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Lyapunov-Based Offset-free Model Predictive Control of Nonlinear Systems

Buddhadeva Das and Prashant Mhaskar

Abstract— **This work considers the problem of control of nonlinear systems subject to uncertainty. To this end, a Lyapunov-based robust model predictive controller (MPC) design is integrated with a moving horizon based offset-free mechanism to enable improved closed–loop performance while still retaining the guarantees provided by the Lypunov-based robust MPC. Simulation results are presented to illustrate the key ideas of the proposed framework.**

I. Introduction

Model predictive control (MPC) has been a key advanced process control system implemented in chemical process industry for the past three decades [1]. An MPC algorithm is based on a predictive mechanism which can reasonably predict the plant outputs in the future given a change in the inputs and an optimization routine that chooses the 'best' control input trajectory. The first 'piece' of the control input trajectory is implemented, and the process repeated.

Several research efforts have explored the issue of handling uncertainty, given the ubiquitous presence of uncertainty (whether using a linear or nonlinear model in the control design). In one approach- the so-called offset-free mechanism approach, the model in the controller is augmented with some form of 'disturbance' states or parameters which are in turn estimated online, effectively turning the controller into an adaptive control design. In one formulation, the current measured process output is compared with the corresponding prediction from the model and the difference is used directly as a bias term added to the model predictions within the MPC [1]. This type of additive feedback is also referred to as a constant output disturbance. Other formulations handle the measurement noise associated with the bias correction term by 'filtering' the measurements/bias term in some form. In general, the offset-free approach has included the use of parameter/state estimation techniques such as Kalman filter [2] and Luenberger Observer [3], to filter the noise and estimate the (assumed constant) plant model mismatch term.

To incorporate constraint handling abilities in the estimation scheme, recent efforts [4], [5] have also considered Moving Horizon Estimation (MHE) as an alternative to Kalman filter and Luenberger Observer. An offset-free nonlinear MPC based on nonlinear MHE has been proposed and applied on an air separation unit [6], where the (constant) process disturbance and measurement noise term is estimated using an MHE formulation and subsequently utilized in the MPC formulation. The existing results for nonlinear systems, however, do not provide any guarantees on the feasibility and robust stabilizability properties of the predictive controller.

Robust MPC designs, on the other hand, are designed to handle worst-case realization of the uncertainty. This is achieved by implementing (in some form) a min-max optimization [7], [8], where the control action is computed to minimize the cost function and satisfy the performance/stability constraints subject to the worst case realization of the uncertainty. While the min-max formulations provide a natural setting within which to address this problem, computational problems with these approaches are well known, and stem in part from the nonlinearity of the model which makes the optimization problem non–convex and in part from performing the min–max optimization over the non-convex problem. Predictive control formulations (including those that do not consider uncertainty) owe their stabilizing properties to some form of 'stability' constraints that essentially require some appropriate measure of the state to decrease or reach a target set by the end of the horizon. The initial feasibility of the stability constraint is typically *assumed* to ensure stability.

Recent advances [9], [10] have utilized Lyapunov-based techniques to design robust MPC formulations which (for system dynamics affine in the control input and disturbance variables) allow an analytical solution to the maximization part of the problem, resulting in a computationally inexpensive formulation. Such formulations have also been shown to possess an explicitly characterized region of robust stability. All such approaches, however, essentially rely (or use) some information about the bounds on the uncertainty (as well as a structure of the uncertainty), and in general do not estimate the uncertain parameter. While the ability to handle the worst case uncertainty provides the stability guarantees, the implementation of robust Lyapunov-based control designs can stand to gain from the augmentation of an offset-free mechanism enabling improved closed–loop performance.

Motivated by the above considerations, this work considers the problem of augmenting Lyapunov-based MPC designs with an offset-free nonlinear model predictive control formulation. The rest of the manuscript is organized as follows: Section II presents the system description, a review of Lyapunov-based robust MPC design, and describes a generic MHE formulation. In Section III, an augmented offset-free Lyapunov-based MPC scheme is presented. In Section IV, simulation results are presented to illustrate the efficacy of

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Buddhadeva Das and Prashant Mhaskar are with the Department of Chemical Engineering, McMaster University, Hamilton, ON L8S 4L7, Canada, dum4@mcmaster.ca, mhaskar@mcmaster.ca.

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the proposed method.

II. Preliminaries

We consider nonlinear systems with uncertain variables and input constraints, described by:

$$\begin{array}{rcl}\dot{x} & = & f(x)+G(x)u+W_b(x)\theta_b(t)+W_u(x)\theta_u(t)\\ u & \in & \mathbf{U},\ \theta_b,\theta_u\in\Theta_b,\Theta_u\\ y_k & = & x_k+\nu_k\end{array}\tag{1}$$

where $x \in \mathbb{R}^n$ denotes the vector of state variables, $u \in \mathbb{R}^m$ denotes the vector of constrained manipulated inputs, taking values in a nonempty convex subset $\mathbf{U}$ of $\mathbb{R}^m$, where $\mathbf{U} = \{u \in \mathbb{R}^m : \|u\| \leq u_{max}\}$, $\|\cdot\|$ is the Euclidean norm of a vector, $u_{max} > 0$ is the magnitude of input constraints, and $\theta_b(t) = [\theta_b^1(t)\cdots\theta_b^{q_b}(t)]^T \in \Theta_b \subset \mathbb{R}^{q_b}$, where $\Theta_b = \{\theta_b \in \mathbb{R}^{q_b} : \theta_b\| \leq \theta_{b,max}\}$, $\theta_{b,max} > 0$ is the bound on the magnitude of the uncertain (possibly time-varying) variables, with $W_b(x)$ known, while $\theta_u(t) = [\theta_u^1(t)\cdots\theta_u^{q_u}(t)]^T \in \Theta_u \subset \mathbb{R}^{q_u}$, denotes the vector of uncertain (possibly time-varying) but bounded variables, with $W_u(x)$ unknown. $y_k \in \mathbb{R}^n$ is the measured state vector, $x_k = x(k\Delta)$ is the state vector at the kth sampling time, with a sampling time interval of Δ and ν_k is the measurement noise.

The vector function $f(x)$ with $f(0) = 0$, the matrices $G(x) = [g^1(x)\cdots g^m(x)]$ where $g^i(x) \in \mathbb{R}^n$, $i = 1,\ldots,m$, $W_b(x) = [w_b^1(x)\cdots w_b^{q_b}(x)]$ where $w^i(x) \in \mathbb{R}^n$, $i = 1,\ldots,q_b$, and $W_u(x) = [w_u^1(x)\cdots w_u^{q_u}(x)]$ where $w_u^i(x) \in \mathbb{R}^n$, $i = 1,\ldots,q_u$, are assumed smooth on their domains of definition. The notation $\|\cdot\|_Q$ refers to the weighted norm, defined by $\|x\|_Q^2 = x'Qx$ for all $x \in \mathbb{R}^n$, where Q is a positive definite symmetric matrix and x' denotes the transpose of x. The notation $L_f h$ denotes the standard Lie derivative of a scalar function $h(\cdot)$ with respect to the vector function $f(\cdot)$. Throughout the manuscript, we assume that for any $u \in \mathbf{U}$ the solution of the system of Eq. 1 exists and is continuous for all t, and we focus on the state feedback problem where measurements of the entire state, $x(t)$, are assumed to be available for all t.

A. Lyapunov-based predictive control

Lyapunov based robust predictive control designs utilize the known effects of the uncertainty and known uncertainty bounds in computing the control action with the help of a suitable control Lyapunov function $V(x)$ [9]. Consider the receding horizon implementation of the control action computed by solving an optimization problem of the form,

$$P(x,t) \quad : \quad \min\{J(x,t,u(\cdot))|u(\cdot)\in S\}\tag{2}$$

$$s.t.\ \dot{x} \quad = \quad f(x)+G(x)u\tag{3}$$

$$L_GVu(t)\leq L_GVu_{bc}(x(t))\tag{4}$$

where u_{bc} =

$$-\left(\frac{\alpha(x)+\sqrt{(\alpha_1(x))^2+(u_{max}\beta(x))^4}}{(\beta(x))^2\left[1+\sqrt{1+(u_{max}\beta(x))^2}\right]}\right)(L_GV)^T\tag{5}$$

when $L_GV \neq 0$ and $u_{bc} = 0$ when $L_GV = 0$, where $L_GV = [L_{g^1}V \ \cdots \ L_{g^m}V]$ is a row vector. With $L_{W_b}V = [L_{w_b{}^1}V \ \cdots \ L_{w_b{}^q_b}V]$, $\alpha(x) = L_fV + (\rho\|x\| + \kappa\theta_{b,max}\|L_{W_b}V\|)\left(\frac{\|x\|}{\|x\|+\phi}\right)$, $\alpha_1(x) = L_fV + \rho\|x\| + \kappa\theta_{b,max}\|L_{W_b}V\|$, $\beta(x) = \|(L_GV)^T\|$, and ρ, κ and ϕ are adjustable parameters that satisfy $\rho > 0$, $\kappa > 1$ and $\phi > 0$, $S = S(t,T)$ is the family of piecewise continuous functions (functions continuous from the right), with period Δ, mapping $[t, t+T]$ into U. Eq.3 is the 'nominal' nonlinear model (without the uncertainty term) describing the time evolution of the state x. A control $u(\cdot)$ in S is characterized by the sequence $\{u[j]\}$ where $u[j] := u(j\Delta)$ and satisfies $u(t) = u[j]$ for all $t \in [j\Delta,(j+1)\Delta)$. The performance index for minimization is given by

$$\begin{array}{c}min(J_{MPC}(u(\cdot))) = \\ \int_t^{t+T}\left[\|x^u(s;x,t)\|_{Q_{MPC}}^2+\|u(s)\|_{R_{MPC}}^2\right]ds\end{array}\tag{6}$$

where Q_{MPC} and R_{MPC} are positive semi-definite, and strictly positive definite, symmetric matrices, respectively, and $x^u(s;x,t)$ denotes the solution of Eq.3, due to control u, with initial state x at time t and T is the specified horizon. The minimizing control $u^0(\cdot) \in S$ is then applied to the plant over the interval $[t, t+\Delta)$ and the procedure is repeated indefinitely.

Theorem 1 [9]: *Consider the constrained system of Eq.1 with $\theta_u,\nu_k \equiv 0$ under the MPC law of Eqs.2–6. Then, given any positive real number d, there exists a positive real number Δ^* such that if $\Delta \in (0,\Delta^*]$ and $x(0) := x_0 \in \Omega$, then the optimization problem of Eqs.2-6 is guaranteed to be initially and successively feasible, $x(t) \in \Omega\ \forall\ t \geq 0$ and $\limsup_{t\to\infty} \|x(t)\| \leq d$.*

Remark 1: The robust MPC design uses the known effect of the unmeasured disturbances ($W_b(x)$), and known bounds on the possibly time varying disturbances ($\theta_{b,max}$) to compute the control effort that would cause at least as much decay in the value of the Lyapunov function as that by the bounded robust controller (which also uses $W_b(x)$ and $\theta_{b,max}$).

B. Nonlinear MHE

In MHE, available measurements at current time k and N prior intervals (beginning from time $k-N$) are used to generate the 'optimal' current state estimate at time k. Existing nonlinear MHE formulations that are used in the context of offset-free MPC estimate the states and disturbance and noise by solving the following optimization problem:

$$\left.\begin{array}{l}min(J(\chi_{k-N|k},\omega|)) = \frac{1}{2}\|(\chi_{k-N|k}-\hat{x}_{k-N|k-1})\|_{P_{k-N}^{-1}}^2\\ \quad +\sum_{i=k-N}^{k-1}\|\omega_{i|k}\|_{Q_k^{-1}}^2+\sum_{i=k-N}^{k}\|\nu_{i|k}\|_{R_k^{-1}}^2\\ \qquad\qquad s.t.\chi_{i+1|k} = F(\chi_{i|k},u_i)+\omega_{i|k}\\ \qquad\qquad\quad y_i = \chi_{i|k}+\nu_{i|k}\end{array}\right\}\tag{7}$$

where $\chi_{i|k}$ is the estimates of the states at time i based on available observations upto time k, $\omega_k \in R^n$ and $\nu_k \in R^n$ are estimates of the process and measurement disturbances (assumed piecewise constant over the sampling time interval) and $\hat{x}_{k-N|k-1}$ is the estimate of χ_{k-N} achieved as a result of the MHE solution at the previous time step. The search variables are $\chi_{k-N|k}$ (which is the same as $\chi(0)$, note that we denote the continuous time variable as $\chi(\cdot)$ and its discrete counterpart as $\chi_{i|k}$ and $\omega_{i|k}$ for $i = k-N, \cdots, k-1$. For more details on nonlinear MHE, and an excellent review, see [11].

Remark 2: Recent results [12], [13] have united nonlinear MHE with rigorous nonlinear state estimation designs to strengthen the state estimation properties of MHE. While the focus of the present work is on using the MHE to estimate the plant-model mismatch and 'filter out' noise (under the assumption of full-state feedback), the results of [12] in conjunction with the proposed approach can be utilized to address the problem under limited availability of state measurements.

III. OFFSET-FREE LYAPUNOV-BASED MPC

In this section, we present the offset-free Lyapunov-based MPC, focussing on the state feedback problem. In particular, the control action at a time t_k is computed as follows: First, state estimates and estimates of the (assumed constant) plant model mismatch are generated by solving the following optimization problem:

$$\left.\begin{aligned} min(J_{MHEMPC}) &= \frac{1}{2}\|(\chi_{k-N} - \hat{x}_{k-N|k-1})\|^2_{P^{-1}_{k-N}} \\ &\quad + \sum_{i=k-N}^{k-1} \|\omega_i\|^2_{Q_k^{-1}} + \sum_{i=k-N}^{k} \|\nu_i\|^2_{R_k^{-1}} \\ s.t.\ \ \dot{\chi}_i &= f_m(\chi_i) + G_m(\chi_i)u_i + \omega_i \\ y_i &= \chi_i + \nu_i \end{aligned}\right\} \tag{8}$$

where $\chi_{i|k}$ is the estimates of the states at time i based on available observations upto time k augmented with the current estimate of the plant model mismatch term θ', $f_m(\chi) = [f(\chi(1,\dots n)) + G_{\theta'}\chi(n+1\dots 2n) \quad \mathbf{0}]'$, where $G_{\theta'}$ is a matrix of appropriate dimensions, $G_m(\chi) = [G(\chi(1,\dots n)), \mathbf{0}]'$, $\omega \in R^{2n}$ and $\nu \in R^n$ (where n is the number of measurements, same as the number of states) are estimates of the process and measurement disturbances (assumed piecewise constant over the sampling time interval) and $\hat{x}_{k-N|k-1}$ is the estimate of χ_{k-N} achieved as a result of the MHE solution at the previous time step. The search variables are $\chi_{k-N|k}$ (which is the same as $\chi(0)$) and piecewise constant values of $\omega_{i|k}$ for $i = k-N, \cdots, k-1$. Effectively, the MHE also yields an estimate of $\chi_{k|k}$, which contains an estimate of the state at time t_k and the plant model mismatch term.

The control action at time t_k is then computed by solving the following optimization problem with $x(0) = \chi(1\dots n)$ and $\theta' = \chi(n+1\dots 2n)$:

$$P(x,t) \quad : \quad \min\{J_{MPCMHE}(x,t,u(\cdot))|u(\cdot) \in S\} \tag{9}$$

$$s.t.\ \ \dot{x} = f(x) + G_{\theta'}\theta' + G(x)u \tag{10}$$

$$L_G V(x)u(t) \le L_G V u_{bc}(x(t)) \tag{11}$$

where $u_{bc} =$

$$-\left(\frac{\alpha(x) + \sqrt{(\alpha_1(x))^2 + (u_{max}\beta(x))^4}}{(\beta(x))^2\left[1 + \sqrt{1 + (u_{max}\beta(x))^2}\right]}\right)(L_G V)^T \tag{12}$$

when $L_G V \neq 0$ and $u_{bc} = 0$ when $L_G V = 0$, where $\alpha(x) = L_{f'}V + (\rho\|x\| + \kappa\theta_b\|L_W V\|)\left(\dfrac{\|x\|}{\|x\| + \phi}\right)$, $\alpha_1(x) = L_{f'}V + \rho\|x\| + \kappa\theta_b\|L_W V\|$, $f'(x) = f(x) + G_{\theta'}\theta'$ $\beta(x) = \|(L_G V)^T\|$, $L_G V = [L_{g^1}V \ \cdots \ L_{g^m}V]$ and $L_W V = [L_{w^1}V \ \cdots \ L_{w^q}V]$ are row vectors, θ_b is a positive real number such that $\|\theta(t)\| \le \theta_b$, for all $t \ge 0$, and ρ, χ and ϕ are adjustable parameters that satisfy $\rho > 0$, $\chi > 1$ and $\phi > 0$. $S = S(t,T)$ is the family of piecewise continuous functions (functions continuous from the right), with period Δ, mapping $[t, t+T]$ into U. A control $u(\cdot)$ in S is characterized by the sequence $\{u[j]\}$ where $u[j] := u(j\Delta)$ and satisfies $u(t) = u[j]$ for all $t \in [j\Delta, (j+1)\Delta)$. The performance index is given by

$$J(x,t,u(\cdot)) = \int_t^{t+T} \left[\|x^u(s;x,t)\|^2_{Q_{MPC}} + \|u(s)\|^2_{R_{MPC}}\right] ds \tag{13}$$

where Q_{MPC} and R_{MPC} are positive semi-definite, and strictly positive definite, symmetric matrices, respectively, and $x^u(s;x,t)$ denotes the solution of Eq.3, due to control u, with initial state x at time t and T is the specified horizon. The minimizing control $u^0(\cdot) \in S$ is then applied to the plant over the interval $[t, t+\Delta)$ and the procedure is repeated indefinitely.

Remark 3: For the case that the source of uncertainty, and its effect on the system dynamics can be fairly well determined (i.e., $\theta_u \simeq 0$), the MHE formulation can be modified to use $W_b(x)$ for G'_θ. In this case, the MHE (and the associated disturbance states) essentially generate estimates of θ_b, which can in turn be used in computing the control action. The robust MPC enables preserving stability while the estimation error converges.

IV. SIMULATION EXAMPLE

In this section we use a chemical reactor example (also used in [14]) to demonstrate the constraint and uncertainty handling capability of the robust predictive controller design. To this end, consider the following model of an irreversible elementary exothermic reaction of the form $A \xrightarrow{k} B$ in a well–mixed continuous stirred tank reactor:

TABLE I

PROCESS PARAMETERS AND STEADY–STATE VALUES.

$V_R = 0.1\ m^3$	$R = 8.314\ J/mol \cdot K$
$C_{A0s} = 1.0\ kmol/m^3$	$T_{A0s} = 310.0\ K$
$\Delta H_n = -4.78 \times 10^4\ J/mol$	$k_0 = 72 \times 10^9\ min^{-1}$
$E = 8.314 \times 10^4\ J/mol$	$c_p = 0.239\ J/g \cdot K$
$\rho = 1000.0\ kg/m^3$	$F = 0.1\ m^3/min$
$T_{Rs} = 395.3\ K$	$C_{As} = 0.57\ kmol/m^3$

$$V_R \frac{d(C_{\mathrm{A}})}{dt} = F(C_{\mathrm{A0}} - C_{\mathrm{A}}) - V_R.k_0 e^{-E/RT_R} C_{\mathrm{A}}$$
$$V_R \frac{d(T_{\mathrm{R}})}{dt} = F(T_{A0} - T_R) + \frac{Q_{in}}{\rho c_p} - \frac{V_R \Delta H}{\rho c_p} k_0 e^{-E/RT_R} C_{\mathrm{A}} \tag{14}$$

where C_A denotes the concentration of species A, T_R, V_R denote the temperature and volume of the reactor, respectively, Q_{in} denotes the rate of heat input to the reactor, k_0, E, ΔH denote the pre–exponential constant, the activation energy, and the enthalpy of the reaction, respectively, and c_p and ρ denote the heat capacity and density of the fluid in the reactor, respectively. Table 1 lists the steady–state values and process parameters. The control objective is to stabilize the reactor at the (open–loop) unstable equilibrium point $x = [0.57; 395.3]$ and also affect set point changes in both concentration and temperature by manipulating both the rate of heat input/removal and the inlet reactant concentration. Defining $x_1 = C_A - C_{As}$, $x_2 = T - T_s$, $u_1 = C_{A0} - C_{A0s}$, $u_2 = Q$, $\theta_{b,1}(t) = T_{A0} - T_{A0s}$, $\theta_{b,2}(t) = \Delta H - \Delta H_n$, where the subscript s denotes the steady–state value and ΔH_n denotes the nominal value of the heat of reaction, the process model of Eq.14 can be cast in the form of Eq.1. In the first set of simulation runs, $\theta_1(t) = \theta_0 sin(3t)$, where $\theta_0 = 0.08 T_{A0s}$ and $\theta_2(t) = 0.5\,(-\Delta H_n)$, and the manipulated input constraints were $|u_1| \leq 1.0\ kmol/m^3$ and $|u_2| \leq 92\ kJ/s$. Subsequently, we also introduce non-zero θ_u disturbances.

Subsequent to stabilization to the nominal equilibrium point, the process is required to go to another steady state which is also open-loop unstable. All the following simulations have $\Delta = 0.005$ min and MPC horizon $N = 12$.

In the first set of simulations, we implement a the robust-Lyapunov-based predictive control design in isolation, using a quadratic Lyapunov function with $\kappa = 1.01$, $\phi = 0.001$, and $\rho = 0.01$ and this is kept unchanged for all subsequent Lyapunov based simulations. The Lyapunov function $V = x'Px$ is chosen with P as a positive-definite matrix that satisfies the Riccati equation,

$$A'P + PA - PBB'P = -Q_R \tag{15}$$

with the positive definite matrix $Q_R = 10^{-3} I_2$, where A and B are obtained by linearizing the model at the equilibrium point, and I_n denotes the identity matrix of dimension n.

We first choose $Q_{MPC} = 0.01 * diag([1, 1000])$ and $R_{MPC} = 100 * diag([0.00000001, 0.1])$. Figure 1 shows the closed loop response under the Lyapunov based MPC in isolation. As can be seen from the profiles, the robust MPC is able to drive the system sufficiently close to the desired equilibrium point but not meeting offset-free requirement.

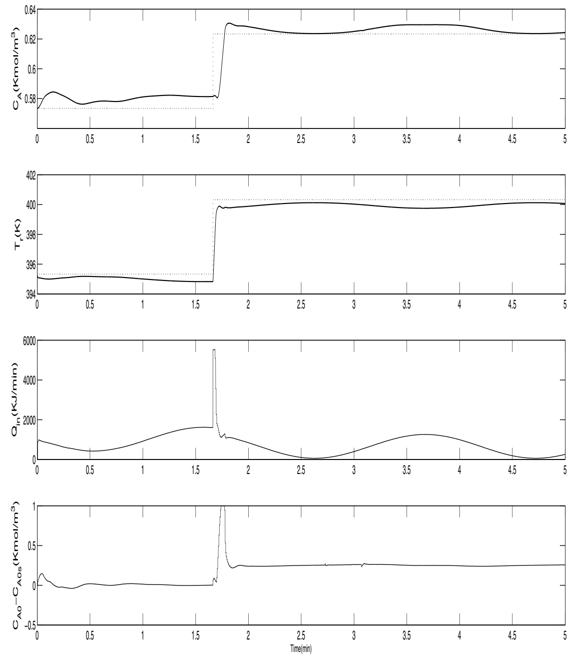

Fig. 1. Closed-loop state and input behavior (solid lines) under the robust Lyapunov-based MPC in isolation. The dashed lines show the setpoint trajectory.

We next illustrate the fact that offset-elimination with stabilization requires a unified framework of robust MPC (or some other form of stabilizing MPC) with an offset free mechanism. To this end, we remove the robust stability constraint from the MPC, and implement that in conjunction with the MHE. As expected, closed-loop offset elimination is not achieved.

Next, we implement the integrated offset-free MPC. Figure 3 shows the closed loop response of the Lyapunov based MPC using MHE based offset-free mechanism. As can be seen, significantly improved performance over that of the robust MPC alone is achieved. Finally, figure 4 shows the improved performance achievable through improved tuning of the MPC parameters. ($Q_{MPC} = diag([1, 1000])$ and $R_{MPC} = diag([0.00000001, 0.1])$).

We next demonstrate the improved flexibility achieved by the integrated framework through testing against additional disturbances (θ_u), for which the corresponding $W_u(x)$ is not known. In particular, a plant model mismatch of 2% on the activation energy E is introduced. As can be seen in Figure 5, the mechanism is able to reject this additional disturbance as well. In the last simulation, we test the performance of the proposed framework subject to measurement noise. (A MATLAB $rand()$ function was used to introduce a zero mean noise of standard variations 0.5% to C_A and 0.125% to T_r, both with respect to their current values). The noisy pro-

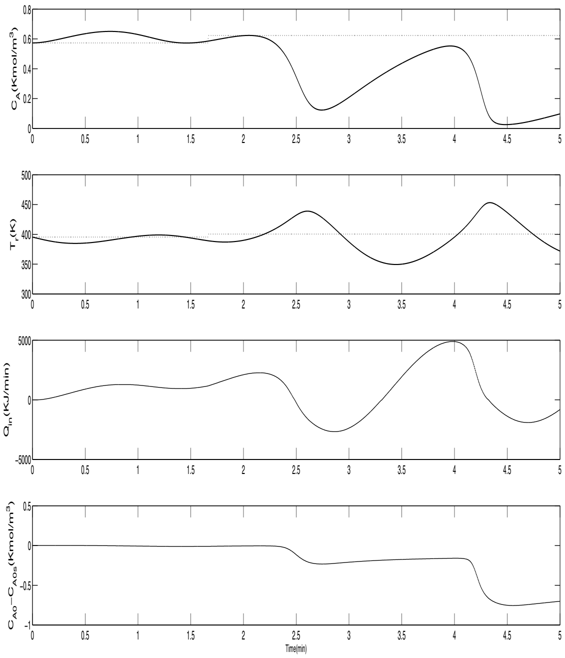

Fig. 2. Closed-loop state and input behavior (solid lines) an MPC without a stability constraint in conjunction with the nonlinear MHE-based offset-free mechanism. The dashed lines show the setpoint trajectory.

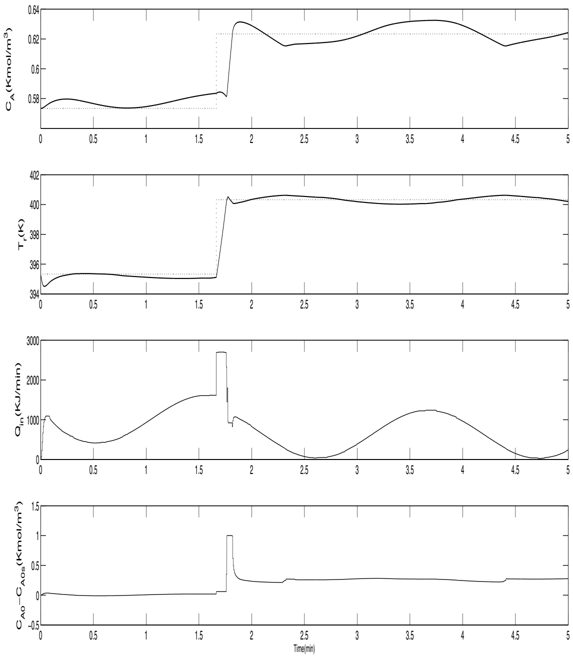

Fig. 3. Closed-loop state and input behavior (solid lines) under the robust Lyapunov-based MPC in conjunction with the nonlinear MHE-based offset-free mechanism. The dashed lines show the setpoint trajectory.

cess values were subsequently filtered to simulate a typical process computer pre-filtering. The final noise added to true measurement had a mean of $[-0.00009, 0.0182]$ and standard deviation of $[0.0027, 0.4735]$). As seen in Figure 6, the MHE mechanism is able to significantly filter the measurement noise, while still achieving offset-free predictions, resulting in significantly less noisy control action.

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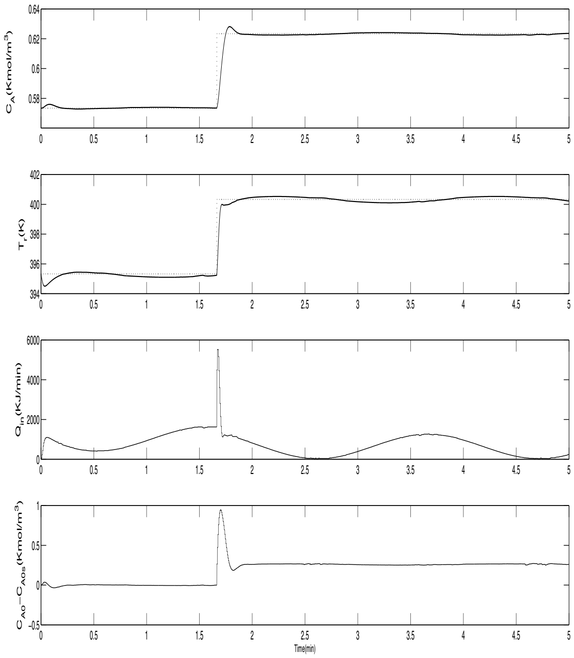

Fig. 4. Improved with tuning, closed-loop state and input behavior (solid lines) under the robust Lyapunov-based MPC in conjunction with the nonlinear MHE-based offset-free mechanism. The dashed lines show the setpoint trajectory.

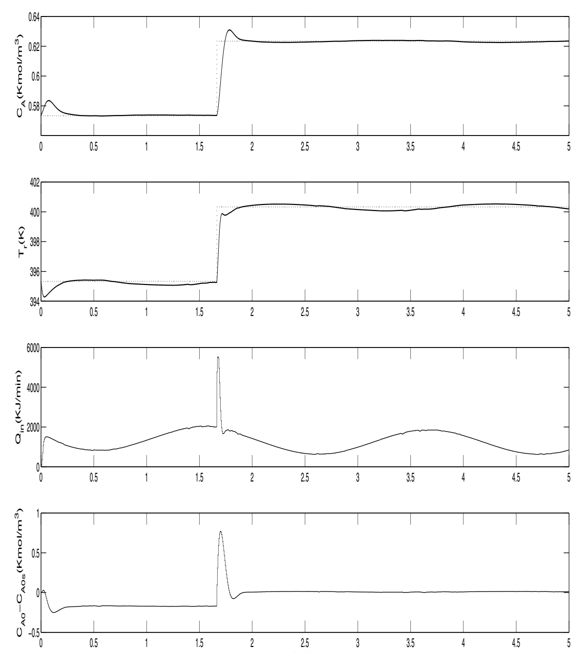

Fig. 5. Closed-loop state and input behavior (solid lines) under the robust Lyapunov-based MPC in conjunction with the nonlinear MHE-based offset-free mechanism subject to additional disturbances.

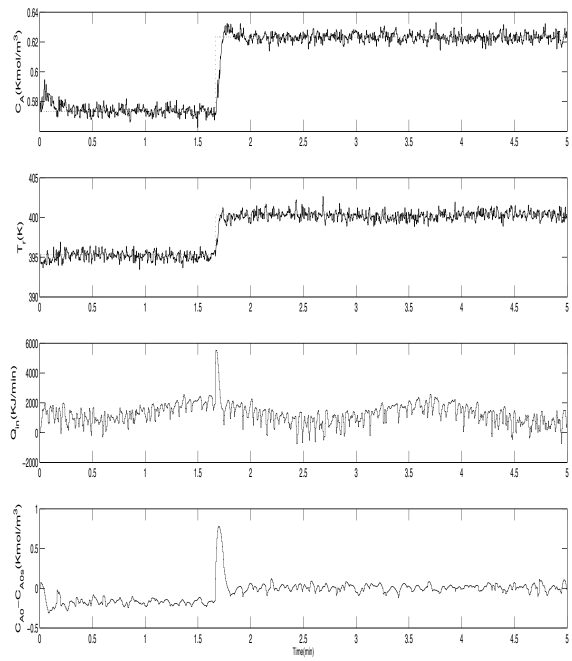

Fig. 6. Closed-loop state and input behavior (solid lines) under the robust Lyapunov-based MPC in conjunction with the nonlinear MHE-based offset-free mechanism subject to additional disturbances with measurement noise.