Particle Simulations of Whistler-mode Chorus and Electromagnetic Ion Cyclotron Waves

Yoshiharu Omura
Research Institute for Sustainable Humanosphere,
Kyoto University
omura@rish.kyoto-u.ac.jp

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Whistling Atmospherics

[Stanford Web] Long whistlers
Outer Ionosphere = Magnetosphere

Appendix: Other atmospherics on audio-frequencies

[Hiss]

[Isolated Rising Whistles]
[Dawn Chorus]

[Santolik et al., JGR, 2003]
VLF Triggered Emissions

[R. A. Helliwell, et al., JGR, 1964]

Rising and falling tones from the Morse code dashes
Simulation Model

\[ B_r = -\frac{r}{2} \frac{\partial B_z}{\partial z} \]

External current source at the equator
Test Particle Simulation of Nonlinear Wave Trapping and Acceleration of Trapped Particles (10 – 100 keV)

\[ \omega - kv_|| = \Omega_e(h)/\gamma \rightarrow V_R = \frac{1}{k} \left( \omega - \frac{\Omega_e}{\gamma} \right) \]

\[ \frac{\partial \omega}{\partial t} = 0 \]

\[ V_R \]

\[ \frac{v_\\parallel}{c} \]

\[ \frac{v_\\perp}{c} \]

Untrapped

Trapped

Equator

Kinetic Energy (\(\gamma - 1\))

Wave

Electrons

[Omura and Summers, JGR, 2006]
Equations of Resonant Particles

\[ \frac{d\theta}{dt} = \omega_t^2 \sin \zeta \]

\[ \frac{d\zeta}{dt} = \theta \]

Trapping Frequency

\[ \omega_t = \sqrt{\frac{k|q_s|E_w}{m_s}} \]
Equations of Motion under a Coherent Electrostatic Wave

\[ \frac{dv}{dt} = \frac{q_s}{m_s} E_x \sin (kx - \omega t + \zeta_o) \]

\[ \frac{dx}{dt} = v \]

\[ \theta = k(v - \frac{\omega}{k}) = kv - \omega \]

\[ \zeta = kx - \omega t + \zeta_o + \pi \quad (q_s > 0) \]

\[ \zeta = kx - \omega t + \zeta_o \quad (q_s < 0) \]

For whistler-mode wave:

\[ E_x \rightarrow v_{\perp} B_w \]
for Longitudinal Wave

\[ \theta = k(v_\parallel - V_R) \]

for Whistler-mode Wave

\[ \theta = -k(v_\parallel - V_p) \]

\[ \frac{d\zeta}{dt} = \theta \]

\[ \frac{d\theta}{dt} = \omega_t^2 (\sin \zeta + S) \]

\[ \frac{\partial \omega}{\partial t} = 0 \]
Formation of Electron Hole by Nonlinear Trapping with Inhomogeneity

$S = -0.4$
Inhomogeneity Ratio

\[ S = -\frac{1}{s_0 \omega \Omega_w} \left( s_1 \frac{\partial \omega}{\partial t} + cs_2 \frac{\partial \Omega_e}{\partial h} \right) \]

\[ s_0 = \frac{\delta V_{\perp 0}}{\xi c} \]

\[ s_1 = \gamma (1 - \frac{V_R}{V_g})^2 \]

\[ s_2 = \frac{1}{2\xi \delta} \left\{ \frac{\gamma \omega}{\Omega_e} \left( \frac{V_{\perp 0}}{c} \right)^2 - \left[ 2 + \frac{\Lambda \delta^2 (\Omega_e - \gamma \omega)}{\Omega_e - \omega} \right] \frac{V_R V_p}{c^2} \right\} \]

\[ \Lambda = \omega / \Omega_e \quad \text{for inhomogeneous density model} \]

\[ \Lambda = 1 \quad \text{for constant density model} \]

[Omura et al., JGR, 2008; 2009]
Electron Hybrid Simulation (cold electrons : fluid)

\[ \frac{\partial \omega}{\partial t} > 0 \]

\[\frac{T_{\perp}}{T_{\parallel}} = 9\]

[Katoh and Omura, GRL, 2007]
Nonlinear Wave Growth due to Formation of Electromagnetic Electron Hole

\[
\frac{\partial B_w}{\partial t} + V_g \frac{\partial B_w}{\partial h} = -\frac{\mu_0 V_g}{2} J_E \\
\frac{\partial \omega}{\partial t} + V_g \frac{\partial \omega}{\partial h} = 0
\]

Maximum \(-J_E\)

\[S_{EQ} = -0.4\]

[Omura, Katoh, Summers, JGR, 2008]
Chorus Equations: Absolute Instability

\[
\frac{\partial \tilde{\Omega}_w}{\partial \tilde{t}} = \tilde{V}_g \left[ \frac{Q\tilde{\omega}^2_{ph}}{2\tilde{U}_{t||}} \left( \frac{\tilde{V}_{10} \delta}{\pi \gamma} \right)^{3/2} \left( \frac{\xi \tilde{\Omega}_w}{\tilde{\omega}} \right)^{1/2} \exp \left( -\frac{\gamma^2 \tilde{V}_{R}^2}{2\tilde{U}_{t||}^2} \right) - \frac{5s_2 \tilde{a}}{s_0 \tilde{\omega}} \right]
\]

\[
\frac{\partial \tilde{\omega}}{\partial \tilde{t}} = \frac{2s_0}{5s_1} \tilde{\omega} \tilde{\Omega}_w
\]

(a) Earth \( L = 4.4 \)

(b) Saturn \( L = 7.0 \)

Threshold of Wave Amplitude \( B_w \)

Time Scale of Chorus \( T_c \)
Nonlinear Wave Growth through Propagation: Convective Instability

Equator

Self-sustaining Mechanism

\[ S = \frac{1}{B_w} \omega_t^2 \delta^2 \left[ \gamma \left( 1 - \frac{V_R}{V_g} \right)^2 \frac{\partial \omega}{\partial t} + \left\{ \frac{k \gamma v_1^2}{2 \Omega_e} - \left( 1 + \frac{\delta^2 \Omega_e - \gamma \omega}{2 \Omega_e - \omega} \right) V_R \right\} \frac{\partial \Omega_e}{\partial h} \right] \]
Formation of Electron Hole and Bump

Hikishima et al., JGR, 2010

[Image of diagrams and graphs with captions highlighting the formation process and parameters such as $h = -100 \, c \Omega_{e0}^{-1}$ and $t = 5980 \, \Omega_{e0}^{-1}$]
Resonance Velocity

\[ V_R = \frac{\omega}{k'} \left( 1 - \frac{\Omega_e(h)}{\omega} \right) \]

\[ \gamma = \left[ 1 - \left( \frac{v_{||}^2 + v_{\perp}^2}{c^2} \right) \right]^{-1/2} \]

\[ \gamma \geq \frac{\Omega_e(h)}{\omega} \Rightarrow V_R \geq 0 \]

\[ K = m_0 c^2 (\gamma - 1) \]

\[ \frac{dK}{dt} = -eE_w v_{\perp} S \]
Trajectories of Resonant Electrons (400 keV)

Relativistic Turning Acceleration (RTA)

\[
\frac{\partial \omega}{\partial t} = 0
\]

\[\omega_p = 2.0\Omega_{e0} \quad \omega = 0.4\Omega_{e0}\]

[Omura, Furuya, and Summers, JGR, 2007]
Trajectories of Resonant Electrons ($>1\text{MeV}$)

Ultra-Relativistic Acceleration (URA)

$\gamma_0 > \Omega_{EQ}/\omega$

$V_{R0} > 0$

[Summers and Omura, GRL, 2007]

$\frac{\partial \omega}{\partial t} = 0$
RTA and URA in the chorus generation process

\[
\frac{\partial \omega}{\partial t} > 0
\]

\[t = 10000 - 15000 \left[ \Omega_{e0}^{-1} \right]\]

EMIC Triggered Emissions

[Omura et al., JGR, 2010]
EMIC Chorus Equations

\[
\frac{\partial \tilde{\Omega}_{w0}}{\partial \tilde{t}} = \tilde{V}_g \left[ \frac{Q\tilde{w}_{ph}^2}{2\tilde{V}_{t||}} \left( \frac{\tilde{V}_{\perp 0}}{\pi} \right)^{3/2} \left( \frac{\tilde{V}_p \tilde{\Omega}_{w0}}{\tilde{\omega}} \right)^{1/2} \exp \left( -\frac{\tilde{V}_R^2}{2\tilde{V}_{t||}^2} \right) - \frac{5\tilde{V}_p s_2 \tilde{a}}{s_0 \tilde{\omega}} \right]
\]

\[
\frac{\partial \tilde{\omega}}{\partial \tilde{t}} = \frac{2s_0}{5s_1} \tilde{\omega} \tilde{\Omega}_{w0}
\]

Solution with a saturation

Cluster Observation

(a) [nT]

(b) [Hz]

\[\text{[Omura et al., JGR, 2010]}\]
Waves in Plasmas (Parallel Propagation)
L-mode EMIC Wave

Cut-off Frequency

$$\tilde{\omega} = \frac{\tilde{n}_H}{1 - \tilde{\omega}} + \frac{\tilde{n}_{He}}{1 - 4\tilde{\omega}} + \frac{\tilde{n}_O}{1 - 16\tilde{\omega}} = 1$$
EMIC Triggered Emissions

Cluster Spacecraft Observation

Hybrid Code Simulation

[Omura et al., JGR, 2010]

[Shoji and Omura, JGR, 2011]
Conclusion

• **Nonlinear trapping** plays an essential role in the generation processes of coherent waves in plasmas such as **ESW** and **chorus emissions**.

• **Inhomogeneity** due to the **rising tone** and the **non-uniform geomagnetic field** breaks the symmetry of the **nonlinear trapping** motion, and causes **nonlinear wave growth and damping**.

• **Nonlinear trapping by chorus emissions** results in efficient acceleration of resonant electrons to relativistic energy.

• **Nonlinear pitch angle scatterings by chorus and EMIC triggered emissions** results in precipitation of electrons and protons, causing auroras.