Cellular automata

- **Cellular automata** system is a grid of cells evolving synchronously according to a discrete global clock.
- Discrete and finite set of states in each cell.
- Computation is exact (and in some cases time-reversible).
- Evolution rules are local; e.g., $x_i^{(t+1)} := F(x_{\{i,i\pm1\}}^{(t)})$. 
Causal structure

states in the past that were influential

$\mathbf{x}_i^{(t)}$

states in the future that can be influenced

states in the past that were influential
Margolus scheme

- Space-filling partition of groups of cells, alternating with time
- Each group updated independently
- Easy to enforce conservation laws, especially number conservation

1d lattice gas: □□ □□
Square lattice gas

- 4-site Margolus tiling, shifted by (1,1) for odd times
- Updates conserve particle number, energy, and momentum (along the links of the dual lattice)
Square lattice gas

- Velocities allowed along only two orthogonal directions
- Leads to many undesirable hidden invariants

\[ \text{momentum conserved independently along every line of the dual lattice} \]
Coarse-grained behaviour

- Long-distance, long-time behaviour is related to Navier-Stokes hydrodynamics but with an artificial anisotropy
- Spurious invariants survive coarse graining
- Cured by three-body interactions on the triangular lattice