The smooth block $B$ of negligible size has a mass $m$ and rests on the horizontal plane. If the board $AC$ pushes on the block at an angle $\theta$ with a constant acceleration $a_0$, determine the velocity of the block along the board and the distance $s$ the block moves along the board as a function of time $t$. The block starts from rest when $s = 0$, $t = 0$.

**SOLUTION**

\[ F + \sum F_x = m\ a_x; \quad 0 = m\ a_B\ \sin\ \phi \]

\[ a_B = a_{AC} + a_{B/AC} \]

\[ a_B = a_0 + a_{B/AC} \]

\[ a_B\ \sin\ \phi = -a_0\ \sin\ \theta + a_{B/AC} \]

Thus,

\[ 0 = m(-a_0\ \sin\ \theta + a_{B/AC}) \]

\[ a_{B/AC} = a_0\ \sin\ \theta \]

\[ \int_0^{v_{B/AC}} dv_{B/AC} = \int_0^t a_0\ \sin\ \theta\ dt \]

\[ v_{B/AC} = a_0\ \sin\ \theta\ t \]

\[ s_{B/AC} = s = \int_0^t a_0\ \sin\ \theta\ t\ dt \]

\[ s = \frac{1}{2} a_0\ \sin\ \theta\ t^2 \]

Ans.
The 5-kg collar $A$ is sliding around a smooth vertical guide rod. At the instant shown, the speed of the collar is $v = 4 \text{ m/s}$, which is increasing at $3 \text{ m/s}^2$. Determine the normal reaction of the guide rod on the collar, and force $P$ at this instant.

**SOLUTION**

$$\sum F_i = ma_i; \quad P \cos 30^\circ = 5(3)$$

$$P = 17.32 \text{ N} = 17.3 \text{ N}$$

$$\sum F_n = ma_n; \quad N + 5(9.81) - 17.32 \sin 30^\circ = 5\left(\frac{4^2}{0.5}\right)$$

$$N = 119.61 \text{ N} = 120 \text{ N}$$
The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point $A$, it is traveling at 9 m/s and increasing its speed at $3 \text{ m/s}^2$. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.

**SOLUTION**

**Geometry:** Here, \( \frac{dy}{dx} = -0.00625x \) and \( \frac{d^2y}{dx^2} = -0.00625 \). The slope angle \( \theta \) at point $A$ is given by

\[
\tan \theta = \left. \frac{dy}{dx} \right|_{x=80} = -0.00625(80) \quad \theta = -26.57^\circ
\]

and the radius of curvature at point $A$ is

\[
\rho = \left. \frac{1 + (\frac{dy}{dx})^2}{{(\frac{d^2y}{dx^2})}^{3/2}} \right|_{x=80} = 223.61 \text{ m}
\]

**Equation of Motion:** Applying Eq. 13–8 with \( \theta = 26.57^\circ \) and \( \rho = 223.61 \text{ m} \), we have

\[
\sum F_x = ma_x; \quad 800(9.81) \sin 26.57^\circ - F_f = 800(3) \quad F_f = 1109.73 \text{ N} = 1.11 \text{ kN} \quad \text{Ans.}
\]

\[
\sum F_y = ma_y; \quad 800(9.81) \cos 26.57^\circ - N = 800 \left( \frac{g^2}{\rho^2} \right) \quad N = 6729.67 \text{ N} = 6.73 \text{ kN} \quad \text{Ans.}
\]
A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path. If the banking angle \( \theta = 15^\circ \), determine the uplift force \( L \) acting on the airplane and the radius \( r \) of the circular path. Neglect the size of the airplane.

**SOLUTION**

**Free-Body Diagram:** The free-body diagram of the airplane is shown in Fig. (a). Here, \( a_n \) must be directed towards the center of curvature (positive \( n \) axis).

**Equations of Motion:** The speed of the airplane is \( v = \left( 350 \frac{\text{km}}{\text{h}} \right) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 97.22 \text{ m/s} \). Realizing that \( a_n = \frac{v^2}{\rho} = \frac{97.22^2}{r} \) and referring to Fig. (a),

\[ + \Sigma F_b = 0; \quad \rightarrow \Sigma F_n = ma_n; \]

\( L \cos 15^\circ - 5000(9.81) = 0 \)

\( L = 50780.30 \text{ N} = 50.8 \text{ kN} \quad \text{(Ans.)} \)

\( 50780.30 \sin 15^\circ = 5000 \left( \frac{97.22^2}{r} \right) \)

\( r = 3595.92 \text{ m} = 3.60 \text{ km} \quad \text{(Ans.)} \)
The 5-lb collar slides on the smooth rod, so that when it is at A it has a speed of 10 ft/s. If the spring to which it is attached has an unstretched length of 3 ft and a stiffness of \( k = 10 \text{ lb/ft} \), determine the normal force on the collar and the acceleration of the collar at this instant.

**SOLUTION**

\[
y = 8 - \frac{1}{2}x^2
\]

\[
- \frac{dy}{dx} = \tan \theta = x \quad \bigg|_{x=2} \quad \theta = 63.435^\circ
\]

\[
\frac{d^2y}{dx^2} = -1
\]

\[
\rho = \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{1}{2}} = \left( 1 + (-2)^2 \right)^{\frac{1}{2}} = 11.18 \text{ ft}
\]

\[
y = 8 - \frac{1}{2}(2)^2 = 6
\]

\[
OA = \sqrt{(2)^2 + (6)^2} = 6.3246
\]

\[
F_s = kx = 10(6.3246 - 3) = 33.246 \text{ lb}
\]

\[
\tan \phi = \frac{6}{2}; \quad \phi = 71.565^\circ
\]

\[
N = 24.4 \text{ lb}
\]

\[
5 \cos 63.435^\circ + 33.246 \cos 45.0^\circ = \left( \frac{5}{32.2} \right) \left( \frac{10^2}{11.18} \right)
\]

\[
a_t = 180.2 \text{ ft/s}^2
\]

\[
a_n = \frac{v^2}{\rho} = \frac{(10)^2}{11.18} = 8.9443 \text{ ft/s}^2
\]

\[
a = \sqrt{(180.2)^2 + (8.9443)^2}
\]

\[
a = 180 \text{ ft/s}^2
\]