

ON ISOTROPY OF QUADRATIC PAIR

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ABSTRACT. Let F be an arbitrary field (of arbitrary characteristic). Let A be a central simple F -algebra endowed with a quadratic pair σ (if $\text{char } F \neq 2$ then σ is simply an orthogonal involution on A). We show that the Witt index of σ over the function field of the Severi-Brauer variety of A is divisible by the Schur index of the algebra A .

1. INTRODUCTION

Let F be a field (of arbitrary characteristic). Let A be a central simple F -algebra endowed with a quadratic pair σ (cf. §2.1, a reader without interest in characteristic 2 may replace σ by an orthogonal involution).

Let X be the Severi-Brauer variety of the algebra A (cf. §2.5) and let $F(X)$ stands for the function field of X . We show (Theorem 3.3) that the Witt index $\text{ind } \sigma_{F(X)}$ of the quadratic pair $\sigma_{F(X)}$ (cf. §2.4) is divisible by the Schur index $\text{ind } A$ of the algebra A .

This result generalizes [6, Theorem 5.3] stating that $\sigma_{F(X)}$ is anisotropic provided that A is a division algebra.

Besides, this result supports the affirmative answer to the following

Question 1.1. *Assume that the quadratic pair $\sigma_{F(X)}$ is isotropic. Does the F -variety Y_d of right σ -isotropic ideals in A of reduced dimension $d = \text{ind } A$ possess a 0-dimensional cycle of degree 1?*

Indeed, $Y_d(F(X)) \neq \emptyset$ by Theorem 3.3.

If $\text{ind } A = 2$, then Question 1.1 is answered in the affirmative for A and moreover $Y_2(F) \neq \emptyset$ by [8, Corollary 3.4]. We recall that in general it is not known whether $Y_d(F) \neq \emptyset$ provided that the variety Y_d has a 0-dimensional cycle of degree 1 (cf. [2, Question after Proposition 4.1]).

2. PRELIMINARIES

A *variety* is a separated scheme of finite type over a field.

2.1. Quadratic pairs. Let A be a central simple F -algebra. A *quadratic pair* σ on A is given by an involution of the first kind $\tilde{\sigma}$ on A together with a linear map σ' of the space of the $\tilde{\sigma}$ -symmetric elements of A to F , subject to certain conditions (cf. [7, Definition (5.4) of Chapter I]).

Key words and phrases. Algebraic groups, projective homogeneous varieties, Chow groups.
2000 Mathematical Subject Classifications: 14L17; 14C25.

Support of the Max-Planck-Institut für Mathematik is gratefully acknowledged.

If $\text{char } F \neq 2$, then $\tilde{\sigma}$ is an arbitrary orthogonal involution on A and the map σ' is determined by $\tilde{\sigma}$. Therefore the notion of quadratic pair is equivalent to the notion of orthogonal involution in characteristic $\neq 2$.

If $\text{char } F = 2$, then the algebra A is of even degree and the involution $\tilde{\sigma}$ is of symplectic type.

In arbitrary characteristic, any quadratic form on a finite-dimensional vector space V over F such that its polar bilinear form is non-degenerate, produces a quadratic pair on the endomorphisms algebra $\text{End}(V)$, called the quadratic pair *adjoint* to the quadratic form (the involution of the adjoint quadratic pair is the involution adjoint to the polar symmetric bilinear form of the quadratic form). This way one gets a bijection of the set of quadratic forms on V (up to a factor in F^\times) having non-degenerate polar forms onto the set of quadratic pairs on $\text{End}(V)$.

A right ideal I of a central simple F -algebra A endowed with a quadratic pair σ is called *isotropic* or σ -*isotropic*, if $\tilde{\sigma}(I) \cdot I = 0$ and σ' is 0 on the part of I where σ' is defined (we mean that σ' is 0 on the set of $\tilde{\sigma}$ -symmetric elements of I).

Dimension over F of any right ideal of A is divisible by the degree $\deg A$ of A ; the quotient is called the *reduced dimension* of the ideal.

Let $r \geq 0$ be an integer. The variety Y_r of the right σ -isotropic ideals in A of reduced dimension r is empty if $r > (\deg A)/2$. For $r \leq (\deg A)/2$, Y_r is a projective homogeneous variety under the action of the linear algebraic group $\text{Aut}(A, \sigma)$. In particular, Y_0 is $\text{Spec } F$ with the trivial action. If $r < (\deg A)/2$ then Y_r is a projective homogeneous variety under the action of the connected linear algebraic group $\text{Aut}(A, \sigma)^\circ$ (connected component of $\text{Aut}(A, \sigma)$); in particular, Y_r is integral for such r . If $r = (\deg A)/2$ (for even $\deg A$) and the discriminant of the quadratic pair σ (cf. [7, §7B of Chapter II]) is trivial, then the variety Y_r has two connected components each of which is a projective homogeneous variety under $\text{Aut}(A, \sigma)^\circ$; these components are isomorphic to each other if and only if the algebra A is split.

If $A = \text{End}(V)$ and σ is adjoint to a quadratic form φ on V , then for any r , Morita equivalence identifies the variety Y_r with the variety of r -dimensional totally isotropic subspaces of V . In particular, Y_1 is the projective quadric of φ .

2.2. Chow groups. Let X be a variety over F . A *splitting field* of X is a field extension E/F such that the Chow motive of X_E is a direct sum of twists of the motive of the point $\text{Spec } E$. Any projective homogeneous (under an action of a linear algebraic group) variety (in particular, each variety Y_r of §2.1) has a splitting field.

Given a variety X over F , we write $\text{Ch}(X)$ for the Chow group modulo 2 (i.e., with coefficients $\mathbb{Z}/2\mathbb{Z}$) of X . As in [4, §72], we write $\text{Ch}(\bar{X})$ for the colimit $\text{colim}_L \text{Ch}(X_L)$ over all field extensions L of F . Note that for any splitting field E/F of X the canonical homomorphism $\text{Ch}(X_E) \rightarrow \text{Ch}(\bar{X})$ is an isomorphism.

We write $\bar{\text{Ch}}(X)$ for the image of the homomorphism $\text{Ch}(X) \rightarrow \text{Ch}(\bar{X})$. An element of $\text{Ch}(\bar{X})$ is called *rational* or *F-rational*, if it is inside of $\bar{\text{Ch}}(X)$.

2.3. Varieties of isotropic ideals. Let A be a central simple F -algebra endowed with a quadratic pair σ . Let $Y = Y_1$ be the variety of right σ -isotropic ideals in A of reduced dimension 1. Note that for any splitting field L/F of the algebra A , the variety Y_L is

isomorphic to a projective quadric. Therefore $\text{Ch}(\bar{Y})$ has an \mathbb{F}_2 -basis given by the elements $h^i, l_i, i = 0, \dots, [\dim Y/2]$, introduced in [4, §68].

For any $i \geq 0$, the element h^i is the i th power of the hyperplane section h (in particular, $h^0 = [Y]$ and $h^i = 0$ for $i > \dim Y$). The element h can be non-rational. It is L -rational, where L is a splitting field of the algebra A .

The element l_i is the class of an i -dimensional linear subspace lying inside of Y_E (where E is a splitting field of Y). If $i \neq \dim Y/2$, then this class does not depend on the choice of the linear subspace. However in the case of even $\dim Y$ and $i = \dim Y/2$, there are (exactly) two different classes of i -dimensional linear subspaces on Y_E (and their sum is equal to h^i). An *orientation* of the variety Y is a choice of one of these two classes.

The basis elements of $\text{Ch}(\bar{Y})$ introduced above satisfy the formula $hl_i = l_{i-1}$ for $i = 1, \dots, [\dim Y/2]$.

2.4. Witt index. Let σ be a quadratic pair on a central simple F -algebra A . There exists an integer $\text{ind } \sigma$ such that $\{0, \text{ind } A, 2 \text{ind } A, \dots, (\text{ind } \sigma)(\text{ind } A)\}$ is the set of the reduced dimensions of the right σ -isotropic ideals in A . We call $\text{ind } \sigma$ the (Witt) index of the quadratic pair σ . It satisfies the inequalities $0 \leq \text{ind } \sigma \leq (\deg A)/(2 \text{ind } A)$.

Now we assume that the algebra A is split (this is in fact the only case where we use the definition of the Witt index of a quadratic pair). Then the quadratic pair σ is adjoint with respect to some non-degenerate quadratic form φ (whose similarity class is uniquely determined by σ) and $\text{ind } \sigma$ is the Witt index of φ . If Y is the variety of right σ -isotropic ideals in A of reduced dimension 1 and i is a nonnegative integer, then the element $l_i \in \text{Ch}(\bar{Y})$ is rational if and only if $i < \text{ind } \sigma$, cf. [4, Corollary 72.6].

2.5. Severi-Brauer varieties. Let A be a central simple F -algebra. Let X be the Severi-Brauer variety of A , that is, the variety of all right ideals in A of reduced dimension 1. Over any splitting field (of A or, equivalently, of X), the variety X is isomorphic to a projective space of dimension $(\deg A) - 1$. For any $i \geq 0$, we write $h^i \in \text{Ch}^i(\bar{X})$ for the i th power of the hyperplane class $h \in \text{Ch}^1(\bar{X})$. Therefore, for any i with $0 \leq i \leq \dim X$, h^i is the only nonzero element of the group $\text{Ch}^i(\bar{X})$. Note that h^i is rational if i is divisible by $\text{ind } A$ (cf. [1]).

Now we assume that the Schur index of A is a power of 2. Then by [5, Proposition 2.1.1], we have $\bar{\text{Ch}}^i(X) = 0$ for all i not divisible by $\text{ind } A$. Let us additionally assume that A is a division algebra. Since the (say, first) projection $X^2 \rightarrow X$ is a projective bundle, we have a (natural with respect to the base field change) isomorphism $\text{Ch}_{\dim X}(X^2) \simeq \text{Ch}(X)$. Passing to $\bar{\text{Ch}}$, we get an isomorphism $\bar{\text{Ch}}_{\dim X}(X^2) \simeq \bar{\text{Ch}}(X) = \bar{\text{Ch}}^0(X)$ showing that $\dim_{\mathbb{F}_2} \bar{\text{Ch}}_{\dim X}(X^2) = 1$. Since the diagonal class

$$h^0 \times h^{(\deg A)-1} + h^1 \times h^{(\deg A)-2} + \dots + h^{(\deg A)-1} \times h^0 \in \bar{\text{Ch}}_{\dim X}(X^2)$$

is nonzero, it follows that this is the *only* nonzero element of the group. This result is generalized in Lemma 3.1 below.

3. WITT INDEX

Let F be a field.

Lemma 3.1. *Let A be a central simple F -algebra such that the Schur index of A is a power of 2. Let $d = \text{ind } A$ and $n = (\text{deg } A)/(\text{ind } A)$. Let X_A be the Severi-Brauer variety of A . Let X be the Severi-Brauer variety of a central division F -algebra Brauer-equivalent to A . Then for any $r = 1, \dots, n$, the element*

$$h^0 \times h^{rd-1} + h^1 \times h^{rd-2} + \dots + h^{d-1} \times h^{(r-1)d}$$

is the only nonzero element of the group $\bar{\text{Ch}}^{rd-1}(X \times X_A)$.

Proof. Let r be an integer satisfying $1 \leq r \leq n$.

Since the projection $X \times X_A \rightarrow X$ is a projective bundle and $\bar{\text{Ch}}^i(X) = 0$ for $i \neq 0$ (cf. §2.5), we have an isomorphism $\bar{\text{Ch}}^j(X \times X_A) \simeq \bar{\text{Ch}}^0(X)$ for any $j = 0, \dots, \dim X_A$ (and, in particular, for $j = rd - 1$). Therefore the group $\bar{\text{Ch}}^j(X \times X_A)$ has only one nonzero element for such j (for $j > \dim X_A$ this group is zero).

We write D for a central division F -algebra Brauer equivalent to A . We fix an isomorphism of A with the tensor product $D \otimes M_n(F)$ where $M_n(F)$ is the algebra of square n -matrices over F . Tensor product of ideals produces a closed embedding $X \times \mathbb{P}^{n-1} \hookrightarrow X_A$ (which is a twisted form of the Segre embedding). Picking up a rational point of \mathbb{P}^{n-1} we get a closed embedding $in : X \hookrightarrow X_A$ such that for any splitting field E of X the image of X_E is a linear subspace of the projective space $(X_A)_E$. The image of the diagonal class under the push-forward with respect to the closed embedding $\text{id}_X \times in : X^2 \hookrightarrow X \times X_A$ is equal to

$$\alpha_n = h^0 \times h^{nd-1} + h^1 \times h^{nd-2} + \dots + h^{d-1} \times h^{(n-1)d}.$$

It follows that α_n is the only nonzero element of the group $\bar{\text{Ch}}^{nd-1}(X \times X_A)$.

A basis of $\text{Ch}^{rd-1}(\bar{X} \times \bar{X}_A)$ is given by the elements

$$h^0 \times h^{rd-1}, h^1 \times h^{rd-2}, \dots, h^{d-1} \times h^{(r-1)d}.$$

Let

$$\alpha_r = a_0 h^0 \times h^{rd-1} + a_1 h^1 \times h^{rd-2} + \dots + a_{d-1} h^{d-1} \times h^{(r-1)d}$$

with some $a_0, a_1, \dots, a_{d-1} \in \mathbb{Z}/2\mathbb{Z}$ be the nonzero element of the subgroup

$$\bar{\text{Ch}}^{rd-1}(X \times X_A) \subset \text{Ch}^{rd-1}(\bar{X} \times \bar{X}_A).$$

Since

$$\alpha_r \cdot (h^0 \times h^{(n-r)d}) = a_0 h^0 \times h^{nd-1} + a_1 h^1 \times h^{nd-2} + \dots + a_{d-1} h^{d-1} \times h^{(n-1)d}$$

is a nonzero element of the group $\bar{\text{Ch}}^{nd-1}(X \times X_A)$ (here we use the fact that the element $h^{(n-r)d} \in \text{Ch}(\bar{X}_A)$ is rational, mentioned in §2.5), it follows that $\alpha_r \cdot (h^0 \times h^{(n-r)d}) = \alpha_n$, i.e., $a_0 = a_1 = \dots = a_{d-1} = 1$. \square

Proposition 3.2. *Let A be a central simple F -algebra. Let $d = \text{ind } A$. Let σ be a quadratic pair on A . Let X be the Severi-Brauer variety of a central simple F -algebra Brauer-equivalent to A . Let Y be the variety of σ -isotropic ideals of reduced dimension 1 in A . If for some integer $r \geq 0$ the cycle $l_{rd} \in \text{Ch}(\bar{Y})$ is $F(X)$ -rational, then (for an appropriately chosen orientation of Y) the cycle $l_{(r+1)d-1} \in \text{Ch}(\bar{Y})$ is F -rational.*

Proof. Let D be a central division F -algebra Brauer-equivalent to A . We may assume that X is the Severi-Brauer variety of D .

Since the algebra A possesses an involution of the first type, the index d of A is a power of 2.

If $d = 1$, then there is nothing to prove. We assume that $d \geq 2$ in the sequel.

For any field extension L/F , the pull-back homomorphism $\mathrm{Ch}(X_L \times Y_L) \rightarrow \mathrm{Ch}(Y_{L(X)})$ with respect to the morphism $Y_{L(X)} = (\mathrm{Spec} L(X)) \times Y \rightarrow X_L \times Y_L$ given by the generic point of X is surjective by [4, Corollary 57.11]. These pull-backs give a surjection

$$f : \mathrm{Ch}(\bar{X} \times \bar{Y}) \rightarrow \mathrm{Ch}(\bar{Y})$$

such that the image of the subgroup of rational cycles in $\mathrm{Ch}(\bar{X} \times \bar{Y})$ is the subgroup of $F(X)$ -rational cycles in $\mathrm{Ch}(\bar{Y})$. Since the external product $\mathrm{Ch}(\bar{X}) \otimes \mathrm{Ch}(\bar{Y}) \rightarrow \mathrm{Ch}(\bar{X} \times \bar{Y})$ is an isomorphism (cf. [4, Proposition 64.3]), the external products of the basis elements of $\mathrm{Ch}(\bar{X})$ with the basis elements of $\mathrm{Ch}(\bar{Y})$ form a basis of $\mathrm{Ch}(\bar{X} \times \bar{Y})$. The homomorphism $\mathrm{Ch}(\bar{X} \times \bar{Y}) \rightarrow \mathrm{Ch}(\bar{Y})$ is easily computed in terms of this basis: for any basis element $\beta \in \mathrm{Ch}(\bar{Y})$, the image of $h^0 \times \beta$ is β and the image of any other basis element of $\mathrm{Ch}(\bar{X} \times \bar{Y})$ is 0.

We fix an integer $r \geq 0$ such that the cycle $l_{rd} \in \mathrm{Ch}(\bar{Y})$ is $F(X)$ -rational. Let s stands for the integer $(\deg A)/2 - 1$. Note that $rd \leq s$ (otherwise, the cycle l_{rd} is not defined at all). Let $\alpha \in \mathrm{Ch}(\bar{X} \times \bar{Y})$ be a rational cycle whose image in $\mathrm{Ch}(\bar{Y})$ under the surjection f is l_{rd} . We have

$$\begin{aligned} \alpha = h^0 \times l_{rd} + a_1 h^1 \times l_{rd+1} + \cdots + a_{s-rd} h^{s-rd} \times l_s \\ + b_s h^{s-rd} \times h^s + b_{s-1} h^{s-rd+1} \times h^{s-1} + \cdots + b_0 h^{2s-rd} \times h^0 \end{aligned}$$

for some $a_1, \dots, a_{s-rd}, b_s, \dots, b_0 \in \mathbb{Z}/2\mathbb{Z}$ and for an arbitrary chosen orientation of Y (in this sum, the summands with the first factor h^i are of course 0 for $i > d - 1$).

The variety Y is a closed subvariety of X_A . The image of α under the push-forward $\mathrm{Ch}(\bar{X} \times \bar{Y}) \rightarrow \mathrm{Ch}(\bar{X} \times \bar{X}_A)$ with respect to the base change $X \times Y \hookrightarrow X \times X_A$ of the closed embedding $Y \hookrightarrow X_A$ is equal to

$$h^0 \times h^{D-rd} + a_1 h^1 \times h^{D-rd-1} + \cdots + a_{s-rd} h^{s-rd} \times h^{D-s},$$

where $D = \dim X_A = \deg A - 1$. It follows by Lemma 3.1 that $d - 1 \leq s - rd$ and $a_1 = \cdots = a_{d-1} = 1$. Choosing an appropriate orientation of Y , we may assume that $b_s = 0$.

It follows that

$$\alpha = h^0 \times l_{rd} + h^1 \times l_{rd+1} + \cdots + h^{d-1} \times l_{(r+1)d-1}.$$

Taking push-forward of α with respect to the projection $X \times Y \rightarrow Y$ we obtain $l_{(r+1)d-1} \in \mathrm{Ch}(\bar{Y})$. Therefore the cycle $l_{(r+1)d-1}$ is rational. \square

Theorem 3.3. *Let A be a central simple algebra over a field F . Let σ be a quadratic pair on A . Then the Witt index of σ over the function field of the Severi-Brauer variety of any central simple F -algebra Brauer-equivalent to A is divisible by the Schur index of A .*

Proof. Let Y be the variety of σ -isotropic ideals of reduced dimension 1 in A . Let $d = \mathrm{ind} A$ and let $r \geq 0$ be the largest integer with $\mathrm{ind} \sigma_{F(X)} \geq rd$. If $\mathrm{ind} \sigma_{F(X)} > rd$, then the cycle

$l_{rd} \in \text{Ch}(\bar{Y})$ is $F(X)$ -rational. Therefore, by Proposition 3.2, the cycle $l_{(r+1)d-1}$ is F -rational (and in particular $F(X)$ -rational) and it follows by §2.4 that $\text{ind } \sigma_{F(X)} \geq (r+1)d$, a contradiction with the choice of r . Consequently, $\text{ind } \sigma_{F(X)} = rd$. \square

Remark 3.4. Proposition 3.2 is stronger than Theorem 3.3, because in the proof of Theorem 3.3 we have only used the fact that the cycle $l_{(r+1)d-1}$ is $F(X)$ -rational while Proposition 3.2 states that this cycle is F -rational.

4. MOTIVIC DECOMPOSITION

Cycles constructed in the previous section produce a motivic decomposition which we now describe. Our motivic category is the category of graded correspondences $\text{CR}(F, \mathbb{Z}/2\mathbb{Z})$ defined in [4, §63]. We write $\mathcal{M}(X)$ for the motive of a smooth projective variety X .

Proposition 4.1. *Let A be a central simple F -algebra endowed with a quadratic pair σ . Let Y be the variety of right σ -isotropic ideals in A of reduced dimension 1. Let $d = \text{ind } A$. Let D be a central division algebra Brauer-equivalent to A . Let X be the Severi-Brauer variety of D . Let w be the Witt index of the quadratic pair $\sigma_{F(X)}$. Then the Ch-motive $\mathcal{M}(Y)$ of Y has a direct summand isomorphic to*

$$S = \mathcal{M}(X) \oplus \mathcal{M}(X)(d) \oplus \cdots \oplus \mathcal{M}(X)(w-d) \\ \oplus \mathcal{M}(X)(m) \oplus \mathcal{M}(X)(m-d) \oplus \cdots \oplus \mathcal{M}(X)(m-w+d)$$

where $m = \dim Y - \dim X$. If $\sigma_{F(X)}$ is hyperbolic (i.e., if $w = \deg A/2$), then $\mathcal{M}(Y) \simeq S$.

Proof. We have seen in the proof of Proposition 3.2 that the cycles

$$\alpha_r = h^0 \times l_{rd} + h^1 \times l_{rd+1} + \cdots + h^{d-1} \times l_{(r+1)d-1} \in \text{Ch}(\bar{X} \times \bar{Y})$$

with $r = 0, 1, \dots, w/d - 1$ are rational. For the same r , taking pull-back with respect to the closed embedding $X \times Y \hookrightarrow X \times X_A$ of the rational cycle

$$h^0 \times h^{(r+1)d-1} + h^1 \times h^{(r+1)d-2} + \cdots + h^{d-1} \times h^{rd} \in \text{Ch}(\bar{X} \times \bar{X}_A)$$

of Lemma 3.1, we get a rational cycle

$$\beta_r = h^0 \times h^{(r+1)d-1} + h^1 \times h^{(r+1)d-2} + \cdots + h^{d-1} \times h^{rd} \in \text{Ch}(\bar{X} \times \bar{Y}).$$

Let E/F be a splitting field of Y . Using the multiplication formula of §2.3, one checks that the morphism

$$(\alpha_0, \alpha_1, \dots, \alpha_{w/d-1}, \beta_0, \beta_1, \dots, \beta_{w/d-1}) : S_E \rightarrow \mathcal{M}(Y_E)$$

is right inverse to the morphism

$$(\beta_0^t, \beta_1^t, \dots, \beta_{w/d-1}^t, \alpha_0^t, \alpha_1^t, \dots, \alpha_{w/d-1}^t) : \mathcal{M}(Y_E) \rightarrow S_E$$

(where t stands for the transposition). Moreover, these are mutually inverse isomorphisms provided that $\sigma_{F(X)}$ is hyperbolic. Nilpotence theorem [4, Theorem 92.4 with Remark 92.3] finishes the proof. \square

Remark 4.2. The summands of S are indecomposable by [5]. The decomposition of $\mathcal{M}(Y)$ into a sum of indecomposable summands which we get in the case of hyperbolic σ (or, at least, of hyperbolic $\sigma_{F(X)}$), is unique by [3].

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