**Mathematics 436 Midterm Examination** Lecturer: Professor G. E. Swaters

28 October 2014

**Instructions.** Please answer all 4 questions. Each question is worth 25 points.

1. Use the Method of Characteristics to show that the solution to (Euler's equations for the isentropic flow of a gas)
$$u_t + u\,u_x = 0,$$
$$\rho_t + u\rho_x + \rho\,u_x = 0,$$
for $-\infty < x < \infty$, $t > 0$ (where $u$ is the velocity and $\rho$ is the density) with the initial condition
$$u\,(x, 0) = f\,(x) \text{ and } \rho\,(x, 0) = g\,(x)\,,$$
can be written in the implicit form
$$u = f\,(x - ut) \text{ and } \rho = \frac{g\,(x - ut)}{1 + t\,f'\,(x - ut)}.$$
HINT: Setup the characteristic equations and first solve for $u$ and then for $\rho$.

2. Suppose the initial data for the linear $1st$-order pde
$$a\,(x, y)\,u_x + b\,(x, y)\,u_y = c\,(x, y)\,u + d\,(x, y)\,,$$
is given by
$$u\,(x, h\,(x)) = f\,(x)\,,$$
where $y = h\,(x)$ is a characteristic. Show that
$$\frac{df}{dx} = \frac{c\,(x, h)\,f + d\,(x, h)}{a\,(x, h)}.$$

3. Show that the solution to the Cauchy problem
$$u_{tt} - u_{xx} = h\,(x, t)\,, \quad -\infty < x < \infty,\ t > 0,$$
$$u\,(x, 0) = f\,(x) \text{ and } u_t\,(x, 0) = g\,(x)\,, \quad -\infty < x < \infty,$$
where $h\,(x, t)$, $f\,(x)$ and $g\,(x)$ are smooth and spatially square-integrable functions is unique. HINT: You may assume that $u\,(x, t)$ and all its derivatives are square-integrable functions with respect to $x$.

4. Classify and reduce to canonical form the pde
$$u_{xx} - 2u_{xz} + u_{yy} + u_{zz} = 0,$$