Classifying Spaces of Algebraic Groups – MATH 682-850
Winter 2023

Lectures: Tuesday Thursday 9:30AM - 10:50AM online.
Lectures will be recorded, recordings and lecture notes will be available to the participants.

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Office hours: online by appointment

Course webpage: eClass

Course Outline: Classifying spaces of algebraic groups do not exist in classical algebraic geometry. However, they can be approximated by algebraic varieties. Using this approach, Burt Totaro defined the Chow ring \( CH(BG) \) of algebraic cycles on the classifying space \( BG \) of an affine algebraic group \( G \). The representation ring \( R(G) \) can be viewed as the Grothendieck ring of \( BG \). It turns out that the rings \( CH(BG) \) and \( R(G) \) are related the same way as the Chow ring and the Grothendieck ring of a smooth algebraic variety are. Namely, for elements of \( R(G) \) one can define their Chern classes with values in \( CH(BG) \). And there is a surjective ring homomorphism of \( CH(BG) \) onto the graded ring associated with certain filtration on \( R(G) \). We discuss these and related objects, consider examples of their computation, review some applications. We also take a look at the rationality problem of \( BG \). We start with a quick introduction into algebraic groups and into the theory of Chow rings of algebraic varieties.

Prerequisites: Any basic course of algebraic geometry.

Textbook (optional):

Guest lectures:
There will be two guest lectures by Dr. Alexander Merkurjev (UCLA) on “Rationality Problem for Classifying Spaces”.

Grading:
The grade will be based on project on a topic of your choice related to the course consisting of a written report and an oral presentation.