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Transform-domain Sparsity Regularization in Reconstruction of Channelized Facies

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SUMMARY

Petrophysical reservoir properties, such as porosity and permeability that determine fluid flow displacement within rocks and oil production, are highly correlated spatial parameters. These parameters are very important for future prediction scenarios. Identification of these input parameters using available knowledge (Library) and observations (data) is essential for optimum characterization of reservoir and proper development of the resources in the region. However, library (Geology, etc) and observations may be among available sources of information that can be used for identifying the high energy bearing basis vectors and estimating their corresponding magnitudes. In this study a few examples are used to show how these information sources can be combined with a reduced representation to provide an efficient reconstruction method. In this paper, compressive sensing is studied for inverse modeling of such ill-posed problems with spatially continuous parameters (channelized features) that are sparse in an incoherent basis (e.g. DCT). Compressive sensing is a recently introduced alternative for exact reconstruction of sparse signals from partial linear observations in an incoherent basis. The suitability of the inversion method is illustrated through synthetic examples in characterization of channelized reservoirs.
Introduction

The problem of estimating patterns and structures is encountered in many engineering and science applications, from earth sciences to computer vision and medical imaging. These problems are often ill-posed, having more unknowns than measurements, which can result in non unique solutions. For example, the problem of estimating subsurface structures such as channels and faults from limited point observations is severely ill-posed when these structures are described in terms of many independent pixel values. In such situations, structural assumptions about the solutions are usually built into the solution algorithm to favour solutions in close proximity to a prior description of the unknown structure. However, it is often difficult to specify the spatial patterns a priori, especially when they are poorly defined and have irregular geometry. In such cases, it is better to introduce implicit structural information through constraints on smoothness or sparsity to produce solutions that naturally reveal patterns present in the data.

A common approach for constraining the solutions of ill-posed inverse problems is through regularization (Tikhonov and Arsenin, 1977). Tikhonov regularization methods have enjoyed widespread application in inverse theory, particularly in the geosciences. Although smoothness is a desirable property in many applications, the choice of a regularization technique ultimately should take into account the underlying physics that has generated the observed quantities. Consequently, the form of the regularization constraint should be consistent with and promote the expected properties of the physical system. For instance, compactness measures are preferred to smoothness when working with models that are believed to have sharp local features (Last and Kubik, 1983; Portniaguine and Zhdanov, 1999; Ajo-Franklin et al., 2007).

Another approach to regularization is to use compression techniques to reduce the number of unknowns in the inverse problem. In spatially-discretized models, these techniques typically transform the original finite set of problem variables into another set of variables that provide an equivalent description of the original spatial field. If the transformation is appropriately chosen, a relatively small subset of the new variables can provide a very good approximation. In such cases, the remaining transformed variables can be neglected in the inversion procedure. This truncation approximation yields a new inverse problem that has fewer unknowns and is better posed. Consequently, regularization is achieved through transformation and truncation of the problem variables rather than through incorporation of smoothness terms in the performance function. Minimization of the $l_1$ norm of the low-order spatial derivatives of the model (Claerbout and Muir, 1979; Bube and Langan, 1997) is one of several regularization techniques that have been proposed for stabilizing inverse problems. In this study the second alternative will be examined (for more details see Jafarpour 2009).

Methodology

Compressed sensing (CS) [Candes, Romberg, and Tao (2006); Donoho (2006); Candbs, Tao (2006) and Candbs, Romberg (2005)] is a recently introduced paradigm for estimation or perfect reconstruction of sparse signals from partial observations in a complementary, "incoherent" domain using convex optimization. It has attracted researcher's attention in several disciplines including signal processing and statistics. This problem can be solved using Basis pursuit algorithm which uses an $l_1$ norm minimization (Chen et al., 2001; Donoho, 2006a).

Assume a sparse signal $x_N$ with sparsity $S$ (a signal with $S$ non-zero coefficients) and its transformation coefficients $y_K$ under the transformation matrix $\phi_{N\times N}$ (In this study due to continuity character of underlying reservoir model, Discrete Cosine Transform selected as transformation matrix):

$$y_{N\times N} = \phi_{N\times N} x_{N\times N}$$

Reconstruction of the signal $x_N$ using only $K << N$ observations of it in the transformed domain ($y_K$) is desired:
where \( x \in \mathbb{R}^N \), \( \phi \in \mathbb{R}^N \) and \( y \in \mathbb{R}^K \).

A simple formulation of the approach is given in equation (1) and more general formulation that is the case for consideration of noise in the observation points is presented in equation (2).

\[
\min_{x \in \mathbb{R}^N} \| x \|_1 \quad \text{Subject to} \quad y_{K \times 1} = \phi_{K \times N} x_{N \times 1}.
\]  

\[
\min_{x \in \mathbb{R}^N} \| (y - \phi x) \|_2 + \gamma \| W x \|_1
\]

Where \( W \) is the weighting matrix and \( \gamma \) is the regularization parameter. The equation (2) is well-known as Linear Mixed Norm (LMN) method. The CS algorithm has two major and usually not applicable in the real world assumptions: The signal must be sparse and the observation points must be random. In this study we will ignore such assumptions. Figure (1) shows a sample permeability field and its corresponding DCT coefficients magnitude (in logarithmic scale) after transformation (first and second columns respectively). As a first step we started with considering CS assumptions. Although the coefficients are not exactly zero, most of them are small and can be zeroed out without a major loss in quality (third column). The third and fourth columns in Figure (1) show the (S=39) largest DCT coefficients and their corresponding approximate representation, respectively. For 100 and 200 observed pixels of this permeability in spatial domain (without noise), reconstruction of the DCT coefficients was carried out following the equation (1). It is clear that when the total space of the DCT domain searched (with 100 observations) the minimization failed in reconstruction, but for 200 observation points the Basis pursuit algorithm resulted in perfect reconstruction. As we know the observation points in the real world are very limited. As a resolution we restricted the model space to low frequency subspace and finally by using of 100 random observation points the Basis pursuit resulted in perfect reconstruction. But we are far from the real world yet. The observation points of the permeability field can be obtained only from wells. In Figure (2) instead of random observation, areal observations considered. In general, limited localized data tend to degrade the performance of the compressed sensing formulation. In the case of trained basis, reconstruction identifies channels but the exact shape of the channels in not captured well. Furthermore the observation points are contaminated by noise and due to the ill-posed nature of the problem it is necessary to evaluate sensitivity of the approach to different noise level.

**Sensitivity to observation error (noisy data)**

The results presented in previous sections assumed perfect observations, which is often not the case in practice. In this section sensitivity of the estimates to different levels of measurement errors is studied. The ill-posed nature of the problem necessitates such sensitivity studies to assess the robustness of the reconstruction formulation to the noise level in the measurements. Figure (3) shows the estimation results for LMN method. In this example additive Gaussian pseudo-random noise with zero mean and standard deviations equivalent to 0, 10, 20, and 50 percent of the observation mean are considered in the reconstruction. Reconstruction results appear to be quite robust for noise levels less than 20 percent while degradation in the quality is observed when higher noise levels are considered. At noise levels of 50 percent or more the reconstruction loses the location of the present channels in the field. However, for this application, since point measurements in the field have very small errors associated with them, higher level of measurement errors does not seem to be a major concern.

**Conclusion:**

An estimation approach was introduced for solving ill-posed inverse problems with unknown parameters that are approximately sparse in a transform domain such as DCT. The formulation has its origin in compressed sensing. In general, it is concluded that the sparsity constraint can improve the solution of the ill-posed problems, in which the unknown parameters have an approximately sparse
representation in compression domains such as DCT. The examples suggested that a large number of localized observations may constrain the reconstruction algorithm less than fewer observations that are distributed through the domain and provide a good global coverage.

References
——, [2006b] For most large underdetermined systems of linear equations the minimal l1-norm solution is also the sparsest solution. Communications on Pure and Applied Mathematics, 59, 797-829.

Figure 1 Interpolation example using compressed sensing formulation (equation (1)): True and DCT representation of the permeability (full and approximated) are shown in (a); (b) and (c) show the reconstructed permeability using K=100 and 200 random observations; Compressed sensing reconstruction using reduced low-frequency subspace (d): For K=100 observation, by including N=52 sub dimensional search space reconstruction yields perfect result.
Figure 2 Evaluating of LMN formulation for spatially fixed observation and non-sparse permeability field: non-sparse permeability field and its DCT coefficients (first and second columns), their sparse (s=39) representations (third and fourth columns), and K=180 fixed observation of non-sparse field (fifth column). Figure show the LMN results with untrained basis (second and third rows) and trained basis (fourth and fifth rows) for small and large amount of regularization parameter, respectively.

Figure 3 Sensitivity of the LMN reconstruction results to errors in the measurements (Observation points are same as figure (2)): effect of different level of errors (0%, 10%, 20% and 50%) on the quality of the estimates is depicted. The degradation in the quality of estimates is most pronounced for noise levels above 20% of the mean value.