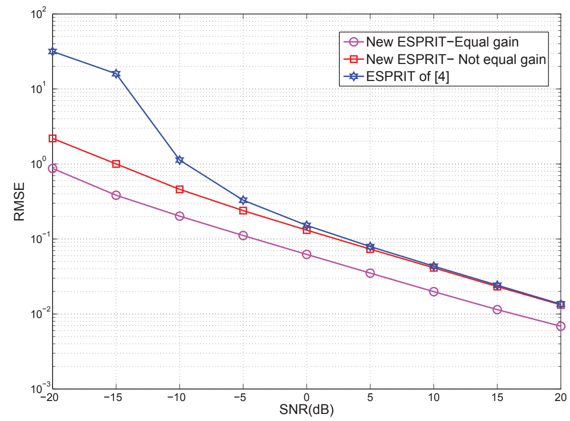

Fig. 2. RMSE versus SNR.

ESPRIT methods, the $2N \times 1$ virtual steering vector is partitioned into two non-overlapping subarrays, i.e., the first subarray contains the first N elements while the second subarray contains the last N elements.

The CVX optimization toolbox [10] is used for solving the relaxed version of (8), that is, the SDP problem. The solution to the original problem is extracted from the optimal solution of the relaxed one through the proposed randomization procedure by generating 1000 candidate vectors.

Fig. 1 shows the normalized transmit beampattern corresponding to the so obtained transmit beamspace matrix. The root-mean-squared error (RMSE) of the three aforementioned DOA estimation methods versus $\text{SNR} = \sigma_\beta^2 / \sigma_z^2$ is shown in Fig. 2. It can be seen from this figure that the ESPRIT DOA estimation based on using the transmit beamspace matrix $\mathbf{W}_0$ outperforms the other methods. It can be also concluded that when the RIP does not hold the performance of DOA estimation can degrade severely.

5. CONCLUSION

The transmit beamspace design problem for colocated MIMO radar has been considered. In addition to the traditional requirements for beamspace design such as approximating the desired beampattern as close as possible and ensuring the uniform power distribution across transmit antenna elements, we also require that the RIP is satisfied at the receive array. The latter enables to use the search-free ESPRIT technique for DOA estimation even in the situation when the receive array has an arbitrary geometry. Moreover, the DOA estimation performance is improved due to the transmit energy focusing. The special case of two orthonormal basis waveforms has been considered. Our simulation results show the improved performance for the proposed method as compared to the existing MIMO radar ESPRIT method without energy focusing.

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