

**University of Alberta**  
**Department of Mathematical & Statistical Sciences**  
**Classifying Spaces of Algebraic Groups – MATH 682-850**  
**Winter 2023**

**Lectures:** Tuesday Thursday 9:30AM - 10:50AM online.

Lectures will be recorded, recordings and lecture notes will be available to the participants.

**Instructor:** Nikita Karpenko

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**Office hours:** online by appointment

**Course webpage:** eClass

**Course Outline:** Classifying spaces of algebraic groups do not exist in classical algebraic geometry. However, they can be approximated by algebraic varieties. Using this approach, Burt Totaro defined the Chow ring  $CH(BG)$  of algebraic cycles on the classifying space  $BG$  of an affine algebraic group  $G$ . The representation ring  $R(G)$  can be viewed as the Grothendieck ring of  $BG$ . It turns out that the rings  $CH(BG)$  and  $R(G)$  are related the same way as the Chow ring and the Grothendieck ring of a smooth algebraic variety are. Namely, for elements of  $R(G)$  one can define their Chern classes with values in  $CH(BG)$ . And there is a surjective ring homomorphism of  $CH(BG)$  onto the graded ring associated with certain filtration on  $R(G)$ . We discuss these and related objects, consider examples of their computation, review some applications. We also take a look at the rationality problem of  $BG$ . We start with a quick introduction into algebraic groups and into the theory of Chow rings of algebraic varieties.

**Prerequisites:** Any basic course of algebraic geometry.

**Textbook (optional):**

Burt Totaro. Group cohomology and algebraic cycles. Cambridge Tracts in Mathematics, 204. Cambridge University Press, Cambridge, 2014.

**Guest lectures:**

There will be two guest lectures by Dr. Alexander Merkurjev (UCLA) on “Rationality Problem for Classifying Spaces”.

**Grading:**

The grade will be based on **project** on a topic of your choice related to the course consisting of a written report and an oral presentation.