Drift-kinetic Vlasov simulations of shear Alfvén waves and electron acceleration

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Theory
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Data Comparisons
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Computational Support
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Summary

• Motivation
  – physics of shear Alfvén waves and their interaction with electrons
• DK1D code
  – Governing equations
  – Assumptions
  – Algorithms
  – Boundary conditions
• Results
  – Auroral electron acceleration in cold plasma
  – Auroral electron acceleration in warm plasma
Shear Alfvén Waves

- Low frequency waves (<1Hz)
- Long wavelengths along the field
- Disturbances propagate along field line at \( \sim v_A \)
- Significant field-aligned component of Poynting Vector
- If perpendicular scales are short →
- Transfer wave energy to electron energy, accelerating electrons in field-aligned direction
Shear Alfvén Waves

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Observational Evidence: Dombeck et al. [2005]; Chaston et al. [2000, 2002, 2003, 2007]; Andersson et al. [2002]; Wygant et al. [2002]; Keiling et al. [2003], etc, etc
Shear Alfvén Waves

\[ \nabla \cdot \mathbf{J} = 0 \]

\[ k_\perp J_\perp + k_\parallel J_\parallel = 0 \]

\[ J_\parallel = \left( \frac{k_\perp}{k_\parallel} \right) J_\perp \]
Shear Alfvén Waves

\[ \delta B \]

\[ \nabla \cdot \mathbf{J} = 0 \]

\[ k_\perp J_\perp + k_\parallel J_\parallel = 0 \]

\[ J_\parallel = \left( \frac{k_\perp}{k_\parallel} \right) J_\perp \]

\[ \mathbf{E}_\parallel \]
Shear Alfvén waves & aurora

Figure 1. Illustration of a magnetic flux tube conjugate to the auroral acceleration region with incident Poynting flux and its conversion to energized particles and joule heating of the ionosphere. Wavelength of wave fluctuations not to scale.

Figure 1, Wygant et al., JGR, 2000
Computational Challenge

- Long wavelengths along the field (~$R_E$)
- Short wavelengths across the field (~10-500km)
- Parallel wave electric fields
- Kinetic electron dynamics are important
Help from shear Alfvén wave physics

- Two oscillating currents: $J_\perp$ and $J_\parallel$
  - $J_\perp$ is carried by ions (electrons tied tightly to field, ion inertia carries them further) $\rightarrow$ polarisation current
  - $J_\parallel$ is carried by electrons (electrons move easily along the field)
- $\omega \ll \Omega_e, \Omega_i$ and $k_\perp \rho_e \ll 1$
  - Drift-kinetic description appropriate
  - $\rightarrow$ 2 dimensions in velocity space
  - $v_\parallel$ and $\mu$
- Describe wave in terms of potentials, not fields
  - $\varphi, A_\parallel$
Change the governing equations...

- Vlasov equation
- Maxwell’s equations
- Drift-kinetic equation
- ion polarisation current equation
- Ampère’s Law

\[ f(x, y, z, v_x, v_y, v_z, t) \]
\[ E(x, y, z, t) \]
\[ B(x, y, z, t) \]
\[ f(p_{||}, \mu, z, t) \]
\[ A_{||}(z, t) \]
\[ \varphi(z, t) \]
Scalar potential equation

- Continuity equation + quasineutrality → \( \nabla \cdot J = 0 \)
- \( J_\perp \) from polarisation current equation
- \( J_\parallel \) from Ampere’s Law (neglect displacement current)
- Assume perpendicular variations: \( \exp(-i k_\perp x) \)

Uniform B

\[
\frac{\partial \varphi}{\partial t} = -v_A^2 \frac{\partial A_\parallel}{\partial z}
\]

Non-uniform B

\[
\frac{\partial \varphi}{\partial t} = -\frac{v_A^2}{k_\perp^2} \frac{\partial (k_\perp^2 A_\parallel)}{\partial z}
\]
Drift-kinetic equation

\[
\frac{\partial f_e}{\partial t} + v \cdot \nabla f_e + a \cdot \nabla_v f_e = 0
\]

Change coordinates → guiding centre

Average over gyrophase

μ is an adiabatic invariant

Ignore perpendicular drifts of electrons due to non-uniform B (in dipole field, small compared to parallel velocities)

\[
\frac{\partial f_e}{\partial t} + v_\parallel \frac{\partial f_e}{\partial z} + \left[ \frac{q_e}{m_e} \left( - \frac{\partial A_\parallel}{\partial t} - \frac{\partial \phi}{\partial z} \right) - \frac{\mu}{m_e} \frac{\partial B_0}{\partial z} \right] \frac{\partial f_e}{\partial p_\parallel} = 0
\]

Change parallel velocity coordinate \( v_\parallel \rightarrow p_\parallel = v_\parallel + (q_e/m_e)A_\parallel \) (Hahm et al., 1988; Jenko, 2000)

\[
\frac{\partial f_e}{\partial t} + \left( p_\parallel - \frac{q_e}{m_e} A_\parallel \right) \frac{\partial f_e}{\partial z} + \left[ \frac{q_e}{m_e} \left( \left( p_\parallel - \frac{q_e}{m_e} A_\parallel \right) \frac{\partial A_\parallel}{\partial z} - \frac{\partial \phi}{\partial z} \right) - \frac{\mu}{m_e} \frac{\partial B_0}{\partial z} \right] \frac{\partial f_e}{\partial p_\parallel} = 0
\]
Ampere’s Law

- Assume no displacement current
- Use change of variables
- Assume perpendicular variations: \( \exp(-ik_\perp x) \)

\[
A_\parallel = \frac{\mu_0 q_e F}{k_\perp^2 + \mu_0 \frac{q_e^2}{m_e} Z'}
\]

\[
Z = \int_0^{\mu_M} \int_{-p_M}^{p_M} \frac{B_0}{m_e} f \, dp_\parallel \, d\mu
\]

\[
F = \int_0^{\mu_M} \int_{-p_M}^{p_M} p_\parallel \frac{B_0}{m_e} f \, dp_\parallel \, d\mu,
\]
Simulation variables

- Scalar Potential $\varphi(z,t)$
- Parallel Vector Potential $A_{||}(z,t)$
- Distribution Function $f(z,p_{||},\mu,t)$

$\rightarrow$ 1D-2V Drift-kinetic simulation code for low-frequency SAW

- NB $\mu$ only required for moment calculation, no $\partial\mu/\partial t$ term, so coarse $\mu$-grid
Algorithms

- Continuous equation $\rightarrow$ finite volume method
  - Corner Transport Upwind method (e.g. Leveque, 2002)
  - Van Leer gradient limiters

- Electron moments for Ampere’s Law $\rightarrow$ “in pairs” method
  - Horne and Freeman, JCP [2001]
Boundary Conditions

- Initialised with kappa/Lorentzian electron distribution function
- Wave is added to scalar potential at top of simulation
- Plasma allowed to flow out of simulation domain at each boundary
- Incoming plasma is kappa distribution with fixed n,T
More info

- Previous incarnations
  - Watt et al., PoP, 2004
  - Watt and Rankin, PPCF, 2008

- Present version
  - Watt and Rankin, JGR, 2010
Electron acceleration in cold plasma

- Nominal auroral acceleration region 1-2\(R_E\) altitude
- FAST observations of acceleration electrons
- Original test-particle analysis Su et al., 2004
- Concluded that to get lower energy enhancement, required large ion densities
DK1D simulations

Simulation

Add gaussian pulse to $\varphi$

Uniform B simulations
DK1D simulations

Simulation

Observation

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DK1D simulations

Simulation

Observation

Simulation Observation
Acceleration height and electron properties

- Local simulations of single pulses
  - energy of electrons depends upon wave amplitude and phase velocity
- [see e.g. Watt et al., JGR, 2005; Watt & Rankin, ASR, 2008]

At \( \sim 5R_E \), \( v_A \sim v_{\text{th},e} \), and electrons can easily be accelerated to \(~\text{keV}\) energies
\( E_\perp \sim 60\text{mV/m}, k_{\perp} \lambda_e = 1.7 \)

At \( \sim 2R_E \), \( v_A \gg v_{\text{th},e} \) and electrons can only be accelerated when \( k_{\perp} \lambda_e > 1 \), which reduces wave phase velocity:
\( E_\perp \sim 250\text{mV/m}, k_{\perp} \lambda_e = 3.4 \)
Electron acceleration in warm plasma

- Non-uniform B
- Uniform initial n, T
  - n = 1.0x10^6/m^3
  - T = 400eV

Watt & Rankin, PRL, 2009
SAW interaction in warm plasma

Watt & Rankin, PRL, 2009
SAW interaction in warm plasma

\[ E_\parallel \propto \left( \frac{k_\perp}{k_\parallel} \right) v_{th}^2/v_A^2 \]

Above \( \sim 4R_E \), strong \( E_\parallel \)

Below \( \sim 4R_E \), weak \( E_\parallel \)

Watt & Rankin, PRL, 2009
Trapping Islands: distribution functions

Figure 9
Janhunen et al., AG, 2004
Polar Observations
\[ r \sim 4.5R_E \]

Figure 6
Wygant et al., JGR, 2002
Polar Observations
\[ r \sim 4.5R_E \]

Time-averaged simulation
distribution function
\[ R \sim 5R_E \]
Trapping Islands: distribution functions

Figure 9
Janhunen et al., AG, 2004
Polar Observations
$r \sim 4.5R_E$

Figure 6
Wygant et al., JGR, 2002
Polar Observations
$r \sim 4.5RE$

Simulation Results
$R \sim 5R_E$
Trapping Islands: distribution functions

Enhancement away from ionosphere

→ $J_\parallel$ for SAW

Enhancement towards ionosphere

→ “Trapping Island” particles
→ Electron precipitation at lower altitudes

Simulation Results
$R \sim 5R_E$

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Poynting flux as wave interacts
Survey of Poynting flux behaviour

Figure 1: Polar Observations; Janhunen et al., SSR 2006
Summary

• Drift-kinetic simulations can be used to effectively study low-frequency space plasma physics
• Self-consistent way to study electron acceleration which causes aurora
• Visualisation of results important
  – How do instruments make measurements?
• Validation studies
  – Case studies good
  – Statistical results better!