**first name** **last name** **id#**

**each page 8 marks** **30 min** **closed book** **no devices** **3 pages** **page 1**

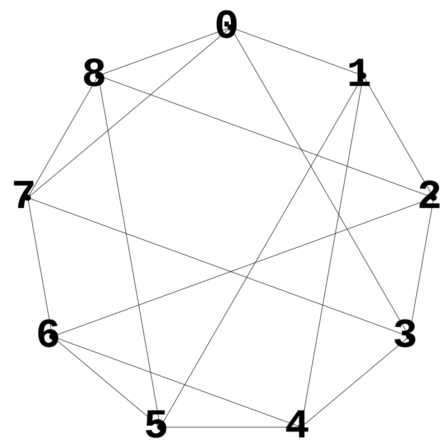
1. Show the missing entries (last column, rows 13 to 21) from the dynamic-program-by-value (DPV) 0-1 knapsack algorithm. values [3, 4, 6, 5, 3]. weights [4, 3, 5, 6, 1].

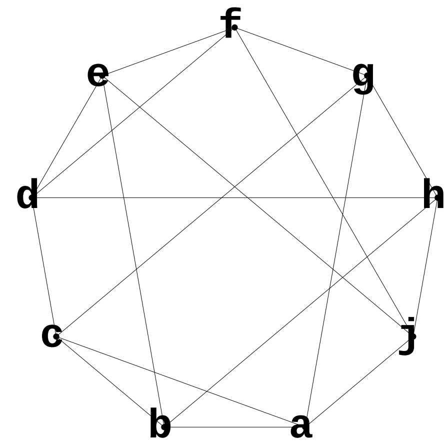
| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | - | - | - | - | - |
| 2 | - | - | - | - | - |
| 3 | 4 | 4 | 4 | 4 | 1 |
| 4 | - | 3 | 3 | 3 | 3 |
| 5 | - | - | - | 6 | 6 |
| 6 | - | - | 5 | 5 | 5 |
| 7 | - | 7 | 7 | 7 | 4 |
| 8 | - | - | - | 10 | 7 |
| 9 | - | - | 9 | 9 | 6 |
| 10 | - | - | 8 | 8 | 8 |
| 11 | - | - | - | 11 | 11 |
| 12 | - | - | - | 13 | 10 |
| 13 | - | - | 12 | 12 | |
| 14 | - | - | - | 15 | |
| 15 | - | - | - | 14 | |
| 16 | - | - | - | - | |
| 17 | - | - | - | - | |
| 18 | - | - | - | 18 | |
| 19 | - | - | - | - | |
| 20 | - | - | - | - | |
| 21 | - | - | - | - | |

2. Complete this definition: we say that a function $f(n)$ is *superpolynomial* if

3. Graphs $G = (V, E)$ (left) and $H = (W, F)$ (right) and bijection $f : V \leftrightarrow W$ are shown below.

| j | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| f(j) | a | b | c | d | e | f | g | h | j |




(a) Is $f$ an isomorphism between $G$ and $H$? (answer yes or no)

(b) Justify your answer.

4. On a 0-1 knapsack instance with $n$ items, explain why the brute force algorithm performs $n2^{n-1} - 2^n$ additions.

5. *Set cover* is this problem:

   *Instance.* A set $U$, a set $S = \{S_1, S_2, \ldots, S_t\}$ of subsets of $U$, and an integer $k$.

   *Query.* Is there a subset $C$ of $S$ of size $k$ whose union equals $U$?

   a) In the set cover instance below, $U = \{0, 1, \ldots, 8\}$ and $S = \{$S0, S1, …, S10$\}$. Does this instance have a cover of size 6? (yes/no)

   b) Justify your answer (use the space below at the right).

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| S0 | - | - | - | - | - | - | - | * | * |
| S1 | - | - | * | - | - | - | - | - | - |
| S2 | - | * | * | - | - | - | * | - | * |
| S3 | * | - | - | * | - | * | - | * | - |
| S4 | - | - | - | * | - | - | - | - | - |
| S5 | * | - | - | - | - | - | - | - | - |
| S6 | - | - | - | - | * | * | * | - | - |
| S7 | * | - | - | - | - | * | - | - | * |
| S8 | - | - | - | - | - | - | - | - | - |
| S9 | - | - | - | - | * | - | - | - | * |
| S10 | - | * | - | - | - | * | * | - | - |

6. Prove/disprove: the set cover problem is in the class NP.

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| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | - | - | - | - | - |
| 2 | - | - | - | - | - |
| 3 | 4 | 4 | 4 | 4 | 3 |
| 4 | - | 3 | 3 | 3 | 3 |
| 5 | - | - | - | 6 | 6 |
| 6 | - | - | 5 | 5 | 5 |
| 7 | - | 7 | 7 | 7 | 6 |
| 8 | - | - | - | 10 | 9 |
| 9 | - | - | 9 | 9 | 8 |
| 10 | - | - | 8 | 8 | 8 |
| 11 | - | - | - | 11 | 11 |
| 12 | - | - | - | 13 | 12 |
| 13 | - | - | 12 | 12 | |
| 14 | - | - | - | 15 | |
| 15 | - | - | - | 14 | |
| 16 | - | - | - | - | |
| 17 | - | - | - | - | |
| 18 | - | - | - | 18 | |
| 19 | - | - | - | - | |
| 20 | - | - | - | - | |
| 21 | - | - | - | - | |

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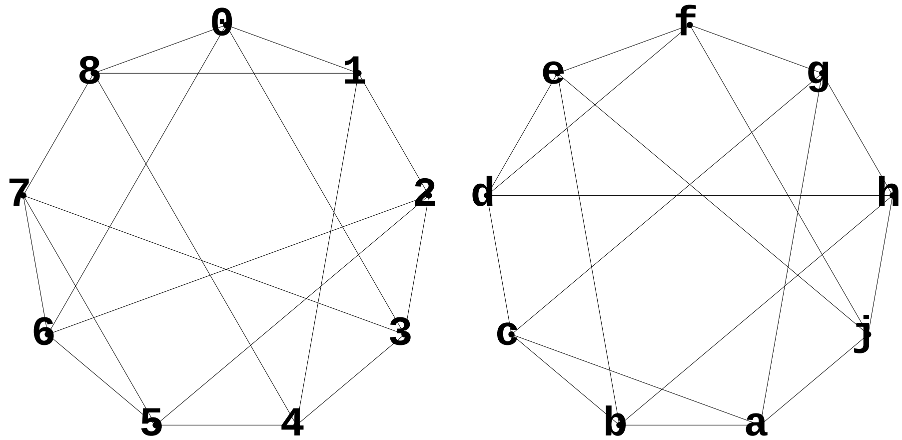
| f(j) | a | b | c | d | e | f | g | h | j |

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```
    0 1 2 3 4 5 6 7 8
S0  - - - * - - - - -
S1  * - - - - - - - -
S2  - - - - * * * - -
S3  * - - - - * - - *
S4  - - - - - - - - -
S5  - - - - * - - - *
S6  - * - - - * * - -
S7  - - - - - - - * *
S8  - - * - - - - - -
S9  - * * - - - * - *
S10 * - - * - * - * -
```

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**first name** **last name** **id#**

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| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | - | - | - | - | - |
| 2 | - | - | - | - | - |
| 3 | 4 | 4 | 4 | 4 | 2 |
| 4 | - | 3 | 3 | 3 | 3 |
| 5 | - | - | - | 6 | 6 |
| 6 | - | - | 5 | 5 | 5 |
| 7 | - | 7 | 7 | 7 | 5 |
| 8 | - | - | - | 10 | 8 |
| 9 | - | - | 9 | 9 | 7 |
| 10 | - | - | 8 | 8 | 8 |
| 11 | - | - | - | 11 | 11 |
| 12 | - | - | - | 13 | 11 |
| 13 | - | - | 12 | 12 | |
| 14 | - | - | - | 15 | |
| 15 | - | - | - | 14 | |
| 16 | - | - | - | - | |
| 17 | - | - | - | - | |
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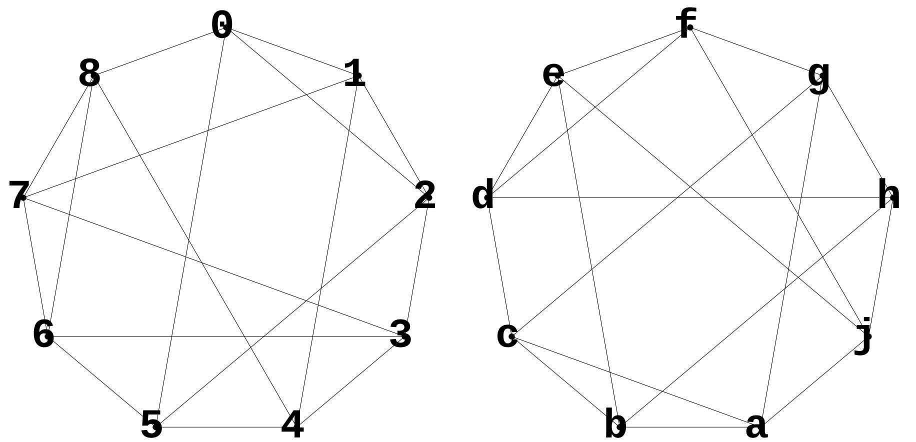


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```
    0 1 2 3 4 5 6 7 8
S0  * - - - - * - - *
S1  - - - - - - - - -
S2  - - - - * - - - *
S3  - * - - - * * - -
S4  - - - - - - - * *
S5  - - * - - - - - -
S6  - * * - - - * - *
S7  * - - * - * - * -
S8  - - - * - - - - -
S9  * - - - - - - - -
S10 - - - - * * * - -
```

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