Processing seismic data in the presence of residual statics
Aaron Stanton*, Nasser Kazemi, and Mauricio D. Sacchi, Department of Physics, University of Alberta

SUMMARY

Many common processing steps are degraded by static shifts in the data. The effects of static shifts are analogous to noise or missing samples in the data, and therefore can be treated using constraints on sparsity or simplicity. In this paper we show that random static shifts decrease sparsity in the Fourier and Radon transforms, as well as increase the rank of seismic data. We also show that the concepts of sparsity promotion and rank reduction can be used to solve for static shifts as well as to carry out conventional processes in the presence of statics. The first algorithm presented is a modification to the reinsertion step of Projection Onto Convex Sets (POCS) and Tensor Completion (TCOM) that allows for the compensation of residual statics during 5D denoising and interpolation of seismic data. The method allows preserving residual statics during denoising, or correction of residual statics in the case of simultaneous denoising and interpolation. An example is shown for a 5D reconstruction of synthetic data with added noise, missing traces and random static shifts, as well as for a 2D stacked section with missing traces and static shifts. While standard reconstruction struggles in the presence of even small static shifts, reconstruction with simultaneous estimation of statics is able to accurately reconstruct the data. The second algorithm presented is a Statics Preserving Sparse Radon transform (SPSR). This algorithm includes statics in the Radon bases functions, allowing for a sparse representation of statics-contaminated data in the Radon domain.

INTRODUCTION

Many processing steps can fail in the presence of even small static shifts ($\leq 10\text{ms}$). Creating robust methods that can handle residual static shifts can benefit early stages of processing workflows, such as denoising, interpolation, and multiple attenuation. This paper is divided into two parts. In the first part a modification to the reinsertion step of Tensor Completion (TCOM) and Projection Onto Convex Sets (POCS) based reconstruction is provided. This step inserts the estimated data into the original data and can be used for both interpolation and denoising. We propose a modification to the reinsertion that allows for the compensation of residual static shifts during this step. In the case of POCS the property being exploited is sparsity in the frequency-wavenumber domain, while in the case of TCOM the property exploited is rank in the frequency-space domain.

Residual statics are typically attributed to near surface lateral velocity and topographical variations (Ronen and Claerbout, 1984). Because these effects are considered to be surface consistent their correction involves a single static shift for each trace. A common practice for residual static estimation is to use the cross-correlation of unstacked data within a CMP gather compared with the stacked trace, taking the maximum lags for each trace as an estimate of the residual statics. A problem with this method is that it is sensitive to the velocity used to NMO correct the input gathers (Erikson and Willen, 1990). Traonnmilin and Gulunay (2011) tackle the problem by simultaneously estimating the statics during projection filtering for the purpose of denoising seismic data. Recently Gholami (2013) showed that sparsity maximization can be used as a criterion for short period statics correction. Building off the work of Stanton and Sacchi (2012) we propose two algorithms that allow for statics to be estimated during 5D trace interpolation and denoising as well as for demultiple using the sparse Radon transform in the presence of statics.

THEORY

5D Reconstruction

Our method begins with the observation that statics appear to have the same character as noise or missing traces in the Fourier domain. Figure 1 shows a 2D synthetic gather (a) with added noise (b), missing traces (c), static shifts (d), and their respective f-k amplitude spectra (e)-(h). The destruction of the signal observed in the f-k amplitude spectra for each of these three cases are remarkably similar.

The assumption of any Fourier reconstruction method is that the desired noise-free, fully-sampled signal can be sparsely represented in the Fourier domain. In this paper we show that this same sparsity relation can be used for the removal of static shifts within the data.

In the case of 5D Projection Onto Convex Sets (POCS) reconstruction a Fourier estimate of the data is found by iteratively thresholding the amplitude spectrum of the data (Abma and Kabir, 2006). For a given temporal frequency, $\omega$, the data in the $\omega - m_x - m_y - h_x - h_y$ domain at the $k^{th}$ iteration of POCS are given by

$$D^k = \alpha_3 D_{obs}^{nobs} + (1 - \alpha_3) S F_D^{-1} T F_D D^{k-1}, \quad k = 1, \ldots, N,$$

where $m_x$, $m_y$, $h_x$, $h_y$ are midpoints and offsets in the $x$ and $y$ directions respectively, $D_{obs}^{nobs}$ are the original data with missing traces, and $F_D$ and $F_D^{-1}$ are the forward and reverse 4D Fourier transforms in the spatial dimensions respectively. In this notation $D^k(\omega, km_x, km_y, kh_x, kh_y) = F_D D^{k}(\omega, m_x, m_y, h_x, h_y)$, and $T$ is an iteration dependent threshold operator that is designed using the amplitudes of the input data (Gao and Sacchi, 2011). $S$ is the sampling operator and is equal to one for points with existing traces and zero for points with unrecorded observations. The scaling factor $\alpha_3 \leq 1$ can be used to simultaneously denoise the data. A choice of $\alpha_3 = 1$ reinserts the noisy original data at each iteration, whereas a lower value of $\alpha_3$ will denoise the volume by averaging the original and reconstructed data.

The modification we propose to allow for statics to be compensated for during the reconstruction is to derive a static shift between the thresholded data and the data from the previous
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The data in the \( \omega - m_x - m_y - h_x - h_y \) domain at the \( k^{th} \) iteration of POCs with statics compensation are given by

\[
D^k = a_k D^{obs} e^{-a_1(1-a_2)\omega} + (1-a_2)S_f D^k + F D^k, \quad k = 1, ..., N,
\]

(2)

where \( a_k(m_x, m_y, h_x, h_y) \) are the estimated static shifts at the \( k^{th} \) iteration and are constant for all frequencies, \( \omega \). The scaling factor \( a_2 \leq 1 \) can be used to control the level of static correction. A choice of \( a_2 = 1 \) can be used for data with little to no residual statics, whereas a value of \( a_2 = 0 \) will remove statics more aggressively by fully applying the estimated static shifts at a given iteration. The time shifts \( \tau^k(m_x, m_y, h_x, h_y) \) are the lags given by the maximum values of the cross correlation of the static corrected input data from the previous iteration, \( D^{obs} e^{-a(1-a_2)\tau} \) (where \( \tau^{-1} = \sum_{n=1}^{N} \tau^n \)), with the thresholded data from the current iteration, \( F D^k \). This allows for the iterative application of noise attenuation, missing trace interpolation, and static correction.

The reinsertion step of POCs is identical to that used during 5D Tensor Completion (TCOM) (Stanton et al., 2012). This implies that we can replace the Fourier estimate of the data, \( F D^k T F D^k \), at a given iteration, \( k \), with a rank-reduced version of the data \( R(D^{k-1}) \). This formulation has the advantage that it can deal with the reconstruction of curved events (Kreimer and Sacchi, 2011). In the case of noise attenuation given data that is fully spatially sampled one may wish to denoise the data while preserving residual statics. The total residual statics applied during the denoising are contained in \( \tau^k(m_x, m_y, h_x, h_y) \) allowing for them to be removed from the data. In the case of interpolation it is preferable to leave the static corrections applied to avoid static shifts between interpolated and original traces. The fact that the algorithm produces a tensor of time-shifts \( \tau(m_x, m_y, h_x, h_y) \) could offer an advantage for other processing steps. Converting this tensor to shot and receiver coordinates, \( \tau(s_x, s_y, g_x, g_y) \), the time shifts could then be used for further processing or to gain a better understanding of the near surface.

Sparse Radon Transform

The Sparse Radon Transform can be written as the linear operation. Data, \( d \) can be generated from a model \( m \) under the action of the Radon operator \( L \) as follows:

\[
d = L m + n,
\]

(3)

where \( d \) is data, \( L \) is the Radon model and \( n \) is the noise content. Thorson and Claerbout (1985) showed that given the data, equation (3) can be solved via damped least squares approach. In other words, the objective function is

\[
\text{minimize } ||m||_2 \quad \text{s.t. } ||d - L m||_2 < \varepsilon
\]

(4)

where \( \varepsilon \) is some estimate of noise level in the data. To increase the resolution of the Radon model, one can adopt \( l_2 \) norm for data misfit and \( l_1 \) norm for the model

\[
\hat{m} = \arg \min_m \frac{1}{2} ||d - L m||_2 + \tau ||m||_1,
\]

(5)

where \( m \) is desired sparse model and \( \tau \) is a regularization parameter that balances the importance of the misfit functional and regularization term. The cost function of equation (5) is complete if data has no statics. As discussed earlier statics introduce artifacts and smears the Radon model. Considering statics changes equation (5) to

\[
\arg \min_m \arg \min_Q \frac{1}{2} ||Q d - L m||_2 + \tau ||m||_1,
\]

(6)

where \( Q \) is shifting operator that corrects statics in data. Equation (6) can also be written as

\[
\arg \min_m \arg \min_Q \frac{1}{2} ||d - Q^T L m||_2 + \tau ||m||_1,
\]

(7)

where \( Q^T \) is the adjoint operator of \( Q \). The minimization of equation (8) can be further reduced to the Radon predicted data. This can be done by solving the following sub-problems:

**m-step**

\[
\hat{m} = \arg \min_m \frac{1}{2} ||d - Q^T L m||_2 + \tau ||m||_1,
\]

(8)

**Q-step**

\[
\hat{Q} = \arg \min_Q \frac{1}{2} ||d - Q^T L \hat{m}||_2.
\]

(9)

By considering the initial shifting operator as the identity matrix, the m-step can be solved using Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) (Beck 2009). The Q-step has a closed form solution but it could be considered a full matrix and without physical meaning. So, instead of solving the minimization of equation (7) instead of solving (6) is that equation (7) preserves statics in the predicted data. The cost function of equation (7) can be minimized by alternatively solving the following sub-problems:

**m-step**

\[
\hat{m} = \arg \min_m \frac{1}{2} ||d - Q^T L m||_2 + \tau ||m||_1,
\]

(8)

**Q-step**

\[
\hat{Q} = \arg \min_Q \frac{1}{2} ||d - Q^T L \hat{m}||_2.
\]

(9)

By considering the initial shifting operator as the identity matrix, the m-step can be solved using Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) (Beck 2009). The Q-step has a closed form solution but it could be considered a full matrix and without physical meaning. So, instead of solving directly the Q-step we update the shifting operator by simply cross correlating the \( d \) and \( L \hat{m} \) vectors. Note that this is also a valid solution by defining the \( Q \) operator space as a combination of some shifting basis functions.

EXAMPLES

To test the reconstruction algorithm we apply the algorithm to a 5D synthetic dataset with dimension 100x12x12x12x12. The data has hyperbolic moveout in all four of the spatial directions as seen in figure 2. This figure shows one central bin location out of a total of 144 bins that comprise the complete data. The complete noise free data is shown in figure 2 (a). Before reconstruction random noise was added to the data giving a signal to noise ratio of 2. Random traces were then decimated from the data leaving 50% of the original traces. Random static shifts between \( \pm 10 \)ms were then applied to the data producing the data seen in figure 2 (b). Standard 5D POCs reconstruction was applied to the data resulting in the data shown in figure 2 (c). The static shifts cause a very low quality reconstruction that smears the signal. 5D POCs Reconstruction with static compensation gives a much higher quality result as seen in figure 2 (d). Figure 2 shows reconstruction results for a stacked inline of data that has been corrupted with random static shifts (a). The result after simultaneous statics computation and reconstruction (c) is of higher quality compared to the result after standard reconstruction (b). To test the Radon demultiple algorithm we show an NMO corrected CMP gather from a Gulf
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Figure 1: a) Original 2D synthetic data. b) Data with random noise. c) Data with random missing traces (50%). d) Data with random ±10ms statics shifts. e-h) Are the f-k amplitude spectra of a-d.

Figure 2: a) A portion of noise-free, static-free, fully sampled 5-D synthetic data. b) Data after adding random noise (SNR = 2), random ±10ms static shifts, and randomly removing traces (50%). c) Data after standard 5D reconstruction d) Data after simultaneous 5D reconstruction and statics computation.

Figure 3: Stacked section (a) with missing traces and +/-5ms static shifts, (b) after standard reconstruction, and (c) after simultaneous reconstruction with statics computation.
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Figure 4: Marine CMP gather with random +/- 10ms static shifts, (a) input data, (b) sparse Radon Transform with no statics computation, (c) sparse Radon transform with statics computation.

of Mexico towed-streamer dataset in figure 4 (a). The gather is corrupted with random +/-10ms static shifts. The sparse Radon transform is shown in (b), while the sparse Radon transform with simultaneous statics computation is shown in (c). The sparsity of the Radon panel is improved through the use of the new algorithm. Figure 5 shows the estimated data using the conventional sparse Radon transform (a), compared with using SPSR. Notice the dimming of amplitudes in the estimate produced using the conventional sparse Radon transform. The SPSR result preserves the statics on the data, allowing for multiple suppression of statics-contaminated data.

CONCLUSION

We presented methods to process seismic data in the presence of residual statics. The methods are able to preserve the static shifts in the case of noise and multiple removal, or to compensate for the shifts in the case of simultaneous denoising and trace interpolation. The methods make use of sparsity or simplicity of the static-free seismic data. A topic of future research is to incorporate surface consistency into the methodology to characterize anomalies in the near surface.

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Figure 5: Marine CMP gather with random +/- 10ms static shifts, (a) data estimated using conventional sparse Radon transform, (b) data estimated using sparse Radon transform with statics computation.
EDITED REFERENCES

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REFERENCES


