Distributed Model Predictive Control

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Introduction

- Incentives for chemical process control

- Need for continuous monitoring and external intervention (control)
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- Incentives for chemical process control

- Need for continuous monitoring and external intervention (control)

- Objectives of a process control system
  - Ensuring stability of the process
  - Suppressing the influence of external disturbances
  - Optimizing process performance
Feedback Loop/Controller Design

- Feedback control loop

![Feedback control loop diagram](image-url)

- Classical control (40s-60s): single-input/single-output (SISO) systems
  - Proportional-integral-derivative (PID) control
    - Simplicity of implementation

- Multi-input/multi-output systems
  - Many SISO PID loops/Decentralized approach
  - Does not account for interactions, constraints, nonlinear behavior
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![Feedback Loop Diagram]

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Model-Based Controller Design

- Controller design is based on a process dynamic model (60s-today)
  - A mathematical process model is constructed from first-principles or identified from input-output data to describe the process dynamics
  - Controllers are synthesized based on the process model
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- Advantages-disadvantages of model-based control
  - Possibility of improved closed-loop performance
    - Model accounts for inherent process characteristics (e.g., nonlinear behavior, spatial variations, multivariable interactions)
  - Characterization of limitations on achievable closed-loop stability, performance and robustness
  - It may be difficult to construct a model for a large-scale process
Model Predictive Control

(Carcia et al., Automatica, 1989; Mayne et al., Automatica, 2000)

Model predictive control (MPC)

\[
\min_{u \in S(\Delta)} \int_{t_k}^{t_k+N} [\ddot{x}(\tau)^T Q_c \ddot{x}(\tau) + u(\tau)^T R_c u(\tau)] d\tau
\]

s.t. 
\[
\begin{align*}
\dot{x}(t) &= f(\ddot{x}(t), u(t), 0) \\
\dot{x}(t_k) &= x(t_k) \\
u(t) &\in U \\
\ddot{x}(t) &\in X
\end{align*}
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Model Predictive Control

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- **On-line optimization-based approach**
  - Incorporate optimization considerations
  - Explicitly address state and control input constraints
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- Approaches to achieve closed-loop stability
  - Infinite prediction horizon
  - Terminal constraint or terminal cost
  - Constraint based on a Lyapunov function
Centralized vs. Distributed Control

- Centralized process control architecture
  - Computational complexity, fault tolerance
- Move towards distributed process control architecture
Centralized vs. Distributed Control

- **Centralized process control architecture**
  - Computational complexity, fault tolerance

- **Move towards distributed process control architecture**

- **Issues need to be addressed when moving to distributed control**
  - Coordination of controllers for stability and performance
  - Communication strategy between distributed controllers
Centralized vs. Distributed Control

- Centralized process control architecture
  - Computational complexity, fault tolerance
- Move towards distributed process control architecture
- Issues need to be addressed when moving to distributed control
  - Coordination of controllers for stability and performance
  - Communication strategy between distributed controllers
- MPC is a natural framework for distributed control system
Control Architectures

Different control architectures

Centralized control system

Decentralized control system

Distributed control system

Classified by communication between controllers

- Decentralized control system
  - No communication between controllers
- Distributed control system
  - Controllers exchange information to coordinate their actions
Classification of DMPC

Non-Cooperative DMPC

- Sequential DMPC
  - One-directional communication
  - Controllers are evaluated in sequence

- Non-iterative parallel DMPC
  - Controllers are evaluated once at a sampling time

- Iterative parallel DMPC
  - A local cost function is used in each controller
Classification of DMPC

Coordinated DMPC

- There is a coordinator to coordinate the actions of distributed controllers

Cooperative DMPC
Classification of DMPC

Coordinated DMPC

- There is a coordinator to coordinate the actions of distributed controllers

Cooperative DMPC

- In each controller, the same global cost function is optimized
- Achieve the performance of centralized MPC when iterate to convergence
Non-Cooperative DMPC

- DMPC for a class of decoupled systems with the distributed controllers are evaluated in sequence (Richards and How, International Journal of Control, 2007)

- DMPC for a class of discrete-time linear systems (Camponogara et al., IEEE Control Systems Magazine, 2002)

- DMPC for systems with dynamically decoupled subsystems (Keviczky et al., Automatica, 2006)

- DMPC scheme for linear systems coupled through the state (Jia and Krogh, ACC, 2001)
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Coordinated DMPC

- Coordinator-based DMPC (Cheng et al., Journal of Process Control, 2007; Marcos et al., ADCHEM 2009)
Cooperative DMPC

- Idea of cooperative DMPC was first introduced in 2005 (Venkat et al., CDC, 2005)

  - System-wide control objective functions
  - The closed-loop performance converges to the corresponding centralized control system as the iteration number increases

  - Well-characterized regions of closed-loop stability
  - Accounting for asynchronous and delayed measurements

- Robust DMPC for linear systems accounting for model uncertainties explicitly (Al-Gherwi et al., Journal of Process Control, 2011)
Cooperative Nonlinear DMPC

System description

\[ \dot{x}(t) = f(x(t)) + \sum_{i=1}^{m} g_i(x(t))u_i(t) + k(x(t))w(t) \]

- Fully coupled nonlinear processes with \( m \) sets of control inputs

Renders the origin of the nominal system asymptotically stable under the control:

\[ u_i(t) = h_i(x(t)) \quad (i = 1, \ldots, m) \]

Satisfies the input constraints on \( u_i(t) \)

Stability region: \( \Omega \subset D \) is a compact set containing the origin
Cooperative Nonlinear DMPC

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- Fully coupled nonlinear processes with \( m \) sets of control inputs

Nonlinear feedback control law, \( u = h(x) = [h_1(x) \ldots h_m(x)]^T \)

\[ \dot{V}(x) = \frac{\partial V(x)}{\partial x}(f(x) + \sum_{i=1}^{m} g_i(x)h_i(x)) < 0 \]

- Renders the origin of the nominal system asymptotically stable under the control: \( u_i = h_i(x) \text{ (} i = 1, \ldots, m \text{)} \)
- Satisfies the input constraints on \( u_i \text{ (} i = 1, \ldots, m \text{)} \)
- Stability region: \( \Omega \subset D \) is a compact set containing the origin
Sequential and Iterative DMPC

(Liu et al., AIChE J., 2009; AIChE J., 2010)

- **$m$** LMPCs will be designed to decide the $m$ sets of control inputs

  ![Diagram of Sequential DMPC and Iterative DMPC](diagram.png)

  **Sequential DMPC**: One-directional communication, each controller is evaluated once at a sampling time

  **Iterative DMPC**: Bi-directional communication, controllers iterate to achieve convergence at a sampling time
Iterative DMPC

Implementation strategy

1. At $t_k$, controllers receive $x(t_k)$ and initialized with input guesses generated by $h(\cdot)$

2. At iteration $c$ ($c \geq 1$):

   2.1. Each controller evaluates its own future input trajectory

   2.2. Controllers exchange information. Based on the latest information, each controller calculates and stores the value of the cost function

3. If a termination condition is satisfied, each controller sends the input trajectory corresponding to the smallest value of the cost function to its actuators; Else, go to Step 2 ($c = c + 1$)
Convergence of the Iterative DMPC

- The optimal cost of the iterative DMPC is upper bounded by the cost of the nonlinear controller \( h(x) \)
  - \( h(x) \) is a feasible solution to the iterative DMPC \( (x(0) \in \Omega) \)
  - Implementation strategy of the iterative DMPC

- Guaranteed convergence for linear systems
  - The optimization problem of LMPC \( j \) is convex
  - Using a suitable input update rule, as \( c \to \infty \), the cost of the iterative DMPC converges to the corresponding centralized MPC

- For general nonlinear systems, the convergence of the iterative DMPC cost to the centralized MPC is not guaranteed
Application to a Chemical Process

Alkylation of benzene with ethylene

- Three distributed LMPC controllers
  - MPC 1: $Q_1$, $Q_2$, $Q_3$
  - MPC 2: $Q_4$, $Q_5$
  - MPC 3: $F_4$, $F_6$
- Input constraints are considered
Application to a Chemical Process

Mean Evaluation Times

- Mean evaluation times for 100 evaluations

<table>
<thead>
<tr>
<th></th>
<th>$N = 1$ (s)</th>
<th>$N = 3$ (s)</th>
<th>$N = 6$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized MPC</td>
<td>2.192</td>
<td>8.694</td>
<td>27.890</td>
</tr>
<tr>
<td>Sequential</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPC 1</td>
<td>0.472</td>
<td>2.358</td>
<td>6.515</td>
</tr>
<tr>
<td>MPC 2</td>
<td>0.497</td>
<td>1.700</td>
<td>4.493</td>
</tr>
<tr>
<td>MPC 3</td>
<td>0.365</td>
<td>1.453</td>
<td>3.991</td>
</tr>
<tr>
<td>Iterative (1 iteration)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPC 1</td>
<td>0.484</td>
<td>2.371</td>
<td>6.280</td>
</tr>
<tr>
<td>MPC 2</td>
<td>0.426</td>
<td>1.716</td>
<td>4.413</td>
</tr>
<tr>
<td>MPC 3</td>
<td>0.185</td>
<td>0.854</td>
<td>2.355</td>
</tr>
</tbody>
</table>

- Sequential DMPC evaluation time is reduced by 36% - 46%
- Iterative DMPC evaluation time (1 iteration) is reduced by more than 70%; 3 - 4 iterations are possible in 1 evaluation of the Centralized MPC
Application to a Chemical Process

Optimality

- **Performance index**

\[
J = \sum_{i=0}^{M} \left[ x(t_i)^T Q_c x(t_i) + \sum_{j=1}^{3} u_j(t_i)^T R_{cj} u_j(t_i) \right]
\]

- **Simulation time:** \( t_M = 1000 \text{ s}, \ N = 1 \)

- The cost of the iterative DMPC **converges** to the centralized MPC
DMPC for Two-Time-Scale Processes


- Slow dynamics is regulated by slow MPC
- Fast dynamics is regulated by fast MPC (or explicit controller)
- No communication between the two MPCs is necessary
- Near optimality of fast-slow MPC system
  - \( J \to J_s^* + J_f^* \) as \( \epsilon \to 0 \)
  - \( \epsilon \) is a parameter that indicates the level of separation between the fast and slow dynamics
Reactor-Separator with Large Recycle

An example of two-time-scale process

- Two reactions:
  \[ A \xrightarrow{r_1, \text{exothermic}} B \]
  \[ B + C \xrightarrow{r_1, \text{exothermic}} D \]

- Fast dynamics: CSTR-2
  - Residence time: \( \frac{F_1}{V_2} = 0.11 \text{ sec} \)

- Control inputs associated with slow dynamics: \( Q_1, Q_3 \)
- Control inputs associated with fast dynamics: \( Q_2 \)
Reactor-Separator with Large Recycle

Simulation results: Performance trajectories

- Control methods
  - centralized MPC, fast-slow MPC, slow MPC with explicit controller
DMPC with Asynchronous/Delayed Feedback

(Liu et al., Automatica, 2010; IEEE Transactions on Automatic Control, 2012)

Proposed approaches

- Modify the implementation strategies to take into account that the control loop may be open
- Redesign the formulations of the LMPCs to take into account asynchronous and delayed feedback explicitly
- In the case of delayed measurements, iterative DMPC has to be used
DMPC for Switched Nonlinear Processes

(Heidarinejad et al., ACC, 2012)

System description

\[ \dot{x} = f_{\sigma(t)}(x) + \sum_{i=1}^{m} g_{i\sigma(t)}(x)u_{i\sigma(t)} \]

- Switching signal \( \sigma : [0, \infty) \to \mathcal{I} = \{1, 2, \ldots, p\} \)
- Frequently arise in process operation (demand changes, phase changes, etc.)

Proposed approach

- Focused on nonlinear processes with scheduled mode transitions
- Initial feasibility is assumed
- A stability constraint based on multiple Lyapunov function is checked at each iteration
Distributed Energy Generation Systems

(Qi et al., IEEE Transactions on Control Systems and Technology, in press)

- **System description**
  - Wind subsystem
  - Solar subsystem
  - Loads of the system
  - DC bus
- **Control system**
  - One MPC for wind subsystem
  - One MPC for solar subsystem
  - Controllers communicate to meet total power demand

![Diagram of distributed energy generation system](image-url)
Conclusions

- **Trends in process control**
  - Control of large-scale complex processes
  - Distributed model predictive control is an appealing approach

- **Our work on DMPC for nonlinear processes**
  - Sequential and iterative DMPC
  - DMPC for two-time-scale processes
  - DMPC for with asynchronous/delayed measurements
  - DMPC for switched nonlinear processes
  - Distributed energy generation systems
Future Research Directions

- Distributed state estimation and integration with DMPC
- DMPC accounting for process topology
- DMPC with asynchronous evaluation
- Performance assessment of DMPC
- Loop partitioning and decomposition for DMPC
- Monitoring and reconfiguration of DMPC
- Applications