What's new in Hex?

ryan hayward@ualberta.ca

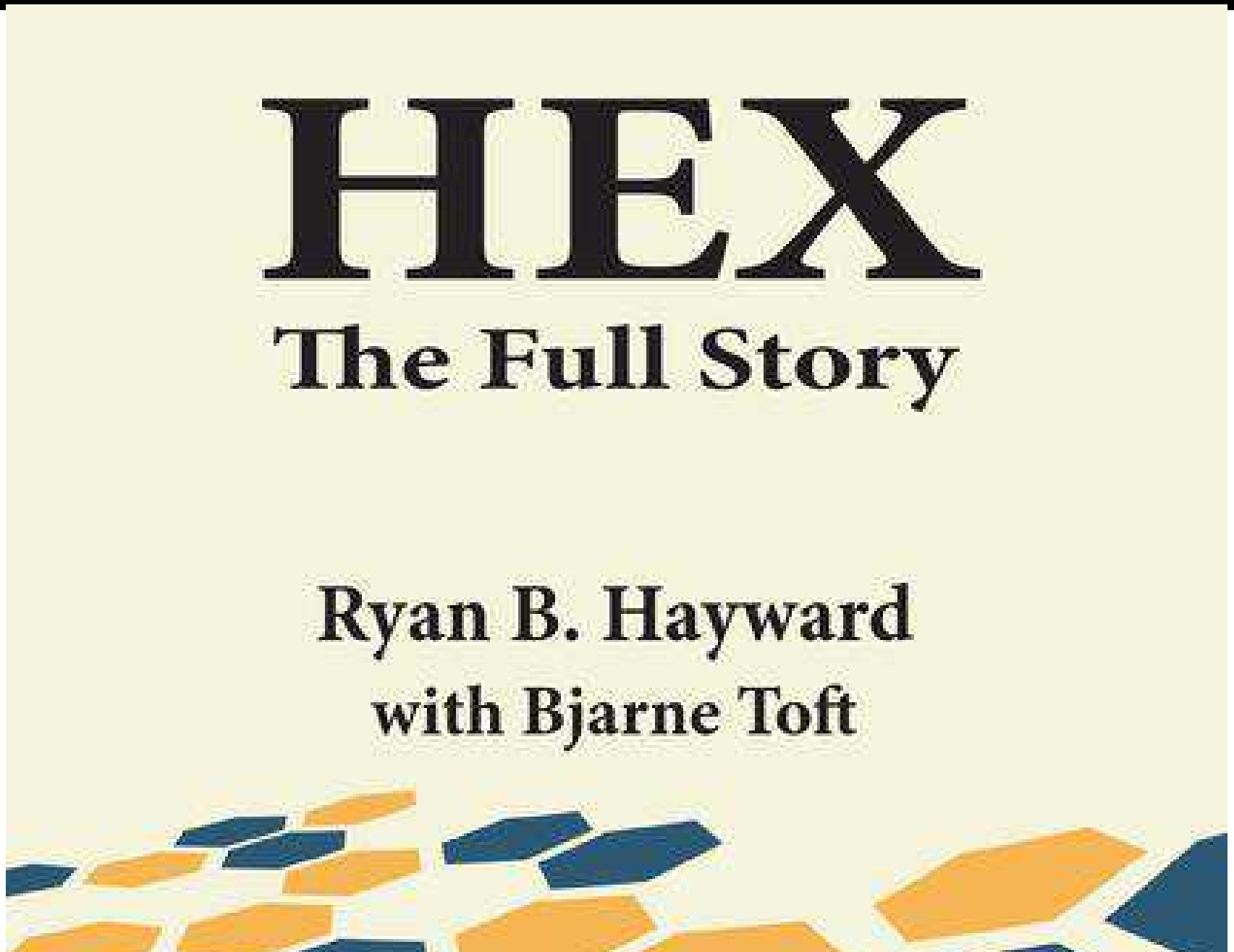
West Chester U Math Colloquium 2026 Oct 7

https://sites.ualberta.ca/~hayward/talks/warsl

- four theorems (old), two books (newish)
- still open: does centermost-opening win?
- recently (Selinger and Demer)

  Hex and combinatorial game theory

Hex: book one – history, 2019 CRC



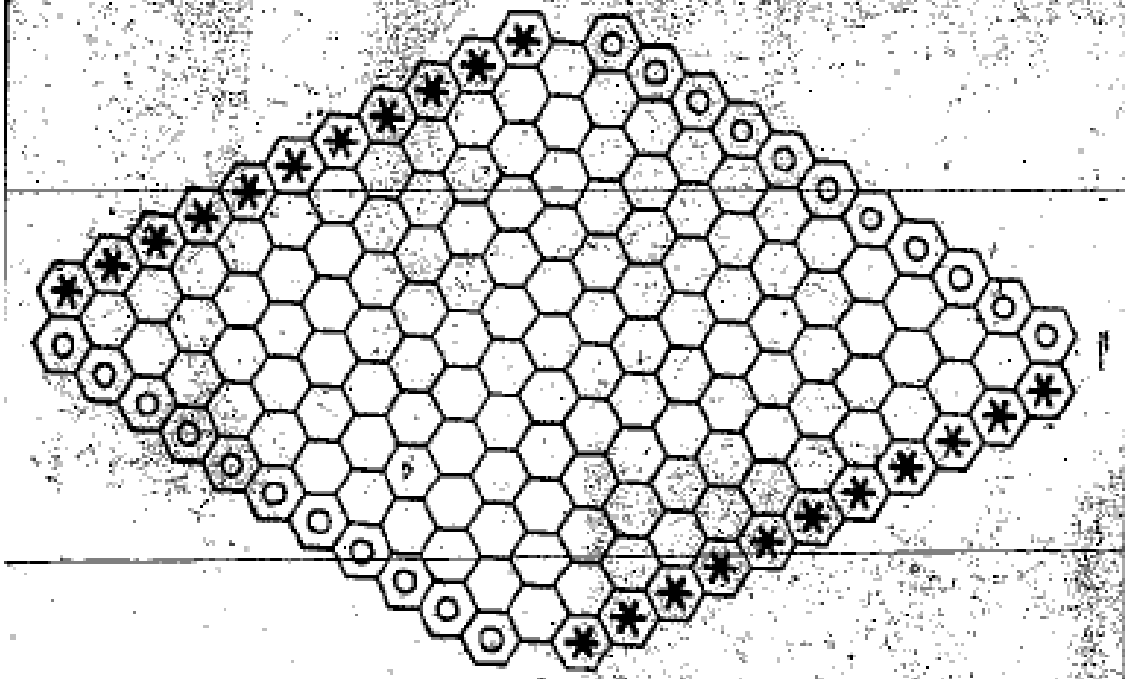
Hex: book one – history, 2019 CRC

TIKEN 26. December 1942

Vil De lære Polygon?

Piet Hein har konstrueret et Spil, der med lige stor Glæde kan dyrkes af Skakeksperten og den, der blot kan holde en Blyant

„Politiken" udskriver i Dag en Præmieopgave, der vil volde Hovedbrud for Begyndere



Saaledes ser Polygon-Spillebrættet ud.

DER er dukket et helt nyt, overordentlig — se Tegningen! — maa kredse hinanden

ten kan afbryde Forbindelsen ved at besætte det mellemliggende Felt, dens Anvendelighed bero paa Brikkernes Placering i den videre Omegn. I det hele taget viser det sig snart nødvendigt at tænke en større Del af Spillebrættet med i Betragtning.

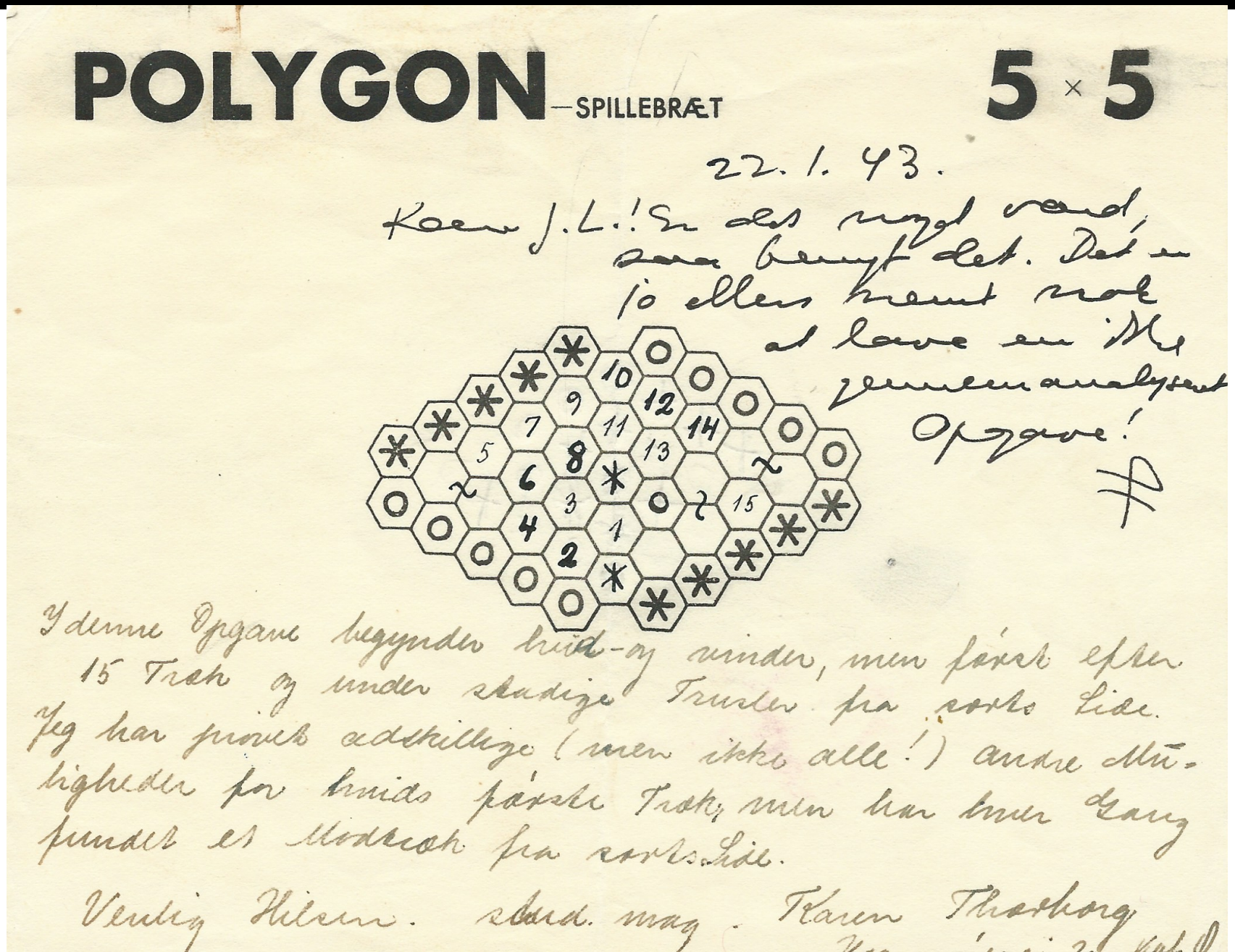
En anden Erfaring, som kommer senere, men som man kan lette Spillets Begyndelse ved at rebe, er, at det betaler sig at begynde i hvert Fald nogenlunde paa Midten. En rimelig, men paa ingen Maade nødvendig Aabning af Spillet er denne:

Paa Spillebrættet i Midten er Hvid begyndt i Midterfeltet. Sea- har Sort sat i Kontaktfeltet til det ned imod Midten af Hvids Front og derved gjort to nyttige Felter, som staar i Vinkelstilling til Midterfeltet, usikre. Hvid har saa valgt et Felt i Kontakt med sit første. Og nu svarer Sort med at besætte et Vinkelfelt, som vilde være meget nyttigt for Hvid. Hvor skal nu Hvid sætte? Der er forskellige gode Muligheder.

Saadan er dette Spil nu begyndt. Nu kan enhver fortsætte. Det er altsaa Hvids Tur. Man skal ikke være udspekuleret fra Begyndelsen. Der er ingen bedre Vej til at lære Spillet end at spille les.

Det er nyttigt at se skiftevis offensivt og defensivt paa Situationen, d. v. s. skiftevis paa sine egne og Modspillernes Muligheder, for at føre en Forbindelse igen-

Hex: book one – history, 2019 CRC



POLYGON—SPILLEBRÆT 5 × 5

22. 1. 43.

Kære J. L.! Er det mon vand, som bevæger det. Det er jo ellers meget nok at have en lille venneanalyseret Opgave!

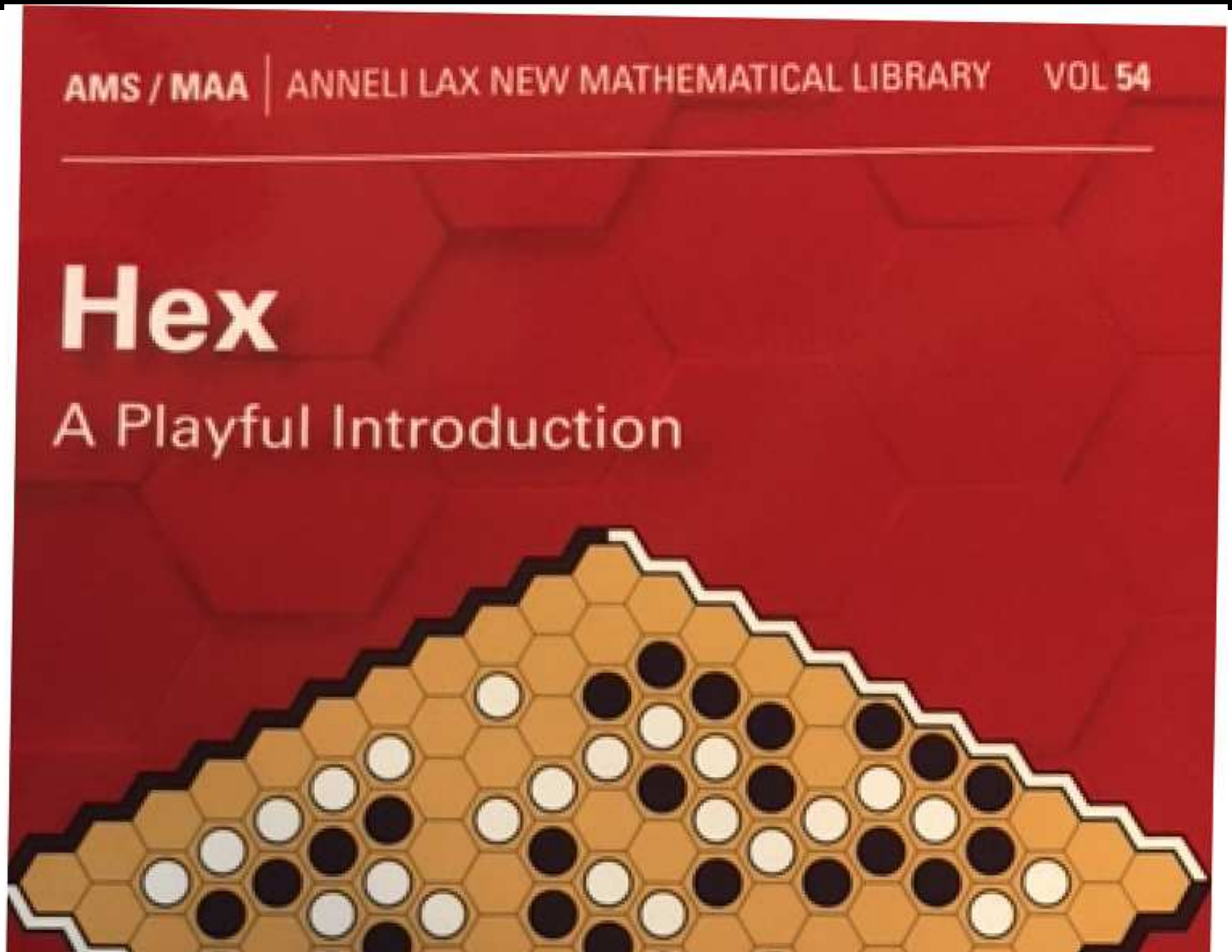
I denne Opgave begynder hvid og vinder, men første efter 15 Træk og under skadige Trusler fra sorts Side. Jeg har prøvet adskillige (men ikke alle!) andre Mulighed for hvids første Træk, men har hver Gang fundet et Modtræk fra sorts Side.

Venlig Hilsen. Hvad mag. Karen Thorborg

Hex: book one – history, 2019 CRC

https://www.routledge.com/hex-Inside-and-Out-

Hex: book two – intro, 2022 MAA



Hex: book two – intro, 2022 MAA

https://bookstore.ams.org/nml-54/

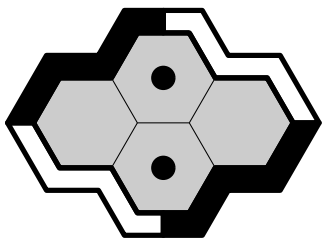
Hex: four theorems

- no draws (Pierce '61)
- nxn board: 1st-player wins (Nash '49)
- nx(n+1) board: closer-sides wins (Milnor '49)
- Pspace-complete (Reisch '81)

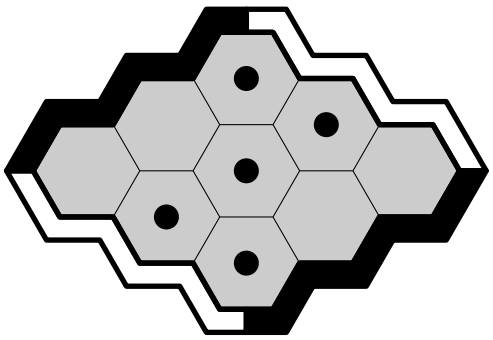
Hex: winning black-1st moves



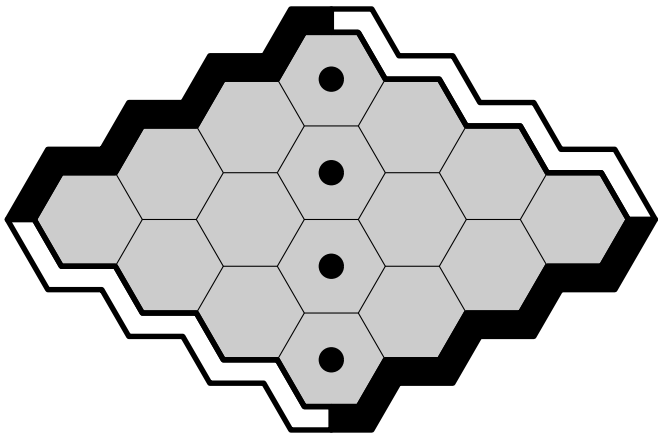
Hex: winning black-1st moves



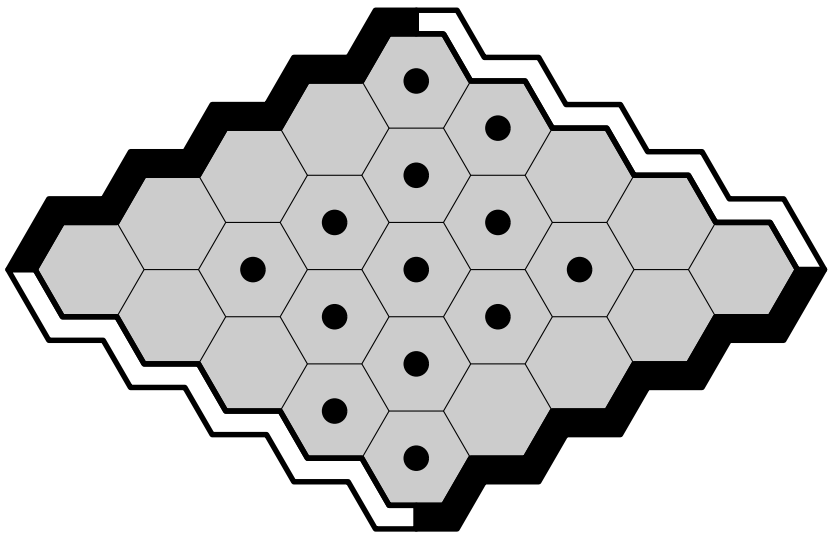
Hex: winning black-1st moves



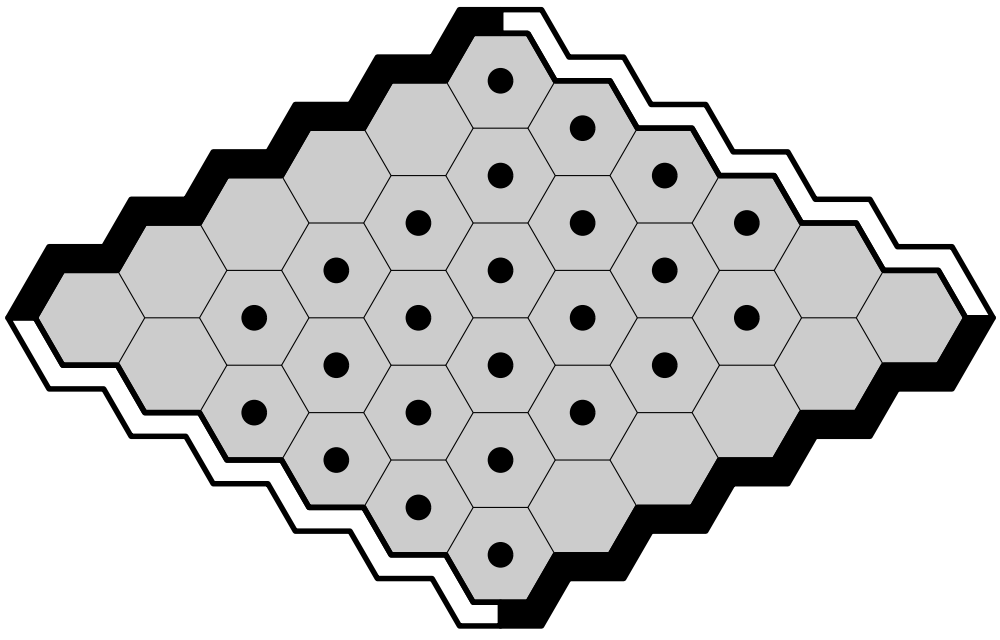
Hex: winning black-1st moves



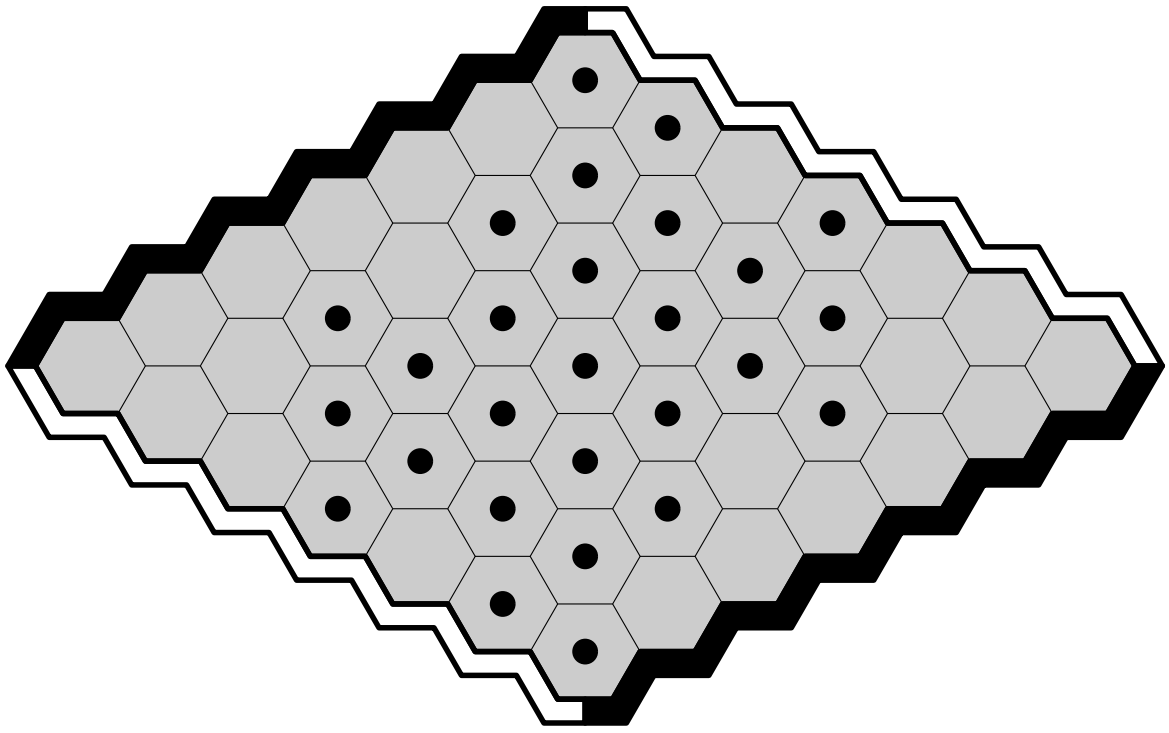
Hex: winning black-1st moves



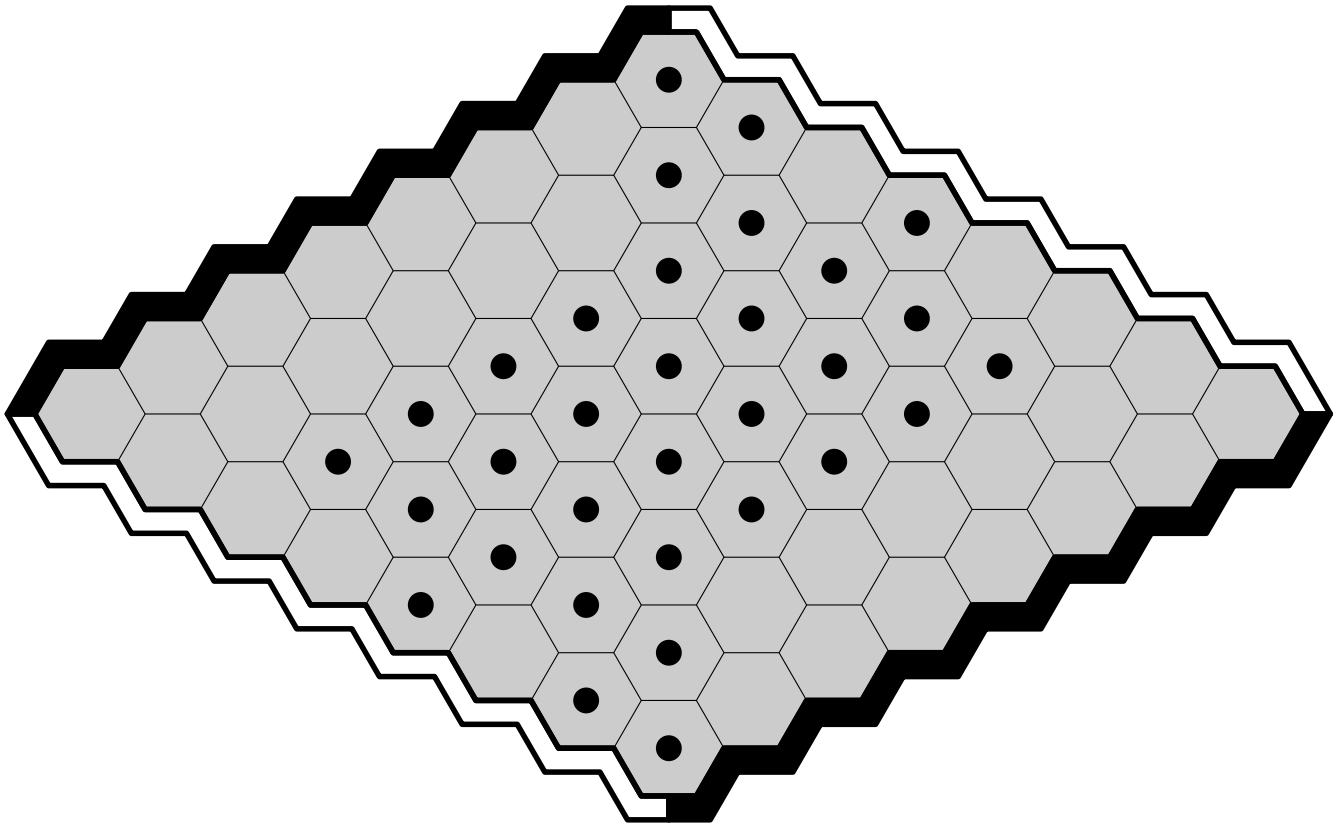
Hex: winning black-1st moves 1995 Enderton



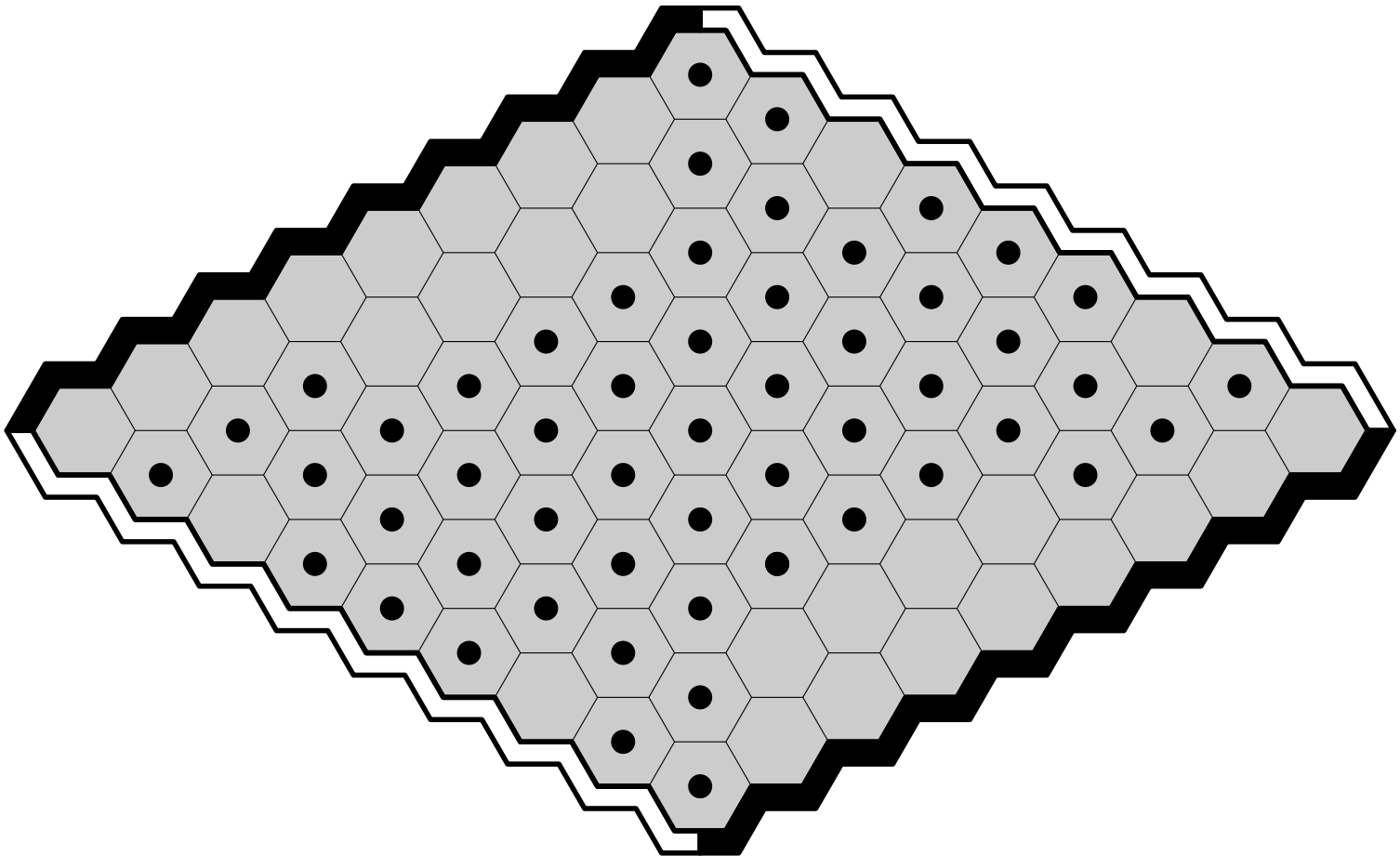
Hex: winning black-1st moves 2004 HBJPvR



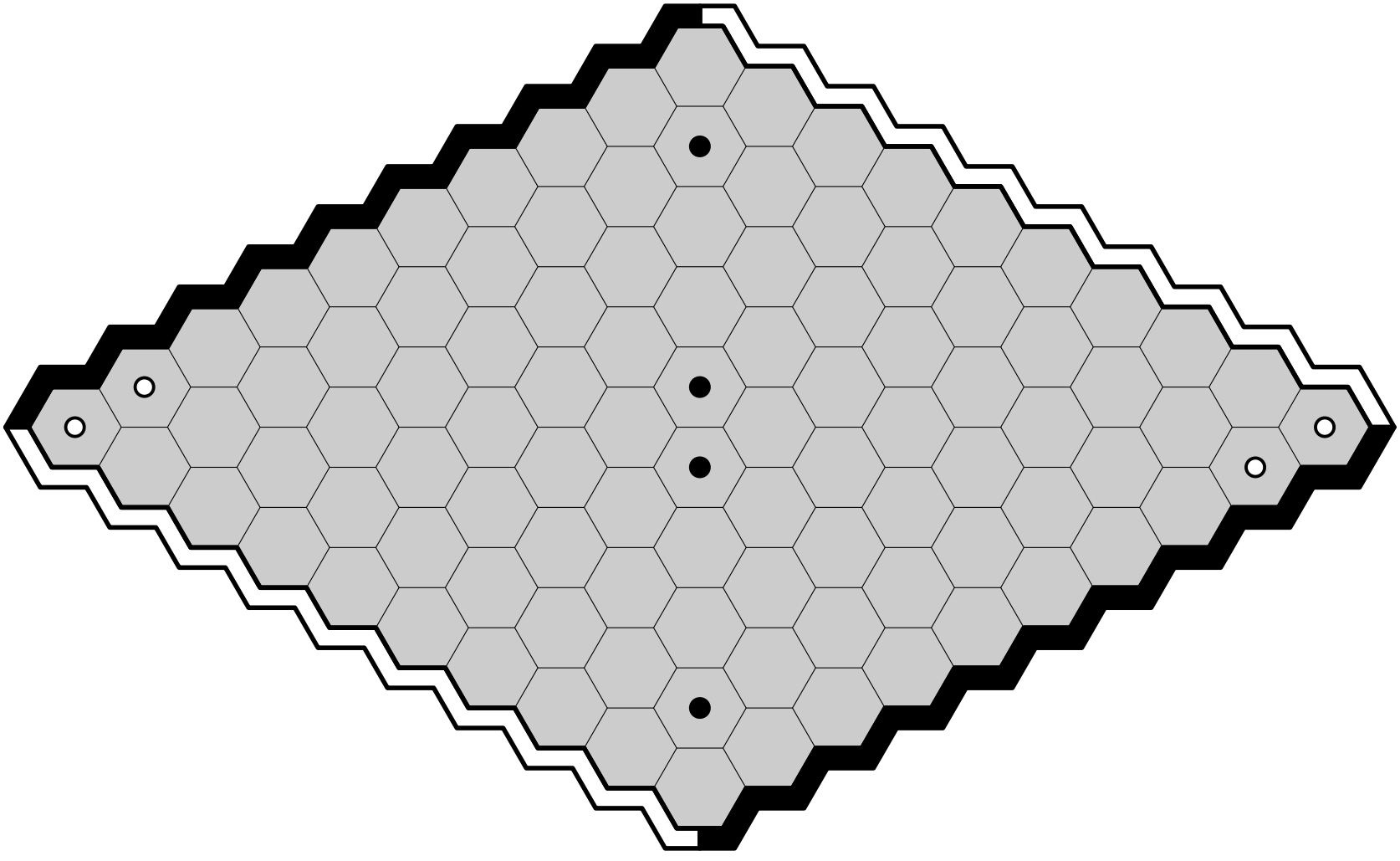
Hex: winning black-1st moves 2009 HAH



Hex: winning black-1st moves 2013 AHHP

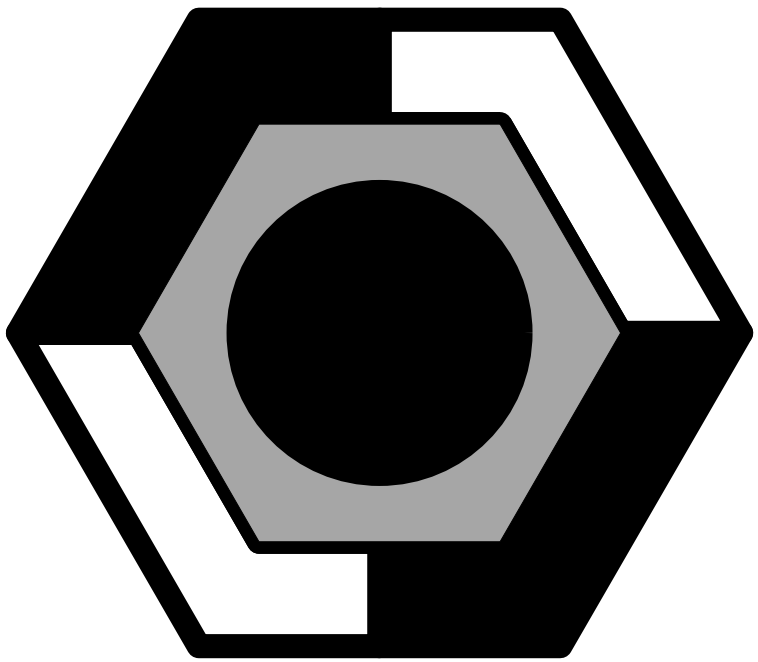


Hex: winning black-1st moves 2014 PH

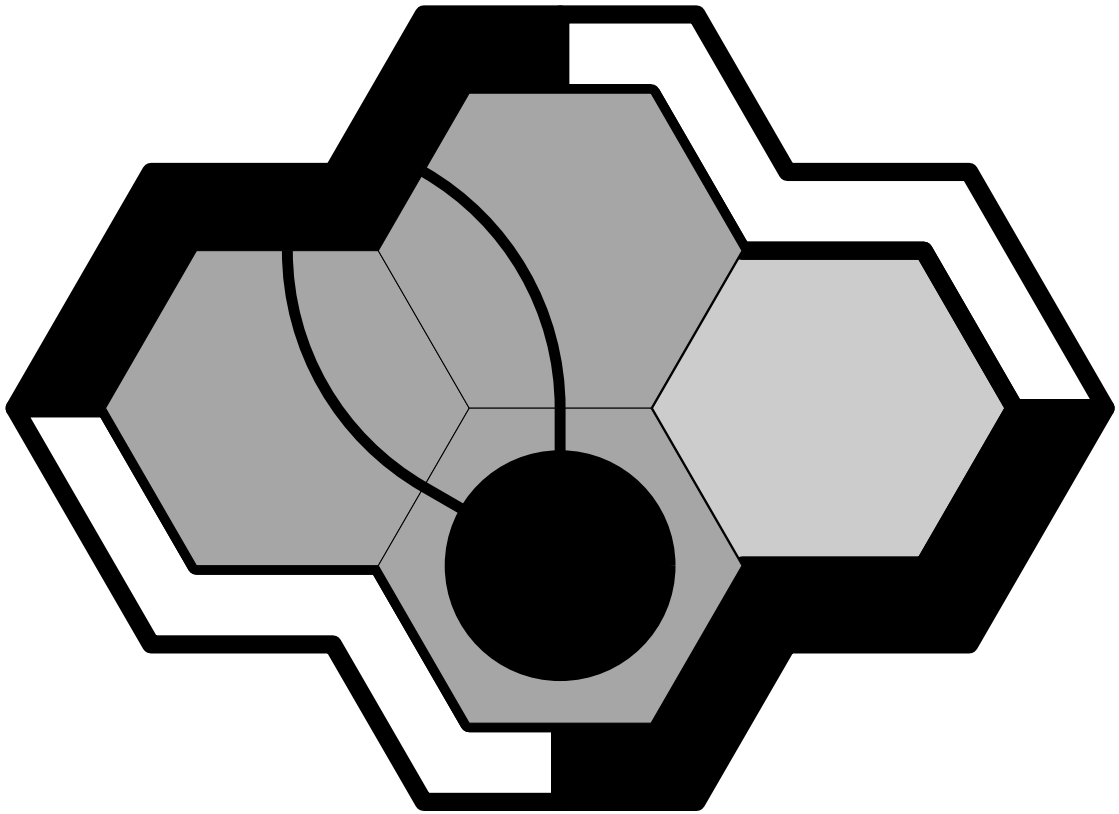


Hex: centermost cell wins (small boards)

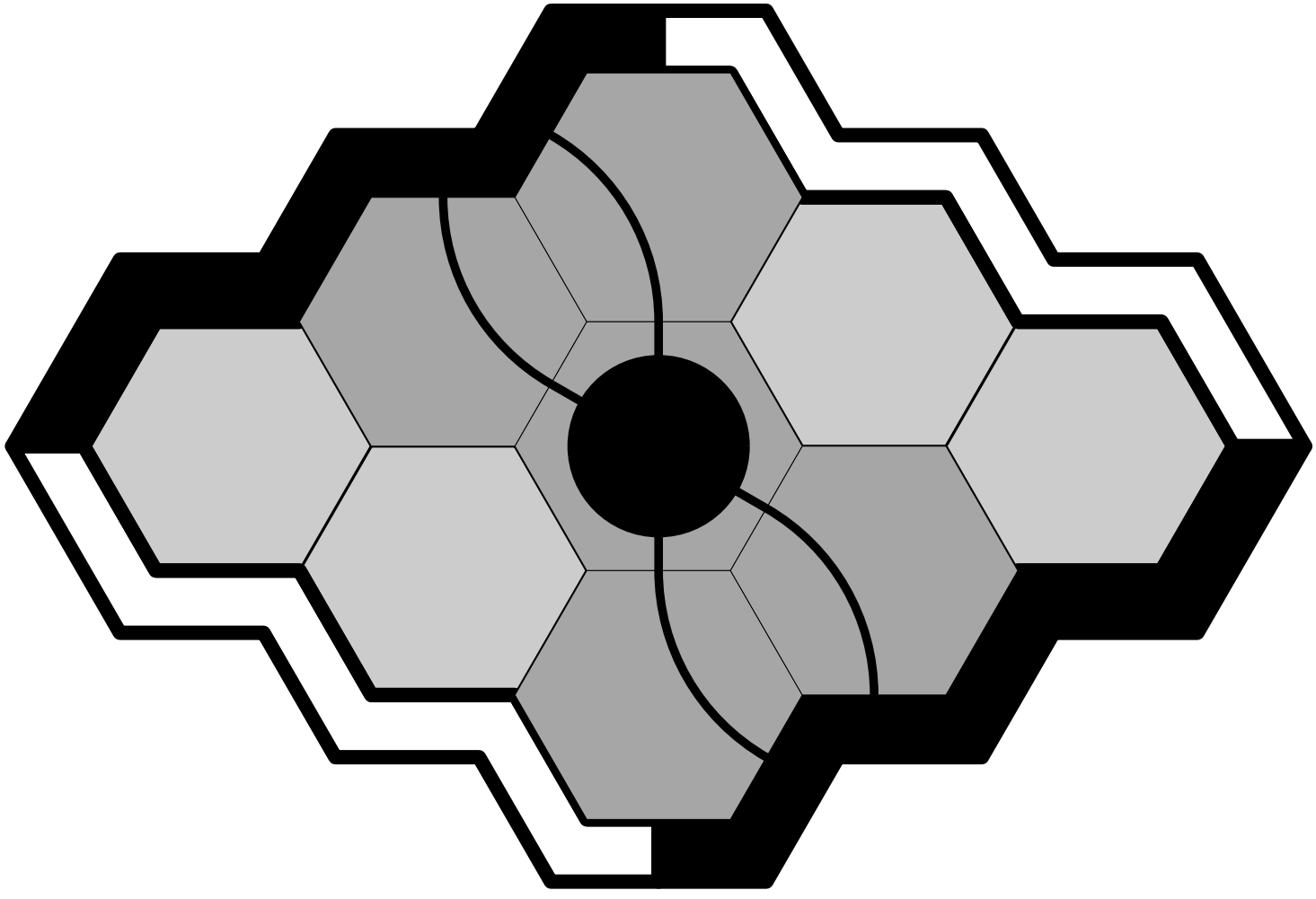
Hex: centermost cell wins (small boards)



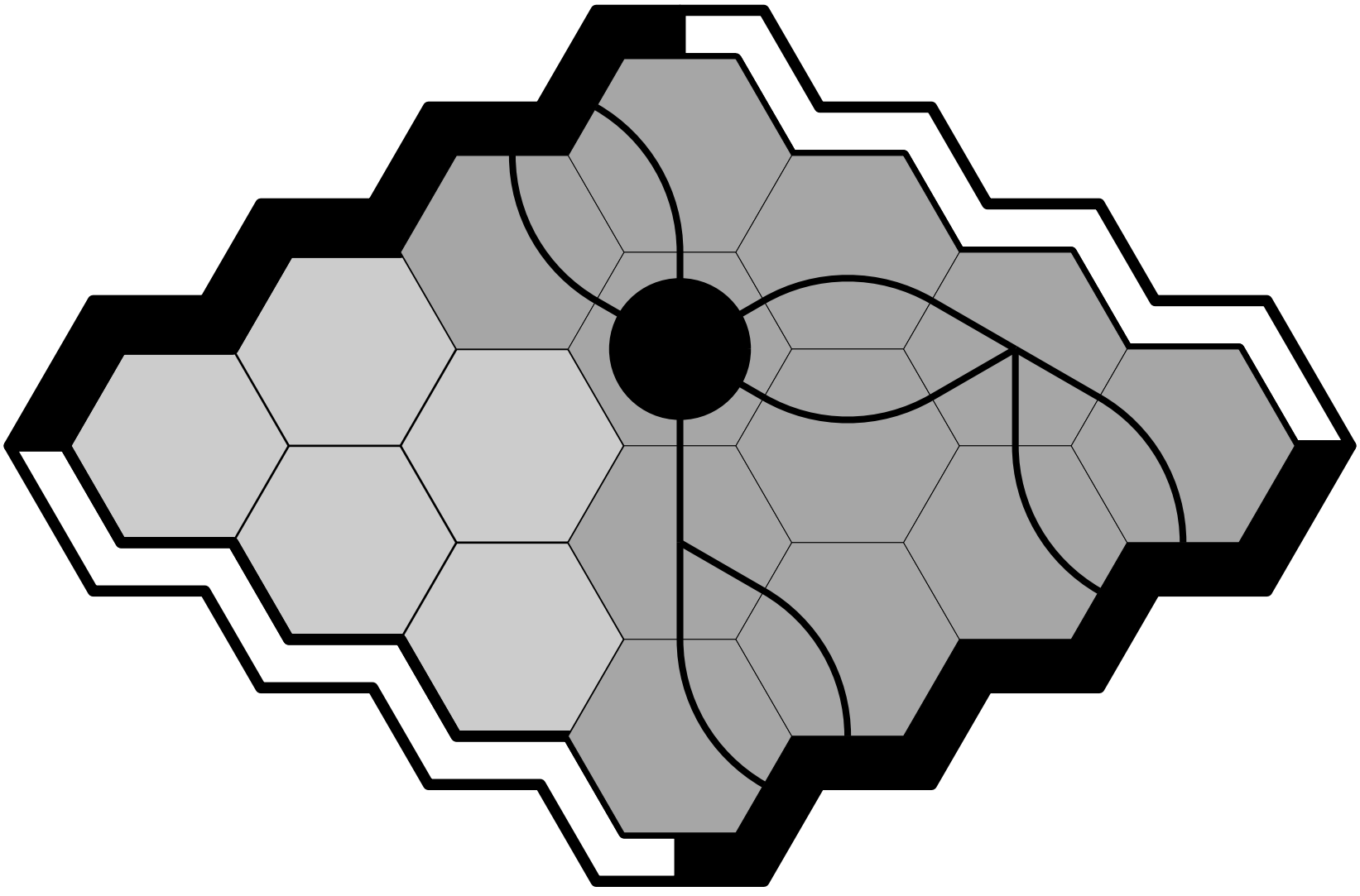
Hex: centermost cell wins (small boards)



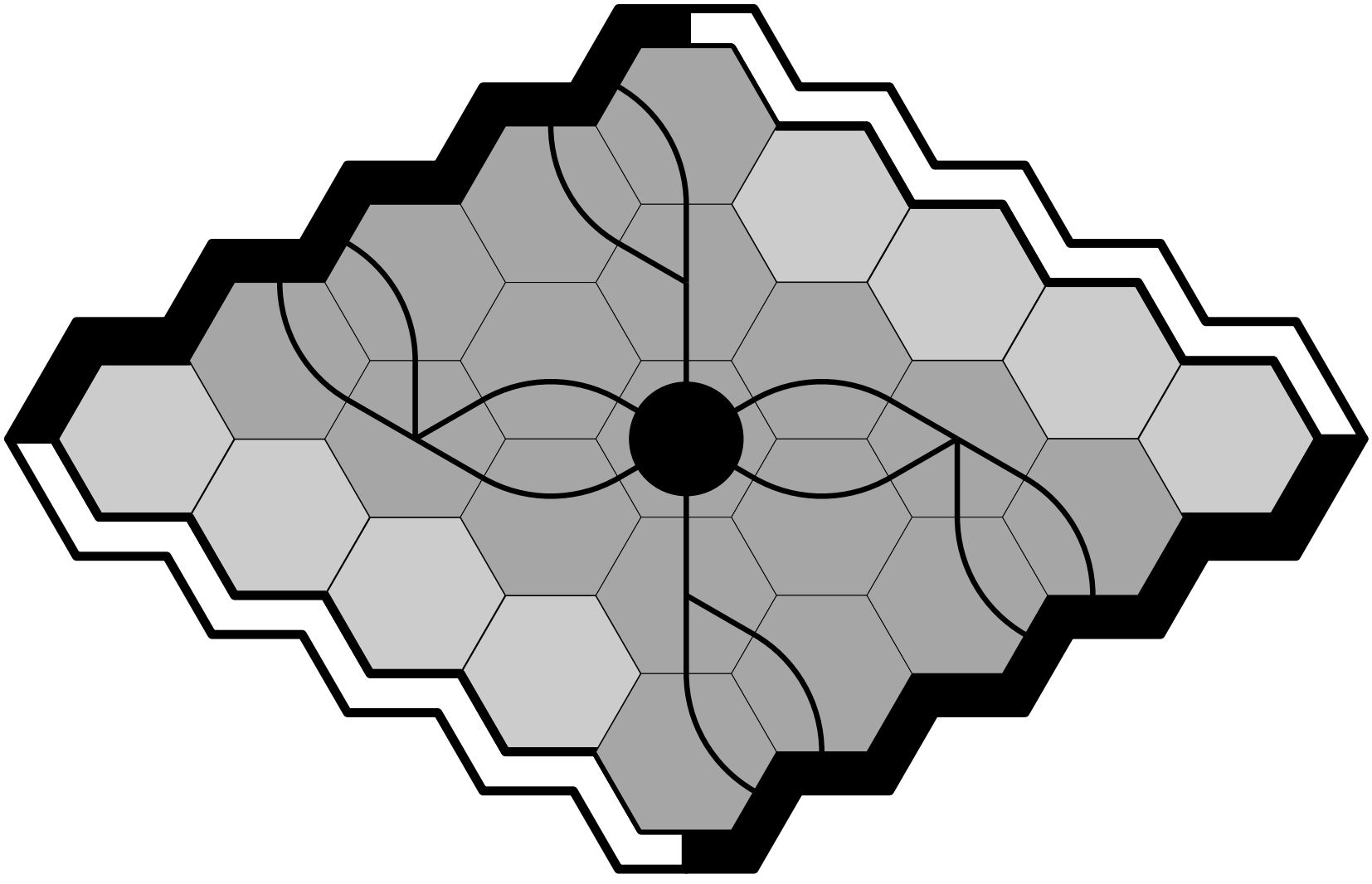
Hex: centermost cell wins (small boards)



Hex: centermost cell wins (small boards)

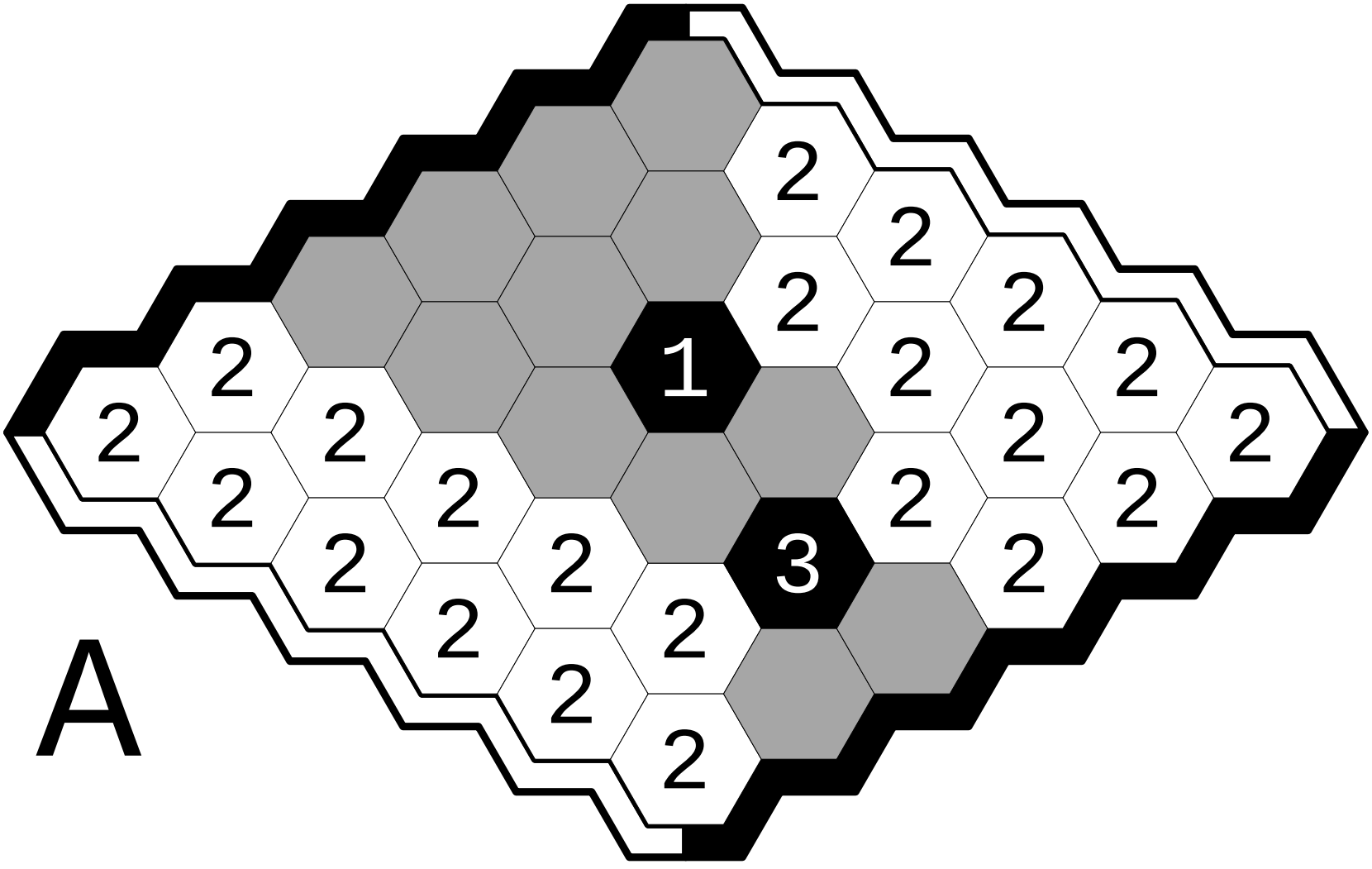


Hex: centermost cell wins (small boards)

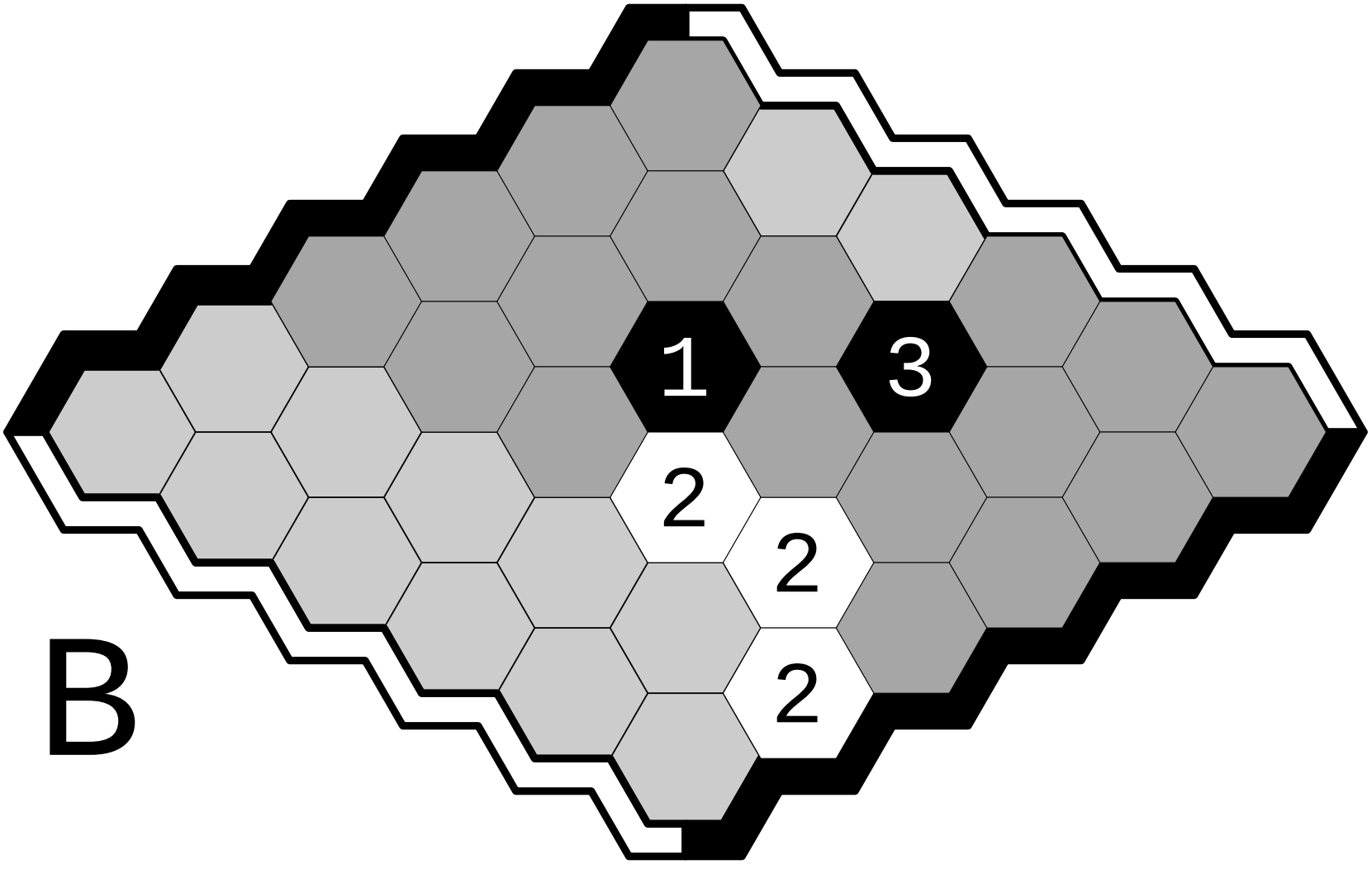


Hex: centermost, 6x6 (Lindhard)

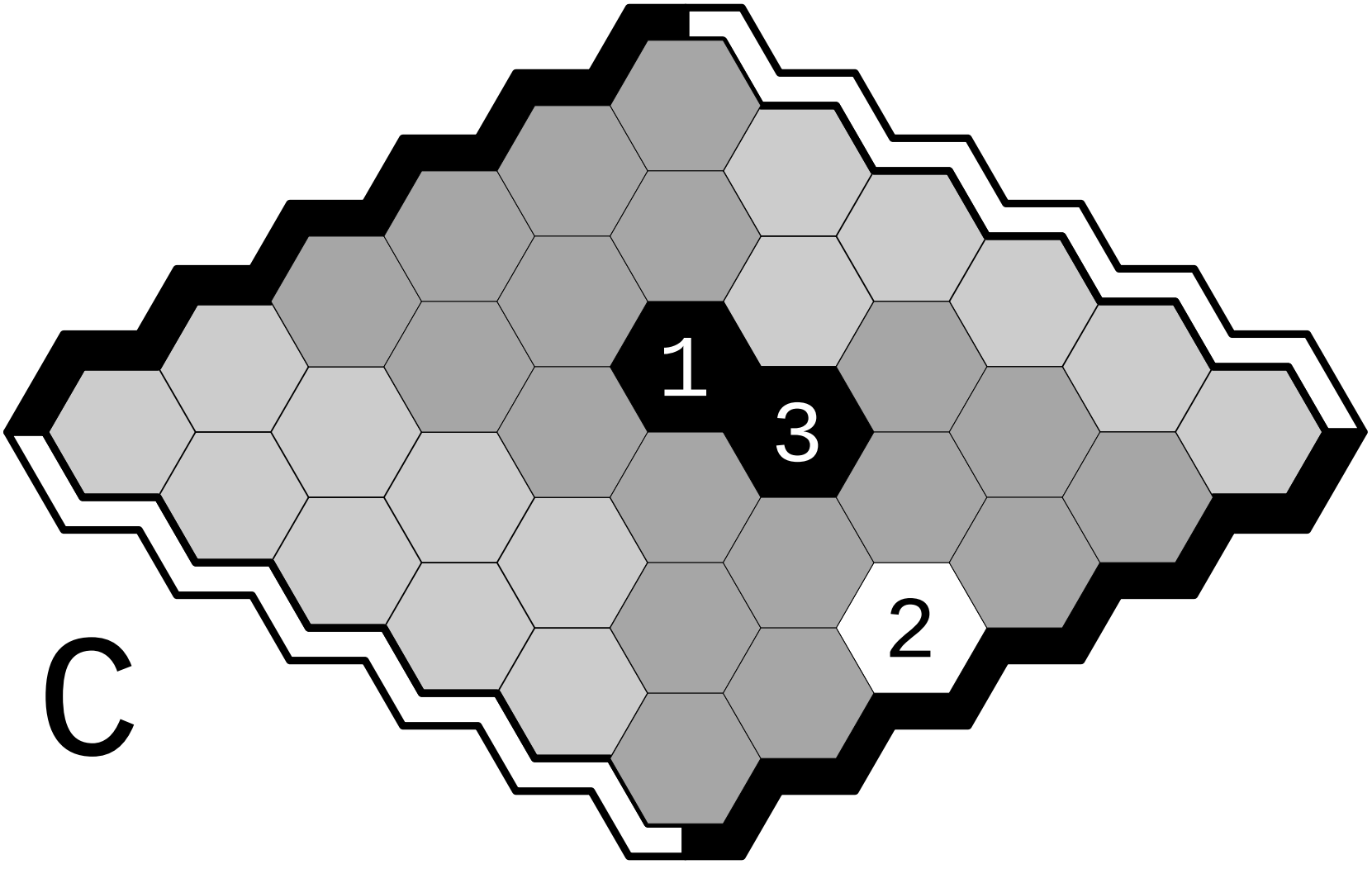
Hex: centermost, 6x6 (Lindhard)



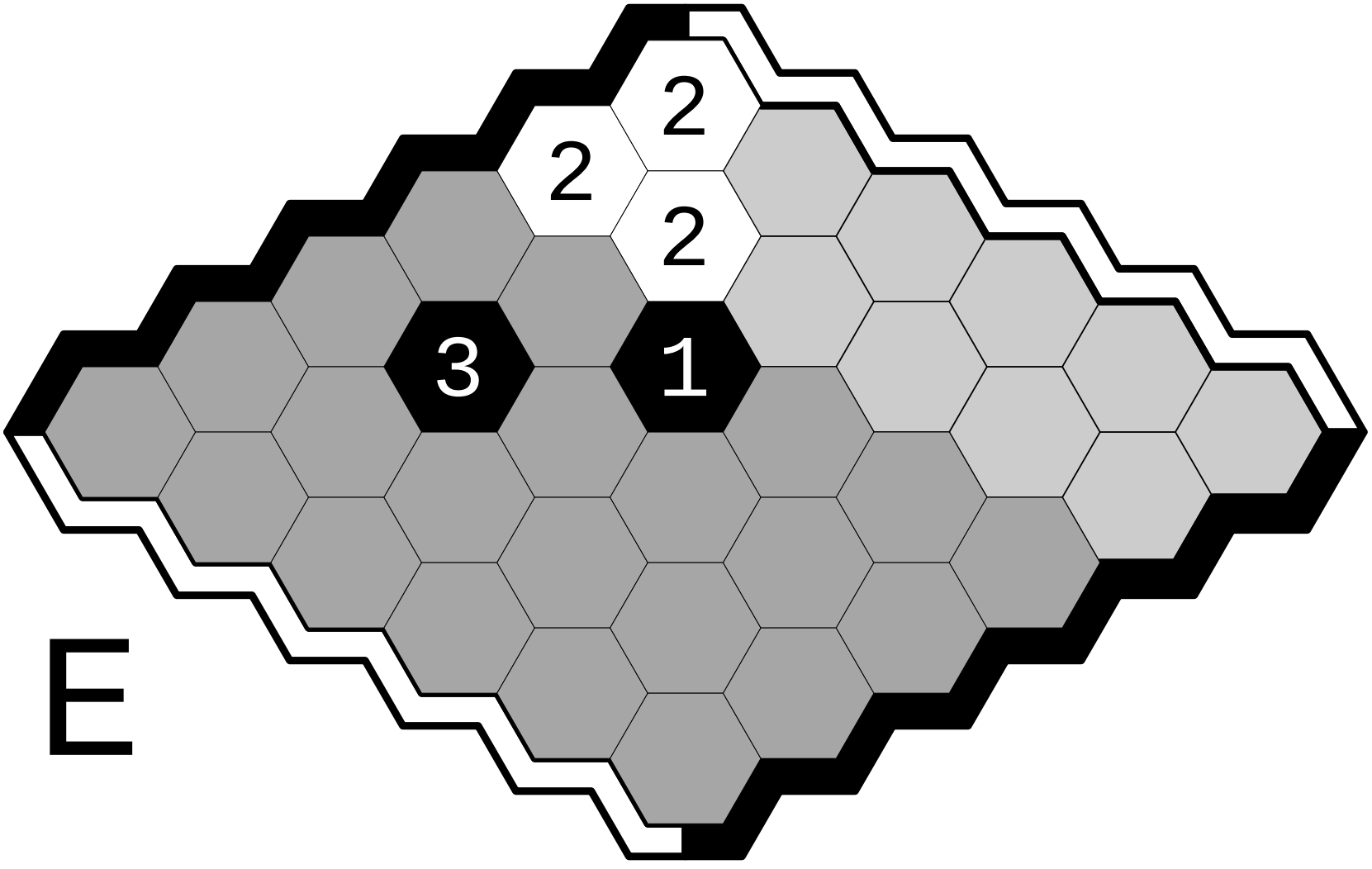
Hex: centermost, 6x6 (Lindhard)



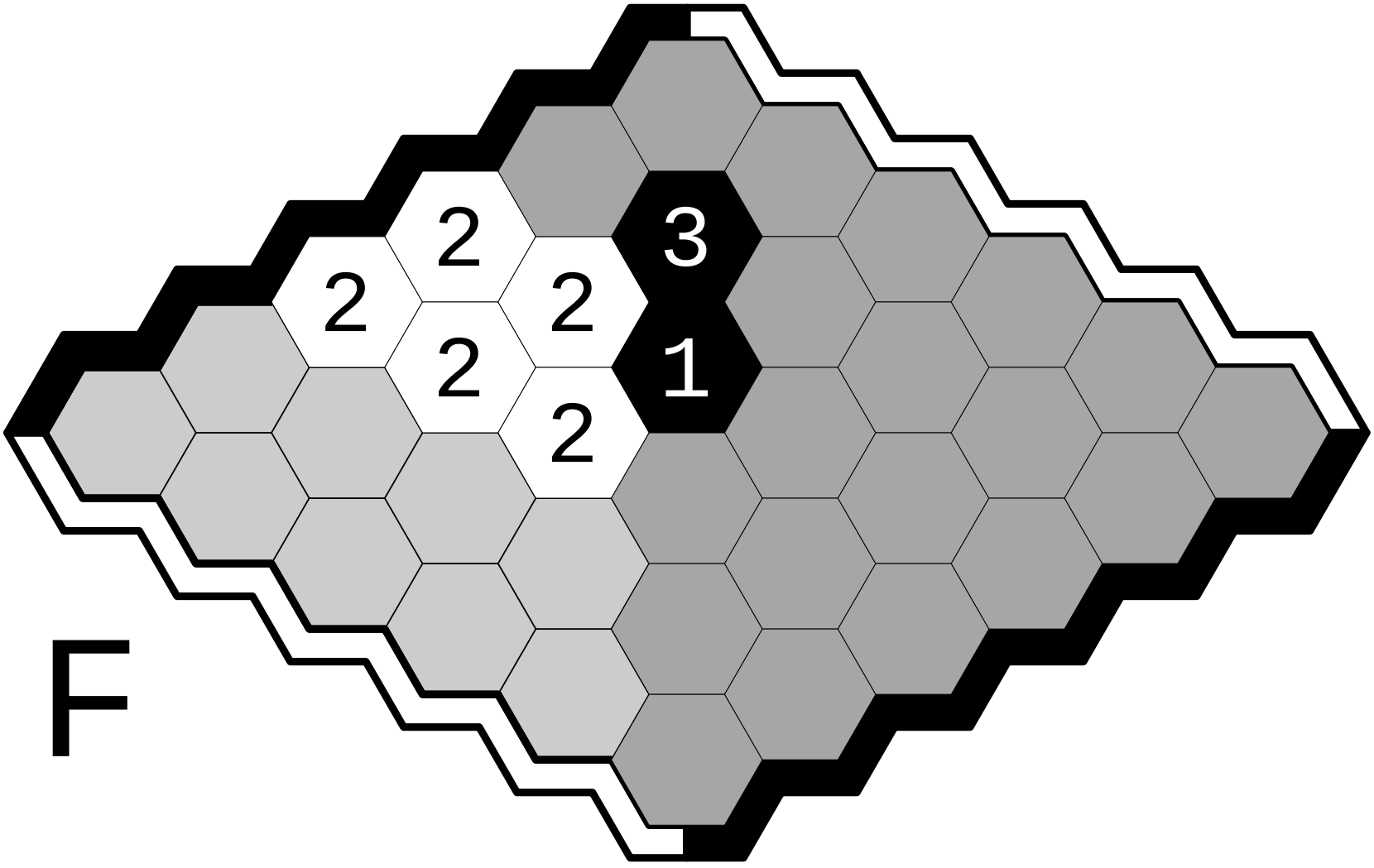
Hex: centermost, 6x6 (Lindhard)



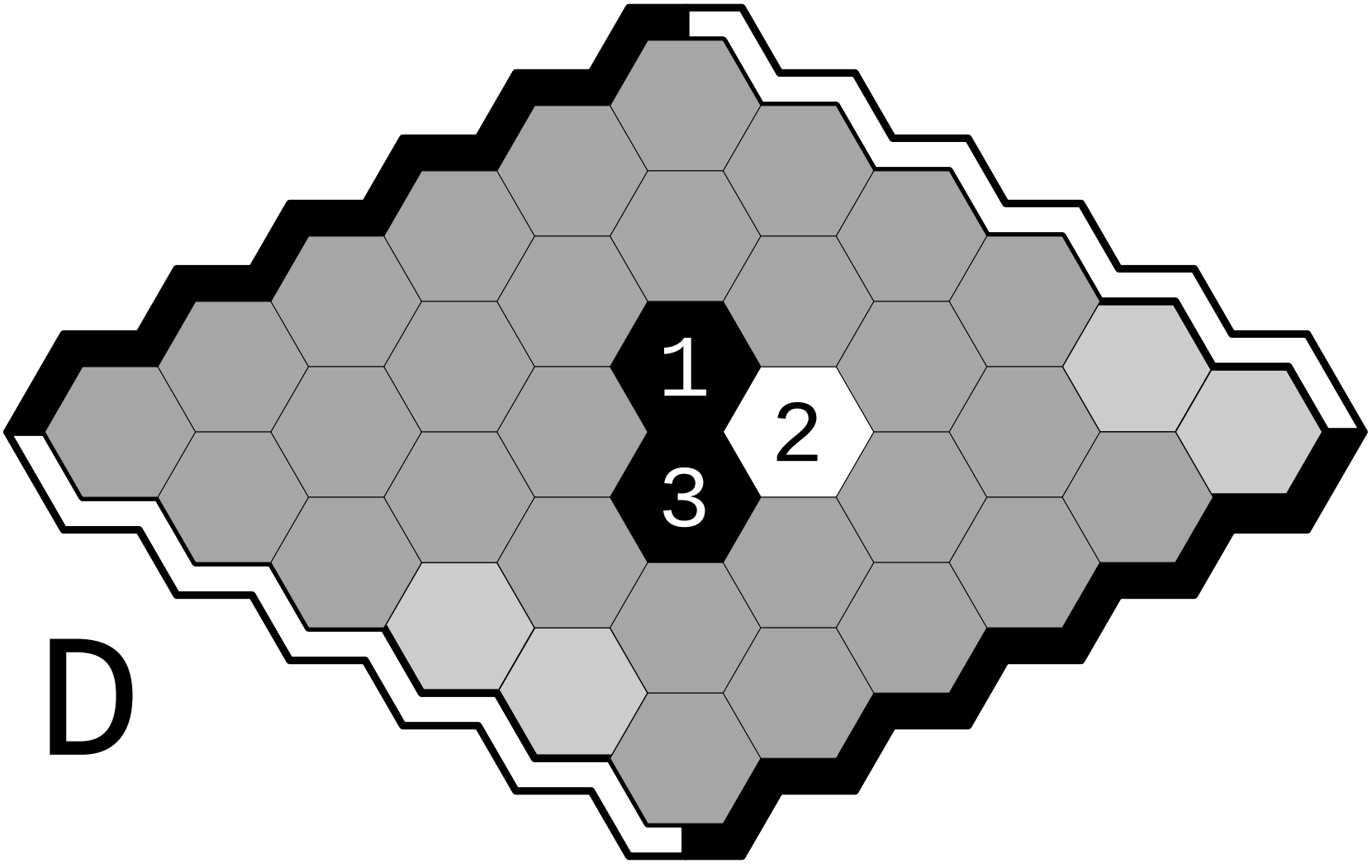
Hex: centermost, 6x6 (Lindhard)



Hex: centermost, 6x6 (Lindhard)



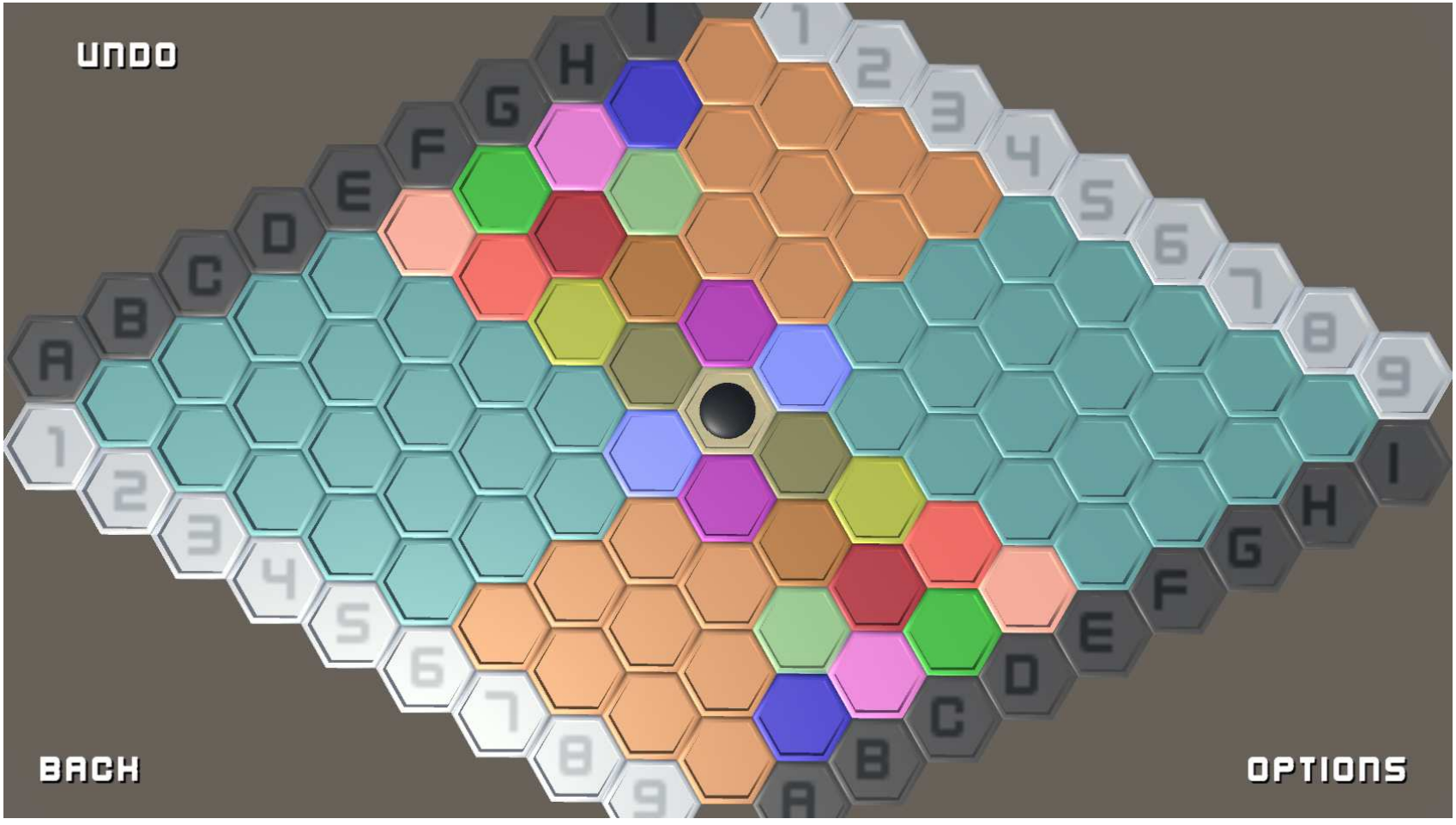
Hex: centermost, 6x6 (Lindhard)



Hex: centermost, 7x7–9x9 (Jing Yang)

http://sites.ualberta.ca/~hayward/355/asn/hexviz/

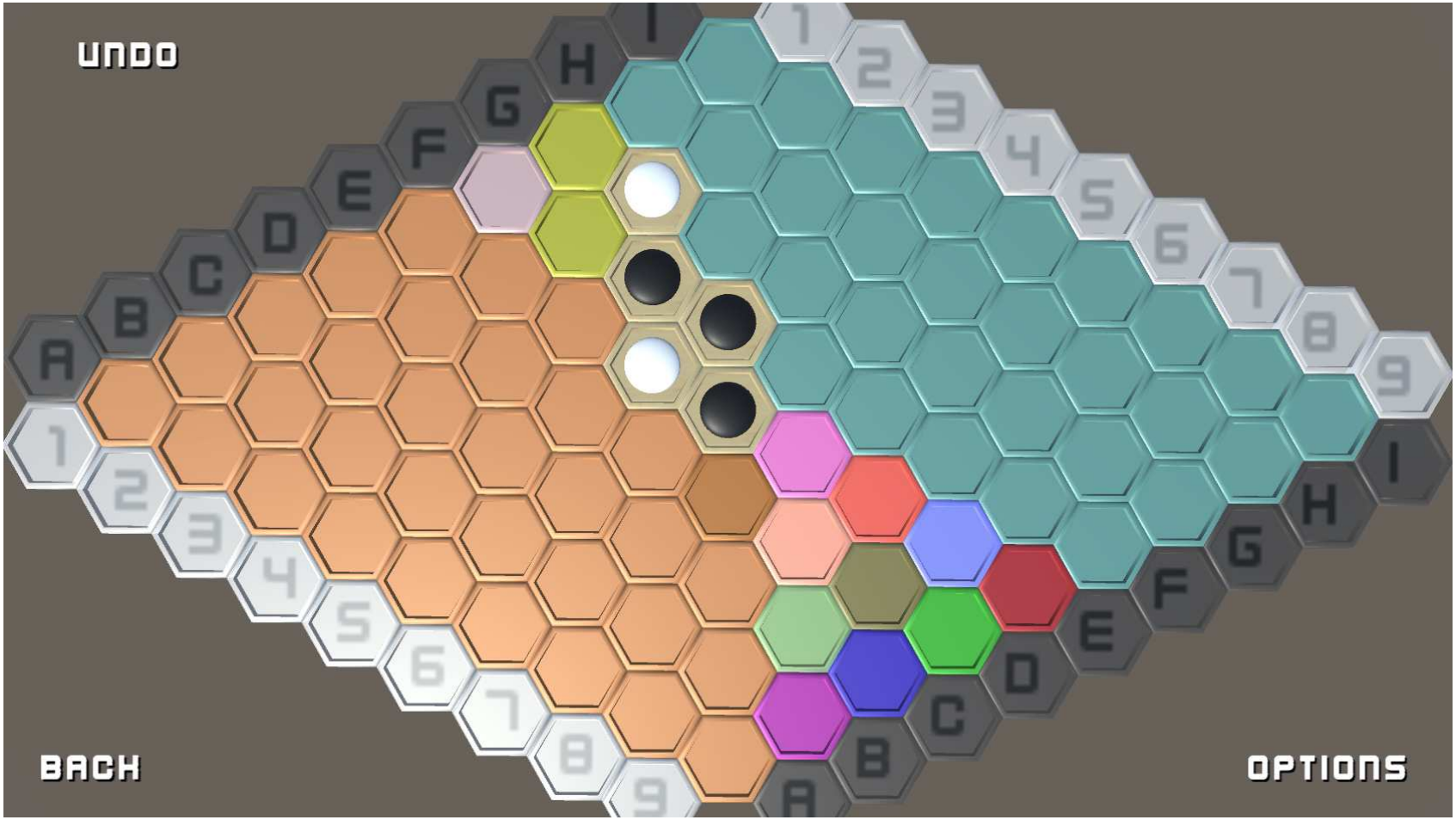
Hex: centermost, 7x7–9x9 (Jing Yang)



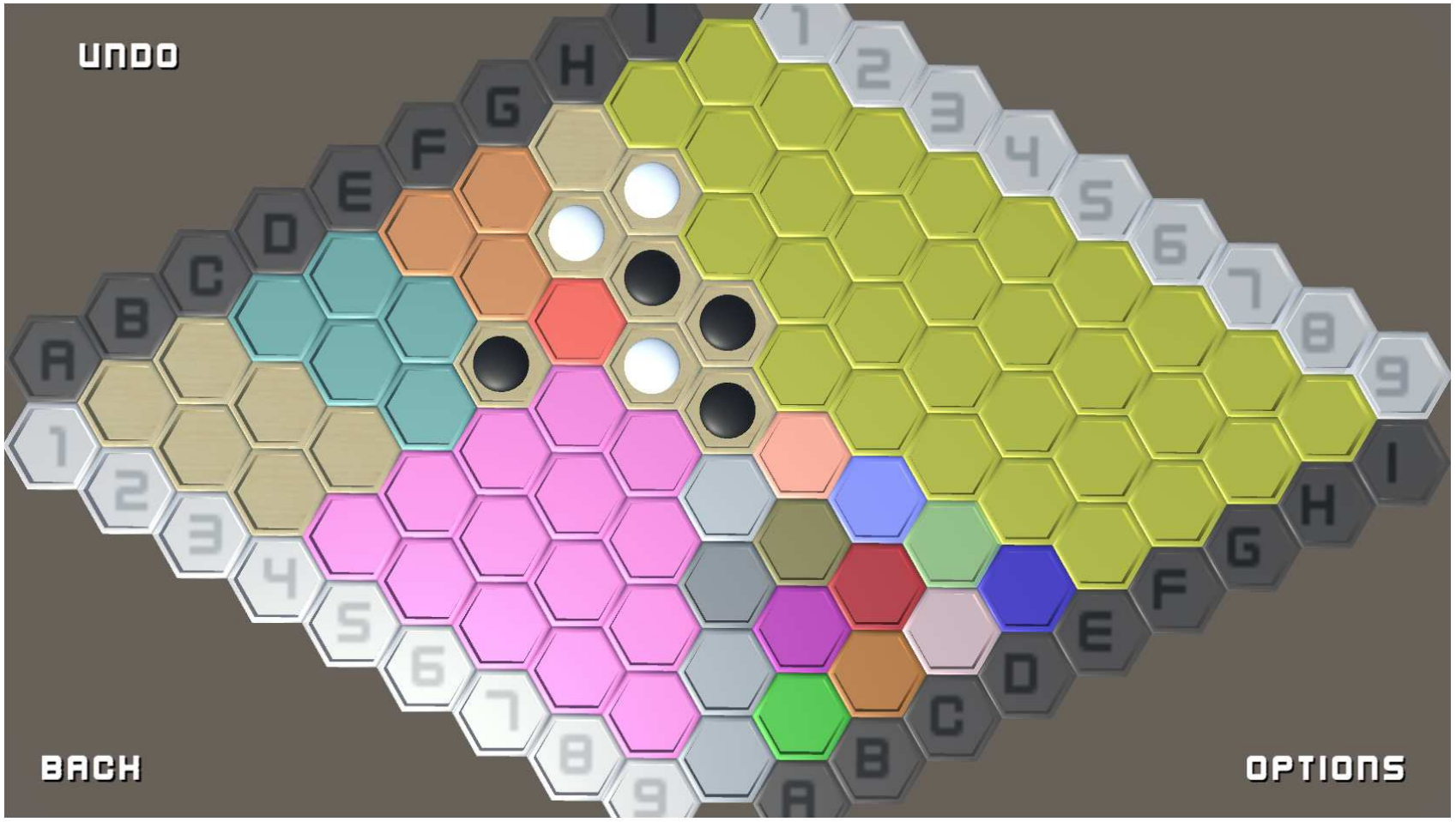
Hex: centermost, 7x7–9x9 (Jing Yang)



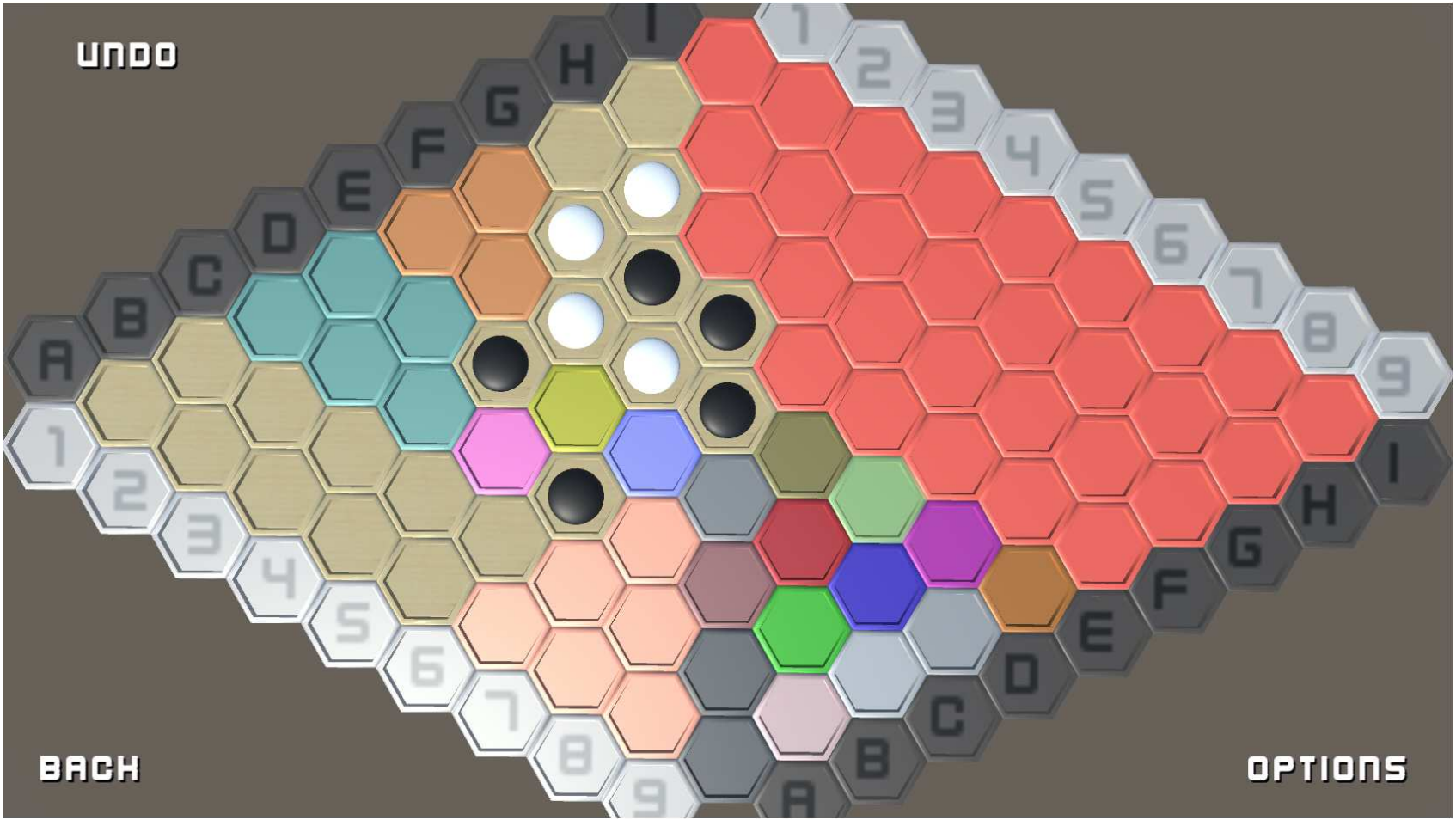
Hex: centermost, 7x7–9x9 (Jing Yang)



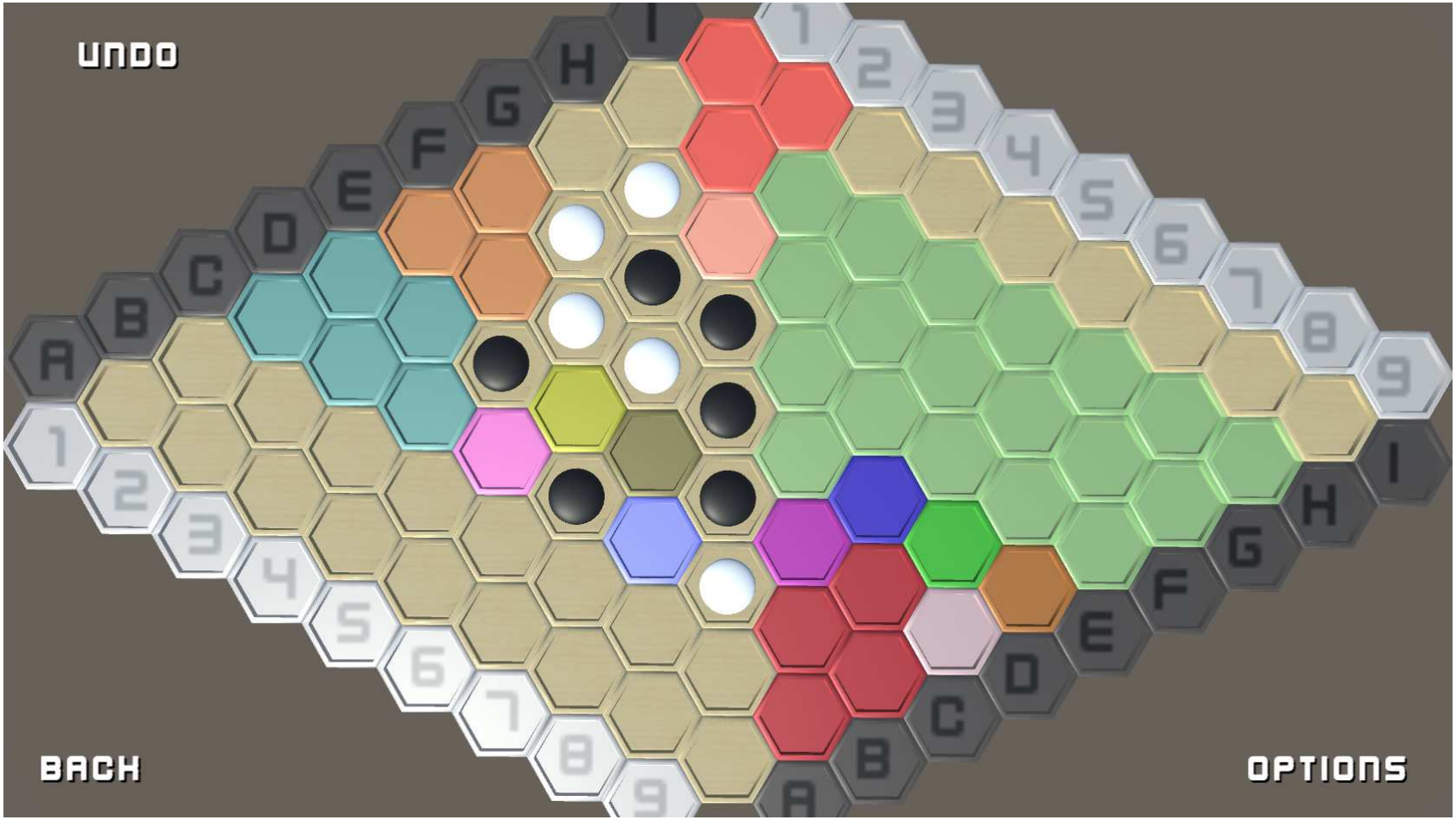
Hex: centermost, 7x7–9x9 (Jing Yang)



Hex: centermost, 7x7–9x9 (Jing Yang)



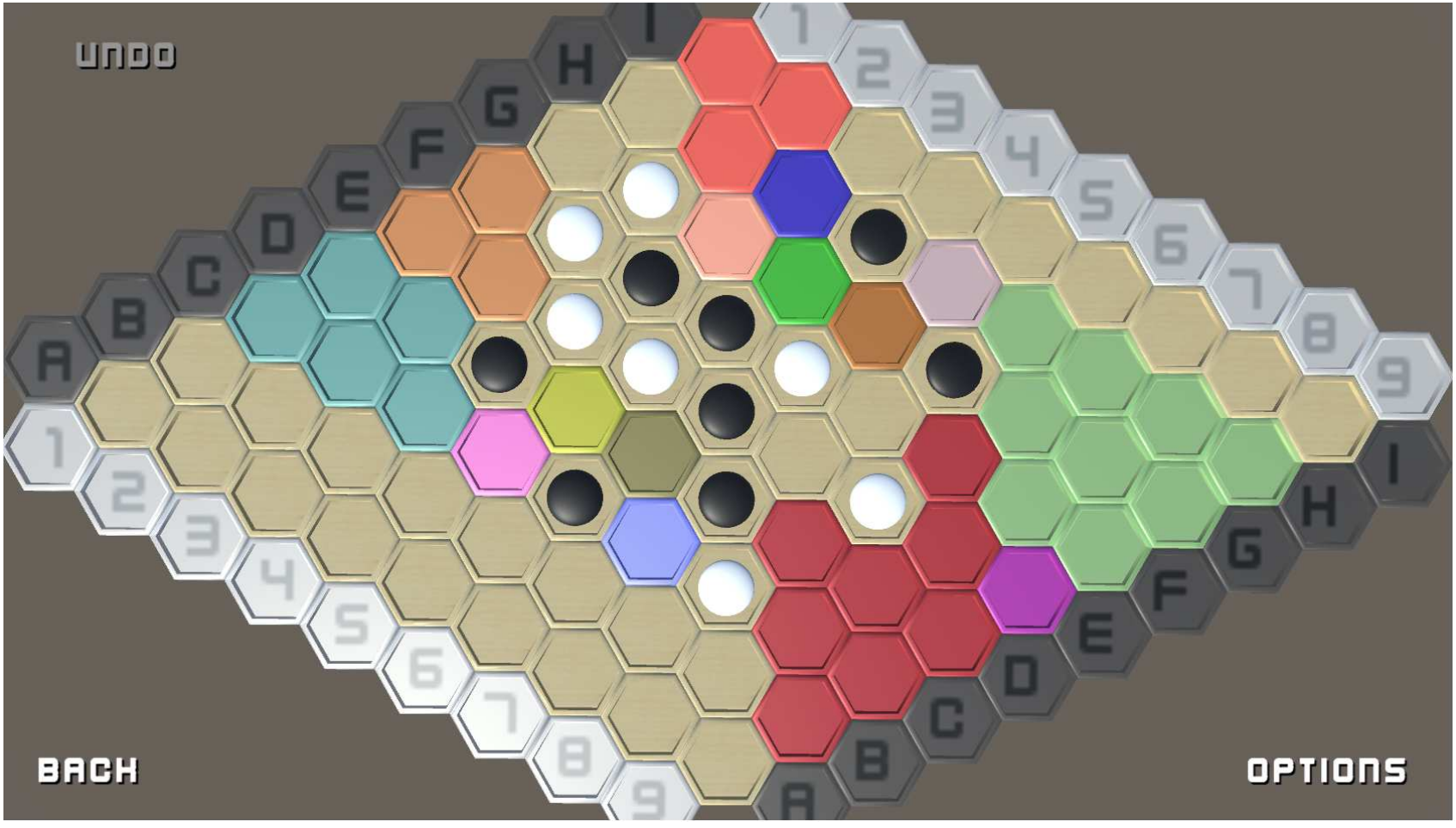
Hex: centermost, 7x7–9x9 (Jing Yang)



Hex: centermost, 7x7–9x9 (Jing Yang)



Hex: centermost, 7x7–9x9 (Jing Yang)



Hex: centermost, 7x7–9x9 (Jing Yang)



Hex and combinatorial game theory

- On combinatorial value of Hex positions

  (Selinger '21)

  `https://arxiv.org/abs/2101.06694`

  `https://pages.cpsc.ucalgary.ca/~robin/FMCS/`
- On 3-terminal positions in Hex

  (Demer/Selinger '25)

  `https://arxiv.org/abs/2507.08247`

Hex: challenge problems

- 19x19: build superhuman bot (11x11 KataHex)

  `https://www.hexwiki.net/index.php/KataHex`

- 10x10, center opening wins: simple proof?

- 11x11, center opening wins?

co-authors

Bjarne Toft, Michael Johanson, Yngvi Björnsson, Morgan Kan, Nathan Po, Jack van Rijswijck, Broderick Arneson, Philip Henderson, Jakub Pawlewicz, Brad Thiessen, Aja Huang, Kenny Young, Noah Weninger, Chao Gao, Martin Müller, Bedir Tapkan, Md-Reza Daliri, Kamillah Hasham, Luke Schultz, Xinyue Chen

thank you

email hayward@ualberta.ca