FORMATION OF ULTRACOMPACT X-RAY BINARIES IN DENSE STAR CLUSTERS


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ABSTRACT

Bright, ultracompact X-ray binaries observed in dense star clusters, such as Galactic globular clusters, must have formed relatively recently, since their lifetimes as persistent bright sources are short (e.g., ~10^6 yr above 10^36 ergs^−1 for a 1.4M_⊙ neutron star accreting from a degenerate helium companion with an initial mass of ~0.2M_⊙). Therefore, we can use the present conditions in a cluster core to study possible dynamical formation processes for these sources. Here we show that direct physical collisions between neutron stars and red giants can provide a sufficient formation rate to explain the observed numbers of bright sources. These collisions produce tight, eccentric neutron star – white dwarf binaries that decay to contact by gravitational radiation on timescales ~ 10^6−10^10 yr, usually shorter and often much shorter than the cluster age.

Subject headings: binaries: close — galaxies: star clusters — globular clusters: general — hydrodynamics — stellar dynamics — X-rays: binaries

1. INTRODUCTION

Ultracompact X-ray binaries (UCXBs) are persistent, bright X-ray sources (L_x ~ 10^{36}−10^{40} ergs^−1) containing a neutron star (NS) accreting from a low-mass, degenerate companion in a very tight orbit of period P ~ 1 hr. UCXBs may well be dominant among the bright low-mass X-ray binaries (LMXBs) observed in old globular clusters (GCs), both Galactic (Deutsch et al. 2000; van der Sluys et al. 2004) and extragalactic (Bildsten & Deloye 2004). It was recognized 30 years ago that the total numbers of LMXBs observed in GCs clearly indicate a dynamical origin, with formation rates exceeding those in field populations by several orders of magnitude (Clark 1975). Indeed, the stellar encounter rate in a cluster core is an excellent predictor for the presence of a bright LMXB (Jordan et al. 2004).

The growing importance of UCXBs is clear from the role they have played recently in a number of different contexts. They may dominate the bright end of the X-ray luminosity function in elliptical galaxies (Bildsten & Deloye 2004). They pose a number of challenges to, and may allow us to test our fundamental physics of, stellar structure for low-mass degenerate or quasi-degenerate objects (Deloye & Bildsten 2003). They may also connect in a fundamental way to NS recycling, as suggested by the fact that three out of six accretion-powered millisecond X-ray pulsars known in our Galaxy are UCXBs (Chakrabarty 2004; Grebenev et al. 2004). Finally, UCXBs may well be the progenitors of the many eclipsing binaries: close — galaxies: star clusters — globular clusters: general — hydrodynamics — stellar dynamics — X-rays: binaries

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3 See http://www.astro.northwestern.edu/StarCrash/.

1. OUTCOME OF COLLISIONS

Using the 3-D Smoothed Particle Hydrodynamics (SPH) code StarCrash we have computed about 40 representative collisions between various RG stars and a 1.4M_⊙ NS (Lombardi et al. 2004). In our models, both the NS and the RG core are represented by point masses coupled to the gas by (softened) gravity only (Rasio & Shapiro 1991) using 3-D hydrodynamic calculation. Nevertheless, if the post-collision NS–WD binaries can decay through gravitational-wave emission all the way to contact, they can still become UCXBs (Davies et al. 1992).
(grazing).

In agreement with previous SPH calculations (Rasio & Shapiro 1991; Davies et al. 1992), we find that all collisions produce bound systems in which the RG core ends up in a high-eccentricity orbit around the NS. However, in contrast to those older studies, our new SPH calculations extend over much longer times (up to $\sim 500$ successive pericenter passages), allowing us to determine accurately the final parameters of the orbit (Lombardi et al. 2004). Typically $\sim 50\%$ of the RG envelope is ejected to infinity, while most of the rest becomes bound to the NS. Only about $\sim 0.1 M_\odot$ remains bound to the RG core, which will eventually cool to a degenerate WD (cf. §4). The material left bound to the NS will attempt to form an accretion disk as it cools. The fate of this material is rather uncertain. It could be accreted onto the NS and spin it up (in $\sim 10^5$ yr at the Eddington limit), or, more likely, it could be ejected if the energy released by accretion couples well to the gas. With an efficiency $\epsilon$, the entire mass of gas could be ejected to infinity in as little as $\tau_{\text{gas}} \sim 500(\epsilon/0.1)^{-1}$ yr. This very short lifetime justifies our assumption that the parameters of the post-collision orbits determined by our SPH calculations are nearly final, i.e., that the orbital parameters are no longer affected by coupling of the orbit to the residual gas.

When we apply the Peters (1964) equations to these post-collision systems, we find that most of them inspiral on rather short timescales (Fig. 1). Therefore, we assume for the rest of this paper that all RG–NS collisions can produce UCXBs.

3. COLLISION RATE

Consider a NS of mass $m_{\text{ns}}$ in the core of a cluster containing $N_\odot$ ordinary stars (here we neglect binaries; see §4). If all these ordinary stars were turn-off stars of radius $R_{\text{to}}$ and mass $m_{\text{to}}$, the collision rate for the NS would be

$$\mathcal{R}_{\text{to}} \equiv 2\pi G(m_{\text{to}}+m_{\text{ns}})N_\odot R_{\text{to}} \sigma^{-1} V_c^{-1}, \quad (1)$$

where $\sigma$ is the relative velocity dispersion, $V_c$ is the core volume. Here we assume that the collision cross section is dominated by gravitational focusing.

To compute the collision rate with RGs, we take into account that the number of RGs, $dN_\text{rg}$, within any small range of radii between $R_{\text{rg}}$ and $R_{\text{rg}}+dR_{\text{rg}}$ is proportional to the time $dt$ spent there by the star as it ascends the RG branch, $dN_\text{rg} = f_{\text{rg}} N_\odot dt / \tau$. Here $f_{\text{rg}}$ is the fraction of stars with masses close enough to the turn-off mass to have become RGs, and $\tau$ is the total lifetime (from the ZAMS to the end of the RG stage) of a turn-off star, only slightly larger than the cluster age. For a simple analytic estimate, we use the following approximate relation between age and radius (eq. A9 of Kalogera & Webbink 1996),

$$R_{\text{rg}}(t) \simeq R_{\text{ZAMS}} \left(1 - \frac{t}{\tau}\right)^{-0.28}, \quad (2)$$

where we use $R_{\text{ZAMS}} \approx 0.7 R_\odot$. Next we replace $dt$ by $dR_{\text{rg}}/(dR_{\text{rg}}/dt) = (R_{\text{to}}/R_{\text{rg}})^{4.6} \tau dR_{\text{rg}}/R_{\text{to}}$. The collision rate for a NS with RGs between $R_{\text{rg}}$ and $R_{\text{rg}}+dR_{\text{rg}}$ is

$$d\mathcal{R} = 2.6\pi G(m_{\text{to}}+m_{\text{ns}}) R_{\text{rg}} \sigma^{-1} V_c^{-1} dN_\text{rg}. \quad (3)$$

Here a collision is defined to be any encounter with a distance of closest approach less than $1.3 R_{\text{rg}}$, consistent with our SPH results. Integrating this over $R_{\text{rg}}$ from the base of the RG branch, defined by setting $R_{\text{rg}} \equiv b R_{\text{to}}$, to the maximum radius of a RG, $R_{\text{max}} \gg b R_{\text{to}}$, we find a total collision rate

$$\mathcal{R}_{\text{UCXB}} \simeq 0.51 f_{\text{rg}}^{2.6} R_{\text{to}}^{-1}. \quad (4)$$

Alternatively, note that we could also directly integrate eq. (3) over time, without changing the variable from $t$ to $R_{\text{rg}}$. Because the collision rate is linearly proportional to radius when gravitational focusing dominates, we can then write

$$\mathcal{R}_{\text{UCXB}} \simeq 2.6\pi G(m_{\text{to}}+m_{\text{ns}}) f_{\text{rg}} R_{\text{avg}} \sigma^{-1} V_c^{-1}, \quad (5)$$

where $R_{\text{avg}}$ is the time-average radius of the RG. Using eq. (2) it is easy to show that eqs. (4) and (5) agree. Eq. (5) has the advantage that any stellar evolution treatment can be used to determine $R_{\text{avg}}$, including fitting formulae more detailed than eq. (2) or numerical results from stellar evolution calculations.

The steep inverse dependence on $b$ in eq. (4) indicates that the collision rate is completely dominated by the smallest RGs: although the cross section increases (linearly) with radius, the faster stellar evolution at larger radii dominates, so that collisions are much more likely to happen when the star is just leaving the main sequence, i.e., on or close to the subgiant branch (Verbunt 1987). The corresponding core mass is also small, typically $m_{\text{c}} \lesssim 0.1 M_\odot$ for $m_{\text{to}} \approx 0.8 M_\odot$.

We now proceed to evaluate $f_{\text{rg}}$. This depends on the mass function of stars in the cluster core, which we expect to be very different from the IMF because of mass segregation. Indeed, observations of cluster cores reveal flat or even slightly rising mass functions (e.g., Richer et al. 2004). Here we assume that the number of stars within $dm$ is proportional to $m^{\alpha} dm$, with $\alpha > -1$, between a minimum $m_{\text{min}}$ and a maximum $m_{\text{to}} + \Delta m$. The spread $\Delta m$ of masses along

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the RG branch is obtained from the mass-dependence of the main-sequence lifetime \( t_{ms} \). Adopting the simple scaling \( t_{ms} = \tau (m_{10}/m)^{2.6} \) (Hurley et al. 2000) we get \( \Delta t_{ms} = 3.6\tau (m_{10}/m)^{2.6} \Delta m/m \). Setting \( \Delta t_{ms} = t_{rg} \), the total time spent on the RG branch, and \( m \approx m_{10} \) gives \( \Delta m \approx 0.28 m_{10} \alpha_{10}^{1.6} \). We can now calculate \( f_{rg} \) directly from IMF. Assuming \( \Delta m \ll m_{10} \) and \( m_{\text{min}} \ll m_{10} \) we get

\[
f_{rg} = (\alpha + 1) \frac{\Delta m}{m_{10}} \approx 0.28(\alpha + 1) \frac{t_{rg}}{\tau} = 0.08(\alpha + 1) t_{rg}^{0.23}. \tag{6}
\]

Combining this with eq (4) we obtain the result,

\[
\mathcal{R}_{UCXB} \approx 0.04 \alpha_{10}^{1.6} \mathcal{R}_{10} \alpha_{10}^{0.6}. \tag{7}
\]

In steady state (justified given the short lifetimes \( t_{UCXB} \ll \tau \) of the bright UCXB phase) we can then estimate the number of UCXBs per 100 NSs at present in a cluster as

\[
N_{100} \approx 100 \mathcal{R}_{UCXB} t_{UCXB}. \tag{8}
\]

The lifetime \( t_{UCXB} \) depends on the minimum luminosity for a system to be classified as an UCXB. For our estimates we adopt a minimum luminosity comparable with the observed minimum in our Galaxy, \( L_{\nu} \approx 10^{36} \text{erg s}^{-1} \). The corresponding lifetime is \( t_{UCXB} \approx 10^{6} \text{yr} \) (e.g., Rasio et al. 2000).

The present mass of a cluster \( M_{\text{cl}} \) is always less than its initial mass \( M_{\text{cl},0} = f_{ML} M_{10} \), where \( f_{ML} \) is the total mass loss fraction. About 40% of the initial mass is lost just through stellar winds and SN explosions, so that \( f_{ML} < 0.6 \). Without tidal mass loss (Joshi et al. 2001), and adopting a lower mass cutoff of 0.1 \( M_{\odot} \) in the IMF of Kroupa (2002), we expect about 1 NS per 65 \( M_{\odot} \) of mass at present (see also Ivanova et al. 2004a). About 5% of these NSs will be retained, depending on the escape velocity and the NS natal kick velocity distribution (Ivanova et al. 2004a). The corresponding minimum number of UCXBs expected (without any tidal mass loss) is then

\[
N_{\text{min}} \approx 8 \times 10^{-4} M_{\odot} \mathcal{R}_{UCXB} t_{UCXB}, \tag{9}
\]

where \( M_{\odot} \) is in \( M_{\odot} \).

In Table 1, we show numerical results for several Galactic clusters: all clusters where a UCXB has been identified, and 47 Tuc (which does not contain any bright LMXB). The probability of finding a bright UCXB in a cluster like 47 Tuc is only about 23%. NGC 6652 has poorly measured parameters (see Note for Table 1), and our numbers for this cluster are necessarily uncertain. Two clusters, NGC 6624 and NGC 6712, are thought to have very eccentric orbits and to be on the verge of complete disruption in the Galactic tidal field (Richtler et al. 1994; Gnedin et al. 1999; Paresce et al. 2000). This suggests that they may have had much higher mass and density in the past. Indeed, observations show that NGC 6712 has a strikingly unusual mass function for stars below the turn-off (Andreuzzi et al. 2001) and this can only be explained if the cluster has lost more than 99% of its initial mass (Takahashi & Portegies Zwart 2000)\(^4\).

4 After significant mass loss, and depending on its initial density profile, the cluster could undergo strong gravothermal oscillations (Takahashi & Portegies Zwart 2000), so that a UCXB could also have formed when the core had a much higher density during a recent, brief episode of core collapse.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>( t_{rg}/\tau )</th>
<th>( R_{rg} )</th>
<th>( \log p_0 )</th>
<th>( \sigma )</th>
<th>( \log M_1 )</th>
<th>( N_{100} )</th>
<th>( N_{\text{min}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 1851</td>
<td>0.071</td>
<td>6</td>
<td>5.7</td>
<td>10.4</td>
<td>6.0</td>
<td>0.11</td>
<td>0.85</td>
</tr>
<tr>
<td>NGC 6624</td>
<td>0.087</td>
<td>4.2</td>
<td>5.6</td>
<td>5.4</td>
<td>5.2</td>
<td>0.14</td>
<td>0.18</td>
</tr>
<tr>
<td>NGC 6652</td>
<td>0.076</td>
<td>5.4</td>
<td>4.8</td>
<td>5.9</td>
<td>5.4</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>NGC 6712</td>
<td>0.070</td>
<td>5.9</td>
<td>3.0</td>
<td>4.3</td>
<td>5.0</td>
<td>0.0005</td>
<td>0.0004</td>
</tr>
<tr>
<td>NGC 7078</td>
<td>0.034</td>
<td>7.4</td>
<td>6.2</td>
<td>12.0</td>
<td>6.1</td>
<td>0.16</td>
<td>0.6</td>
</tr>
<tr>
<td>Terzan 5</td>
<td>0.10</td>
<td>4.3</td>
<td>6.1</td>
<td>10.6</td>
<td>5.6</td>
<td>0.27</td>
<td>0.87</td>
</tr>
<tr>
<td>47 Tuc</td>
<td>0.081</td>
<td>4.9</td>
<td>5.1</td>
<td>11.5</td>
<td>6.1</td>
<td>0.23</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Note. — The RG lifetime fraction \( t_{rg}/\tau \) and the average RG radius \( R_{rg} \) (in \( R_{\odot} \)) are calculated directly from our stellar evolution code and used in eq. (6) (with \( \alpha = 0 \) and eq. (5)). \( p_0 \) is the cluster core density (in \( M_\odot \text{pc}^{-3} \), \( \sigma \) is the (1-D) velocity dispersion (in km s\(^{-1} \)) and \( M_1 \) is the total cluster mass (in \( M_\odot \)). For Ter 5 and NGC 6652, \( \log p_0 \) is based on the luminosity density from Djorgovski (1993) and an adopted mass-to-light ratio of 2. The value of \( \log p_0 \) for NGC 6652 appears rather uncertain (see, e.g., Pryor & Meylan 1993; Djorgovski 1993). Values of \( \log M_1 \) for NGC 6652 and Ter 5 are from Gnedin et al. (2002); \( \sigma \) for Ter 5 is from Gnedin et al. (2002) and for NGC 6652 from Webbink (1985). Otherwise \( \log p_0 \), \( \sigma \) and \( \log M_1 \) are from Pryor & Meylan (1993).
short. However, one can also see directly from Fig. 1 that, even if all binaries were able to circularize quickly (compared to the GR merger time), a large fraction of post-collision systems would still merge in less than the cluster age. Based on the results of Sec. 2 and the relation between post-collision semimajor axis and collision parameters derived from our SPH simulations, we estimate this fraction to be about 70%. Thus, even under the extreme assumption that all systems circularize, the rate of UCXB formation would still be within a factor of 2 of the total RG–NS collision rate. One possible further complication could come from the residual gas left bound to the RG core. All our collision calculations suggest that the mass left bound to the RG core is $\sim 0.1M_\odot$. Although there are many theoretical uncertainties, it is possible that this is sufficient to reconstitute a RG envelope (Castellani et al. 1994). In this case, the orbit would likely circularize, and stable MT from the reconstituted RG onto the NS would occur. However, the Roche lobe in the post-collision binary is smaller than the equilibrium radius of the RG, so that the MT proceeds on a thermal timescale and the corresponding bright LMXB phase lasts only $\sim 10^5$ yr, making detection unlikely. In addition, the total mass accreted by the NS will be only $\sim 10^{-3}M_\odot$, which is not sufficient to produce a recycled millisecond pulsar.

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