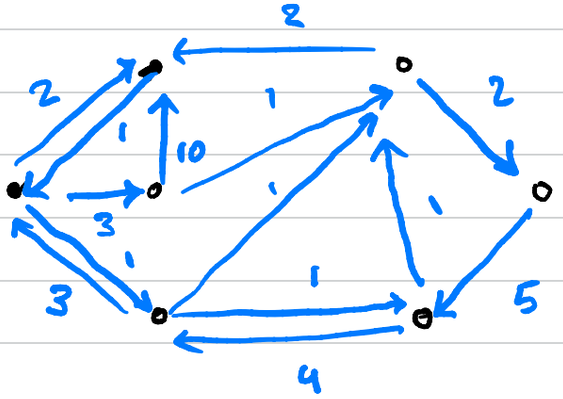
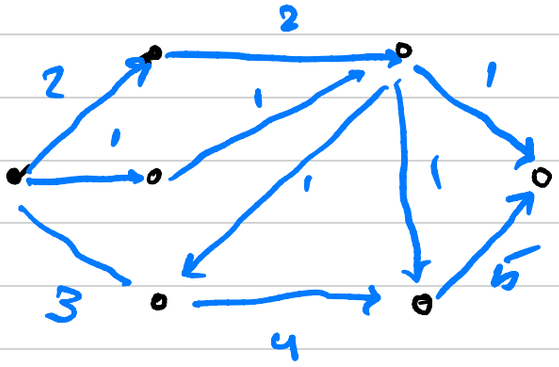
Quiz 6

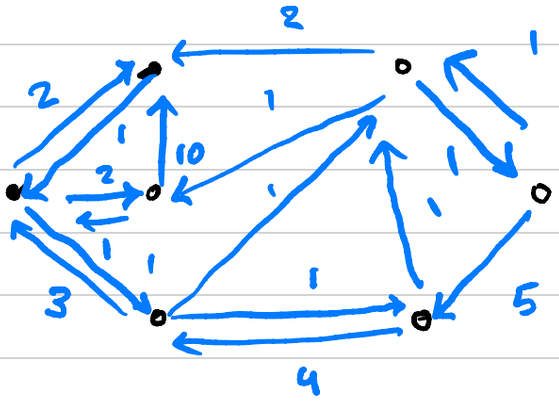
Question 1, 2



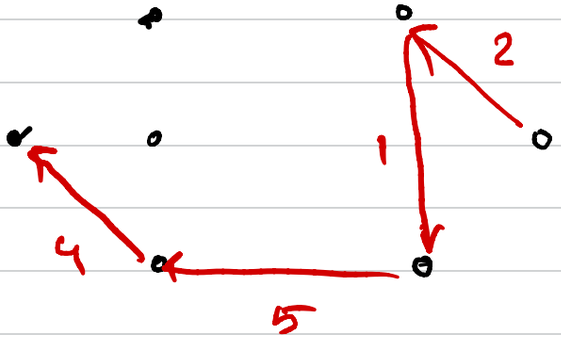
Augmenting path in residual network and pushing flow = 1



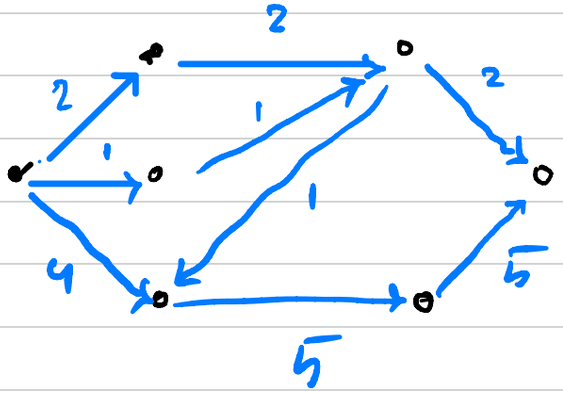
the result flow = 6



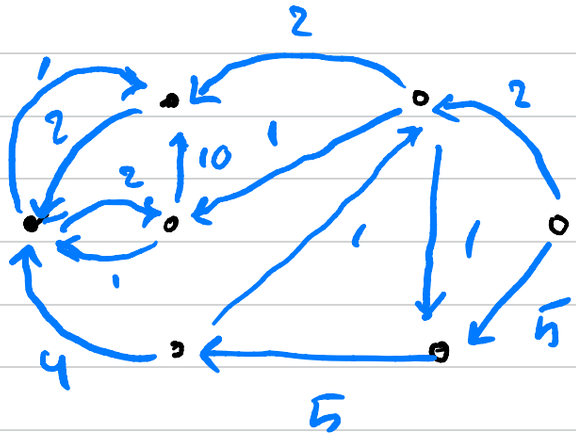
new res. network



new augmenting path



new flow = 7



new res. network

Since there are no augmenting path in the final res. net. $\Rightarrow$ we reached max-flow

$\Rightarrow$ max-flow = min-cut $\Rightarrow$ both are 7

7 {S} {a,b,c,d,e,t}

Question 3 if price BC = 4, BD = 5, CD = 3

the LP =

$$\max \quad 4(x_{BC} + y_{BC}) + 5(x_{BD} + y_{BD}) + 3(x_{CD} + y_{CD})$$

s.t.

$$x_{BC} + y_{BC} + x_{BD} + y_{BD} \leq 18 \qquad \text{(Edge bB)}$$

$$x_{BC} + x_{BD} + y_{CD} \leq 19 \qquad \text{(Edge bc)}$$

⋮

(there are 5 more such constraints)

$$x_{BC} + y_{BC} \geq 7$$

$$x_{BD} + y_{BD} \geq 7$$

$$x_{CD} + y_{CD} \geq 7$$

$$x, y \geq 0$$

A feasible solution $x_{BD} = 0 \quad y_{BD} = 7$

$x_{BC} = 7 \quad y_{BC} = 0$

$x_{CD} = 7 \quad y_{CD} = 0$

Question 5

a) {4,8}, {2,5}, {0,6}, {1,7}, {3,9}

Note : this is a perfect matching

b) Note : This similar to a since in final flow graph each node (excluding for s,t) have at most one incoming edge and one outgoing.

⇒ max flow in this graph is: matching pairs = 1

rest = 0

max flow = 5

⇒ min-cut = 5 {S}

C) Note that in a all nodes are matched [Perfect matching]

⇒ we cannot possibly add another edge to the matching

⇒ it is max