A SIMPLIFICATION OF [1, PROOF OF THEOREM 4.1]

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The second statement $v_p(\operatorname{rk} M_{m,D}) = n-m$ of [1, Theorem 4.1] can be proved separately and without induction as follows. Since the degree of any closed point on the variety $X(p^m, D)$ is divisible by p^{n-m}, we have $v_p(\operatorname{rk} M_{m,D}) \geq n-m$ by [1, Lemma 2.21]. By [1, Example 2.18], the rank of the total motive $M(X(p^m, D))$ of the generalized Severi-Brauer variety $X(p^m, D)$ is given by the binomial coefficient $\binom{p^n}{p^m}$. Therefore

$$v_p\Big(\operatorname{rk}\big(M(X(p^m, D))\big)\Big) = v_p\binom{p^n}{p^m} = n-m.$$

Since by [1, Theorem 3.8] the motive $M(X(p^m, D))$ is a direct sum of shifts of $M_{l,D}$ with $l \leq m$, it follows that $v_p(\operatorname{rk} M_{m,D}) = n-m$.

The proof of the first statement of [1, Theorem 4.1] ends now with the second full paragraph on Page 195. The remainder of [1, Proof of Theorem 4.1] can be omitted.

References

[1] Karpenko, N. A. Upper motives of algebraic groups and incompressibility of Severi-Brauer varieties. *J. Reine Angew. Math. 677* (2013), 179–198.

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