Linear MOC example

The pde and initial condition are given by

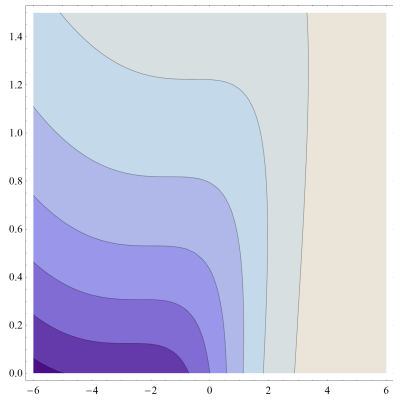
$$u_t + 2u_x = -u, \ -\infty < x < \infty, \ t > 0, \text{ with } u(x,-x) = \frac{1}{1+x^2}.$$

Using the method of characteristics (MOC), the solution is given by

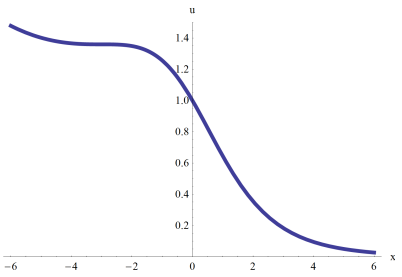
$$u = \frac{\exp\left[-(x+t)/3\right]}{1+\left(\frac{x-2t}{3}\right)^2}.$$

The characteristics are the straight lines in space-time given by

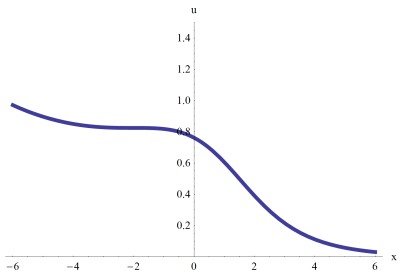
$$\tau = (x-2t)/3 \Longleftrightarrow x = 2t + 3\tau \text{ for fixed } \tau.$$



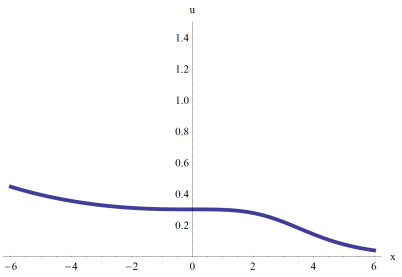
Contour plot of $u(x,t)$



$u(x,0)$



$u(x,0.5)$



$u(x,1.5)$