

A BEAUTIFUL GAME FROM THE WAR

HOW HEX BEGAN

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computing.science university of alberta supported by NSERC

2019

THANK YOU

- Hex research team
Michael Johanson, Yngvi Björnsson, Morgan Kan, Nathan Po, Jack van Rijswijck, Broderick Arneson, Philip Henderson, Jakub Pawlewicz, Aja Huang (AlphaGo), Kenny Young, Noah Weninger, Chao Gao, Martin Müller (Fuego)
- Bjarne Toft
- Liz Greenaway and Audreys Books Edmonton
- UAlberta FacSci events

1942-43 DENMARK: POLYGON
1949-50 PRINCETON-NYC: HEX
1957 SCIENTIFIC AMERICAN

HEX, THE FULL STORY

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1957 SCIENTIFIC AMERICAN

HEX, THE FULL STORY



HEX, THE FULL STORY

<https://webdocs.cs.ualberta.ca/~hayward/hexbook>

HEX, THE FULL STORY

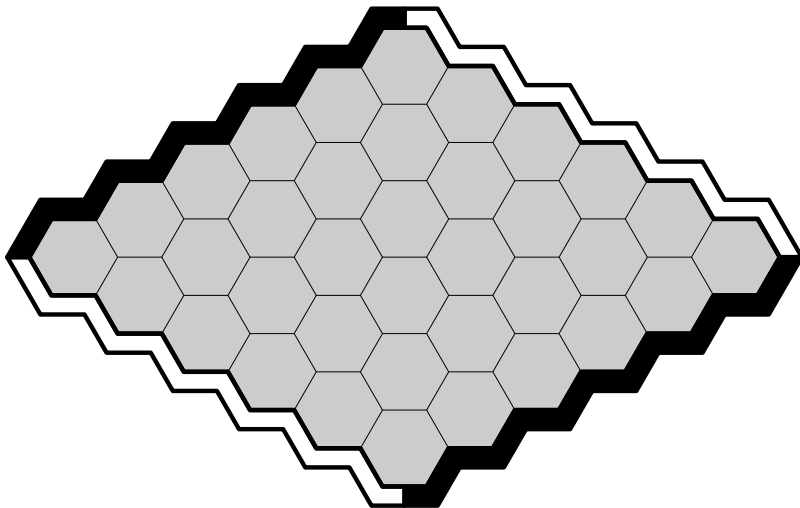
sources

- Berge *L'Art subtil du Hex* 1977
- *Politiken* 1942 – 1943
- Thomas Maarup MSc thesis 2005
- Martin Gardner archive, Stanford U
- Piet Hein archive, collection Hugo Hein
- Jens Lindhard archive, Aarhus University

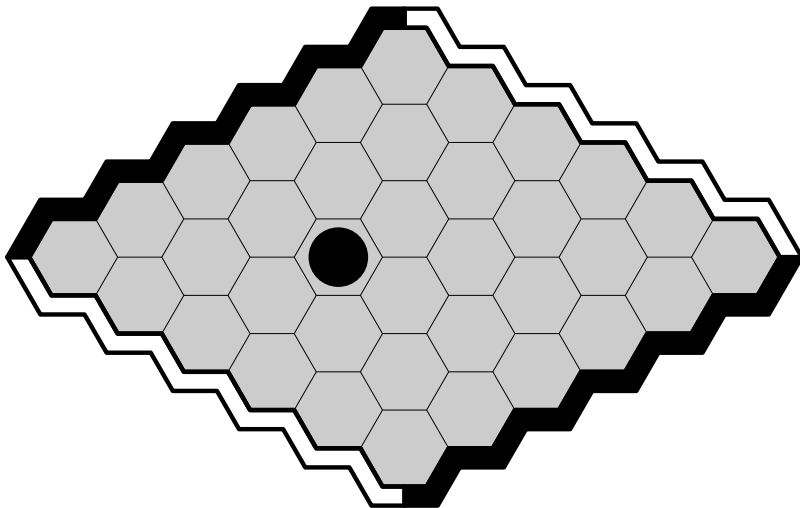
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HEX

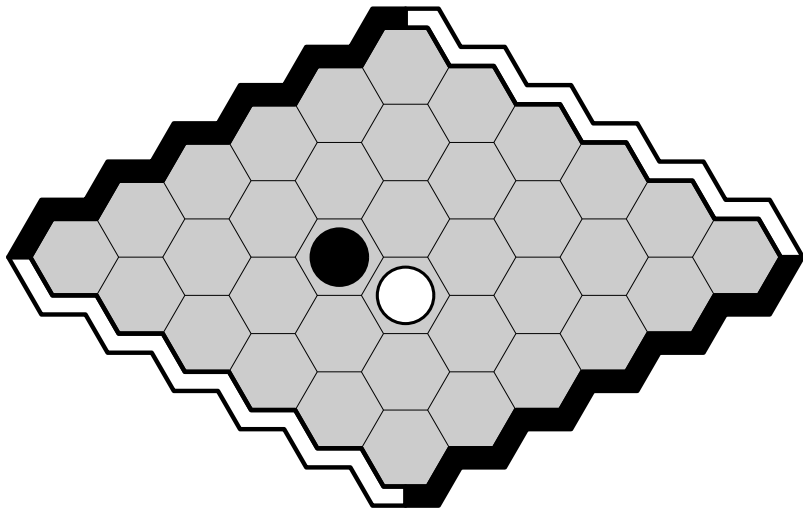
HEX



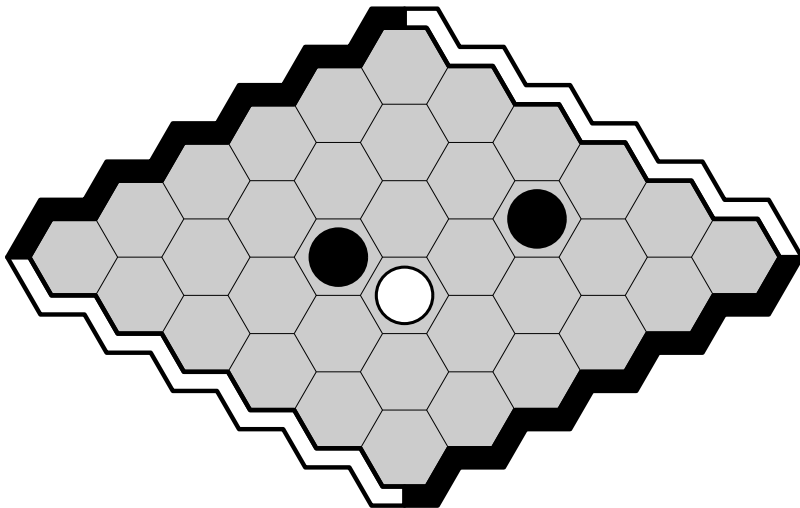
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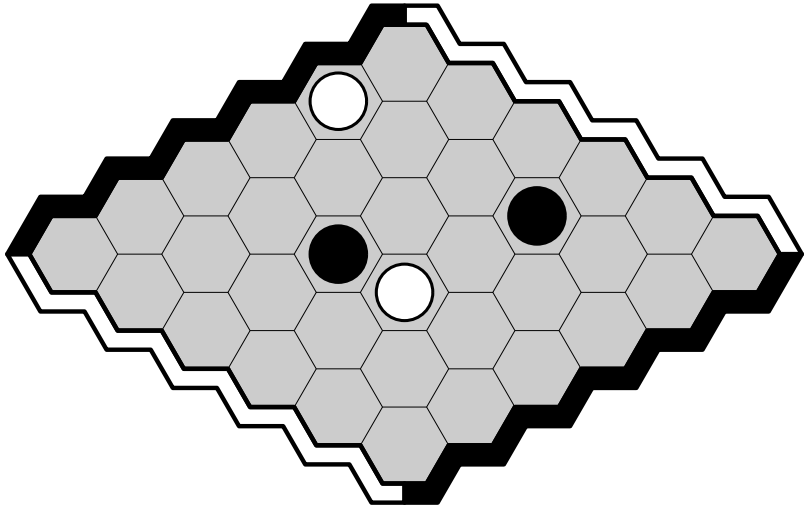
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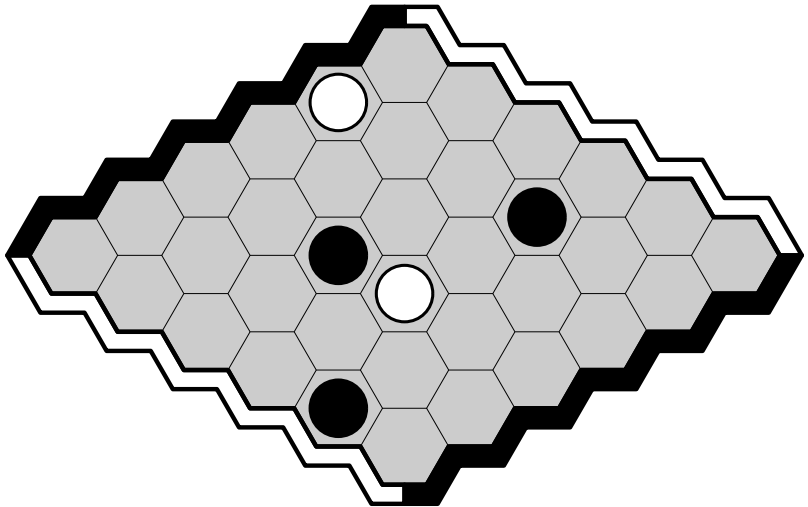
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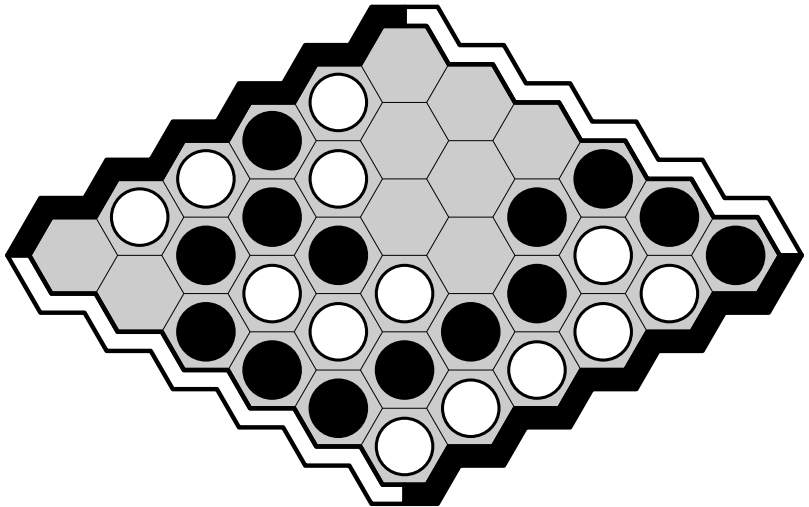
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HEX



HEX



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HOW HEX BEGAN

HOW HEX BEGAN

- WWII
- cold war
- post-war

how came the game?

how came the game again?

how spread the game?

HOW HEX BEGAN

- Hein design 1942
- Lindhard drafts, Thorborg puzzle 1943
- Nash theory, Gale board 1949
- Shannon machine 1950
- Parker Brothers game 1950
- Gardner column 1957

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PIET HEIN

PIET HEIN



PIET HEIN



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WAR

WAR

- 1940 April 9

Germany invades Denmark, Norway

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GROOKS

GROOKS

CONSOLATION GROOK

Losing one glove
is certainly painful,
but nothing
 compared to the pain,
of losing one,
throwing away the other,
and finding
 the first one again.

KUMBEL KUMBELL

GROOKS

PROBLEMS

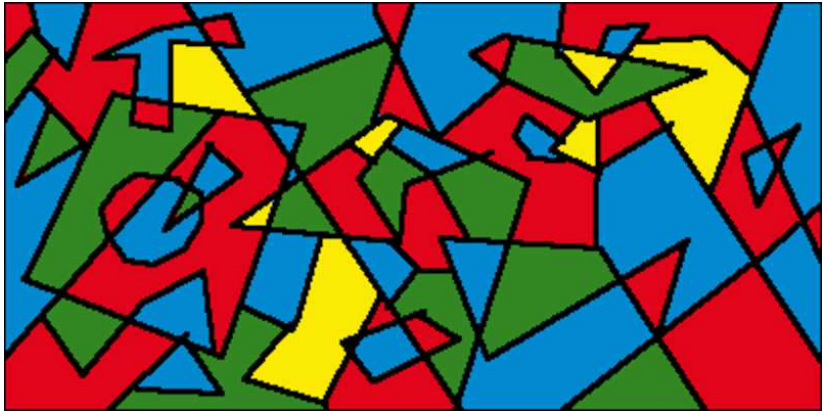
Problems worthy
of attack
prove their worth
by hitting back.

KUMBEL

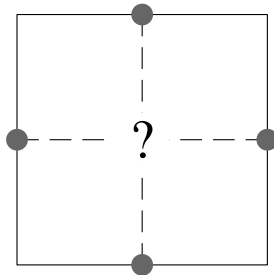
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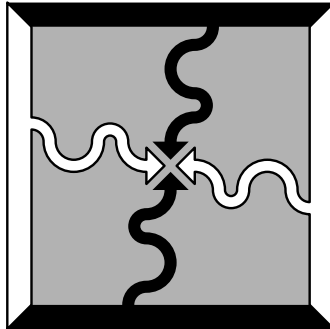
NEW GAME IDEA

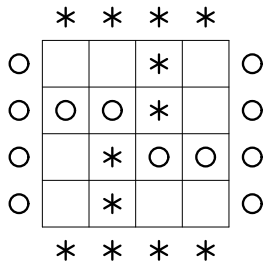
NEW GAME IDEA



©<https://www.kleemans.ch/four-color-theorem->







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1942 POLYGON [HEX]

1942 POLYGON [HEX]

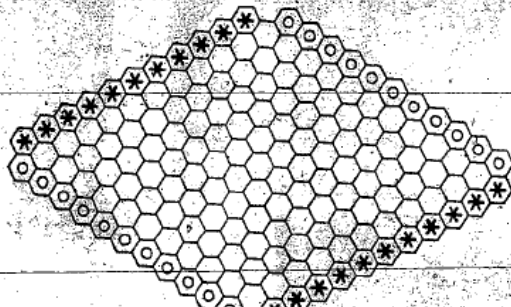
TIKEN

26. December 1942

Vil De lære Polygon?

Piet Hein har konstrueret et Spil, der med lige stor Glæde kan dyrkes af Skakeksperten og den, der blot kan holde en Blyant

„Politiken“ udskriver i Dag en Præmieopgave, der vil volde Hovedbrud for Begyndere



ten kan afbryde Forbindelsen ved at besætte det mellemste gode Felt, dens Angværelighed bliver som Blykammeret Placering i den videre Omegn. I det hele taget viser det sig snart nødvendigt at tage en større Del af Spillebrættet med i Betragtning.

En anden Erfaring, som kommer senere, men som man kan lette Spilleets Begyndelse ved at se, er, at det betaler sig at begynde i hvert Fald nogenlunde paa Midten. En rimelig, men paa ingen Maade nødvendig Aabning af Spillet er denne:

Paa Spillebrættet i Midten er Hvid begyndt i Midterfeltet. Sort har Sæt i Kontaktfeltet til det ned imod Midten af Hvids Front og derved gjort to nyttige Felter, som staar i Vinkelstilling til Midterfeltet, usikre. Hvid har saa vaegt et Felt i Kontakt med de første. Og nu svarer Sort med at besætte et Vinkelstelt, som vilde være meget nyttigt for Hvid. Hvor skal nu Hvid sætte? Der er forskellige gode Muligheder.

Saa dan er dette Spil nu begyndt. Nu kan enhver fortsætte. Det er altsaa Hvids Tur. Man skal ikke være udspekuleret fra Begyndelsen. Der er ingen bedre Vej til at lære Spillet end at spille det. Det er nyttigt at se skitserne offentlig

1942 POLYGON [HEX]

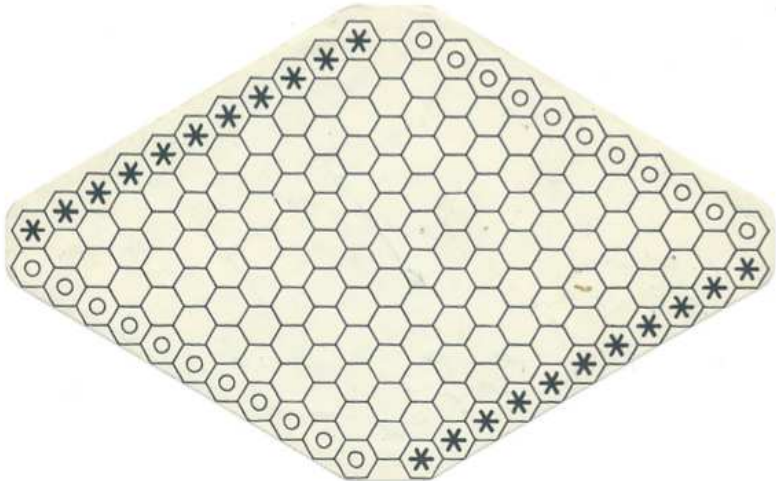
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1942 POLYGON [HEX]



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1942 POLYGON [HEX]



1942 POLYGON [HEX]



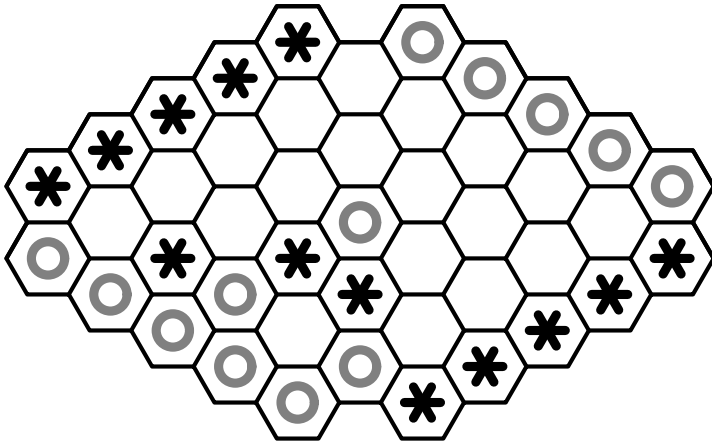
1942 POLYGON [HEX]



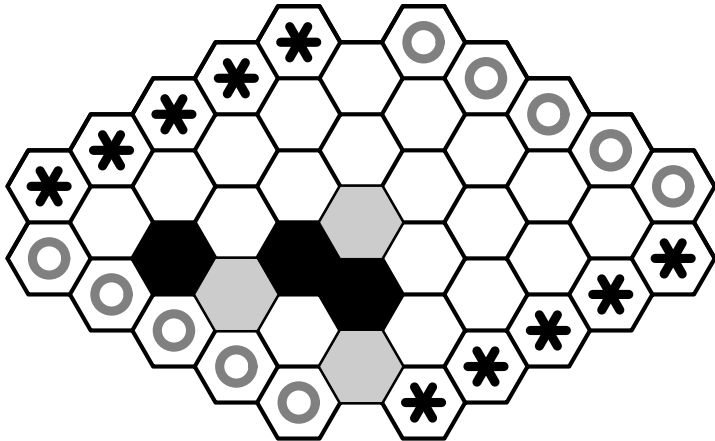
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PUZZLE 16 WHITE TO PLAY

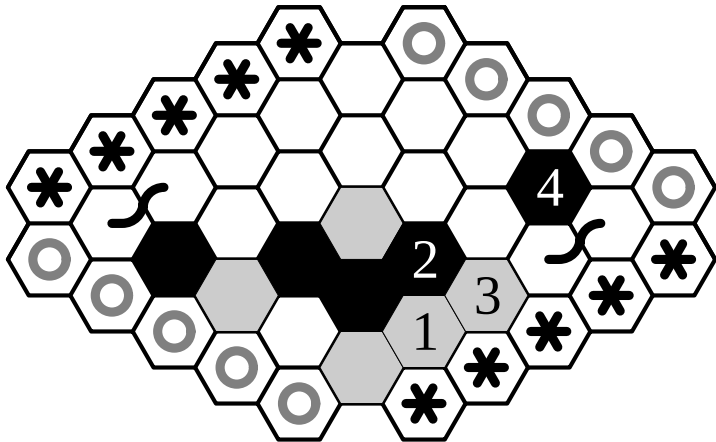
PUZZLE 16 WHITE TO PLAY



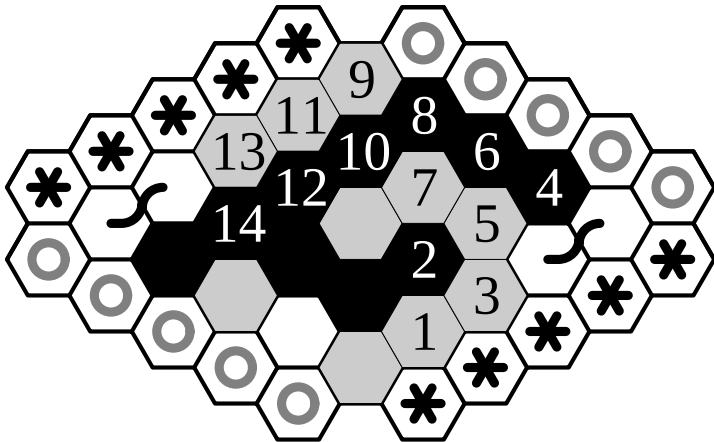
PUZZLE 16 WHITE TO PLAY



PUZZLE 16 WHITE TO PLAY



PUZZLE 16 WHITE TO PLAY



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WHO POSED THE PUZZLES?

WHO POSED THE PUZZLES?

- *I am by no means an excellent Polygon player and I do not feel obliged to be so merely because I happen to be the inventor of the game.*

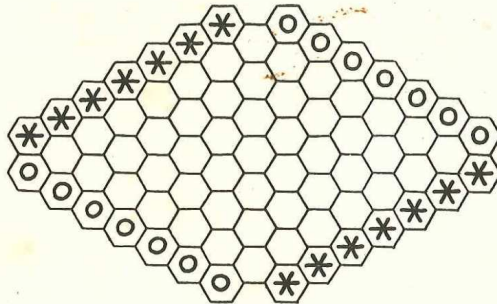
Piet Hein

- puzzle drafts ?

WHO POSED THE PUZZLES?

POLYGON — SPILLEBRÆT

7 × 7



WHO POSED THE PUZZLES?

SPILLETS ENESTE REGEL

POLYGON spilles af to Spillere: Hvid og Sort, som har Mærkerne henholdsvis en Cirkel og en Stjerne. Hver Spiller har to modstaaende Fronter, afmærkede i Felterne. — Man skiftes til at sætte sit Mærke i et hvilket som helst tomt Felt — og fortsætter indtil den ene har vundet. — Det gælder for hver Spiller om at besætte Felter paa saadan en Maade, at de danner en sammenhængende omend nok saa snoet Forbindelse mellem vedkommende Spillers to Fronter.

EFTERLYSNING

For at POLYGON-Spillet kan lanceres for en videre Kreds — hvilket er en Forudsætning for, at der fortsat kan fremstilles Spillediagrammer — behøves et ret stort Opgave-Materiale.

Jeg opfordrer derfor alle, som har Interesse for Spillet, til at ha Opmærksomheden henvendt paa Ideer til POLYGON-Opgaver. Saadan Ideer opstaar ofte under Spillet. Der er Brug for Opgaver paa Spillebrætter af alle Størrelserne fra 5×5 Felter indtil den normale Spillestørrelse 11×11 — især paa Brætter af de smaa Antal Felter. POLYGON-Opgaver med Løsninger bedes indsendt til mig. POLYGON-Spillebrætter tilsendes Opgaveindsendere gratis, saa vidt Oplaget rækker.

Blandt de bedste af de hver Uge indtil videre indkomne Opgaver udtrækkes én, som belønnes med en *Præmie* — i Ugen indtil Lørdag d. 12. December 1942: en Julegaas.

En POLYGON-Opgave bestaar i et POLYGON-Spillebræt, paa hvilket nogen af Felterne er besat med Hvids og Sorts Mærker. Denne Afmærkning behøver ikke at kunne være fremkommet under et normalt Spil, specielt kan der udmærket være mange flere af den ene Spillers Mærker end af den andens. Til en Opgave hører 3 Oplysninger: 1. *hvilken Spiller, der skal begynde*, og 2. *hvilken* (ikke nødvendigvis altid den samme) *Spiller, der kan vinde*, og 3. (helst) i *hvor mange Træk*, hvis begge spiller stærkest muligt. I Overensstemmelse med den Maade, hvorpaa en Løsning angives (se nedenfor), regnes Antallet at Træk efter begge Spilleres samlede Antal.

En Opgave maa være nogenlunde éntydig, dvs. naar begge spiller bedst muligt, skal der praktisk talt kun være én Fremgangsmaade, især skal det første Træk ligge fast. En Opgave bør ikke indeholde overflødige Mærker.

En Opgaves Løsning noteres saaledes paa Spillebrættet: de Mærker, som hører med til den stillede Opgave, sættes paa sædvanlig Maade; de Mærker, som hører til Løsningen, erstattes i kronologisk Orden med Tallene 1, 2, 3 osv., saaledes at den som trækker først altsaa har de ulige Tal, den anden de lige. For Overskueligheden kan Sorts Tal gøres særlig fede. Naar Udfyldningen af de sidste Felter i en Løsning er rent rutinemæssig, er det en Fordel, at den ikke udføres. Naar (hvad der ofte sker) to Nabo-Felter, som er blevet besat af henholdsvis Hvid og Sort, ligesaa godt kunde være blevet besat at dem omvendt, kan det angives ved, at de to Felter forbindes med dette Tegn: ~ .

Venlig Hilsen.

WHO POSED THE PUZZLES?

SPILLET ENEST

POLYGON spilles af to Spillere: Hvid og Sort, som har
Hver Spiller har to modstaaende Fronter, afmærkede i
i et hvilket som helst tomt Felt — og fortsætter indtil
Spiller om at besætte Felter paa saadan en Maade, at
saa snoet Forbindelse mellem vedkommende Spillers to

EFTERLYSN

For at POLYGON-Spillet kan lanceres for en videre K
fortsat kan fremstilles Spillediagrammer — behøves et

Jeg opfordrer derfor alle, som har Interesse for Spille
Ideer til POLYGON-Opgaver. Saadan Ideer opstaar ofte
Spillebrætter af alle Størrelserne fra 5×5 Felter indtil
Brætter af de smaa Antal Felter. POLYGON-Opgaver med
Spillebrætter tilsendes Opgaveindsendere gratis, saa vid

Blandt de bedste af de hver Uge indtil videre indkom
en *Præmie* — i Ugen indtil Lørdag d. 12. December 19

WHO POSED THE PUZZLES?

THE GAME'S ONLY RULE. Polygon is played by two players: White and Black, whose symbols are a circle and a star. Each player has two opposing fronts, marked with their symbols. Each player in turn marks any empty field with their symbol, The game continues until one player wins, A player wins by joining their two sides with a path, which can twist and turn.

WHO POSED THE PUZZLES?

MISSING PERSON'S REPORT. In order to bring Polygon to a larger audience — which we all want — we need a large supply of Polygon puzzles. So I urge anyone with an interest in the game to construct such puzzles. ... Send POLYGON puzzles, with commentary, to me. Among the best of those puzzles submitted this week, one will be picked at random on December 12, 1942 to receive a special prize: a Christmas goose.

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IF NOT HEIN ... ?

IF NOT HEIN ... ?

Politiken Jan 26 1943 Mon Feb 1 8pm: Polygon salon. In response to the spreading Polygon-mania, the game's creator Piet Hein, with expert player Jens Lindhard, will demonstrate the game's strategy and finer points. They will use a giant game board to play members of the audience and answer questions. Tickets at "Polygon" ticket office, Town Hall Square, today 9am to 6pm. Free subscriber admission with Coupon 1 from subscription receipt. Cloakroom payment 25 cents.

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... MAYBE THE SALON EXPERT ?

... MAYBE THE SALON EXPERT ?

- at salon, played simultaneous Polygon games
- chess expert, studied with Aage Bohr under Niels Bohr, physics prof Aarhus U (while Toft student)
- never married, post-retirement to office each day
- any papers left?
- 2017 Nov Toft goes to Aarhus

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JENS LINDHARD, POLYGON PUZZLIST

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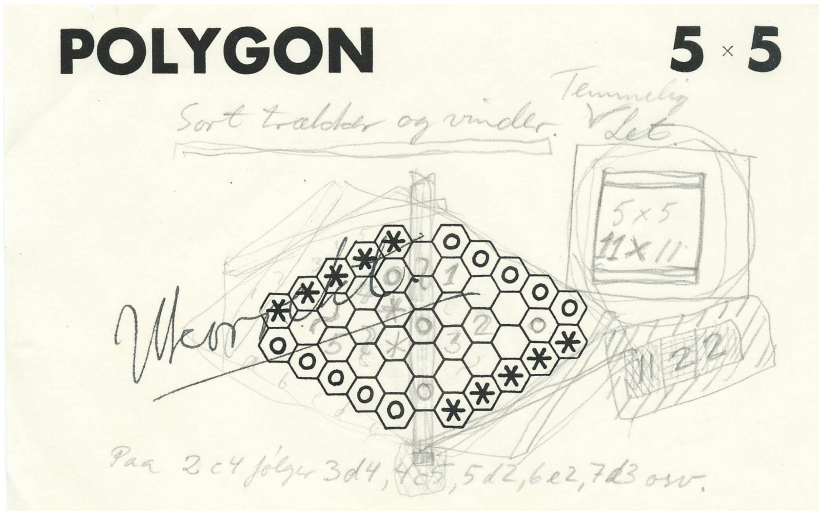
JENS LINDHARD, POLYGON PUZZLIST



JENS LINDHARD, POLYGON PUZZLIST



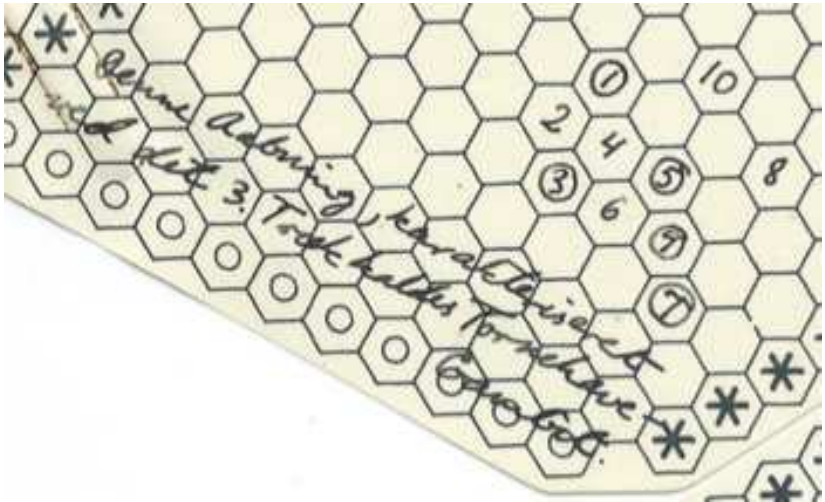
JENS LINDHARD, POLYGON PUZZLIST



JENS LINDHARD, POLYGON PUZZLIST

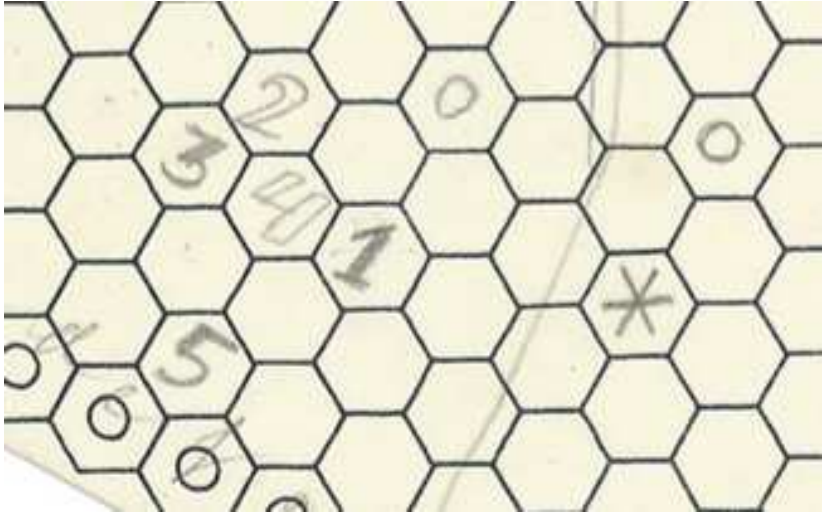


JENS LINDHARD, POLYGON PUZZLIST



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JENS LINDHARD, POLYGON PUZZLIST

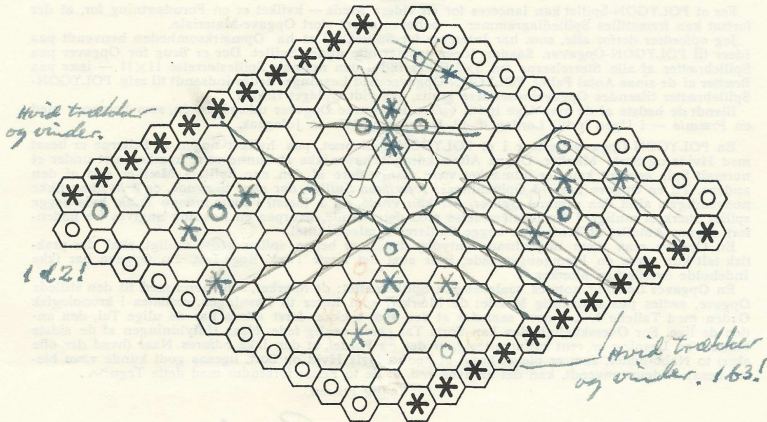


JENS LINDHARD, POLYGON PUZZLIST

POLYGON — SPILLEBRÆT

11 × 11

Sont trækker og vinder: 165, 212, 305.

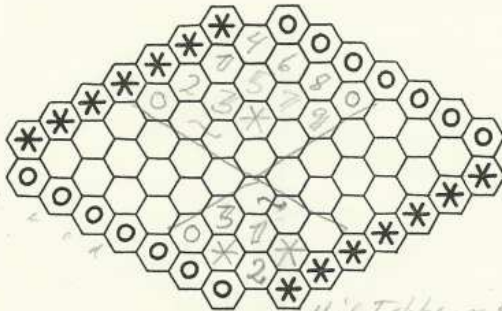


JENS LINDHARD, POLYGON PUZZLIST

POLYGON — SPILLEBRÆT

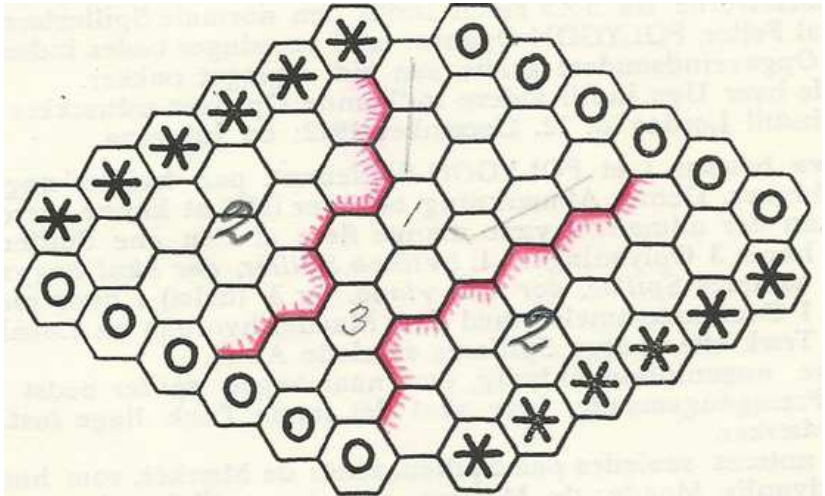
7 × 7

Heidi Takker: Kan Heidi vinde? ja!

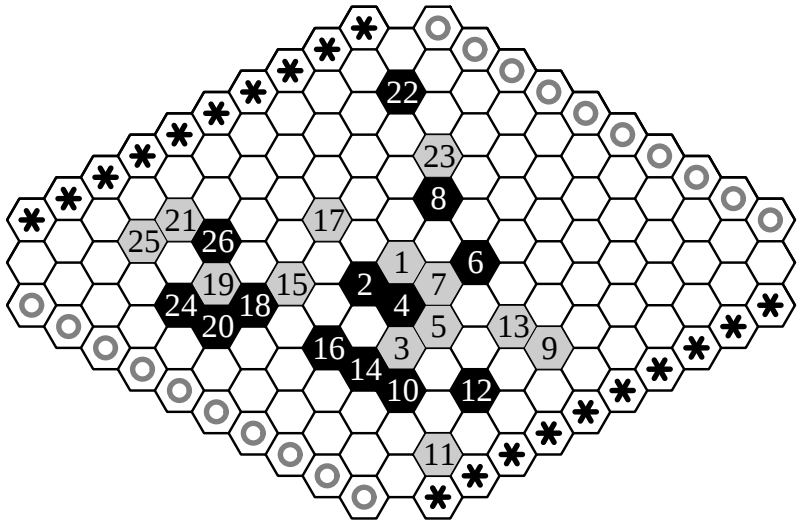


Heidi Takker, og vinder

JENS LINDHARD, POLYGON PUZZLIST



JENS LINDHARD, POLYGON PUZZLIST



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KAREN MARGRETHE THORBORG

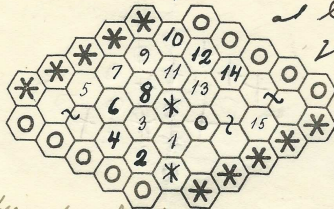
KAREN MARGRETHE THORBORG

POLYGON — SPILLEBRÆT

5 × 5

22. 1. 43.

Kære J.L.! Se det meget vord,
som benyt det. Det er
jo ellers meget nok
at have en lille
vennemæssig
Opgave!



F

Denne Opgave begynder lidt-og vinder, men først efter
15 Træk og under skadige Træk fra røde Side.
Jeg har prøvet adskillige (men ikke alle!) andre Må-
ligheder for hvids første Træk, men har kun fået
fundet et Modstykke fra røde Side.

KAREN MARGRETHE THORBORG

White starts and wins, but only after 15 moves, and under constant threat from Black. I tried very many (but not all!) other possibilities for White's first move, but in each case I found a winning Black reply.

Kind regards, univ. stud. Karen Thorborg
Horserødvej 20 Copenhagen 0

KAREN MARGRETHE THORBORG

22.1.43. Dear JL! If worthwhile, use this puzzle.
But of course it is easy enough to construct an
unanalyzed puzzle!

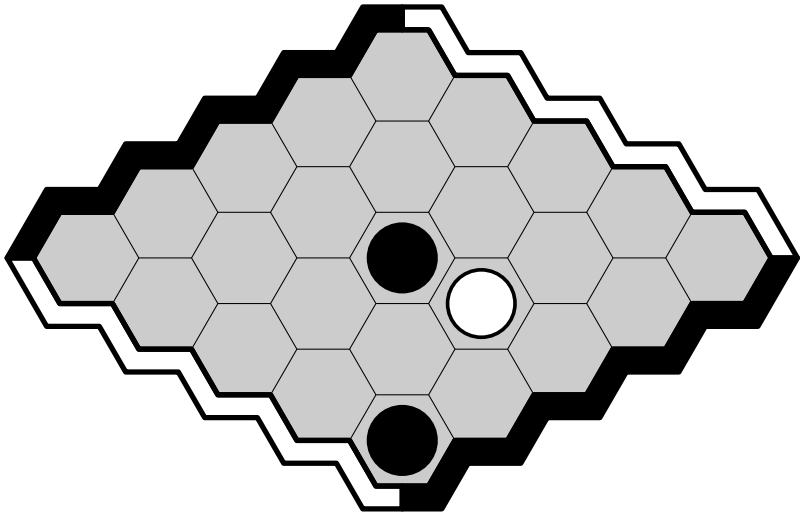
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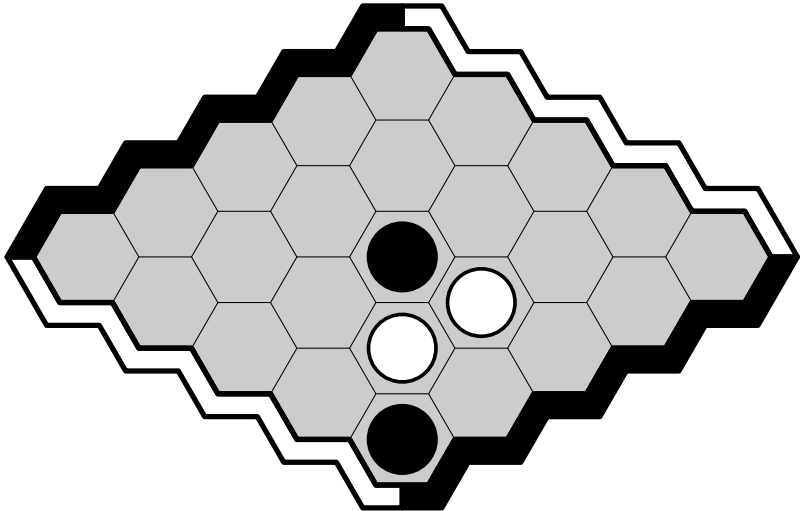
THORBORG'S PUZZLE

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THORBORG'S PUZZLE



THORBORG'S PUZZLE: DOES WHITE WIN?



WAR

- 29 August 1943

Germany declares martial law in Denmark

GAME THEORY

- chess: can 1st player always win or draw?

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JOHN F NASH DAVID GALE

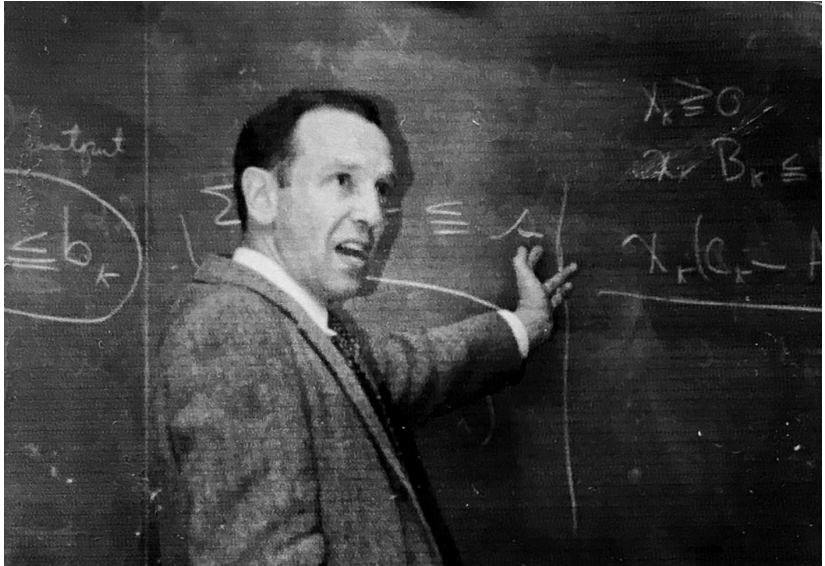
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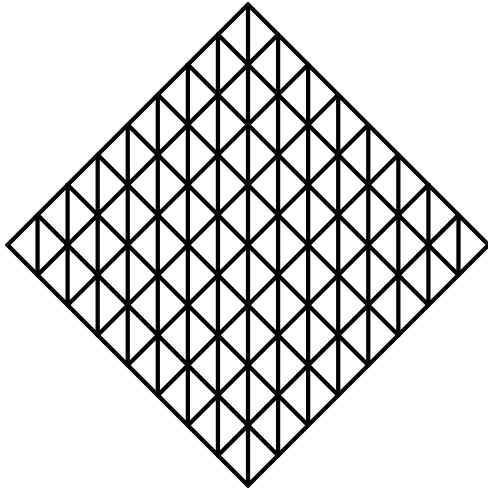
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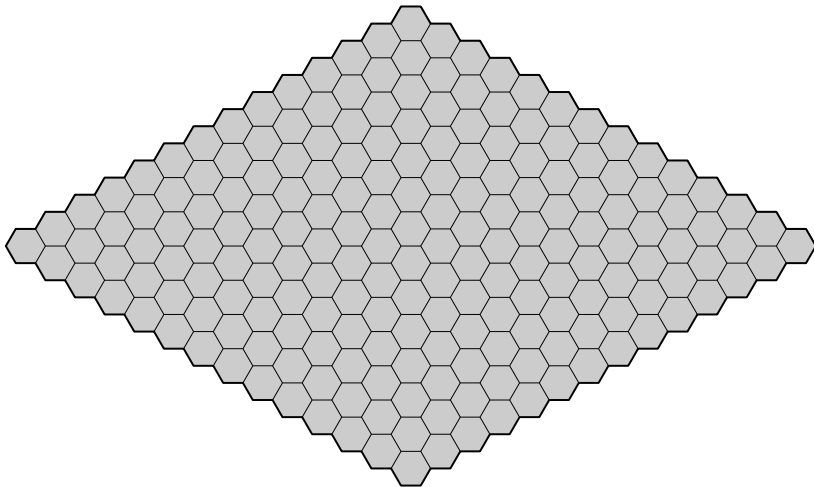
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JOHN F NASH DAVID GALE



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JOHN F NASH DAVID GALE



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FIRST PLAYER WINS

FIRST PLAYER WINS

- ① ~~When the board is~~ ~~field~~ filled one ~~on~~ the other of the players will have connected but not both.
- ② ~~One~~ Either the first player or the second will have a winning strategy.
- ③ Suppose the second player could force a win.
- ④ Consider a defensive strategy

FIRST PLAYER WINS

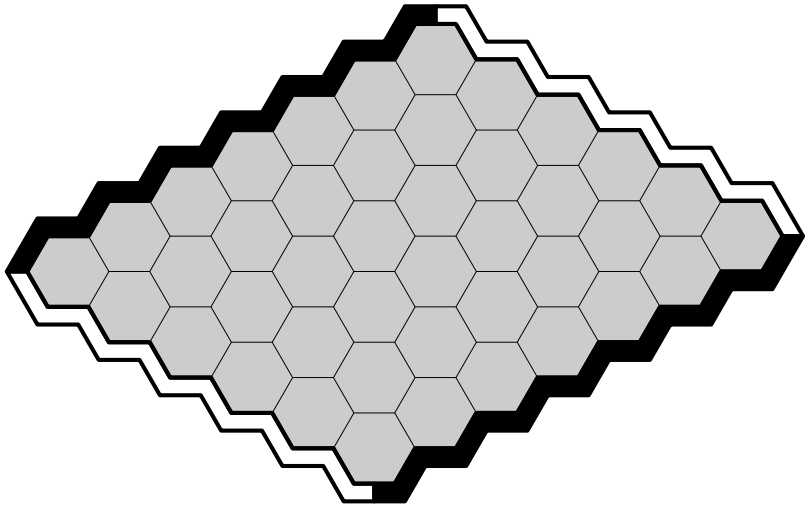
the board is always an asset,
never a handicap in connecting,
at the end of the game
first player will be ^{assumed} better off
~~using~~ the adapted second player
strategy than he would have
been if simply playing as
second player. So he will
win.

⑥ Since this contradicts the
hypothesis ③ that second
player can win it follows that

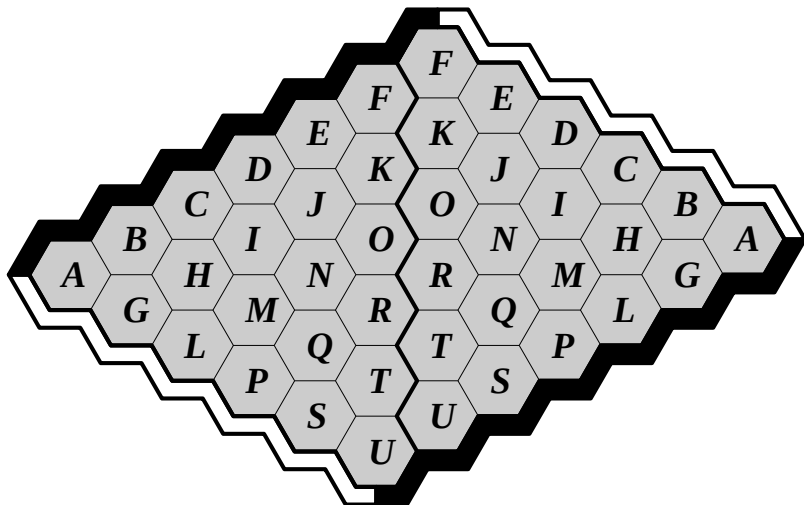
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SHORTER DISTANCE WINS

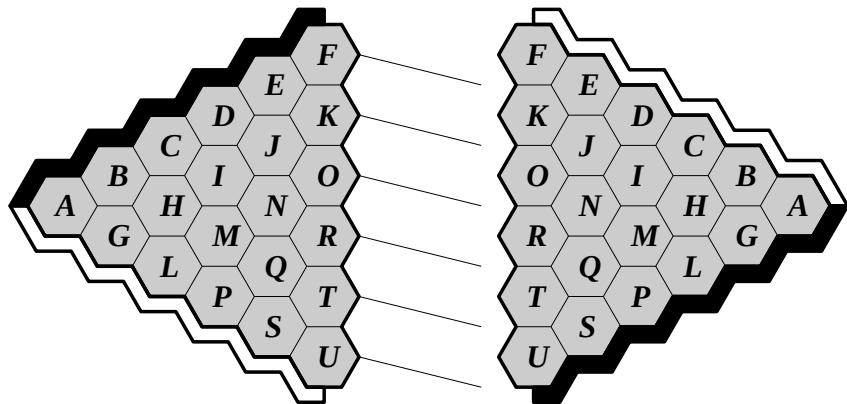
SHORTER DISTANCE WINS



SHORTER DISTANCE WINS



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1942-43 DENMARK: POLYGON
1949-50 PRINCETON-NYC: HEX
1957 SCIENTIFIC AMERICAN

CLAUDE SHANNON

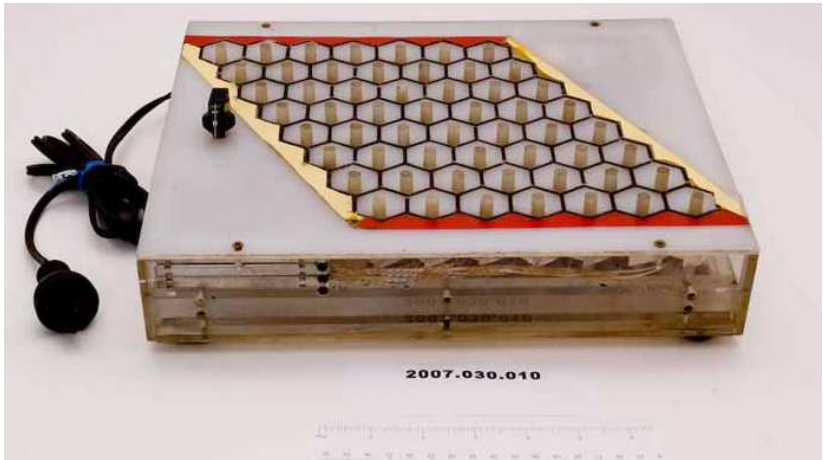
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MARTIN GARDNER

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MARTIN GARDNER



MARTIN GARDNER

- Scientific American
- 1956 Dec hexaflexagons
- 1957 monthly Mathematical Games
- 1957 July *Concerning the game of Hex, which
may be played on the tiles of the bathroom floor*
- find 1pw ? find win/loss for each 1st move ?
- 4x4 easy 5x5 harder 6x6 ?

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THANK YOU



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A BEAUTIFUL GAME FROM THE WAR

