

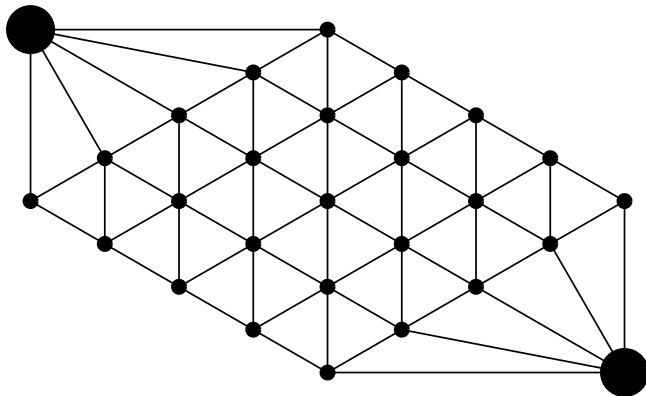
Hex: centermost opening wins?

ryan hayward@ualberta.ca

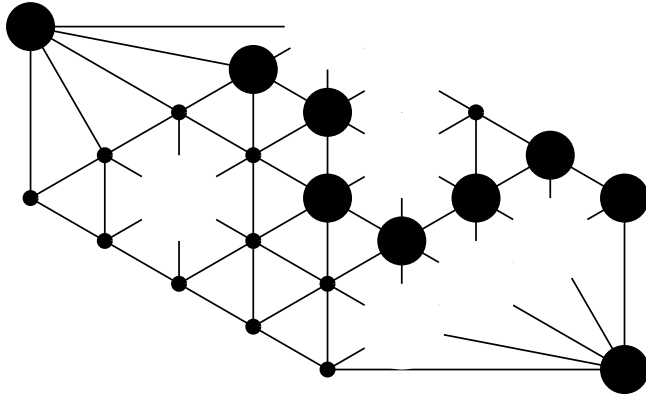
NCM29 2023 Jul 3-7 Aalborg DK

a game on a graph

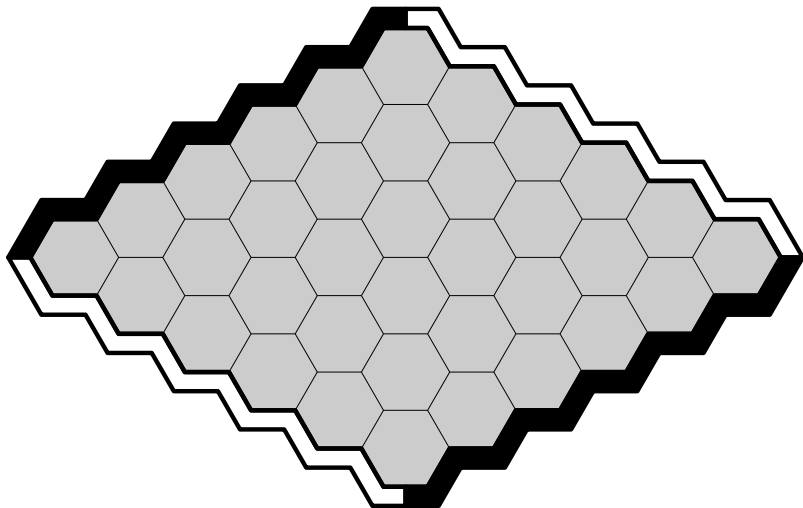
a game on a graph



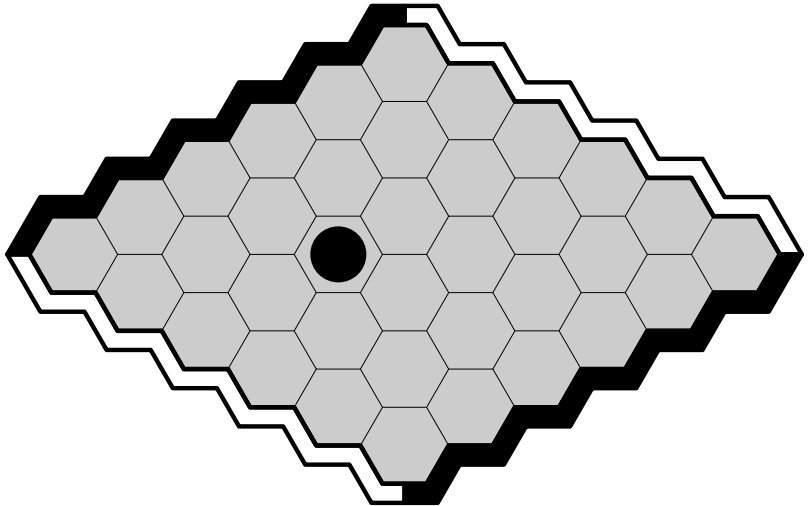
a game on a graph



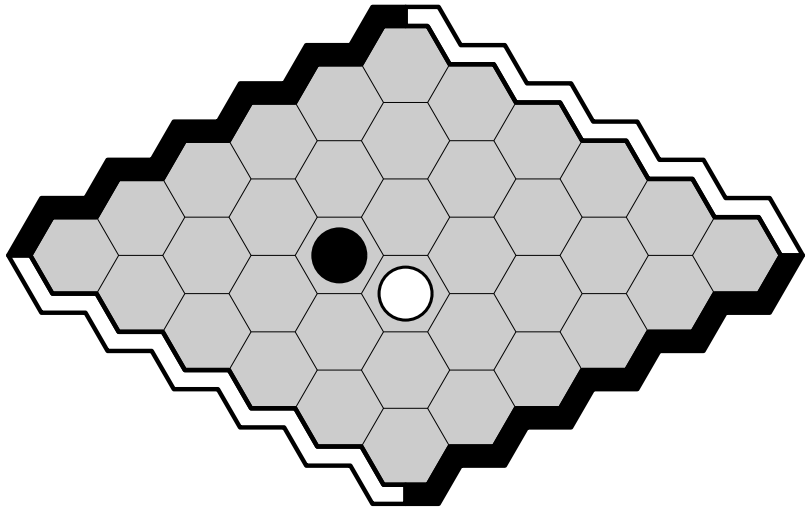
Hex (polygon) Hein-Lindhard 1942



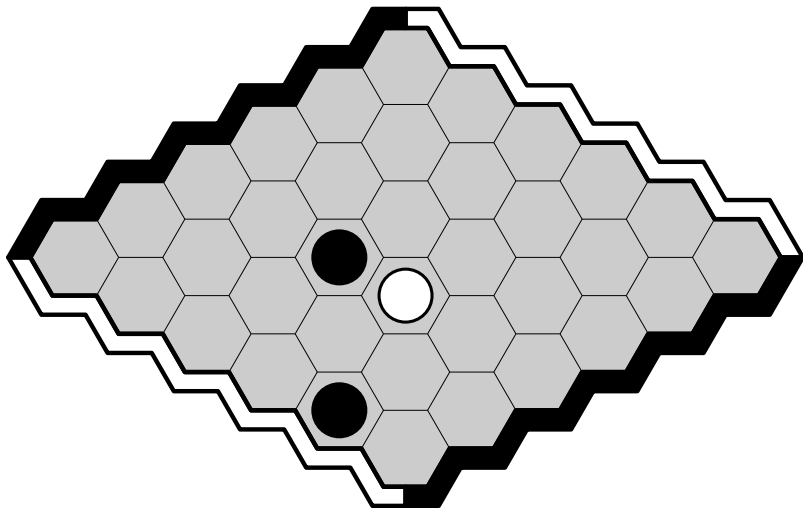
Hex (polygon) Hein-Lindhard 1942



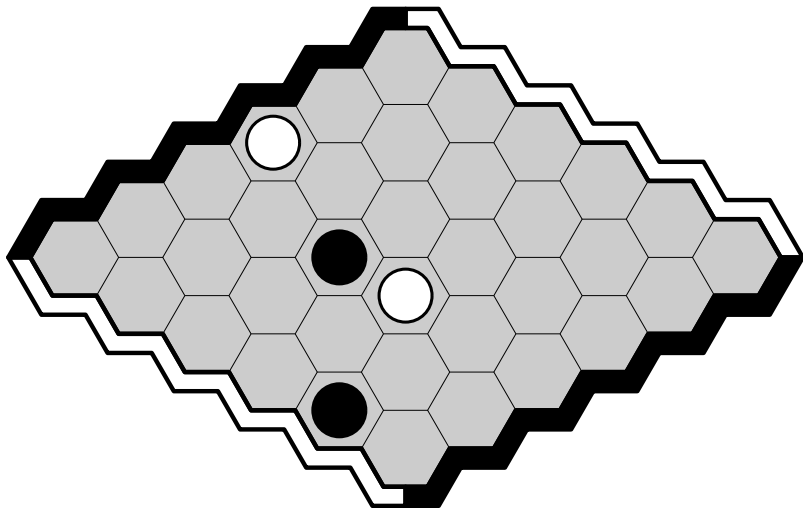
Hex (polygon) Hein-Lindhard 1942



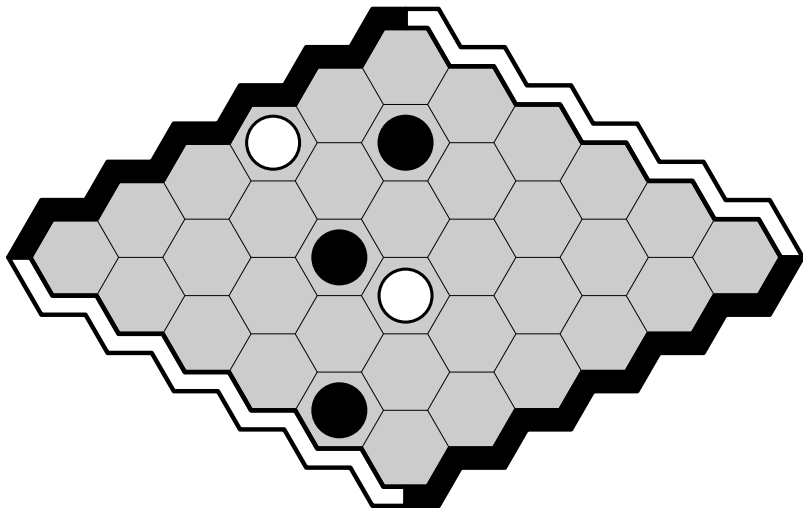
Hex (polygon) Hein-Lindhard 1942



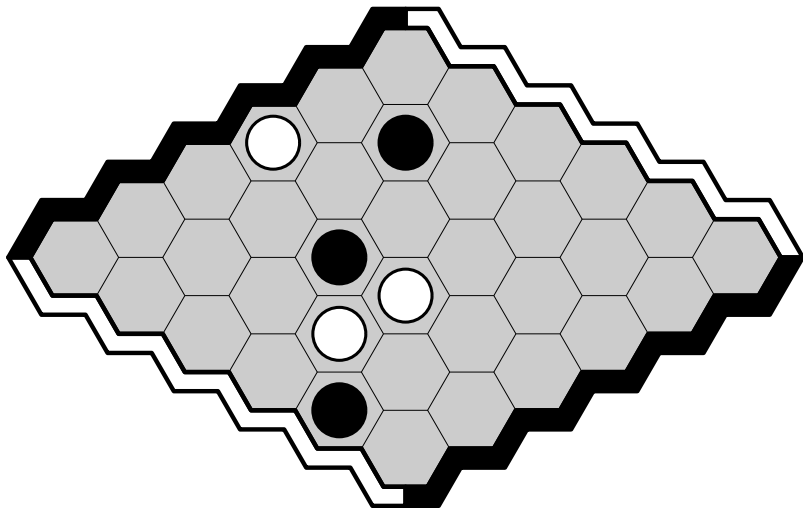
Hex (polygon) Hein-Lindhard 1942



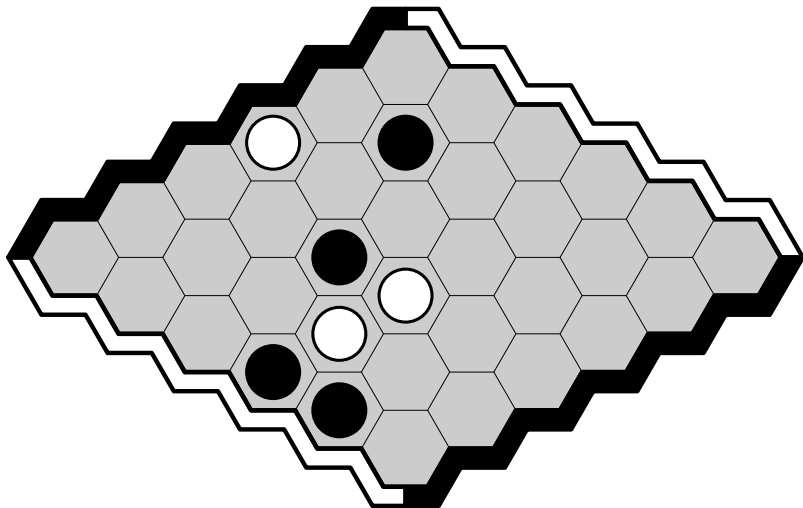
Hex (polygon) Hein-Lindhard 1942



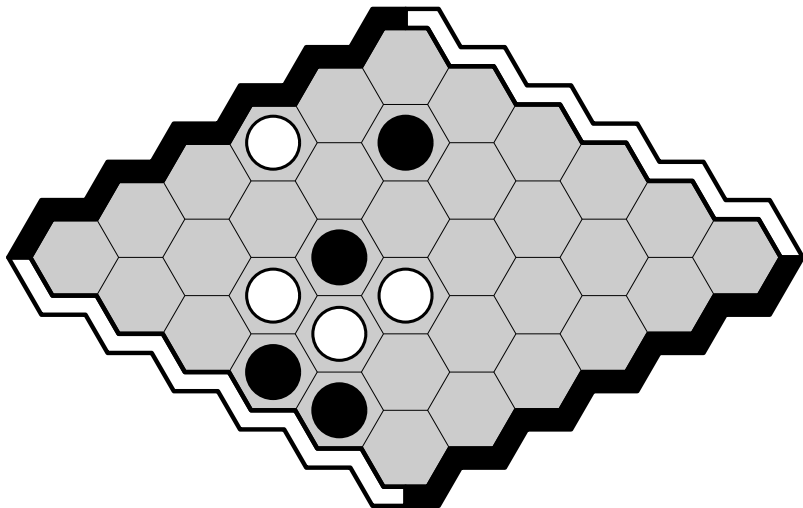
Hex (polygon) Hein-Lindhard 1942



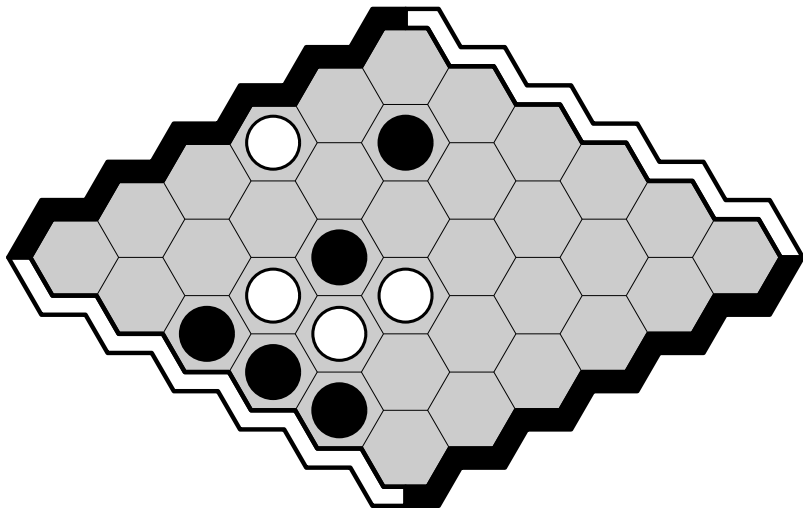
Hex (polygon) Hein-Lindhard 1942



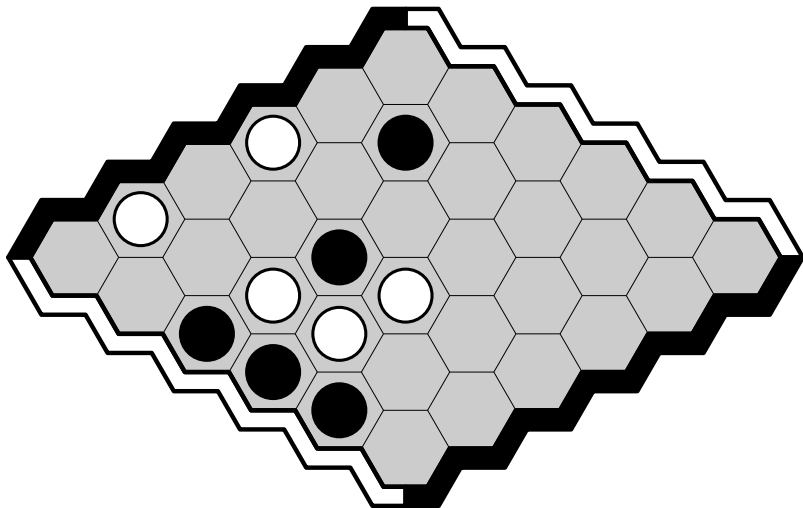
Hex (polygon) Hein-Lindhard 1942



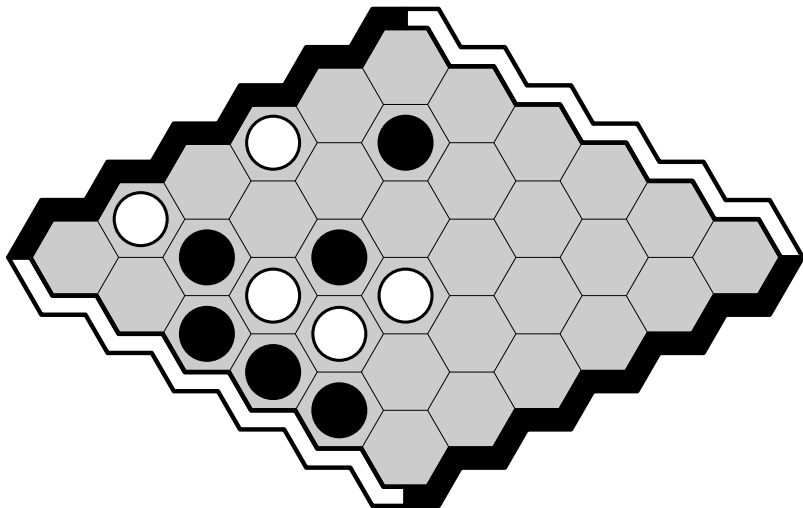
Hex (polygon) Hein-Lindhard 1942



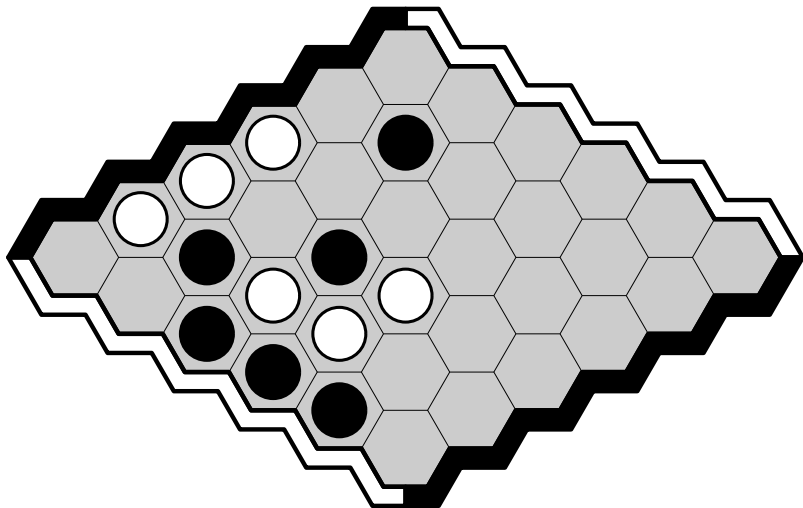
Hex (polygon) Hein-Lindhard 1942



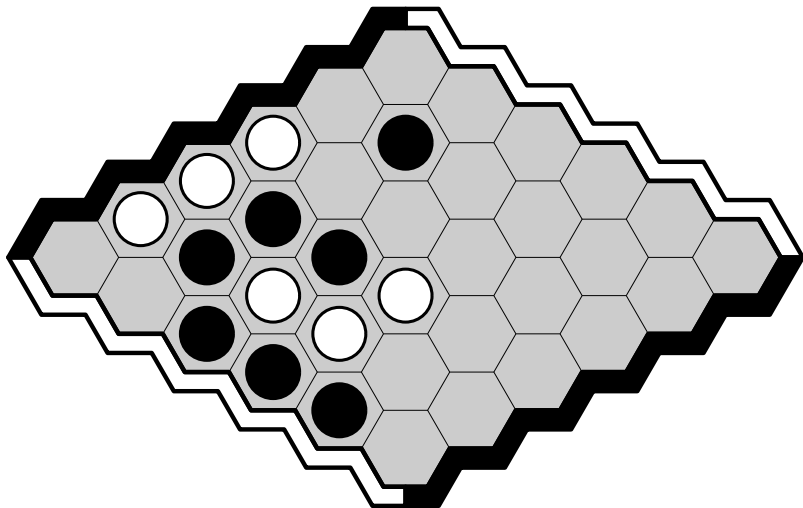
Hex (polygon) Hein-Lindhard 1942



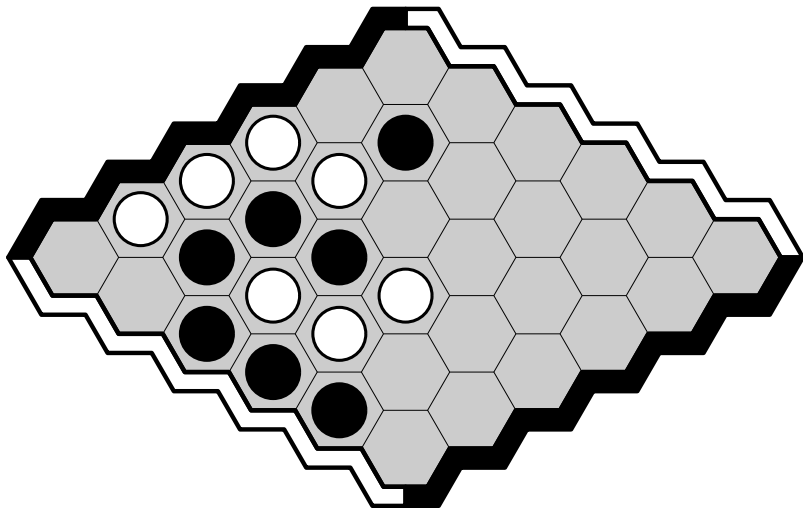
Hex (polygon) Hein-Lindhard 1942



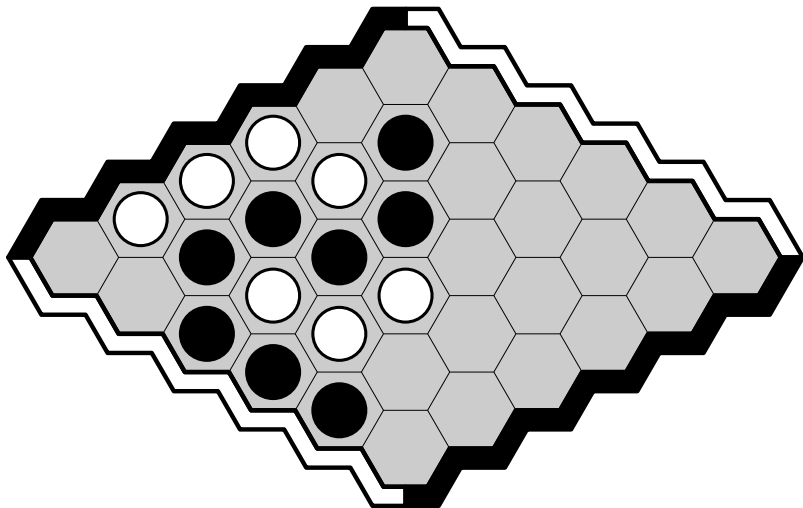
Hex (polygon) Hein-Lindhard 1942



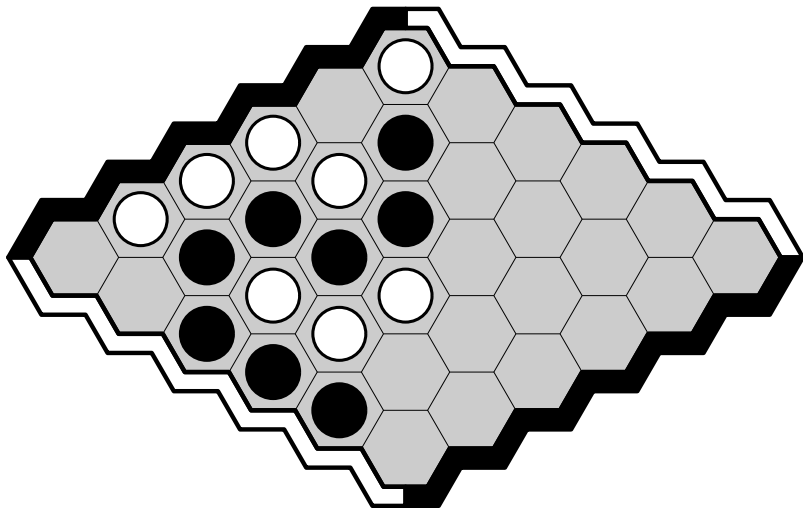
Hex (polygon) Hein-Lindhard 1942



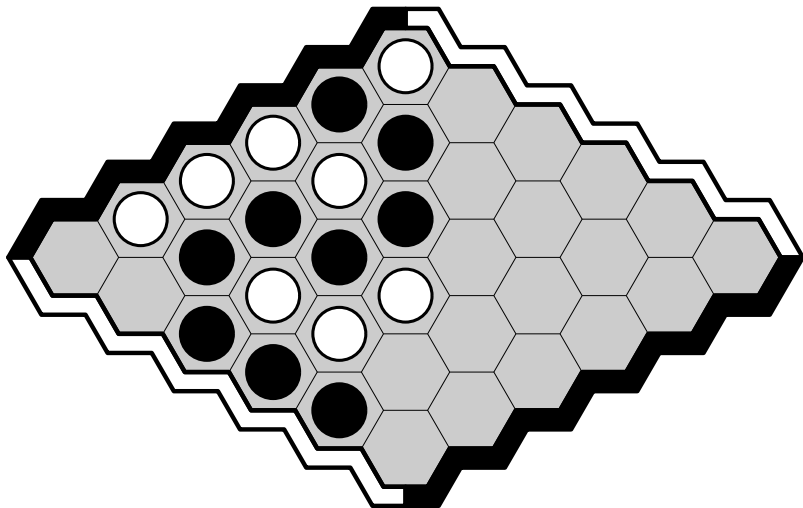
Hex (polygon) Hein-Lindhard 1942



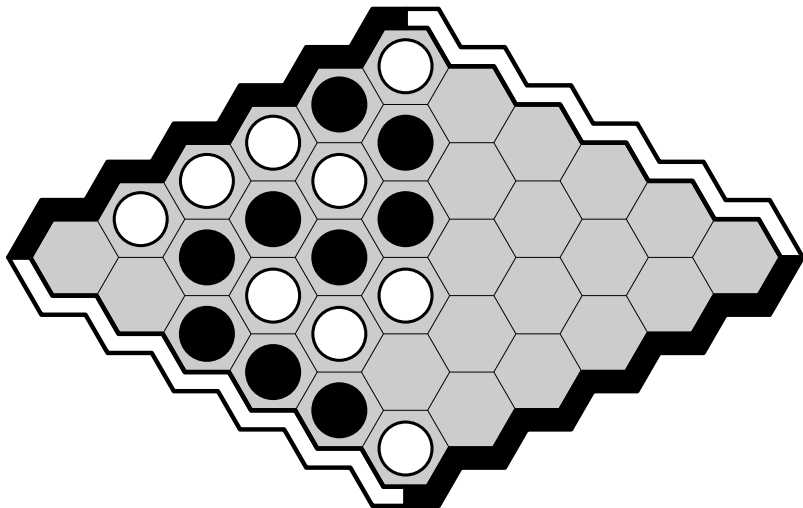
Hex (polygon) Hein-Lindhard 1942



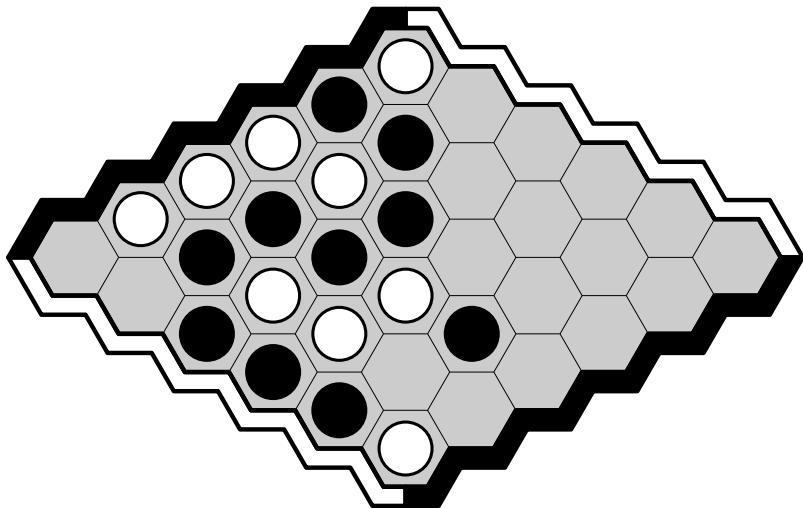
Hex (polygon) Hein-Lindhard 1942



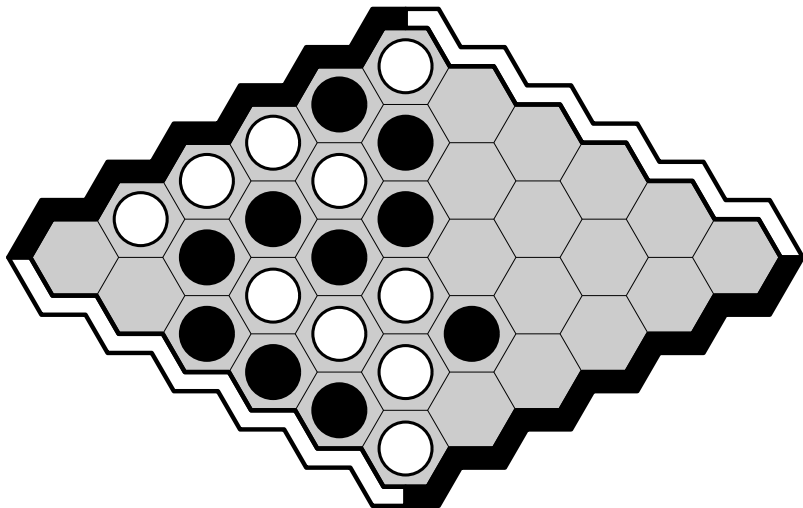
Hex (polygon) Hein-Lindhard 1942



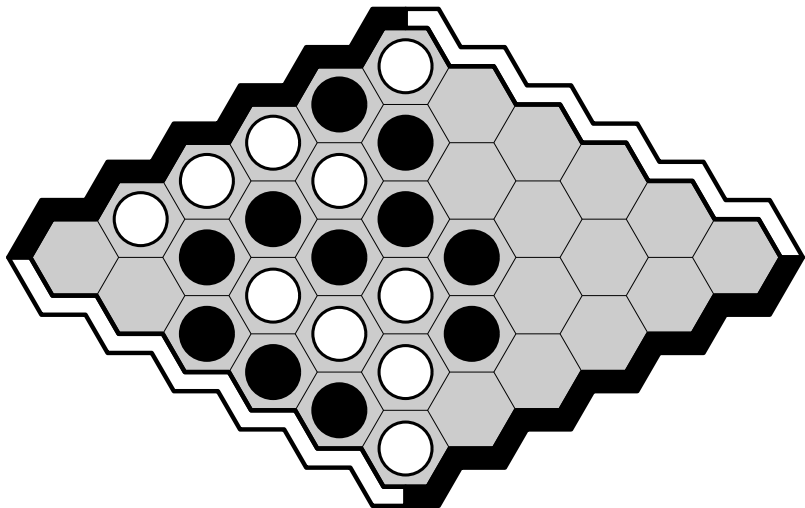
Hex (polygon) Hein-Lindhard 1942



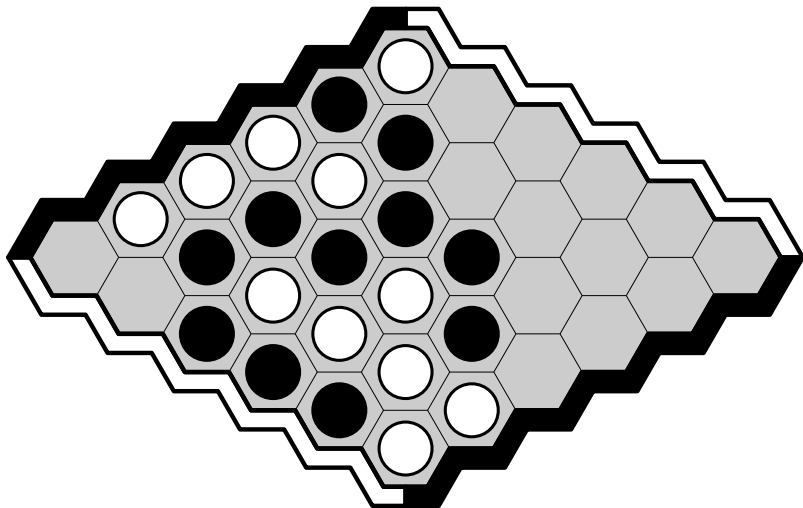
Hex (polygon) Hein-Lindhard 1942



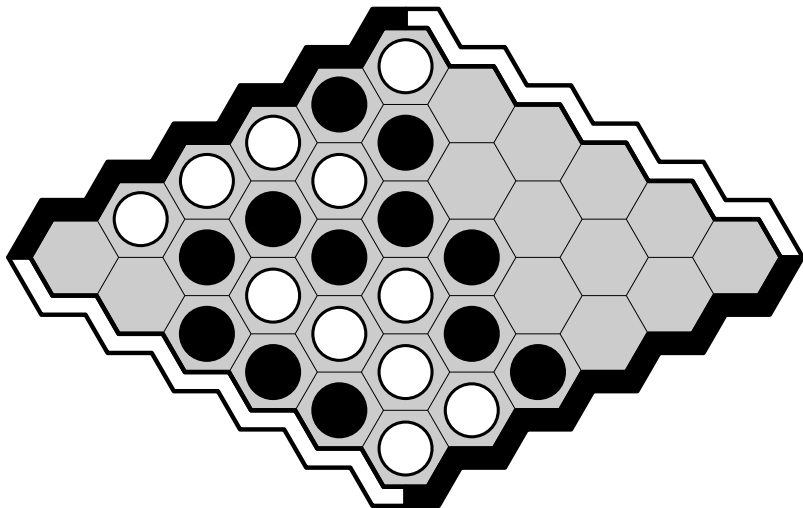
Hex (polygon) Hein-Lindhard 1942



Hex (polygon) Hein-Lindhard 1942



Hex (polygon) Hein-Lindhard 1942



HEX

The Full Story

Ryan B. Hayward
with Bjarne Toft



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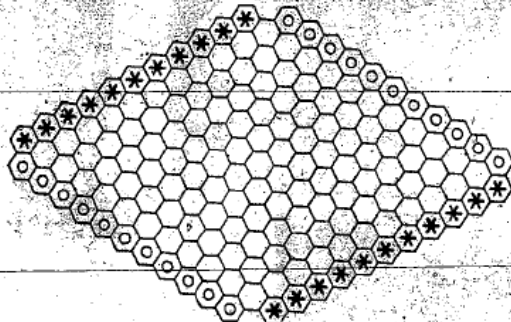
TIKEN

26. December 1942

Vil De lære Polygon?

Piet Hein har konstrueret et Spil, der med lige stor Glæde kan dyrkes af Skakeksperten og den, der blot kan holde en Blyant

„Politiken“ udskriver i Dag en Præmieopgave, der vil volde Hovedbrud for Begyndere



Saaledes ser Polygon-Spilbrættet ud.

ten kan afbryde Forbindelsen ved at besætte det mellemste gode Felt dens Angrebsside bærer paa Blykorpens Placering i den vidtre Omegn. I det hele taget vaser det sig snart nødvendigt at tage en stærk Del af Spillebrættet med i Betragtning.

En anden Erfaring, som kommer senere, men som man kan lide Spilleets Begyndelse ved at se, er, at det betaler sig at begynde i hvert Fald nogenlunde paa Midten. En rimelig, men paa ingen Maade nødvendig Aabning af Spillet er denne:

Paa Spillebrættet i Midten er Hvid begyndt i Midterfeltet. Sort har Sort sat i Kontaktfeltet til det ned imod Midten af Hvids Front og derved gjort de nyttige Felter, som staar i Vinkelstilling til Midterfeltet, usikre. Hvid har saa vaegt et Felt i Kontakt med sin første. Og nu svarer Sort med at besætte et Vinkelfelt, som vilde være meget nyttigt for Hvid. Hvor skal nu Hvid sætte? Der er forskellige gode Muligheder.

Saa dan er dette Spil nu begyndt. Nu kan enhver fortsætte. Der er ogsaa Hvids Tur. Man skal ikke være udspekulerende fra Begyndelsen. Der er ingen bedre Vej til at lære Spillet end at spille det.

Det er nyttigt at se skiftevis offensivt og defensivt paa Situationen, d. v. s. skiftevis paa sine egne og Modspillernes Muligheder, for at føre en Forbindelse igen.

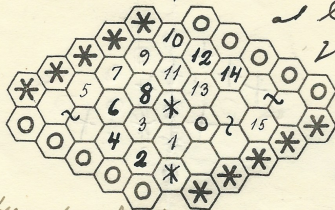
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POLYGON – SPILLEBRÆT

5 × 5

22. 1. 43.

Kære J.L.! En det meget vord,
som berøgt det. Det er
10 eller mere nok
at lave en 10
veemmenalyse
Opgave!



7

I denne Opgave begynder brud- og vinder, men først efter
15 Træk og under skellige Træk fra røde Side.
Jeg har prøvet adskillige (men ikke alle!) andre An-
ledigheder for bruds første Træk, men har kun fået
fundet et Modstrik fra røde Side.

Venlig Hilsen. skad. mag. Klaus Thorsborg

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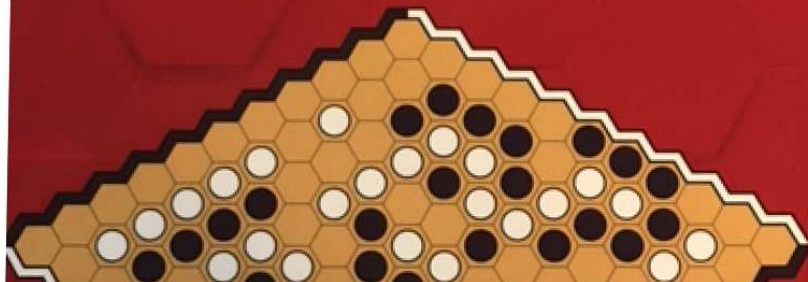
<https://www.routledge.com/hex-Inside-and-Out->

hex book two – intro, 2022 MAA

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Hex

A Playful Introduction



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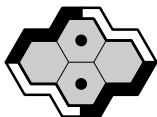
Hein-Lindhard 1942 Hex (polygon)

- draw not possible
- $n \times n$ board: exists 1st-player-wins strategy
- $n \times (n-1)$ board: simple closer-sides-wins strategy
- arbitrary positions Pspace-complete
- variants: reverse hex, dark hex, . . .

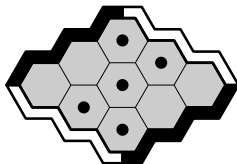
Hex: winning black-1st-moves



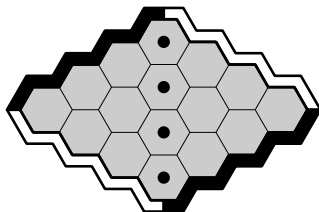
Hex: winning black-1st-moves



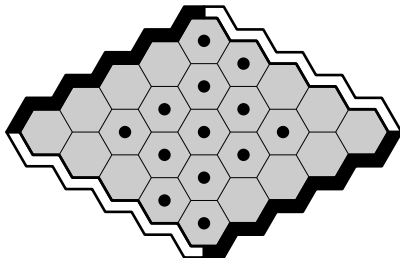
Hex: winning black-1st-moves



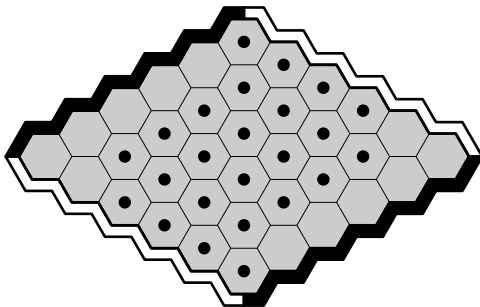
Hex: winning black-1st-moves



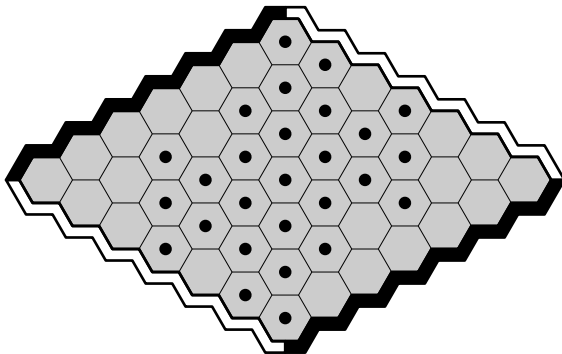
Hex: winning black-1st-moves



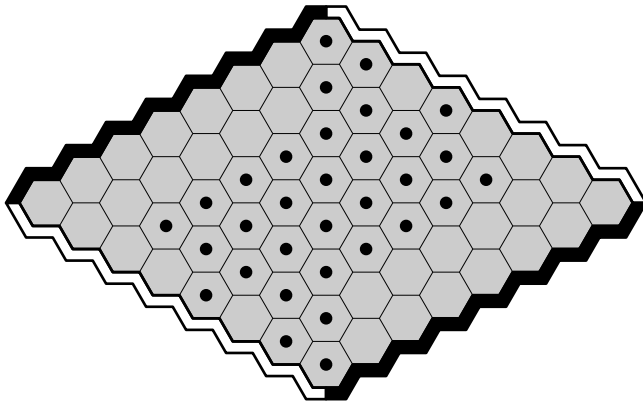
winning 1st-moves 1995 Enderton



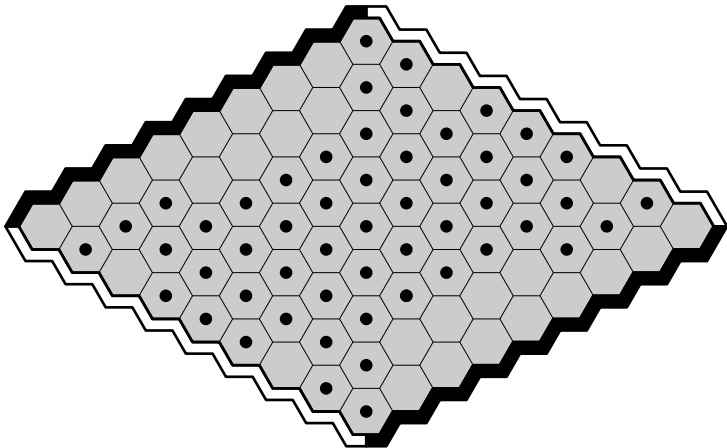
winning 1st-moves 2004 HBJPvR



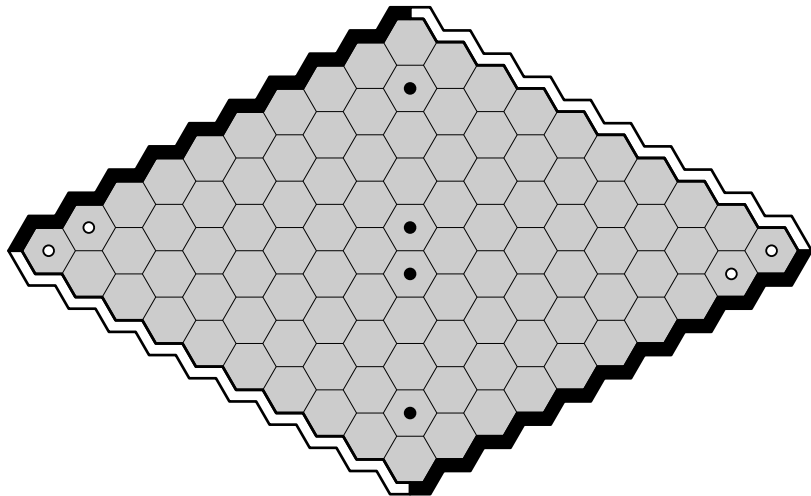
winning 1st-moves 2009 HAH



winning 1st-moves 2013 AHHP



winning 1st-moves 2014 PH



Hex conjectures: winning 1st moves

Hex conjectures: winning 1st moves

- short diagonal: centermost cell

Hex conjectures: winning 1st moves

- short diagonal: centermost cell
- short diagonal: each cell

Hex conjectures: winning 1st moves

- short diagonal: centermost cell
- short diagonal: each cell
- short diagonal: obtuse corner barely wins

strategy needs all empty cells

Hex conjectures: winning 1st moves

- short diagonal: centermost cell
- short diagonal: each cell
- short diagonal: obtuse corner barely wins

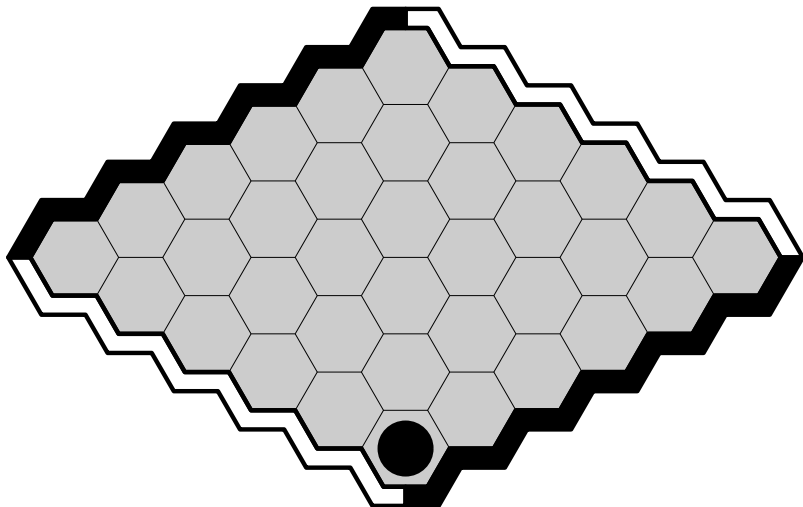
strategy needs all empty cells

- short diagonal: centermost cell strongly wins

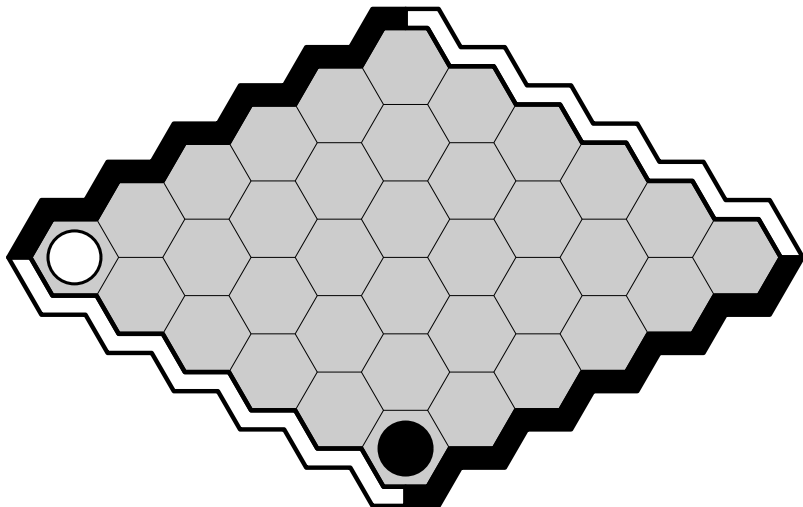
1st row: strategy needs only obtuse corner

conj.: obtuse corner barely wins

conj.: obtuse corner barely wins

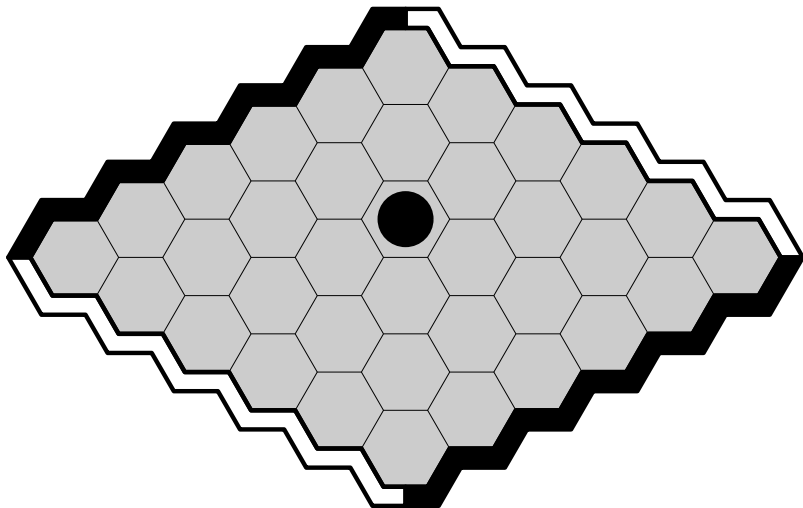


conj.: obtuse corner barely wins

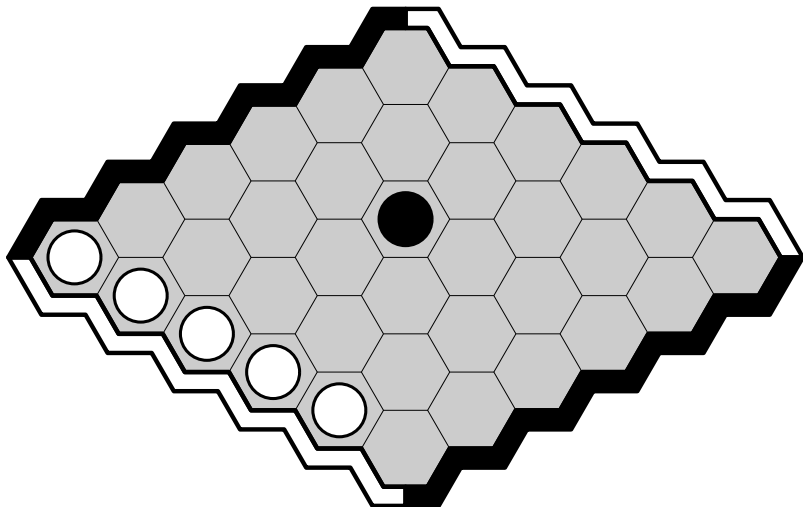


conj.: centermost cell strongly wins

conj.: centermost cell strongly wins

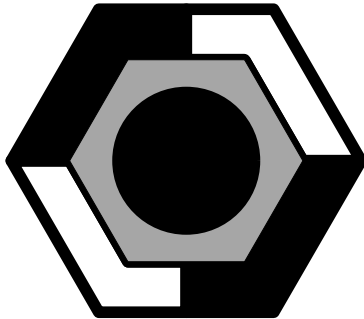


conj.: centermost cell strongly wins

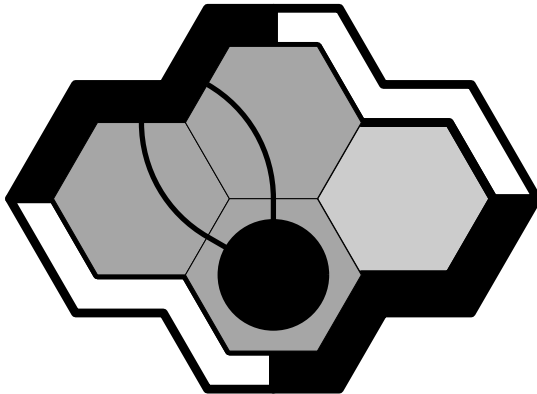


conj.: centermost cell wins

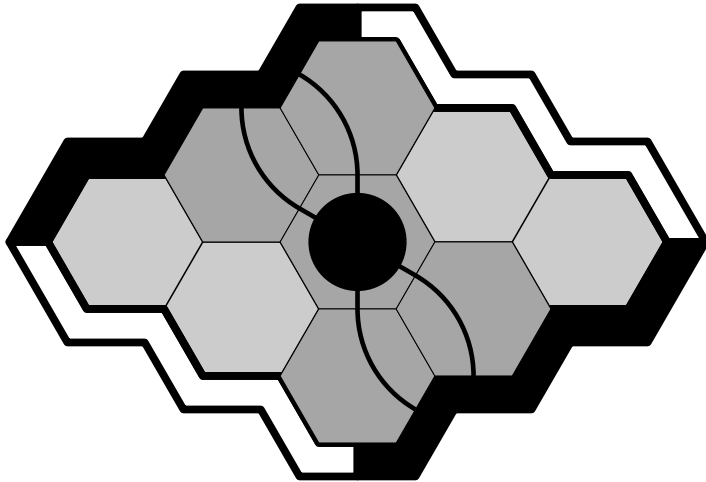
conj.: centermost cell wins



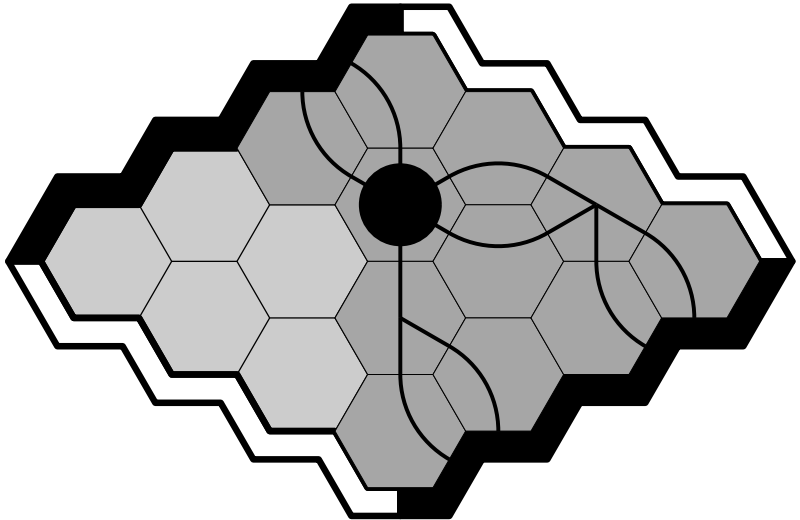
conj.: centermost cell wins



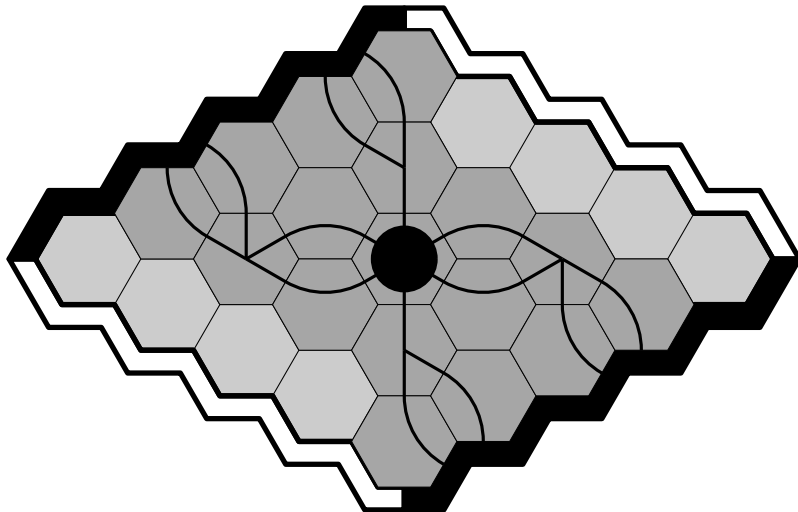
conj.: centermost cell wins



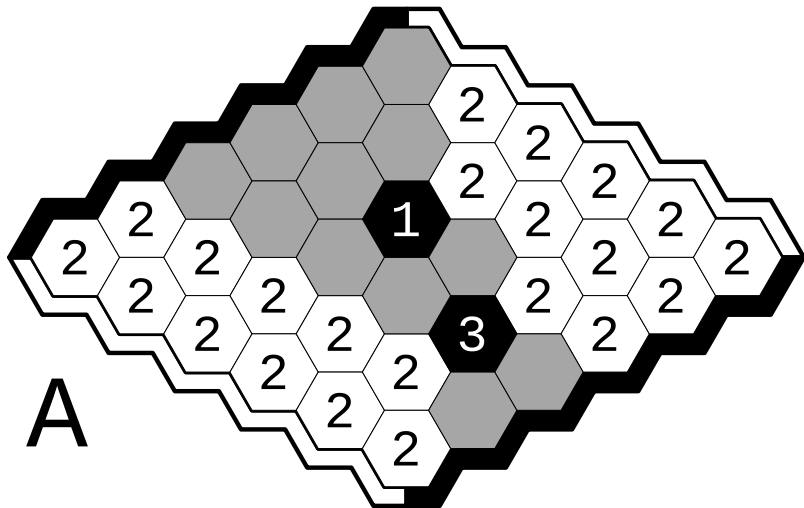
conj.: centermost cell wins

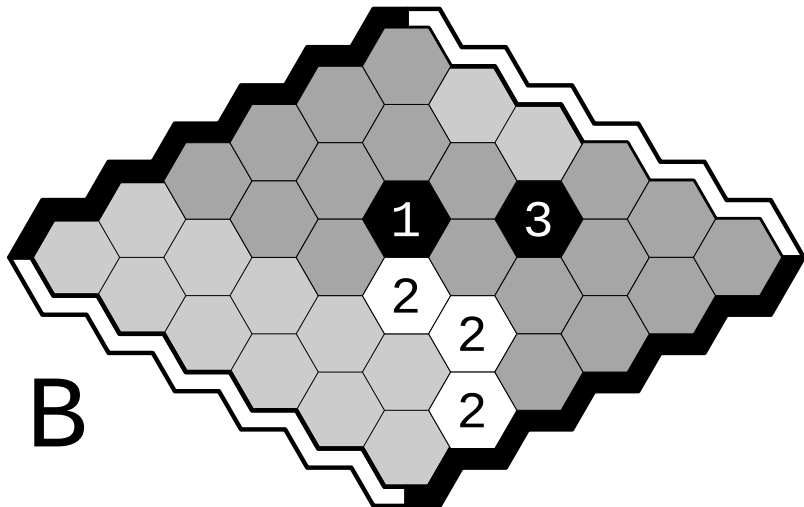


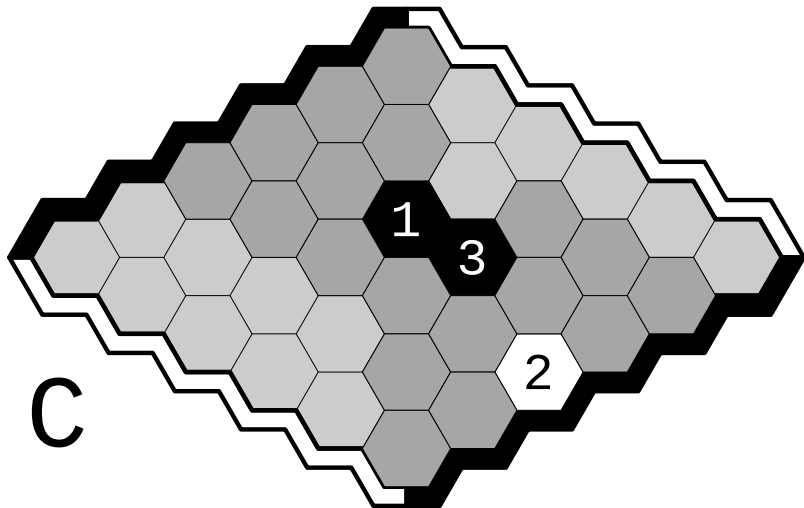
conj.: centermost cell wins

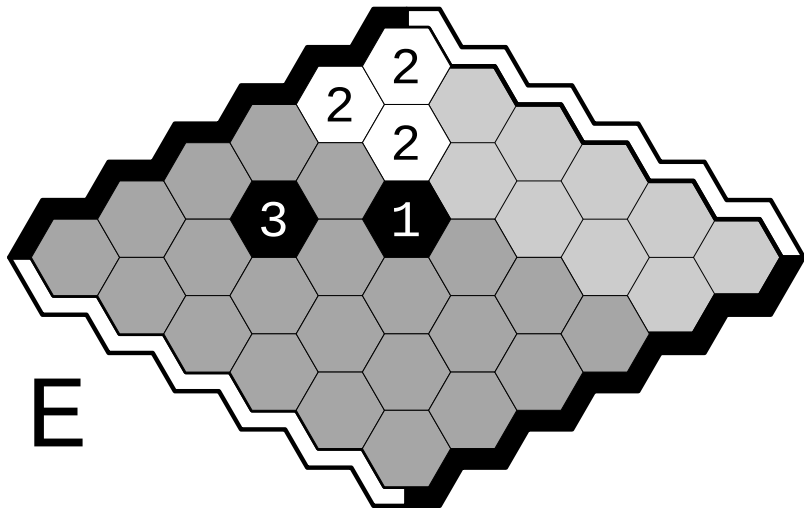


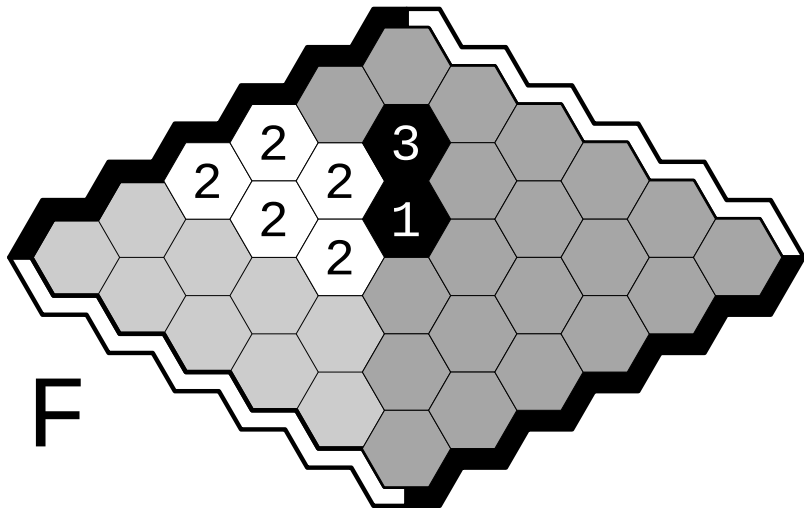
conj.: centermost Lindhard

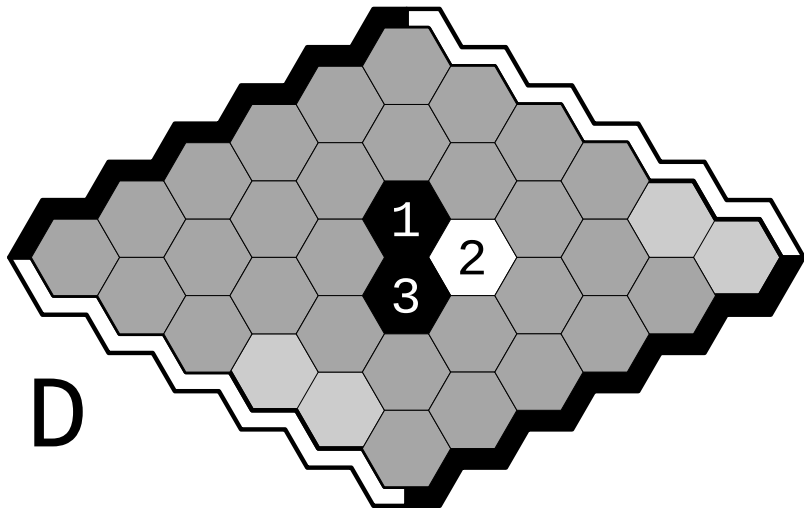






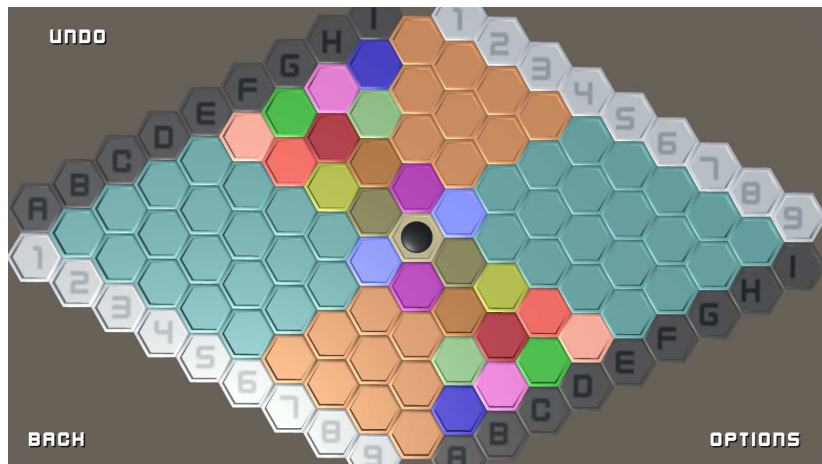






<http://webdocs.cs.ualberta.ca/~hayward/355/asn/hexviz/>

conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



Bjarne Toft, Michael Johanson, Yngvi Björnsson,
Morgan Kan, Nathan Po, Jack van Rijswijk,
Broderick Arneson, Philip Henderson, Jakub
Pawlewicz, Brad Thiessen, Aja Huang, Kenny
Young, Noah Weninger, Chao Gao, Martin Müller,
Bedir Tapkan, Md-Reza Daliri, Kamillah Hasham,
Luke Schultz, Xinyue Chen



thank you

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