Mathematics 436 Midterm Examination Lecturer: Professor G. E. Swaters

27 October 2015

Instructions. Please answer all 4 questions. Each question is worth 25 points.

1. Consider the $1st$-order pde
$$u_t + c\,u_x = -u^2,$$
where $c > 0$ is a constant.
   (a) Find the general solution $u\,(x,t)$.
   (b) Use your result from Part (a) to show that the solution subject to the initial condition $u(x,0) = x$ is given by
$$u\,(x,t) = \frac{x - ct}{1 + t\,(x - ct)}.$$

2. Find all $f\,(x)$ so that
$$tv_x + xv_t = cv \text{ subject to } v\,(x,x) = f\,(x)\,,$$
where $c > 0$ is a constant, is a *characteristic initial value problem.*

3. *Classify* and reduce to the *characteristic normal form equations* the quasi-linear shallow water equations
$$h_t + uh_x + hu_x = 0,$$
$$u_t + u\,u_x + gh_x = 0,$$
where $g > 0$ is a constant.

4. *Classify* and *reduce to canonical form* the linear pde
$$u_{xx} + 4u_{xy} + 3u_{yy} + u = 0.$$