SCATTERING OF ELECTROMAGNETIC WAVES ON TURBULENT PULSATIONS INSIDE A PLASMA SHEATH

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Overview

• Introduction

• Shear flow instability in a partially ionized compressible plasma sheath around a fast moving vehicle

• Scattering and transformation of electromagnetic waves on excited density perturbations

• Conclusions
The plasma formed around hypersonic or reentry vehicles can interfere with antenna performance at microwave frequencies even at electron densities below the overdense level.

Turbulence excited inside a plasma sheath leads to appearance of EM fields modulated in amplitude and phase and can have significant impact on phase-locking-in processes, in particular for GPS signals.

Currently no existing computational fluid dynamic (CFD) code can be used to predict the plasma turbulence around the vehicle.

We analyze growth rates of excited low frequency ion-acoustic type oscillations in a compressible plasma flow with velocity shear in the presence of neutral particles.

We examine influence of such turbulent pulsations on scattering and transformation of high frequency electromagnetic waves used for communication purposes.
Instability Of Plasma Flow Around A Fast Moving Vehicle

We are interested in the growth rates of instability of a flow with velocity shear for different flow profiles in the presence of neutrals.

\[ 10^6 < N_{0e} < 10^{12} \text{ cm}^{-3} \quad 10^{11} < N_n < 10^{16} \text{ cm}^{-3} \quad C_{s,air} < V_0 < 15 \times C_{s,air} \]

\[ 5.4 \times 10^7 < \omega_{pe} < 5.4 \times 10^{10} \text{ rad/s} \quad 0.1 < T_e < 1.0 \text{ eV} \]
Linearized Equations For Compressible Plasma Flow With Velocity Shear

Linearized momentum equation for the ions

\[
\frac{\partial \mathbf{u}_{li}}{\partial t} + u_{o iy} \frac{\partial \mathbf{u}_{li}}{\partial y} + \frac{\partial u_{0 iy}}{\partial x} \mathbf{e}_y = -\frac{Ze}{m_i} \nabla \Phi - \frac{V_{Ti}^2}{N_{0 i}} \nabla n_{li} - \nu_{ie} (\mathbf{u}_{li} - \mathbf{u}_{1 e}) - \nu_{in} (\mathbf{u}_{li} - \mathbf{u}_{1 n})
\]

Linearized momentum equation for the electrons

\[
\frac{\partial \mathbf{u}_{1 e}}{\partial t} + u_{0 ey} \frac{\partial \mathbf{u}_{1 e}}{\partial y} + \frac{\partial u_{0 ey}}{\partial x} \mathbf{e}_y = \frac{e}{m_e} \nabla \Phi - \frac{V_{Te}^2}{N_{0 e}} \nabla n_{1 e} - \nu_{ei} (\mathbf{u}_{1 e} - \mathbf{u}_{1 i}) - \nu_{en} (\mathbf{u}_{1 e} - \mathbf{u}_{1 n})
\]

Linearized momentum equation for the neutrals

\[
\frac{\partial \mathbf{u}_{1 n}}{\partial t} + u_{0 ny} \frac{\partial \mathbf{u}_{1 n}}{\partial y} + \frac{\partial u_{0 ny}}{\partial x} \mathbf{e}_y = -\frac{V_{Tn}^2}{N_{0 i}} \nabla n_{1 n} - \nu_{ne} (\mathbf{u}_{1 n} - \mathbf{u}_{1 e}) - \nu_{ni} (\mathbf{u}_{1 n} - \mathbf{u}_{1 i})
\]
Basic Equations For Compressible Plasma Flow With Velocity Shear

Linearized mass conservation equation for ions, electrons and neutrals

\[
\frac{\partial n_{1\alpha}}{\partial t} + u_{0\alpha y} \frac{\partial n_{1\alpha}}{\partial y} + N_{0\alpha} \nabla \cdot \mathbf{u}_{1\alpha} = 0 \quad j = i, e, n
\]

Poisson equation for electrostatic potential in oscillations

\[
\Delta \varphi = 4\pi e \left( n_e - Zn_i \right)
\]

Flow velocity can be presented in the following form

\[
\mathbf{v}_i = V_{0y}(x)\mathbf{e}_y + \delta \mathbf{v}_i(x, y, t)
\]
Dimensionless Variables

Dimensionless space variables x and y

\[ \tilde{x} = \tilde{\Delta}_i \times x \quad \tilde{y} = \tilde{\Delta}_i \times y ; \quad \tilde{\Delta}_i = \tilde{r}_{Di} \]

Dimensionless time

\[ t = \tilde{\omega}_{pi} \tilde{t} \]

Dimensionless speed

\[ \tilde{u}_i (\tilde{x}, \tilde{y}) = \tilde{\omega}_{pi} \tilde{\Delta}_i \tilde{u}_i (x, y) \]

Dimensionless electrostatic potential in IA wave

\[ \tilde{\Phi} (\tilde{x}, \tilde{y}) = \frac{\tilde{M}_i \tilde{\omega}_{pi}^2 \tilde{\Delta}_i^2}{Z \tilde{\varepsilon}} \Phi (x, y) \]
# Plasma And EM Wave Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron density</td>
<td>$N_{0e} = 10^{10} \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>Neutral density</td>
<td>$N_{0n} = 10^{12} \text{ cm}^{-3}$</td>
</tr>
<tr>
<td>Electron temperature</td>
<td>$T_e = 0.5 \text{ eV}$</td>
</tr>
<tr>
<td>Debye radius</td>
<td>$r_{De} = 5.25 \times 10^{-2} \text{ cm}$</td>
</tr>
<tr>
<td>Electron plasma frequency</td>
<td>$\omega_{pe} = 5.6 \times 10^8 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Ion plasma frequency</td>
<td>$\omega_{pi} = 2.1 \times 10^6 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Frequency of GPS signal</td>
<td>$\omega_{GPS} = 6.3 \times 10^9 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Atomic number of Potassium ions</td>
<td>$\mu_{pot} = 39$</td>
</tr>
<tr>
<td>Wave number of incident EM wave</td>
<td>$k_0 = 2.1 \times 10^{-1} \text{ cm}^{-1}$</td>
</tr>
<tr>
<td>Minimum wave number in IAW</td>
<td>$k_{A\text{min}} = 9.3 \times 10^{-1} \text{ cm}^{-1}$</td>
</tr>
<tr>
<td>Ion Thermal speed</td>
<td>$V_{Ti} = 1.1 \times 10^5 \text{ cm/sec}$</td>
</tr>
<tr>
<td>Vehicle speed</td>
<td>$V_0 = 2.8 \times 10^5 \text{ cm/sec}$</td>
</tr>
<tr>
<td>Doppler shift</td>
<td>$\omega_{Dop} = k_0 V_0 = 7.8 \times 10^2 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Electron-neutral collision frequency</td>
<td>$\nu_{en} = 6.3 \times 10^2 \text{ sec}^{-1}$</td>
</tr>
</tbody>
</table>
Equation For Eigenfunctions

\[ F(x, y, t) \sim f(x) \exp(ik_y y - i\omega t) \]

\[ L_e \left[ \frac{d^2 \phi}{dx^2} - k^2 \phi - \phi + n_i \right] = m\Omega \Omega_e \left[ \frac{d^2 \phi}{dx^2} - k^2 \phi + n_i \right] \]

\[ L_e \equiv \frac{d^2}{dx^2} + \frac{2k}{\Omega_e} \frac{dV_0}{dx} \frac{d}{dx} - k^2. \] \quad \frac{\Omega \Omega_e}{k^2} \ll 1, \quad \frac{m}{k^2} \ll 1, \]

Decoupling of electron and ion dynamics

\[ L_e \left[ \frac{d^2 \phi}{dx^2} - k^2 \phi - \phi + n_i \right] = 0 \]
Equation For Eigenfunctions

\[ D_2 \frac{\partial^2}{\partial x^2} \Phi + D_1 \frac{\partial}{\partial x} \Phi + D_0 \Phi = 0 \]

\[ D_2 = \varepsilon_i = 1 - \frac{1}{\Omega \Omega_i} \quad D_1 = -2k \frac{dV_0}{dx} \left( 1 - \frac{1}{2} \frac{\nu_{in} \nu_{ni}}{\Omega_i^2} \right) \quad D_0 = -\left( k^2 \varepsilon_i + 1 \right) \]

\[ \Omega \equiv \omega - kV_0(x) \quad \Omega_i \equiv \omega - kV_0(x) + i\nu_{in} \left( 1 - \frac{i\nu_{ni}}{\Omega_n} \right) \quad \Omega_n \equiv \omega - kV_0(x) + i\nu_{ni}. \]
Boundary Conditions

At the conducting surface of the vehicle we require

\[ \phi(x = 0) = 0 \]

In the free space beyond the sheath edge at \( x = L \) the potential

\[ \phi(x) = \phi(L) \exp[-k(x - L)] \]

Boundary conditions on the sheath edge

\[ \left[ \frac{d\phi}{dx} + k\phi \right]_{x=L} = 0 \]
$u_{0y} = c_0 x$

Normalized to ion sound speed flow velocity

Normalized to ion Debye length coordinate across plasma sheath
Growth Rate Of Instability With Linear Velocity Profile

(a) Real part of the frequency $\omega$  
(b) The growth rate $\gamma$ as a function of $kL$
Growth Rate Of Instability With Linear Velocity Profile

Peak growth rate as a function of neutral density for linear velocity shear with $c_0 = 0.5/L$ and $c_0 = 1.0/L$ in the collisionless limit.
Gaussian Velocity Profile

\[ u_{0y}(x) = u_{0\text{max}} \exp\left(-\frac{(x - x_m)^2}{2x_w^2}\right) \]

- Normalized to ion sound speed flow velocity
- Normalized to ion Debye length
- Coordinate across plasma sheath
Growth Rate Of Instability
With Gaussian Velocity Profile

Growth rate $\gamma$ as a function of $kL$
Velocity Profiles
Produced By CFD Code Vulkan

\[ V_0(x) = \frac{u_{0y}(x)}{V_s} = \frac{\tanh(x/\alpha)}{\tanh(L/\alpha)} \]

Normalized to ion sound speed flow velocity

Normalized to ion Debye length coordinate across plasma sheath
Growth Rate Of Instability With Velocity Shear Profiles From Vulkan Simulations

Growth rate normalized to ion plasma frequency

\[ \text{Normalized wave vector} \]

\[ k \]

\[ \text{Im}(\omega) \]

\( \alpha = 120 \)

\( \alpha = 180 \)

\( \alpha = 500 \)

\( \alpha \rightarrow \infty \)
Growth Rate Of Instability
With Velocity Profiles From Vulcan Simulations

Peak growth rate as a function of neutral density for the velocity shear profiles obtained from CFD Vulcan simulations.
Air Density

From G. Abell, Exploration of the Universe, 1982
Nonlinear Equations For Compressible Plasma Flow With Velocity Shear

Equation of motion for ions

\[m_i n_i \left( \frac{\partial \tilde{v}_i}{\partial t} + (\tilde{v}_i \cdot \nabla) \tilde{v}_i \right) = -Z e n_i \tilde{\nabla} \phi - \tilde{\nabla} P_i + m_i n_i \nu_i \Delta \tilde{v}_i - \mu_{ia} n_i \nu_{ia} (\tilde{v}_i - \tilde{v}_a)\]

Equation for adiabatic electrons

\[T_e \tilde{\nabla} n_e = e n_e \tilde{\nabla} \phi\]

Mass conservation equation for ions

\[\frac{\partial n_i}{\partial t} + \text{div}(n_i \tilde{v}_i) = 0\]
Nonlinear Equations For Compressible Plasma Flow With Velocity Shear

Poisson equation for electrostatic potential in oscillations

\[ \Delta \varphi = 4\pi e(n_e - n_i) \]

Flow velocity can be presented in the following form

\[ \vec{v}_i = V_{0y}(x)\vec{e}_y + \delta \vec{v}_i(x, y, t) \]
Gaussian Velocity Profile

\[ V_{0y}(x) = \exp\left[-\frac{(x - x_{\text{mid}})^2}{2x_{\text{wid}}^2}\right] \]
Nonlinear Stage Of Instability

Potential at $t = 500$

Potential at $t = 4000$
Density Spectrum Of Excited IAW Turbulence

\[ \delta N_{k,\omega} \sim \frac{1}{k^2} \frac{1}{\omega} \]

\[ k^2 = k_x^2 + k_y^2 \]

Sotnikov et al., TPS IEEE, 2010
Dispersion of waves inside a plasma sheath

1 – Electromagnetic wave (EM)
2 – Langmuir wave (L)
3 – Ion acoustic wave (IA)

EM wave scattering

$EM + IA \rightarrow EM$

EM wave transformation

$EM + IA \rightarrow L$
Electromagnetic Wave Scattering

**EM wave scattering**

\[ EM + IAW \rightarrow EM \]

Nonresonant \( \varepsilon^t(k, \omega) \neq 0 \)

**EM wave transformation**

\[ EM + IAW \rightarrow L \]

Resonant \( \varepsilon^L(k, \omega) = 0 \)
Interaction of EM wave with the excited spectrum of IAW results in appearance of scattered EM waves.

Incident EM wave:
\[ E_0(r, t) = E_{k_0} \cos(k_0 r - \omega_0 t) \]

\[ \omega_- = \omega_0 - \omega_A \]
Stokes component

\[ \omega_+ = \omega_0 + \omega_A \]
Antistokes component

Nonresonant scattering: \( \epsilon^t(k_\pm, \omega_\pm) \neq 0 \)

Scattered EM waves in frequency space.
Scattering Of EM Wave On IA Waves

Scattered EM waves in k-space

$k_- = k_0 - k_A$

$k_+ = k_0 + k_A$

$k_0 \ll k_A$

Nonresonant scattering $\varepsilon^t(k_\pm, \omega_\pm) \neq 0$
Nonlinear Coupling Model

Nonlinear coupling of the GPS signal and IAW causes a beat-wave field at the combination frequencies:

$$\omega_\pm = \omega_{k_1} \pm \omega_{k_2}$$

The scattered wave numbers are matched according to:

$$k_\pm = k_1 \pm k_2$$

Eigenmode frequency matching condition is not necessarily satisfied

$$\omega_\pm \neq \omega_{k_1 \pm k_2}$$
Generation Of Scattered Wave Fields

\[ \varepsilon^t(\omega_{\pm}, k_{\pm}) E^t_{k_{\pm}} = -i \frac{4\pi}{\omega_{\pm}} j^{NL}_{k_{\pm}} \]

\[ \varepsilon^t_{k,\omega} = \varepsilon^t_{1k,\omega} + i\varepsilon^t_{2k,\omega} \]

\[ \varepsilon^t_{1k,\omega} = 1 - \frac{\omega^2_{pe}}{\omega^2} - \frac{k^2c^2}{\omega^2} \]

\[ \varepsilon^t_{2k,\omega} = \frac{\nu_{en}\omega^2_{pe}}{\omega^3} \]

\[ \delta j^{N}_{k,\omega} = \frac{\alpha\omega^2_{pe}}{8\pi n_0\omega_{-}} E_0 \delta n^*_0 \]

Spectral component of the scattered EM wave field

\[ E^-_{k_{-},\omega} = \frac{1}{2i} \frac{\alpha\omega^2_{pe}}{n_0\omega_{-}^2} \frac{1}{\varepsilon^t_{k,\omega}} E_0 \delta n^*_{k_A,\omega_A} \]
Scattered Power

Power going into the scattered EM waves

\[
\delta Q^- = -\text{Re} \int < \delta j_N^- (\mathbf{r}, t) \cdot E^-^*(\mathbf{r}, t) > d^3 r
\]

\[
\delta Q^- = -\frac{\pi^3 \alpha^2 \omega^4}{n_0^2} E_0^2 \text{Re} \int d\mathbf{k} d\omega_+ \frac{i}{\epsilon_{k_-,\omega_-}^t \omega_-^3} |\delta n_{k_0-k_-,\omega_0-\omega_-}|^2
\]

\[
\text{Re}(\frac{i}{\epsilon_{k,\omega}^t}) = -\frac{\epsilon_{1,k,\omega}^t}{(\epsilon_{1,k,\omega}^t)^2 + (\epsilon_{2,k,\omega}^t)^2}
\]

\[
\frac{\epsilon_{2,k,\omega}^t}{|\epsilon_{1,k,\omega}^t|^2 + |\epsilon_{2,k,\omega}^t|^2} \neq \pi \frac{\delta(\bar{\omega}_- - \bar{\omega}_{k_-})}{\left| \frac{\partial \epsilon_{1,k,\omega}^t}{\partial \bar{\omega}_-} \right|_{\bar{\omega}_- = \bar{\omega}(\bar{k}_-)}}
\]
Scattering Cross Section

\[ \sigma^- = \frac{\text{scattered power}}{\text{energy flux of the incident wave}} = \frac{\delta Q^-}{\frac{E_0^2}{c} \frac{4\pi}{4\pi}} \]

\[ \sigma^- \sim \int dk_{Ax} dk_{Ay} d\omega_A \frac{\epsilon_2^t}{|\epsilon_1^t|^2 + |\epsilon_2^t|^2} \frac{1}{(\omega_0 - \omega_A)^3} |\delta n_{k_{Ax}, k_{Ay}, \omega_A}|^2 \]

Spectrum of the excited IA waves

\[ |\delta n_{k_{Ax}, k_{Ay}, \omega_A}|^2 = \frac{A}{(k_{Ax}^2 + k_{Ay}^2)^2} \frac{1}{\omega_A^2} \]
Amplitudes Of Scattered Waves

Density inside a plasma sheath

\[ 10^6 < n_0 < 10^{14} \text{ cm}^{-3} \]

Width of a plasma sheath

\[ 3 < L_x < 15 \text{ cm} \]

Amplitudes of scattered waves

\[ 10^{-5} < \frac{E^\pm}{E_0} < 10^{-1} \]

Scattering cross section

\[ 10^{-17} < \sigma^\pm < 10^{-8} \text{ cm}^2 \]

Phase changes in scattered waves can impact phase-locking-in process for signals from GPS satellites

\[ \phi \sim \int k_A \, dx \quad k_0 << k_A \]

Phase modulation of a data signal onto a carrier
Nonlinear coupling of the GPS signal and IAW causes a beat-wave field at the combination frequencies:

\[ \omega_{\pm} = \omega_{k_1} \pm \omega_{k_2} \]

The scattered wave numbers are matched according to:

\[ k_{\pm} = k_1 \pm k_2 \]

For Langmuir waves eigenmode frequency matching condition is satisfied

\[ \omega_{\pm} = \omega_{k_1 \pm k_2} \]
Amplitudes Of Scattered Waves

Density inside a plasma sheath

\[ n_0 \sim 10^{10} \text{ cm}^{-3} \]

Width of a plasma sheath

\[ 3 < L_x < 15 \text{ cm} \]

Amplitudes of scattered Langmuir waves

\[ 10^{-2} < \frac{E^L_{\pm}}{E_0} < 1 \]

Scattering cross section

\[ 10^{-5} < \sigma^\pm_L < 10^{-3} \text{ cm}^2 \]

Phase changes in scattered waves can impact phase-locking-in process for signals from GPS satellites

\[ \varphi \sim \int k_A dx \quad k_0 \ll k_A \]
Symmetric very low frequency (VLF) scattered emissions were observed by wave instrument near the rocket. They can be explained by nonlinear coupling mechanism due to interaction between the VLF and ELF waves.
A wave power spectrum diagram from the VLF wave instrument mounted aboard the deployable free flyer (DFF) with magnetic field shown versus frequency.

Sotnikov et al., J. Geophys. Res, 1994
Areol 3 Satellite Data

Analysis of Electron Acoustic Wave Propagation and Transformation in a Two-Electron-Temperature Plasma Layer
Excitation Of Waves By Dipole Antenna In Isotropic Plasma

\[ \omega = k c \]

\[ 2 \omega_{pe} \]

\[ \omega_{pe} \]

\[ \omega (k) \]

\[ k \]

\[ 18 \text{ GHz} \]

\[ 9 \text{ GHz} \]

\[ n \sim 10^{12} \text{ cm}^{-3} \]

\[ f_{pe} \sim 9 \times 10^9 \text{ Hz} \]

1 - transverse electromagnetic wave

\[ \omega_k = \sqrt{\omega_{pe}^2 + k^2 c^2} \]

2 - Langmuir wave

\[ \omega_k = \omega_{pe} \left(1 + \frac{3}{2} k^2 r_{de}^2 \right) \]

3 - electron acoustic wave \( (T_{eh} \gg T_{ec}) \)

\[ \omega_s (k) = \omega_{pe} \frac{kr_{dh}}{\sqrt{1 + k^2 r_{dh}^2}} \]

Ratio of the energy in transverse electromagnetic and Langmuir waves

\[ \frac{W_{EM}}{W_L} \sim 3.7 \frac{V_{te}^3}{c^3} << 1 \]

\[ for \ \omega \geq \omega_{pe} \]
Dispersion Equation

\[ D(\zeta) = 1 + \frac{k_{dc}^2}{k^2} [1 + \zeta_c Z(\zeta_c)] + \frac{k_{dh}^2}{k^2} [1 + \zeta_h Z(\zeta_h)] + \frac{k_{db}^2}{k^2} [1 + \zeta_b Z(\zeta_b)] = 0 \]

\[ \zeta_c = \frac{\omega}{\sqrt{2} k V_{Tc}}; \quad \zeta_h = \frac{V_{Tc}}{V_{Th}} \zeta_c; \quad \zeta_b = \frac{V_{Tc}}{V_{Tb}} \zeta_c - \frac{1}{\sqrt{2}} \frac{V_{0b}}{V_{Tb}} \cos \theta \]

\[ Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{\zeta}^{\infty} \frac{\exp(-y^2)}{\zeta - y} dy \]
Plasma And Beam Parameters Inside The Sheath

\[ n_{0c} = 0.4 \times 10^{11} \text{ cm}^{-3} \quad n_{0h} = 0.05 \times 10^{11} \text{ cm}^{-3} \quad n_{0b} = 0.09 \times 10^{11} \text{ cm}^{-3} \]

\[ T_{ec} = 0.2 \text{ eV} \quad T_{eh} = 20 \text{ eV} \quad T_{eb} = 0.1 \text{ eV} \]

\[ V_{0b} = 6.5 \times 10^8 \text{ cm / sec} \]

\[ \omega_{pc} = 5.64 \times 10^{10} \text{ rad / sec} \quad \omega_{ph} = 1.7 \times 10^{10} \text{ rad / sec} \]
Damping Of Electron Acoustic Waves

Real frequency (blue) normalized to cold electron plasma frequency and attenuation rate (green) in 1/cm units. Wave vector k is normalized to a hot electron frequency Debye radius sw2.out
Influence Of Inhomogeneity On The Damping Of Electron Acoustic Waves

\[ n_{0e}(x) \]

Plasma sheath

\[ \omega_k \sim \omega_{pc}(x)kr_{Dh} = \sqrt{\frac{n_c}{n_h}}kV_{Th} \]

\[ k(x) \sim \sqrt{\frac{n_h(x)}{n_c(x)}} \sim \text{const} \]

\[ V_f = \frac{\omega}{k} \]
Influence Of Collisions On The Damping Of Electron Acoustic Waves

\[ 1 + \delta \varepsilon_{eh} + \delta \varepsilon_{ec} + \delta \varepsilon_{eb} = 0 \]

\[ \delta \varepsilon_{ec} = -\frac{\omega_{pc}^2}{\omega^2} + i \sqrt{\frac{\pi}{2}} \frac{\omega \omega_{pc}^2}{k^3 V_{Tc}^3} \exp\left\{ -\frac{\omega^2}{2k^2V_{Tc}^2} \right\} + i \frac{\omega_{pc}^2 \nu_{en}}{\omega^3} \]

\[ \delta \varepsilon_{eh} = \frac{1}{k^2 r_{Dh}^2} \]

\[ \gamma_{col} = -\frac{\text{Im} \varepsilon_{col}}{\text{Re} \varepsilon} = -\frac{\nu_{en}}{2} \]

\[ \kappa_{col} = \frac{\gamma_{col}}{V_g} \sim -\frac{1}{2} \sqrt{\frac{n_h}{n_c}} \frac{\nu_{en}}{V_{Th}} \]

\[ E(x) \sim E_0 \exp(-\kappa x) \]

\[ \nu_{en} < 2 \sqrt{\frac{n_c}{n_h}} \frac{V_{Th}}{\Delta x} \]

\[ \nu_{en} < 2 \times 10^9 \text{ sec}^{-1} \]
Incident and reflected EA waves (in blue with index s) and incident, transformed and reflected EM waves (in red with index t) of P-polarization \((E_x, E_z, H_y)\).
Basic Equations

\[ \frac{d^2 H_y}{dx^2} + \left( \frac{\omega^2}{c^2} \varepsilon_c - k_{\text{tz}}^2 \right) H_y = 0 \]

\[ \varepsilon_c \frac{\gamma_0 V_{Th}^2}{c (\omega + iv_{eh})} \frac{\partial^2 E_x}{\partial x^2} + \frac{\omega}{c} \left[ \varepsilon_c - \frac{\omega_{ph}^2}{\omega (\omega + iv_{eh})} \right] (1 - \frac{\gamma_0 V_{Th}^2 k_{\text{tz}}^2}{\omega (\omega + iv_{eh})}) E_x = (1 - \frac{\gamma_0 V_{Th}^2 \omega_{ph}^2}{c^2 (\omega + iv_{eh})^2}) k_{\text{tz}} H_y \]

\[ \left[ \frac{\omega_{ph}^2}{\omega (\omega + iv_{eh})} - \varepsilon_c \right] \frac{\gamma_0 V_{Th}^2}{c (\omega + iv_{eh})} \frac{\partial^2 E_z}{\partial x^2} + \left[ \frac{\omega_{ph}^2}{(\omega + iv_{eh})^2} - \varepsilon_c (\omega - \frac{\gamma_0 V_{Th}^2 k_{\text{tz}}^2}{\omega (\omega + iv_{eh})}) \right] E_z - ik_{\text{tz}} \frac{\gamma_0 V_{Th}^2 \omega_{ph}^2}{\omega (\omega + iv_{eh})^2} \frac{\partial E_x}{\partial x} = -ic \frac{\partial H_y}{\partial x} \]

\[ \varepsilon_c = 1 - \frac{\omega_{pc}^2}{\omega^2} \]
Solutions Of Wave Equations

$$H_{1y} = -\frac{\omega}{ck_{1zt}} \varepsilon_{tot} GE_1 \exp(-\kappa'_{1tx} x)$$

$$E_{1x} = E_1 \exp(-iq_1 x) - GE_1 \exp(-i\kappa'_{1tx} x)$$

$$E_{1z} = -E_1 \frac{k_{1sz}}{q_1} \exp(-iq_1 x) + iG \frac{\kappa'_{1tx}}{k_{1tz}} E_1 \exp(-i\kappa'_{1tx} x)$$

$$H_{2y} = -\frac{\omega}{ck_{1zt}} \varepsilon_{tot} GE_2 \exp(-\kappa'_{1tx} x)$$

$$E_{1x} = E_{21} \exp(iq_1 x) - GE_2 \exp(-i\kappa'_{1tx} x)$$

$$E_{2z} = E_2 \frac{k_{1sz}}{q_1} \exp(iq_1 x) + iG \frac{\kappa'_{1tx}}{k_{1tz}} E_1 \exp(-i\kappa'_{1tx} x)$$
Transformation Coefficient

Coefficient of transformation of electron acoustic wave into an electromagnetic wave can be defined as the ratio of the of the normal component of the energy density flux of an electromagnetic wave:

$$S_{EM} = c \frac{E_3^2}{4\pi} \cos \theta_3$$

to the normal component of the energy density flux of an electron acoustic wave:

$$S_{EAW} = \frac{1}{k_s r_{Dh}} \frac{\omega_{pc}}{k_s} \frac{E_s^2}{4\pi}$$

**Transformation coefficient** $W_T$

$$W_T = 16\pi |\varepsilon_c| \frac{V_{Teh}}{c} \frac{\omega_{pc}}{\omega_{ph}} \frac{\cos \theta_3 \sin^2 \theta_3}{c q_1 G \sin^2 \theta_3 - |\varepsilon_{tot}| \cos \theta_3} + (|\varepsilon_c| + \sin^2 \theta_3)$$
Coefficient Of Transformation
On The Sheath Boundary

Coefficient of transformation

\[ W_{TR} \]

\[ 0 \quad \theta \quad \pi/2 \]
Conclusions

- In the plasma sheath around a hypersonic vehicle flow with velocity shear can excite intense ion-acoustic type perturbations.

- Presence of neutrals can significantly influence the growth rate. Suppression of the instability with different velocity profiles takes place at neutral densities:

\[ N_n \sim 10^{13} \div 10^{14} \text{ cm}^{-3} \]

- EM scattering on density perturbations excited by the flow leads to appearance of frequency wave spectra shifted above and below the frequency of the incident wave.

- Integrated phase shift and scattering cross section can be significantly affected due to interaction with ion acoustic density perturbations.
Conclusions

- Damping of electron acoustic waves propagating inside the plasma sheath in the presence of an electron beam is small.

- Presence of inhomogeneity does not increase EAW damping inside the plasma sheath.

- Damping of EAW due to electron-neutral collisions for the plasma parameters inside the sheath is small.

- EAW transformation into EM wave on the sheath boundary can lead to significant attenuation of the EM wave amplitude.
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References
