DISPERSION IN SHEARED GAUSSIAN HOMOGENEOUS TURBULENCE

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Abstract. By integrating the Fokker-Planck equation corresponding to a Lagrangian stochastic trajectory model, which is consistent with the selection criterion of Thomson (1987), an analytical solution is given for the joint probability density function $p(x_i, u_i, t)$ for the position (x_i) and velocity (u_i) at time t of a neutral particle released into linearly-sheared, homogeneous turbulence. The solution is compared with dispersion experiments conforming to the restrictions of the model and with a short-range experiment performed in highly inhomogeneous turbulence within and above a model crop canopy. When the turbulence intensity, wind shear and covariance are strong, the present solution is better than simpler solutions (Taylor, 1921; Durbin, 1983) and as good as any numerical Lagrangian stochastic model yet reported.

1. Introduction

Counter-gradient fluxes are often observed in crop canopies (Denmead and Bradley, 1985), a consequence of non-diffusive contributions from nearby sources ("near field" effects). To relate scalar concentrations to source distributions in canopies, Raupach (1989) applied Taylor's (1921) Lagrangian solution to calculate the "near field" contributions. His necessary (and justified) assumption of local homogeneity highlights the paucity of analytical Lagrangian solutions more flexible than Taylor's.

Durbin (1983) gave a Gaussian solution for cross-stream diffusion in uniformly-sheared homogeneous turbulence. The cross-stream velocity was modeled by the Langevin equation, while the along-stream velocity fluctuation was omitted. We have extended that solution by including the along-stream fluctuation and its covariance with cross-stream velocity (these additions were recognised as straightforward by Durbin).

2. Solution for Shear Dispersion

Consideration of scalar dispersion in uniformly sheared homogeneous turbulence goes back at least as far as Corrsin (1952). The diffusion solution has been provided (e.g. Okubo and Karweit, 1969) but of course fails in the near field.

Our solution is two-dimensional. Motion along the (x, z) axes takes place with instantaneous total velocities $(u(t), w(t))$ that have mean values $(U(z), 0)$. The fluctuating velocities $(u', w') = (u - U, w)$ may have distinct time scales τ_u, τ_w

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and velocity scales σ_u, σ_w, and have covariance $-u_*^2$. The mean velocity field is $U(z) = U_o(1 + \alpha z)$, and the Eulerian velocity pdf is:

$$g_a = \frac{1}{2\pi\sigma} exp(-\frac{(u-U)^2\sigma_w^2 + w^2\sigma_u^2 + 2(u-U)wu_*^2}{2\sigma^2})$$

where $\sigma^2 = \sigma_u^2\sigma_w^2 - u_*^4$.

We assume modified Langevin equations for the increments in velocity[1]:

$$du = -\frac{(u-U)}{\tau_u}dt + b_{uu}d\xi_u + b_{uw}d\xi_w$$

$$dw = -\frac{w}{\tau_w}dt + b_{wu}d\xi_u + b_{ww}d\xi_w$$

where the $d\xi_i$ are Gaussian, with vanishing mean and variance dt, and have vanishing expectation $< d\xi_i(t_1)d\xi_j(t_2) >$ for distinct (i, j) and/or distinct (t_1, t_2). Shear stress is forced to arise through the random accelerations, and our model is therefore inconsistent with Kolmogorov's theory of local isotropy (the covariance $< dudw >$ does not vanish for time intervals dt lying in the inertial subrange). We are not interested in times so short.

The joint position-velocity probability density function $p(x, z, u, w, t)$ evolves according to the Fokker-Planck equation corresponding to our Langevin model:

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial x}[u\,p] - \frac{\partial}{\partial z}[w\,p] - \frac{\partial}{\partial u}[-\frac{(u-U)}{\tau_u}p] -$$

$$-\frac{\partial}{\partial w}[-\frac{w}{\tau_w}p] + B_{uu}\frac{\partial^2 p}{\partial u^2} + (B_{uw} + B_{wu})\frac{\partial^2 p}{\partial u \partial w} + B_{ww}\frac{\partial^2 p}{\partial w^2}.$$

By enforcing Thomson's (1987) well-mixed constraint (i.e. insisting that g_a be a steady-state solution to this Fokker-Planck equation) we obtain for the model coefficients $B_{ij} = (1/2)b_{ik}b_{jk}$ the prescription:

$$B_{uu} = \frac{\sigma_u^2}{\tau_u} - u_*^2\frac{\partial U}{\partial z}, \qquad B_{ww} = \frac{\sigma_w^2}{\tau_w},$$

$$B_{uw} + B_{wu} = -u_*^2(\frac{1}{\tau_u} + \frac{1}{\tau_w}) + \sigma_w^2\frac{\partial U}{\partial z}.$$

Solution of the Fokker-Planck equation for the time evolution of the mean state has been performed along the lines suggested by Risken (1985). For particles

[1] We also solved the Fokker-Planck equation corresponding to an alternative treatment of the alongstream velocity $du' = -\frac{u'}{\tau_u}dt + b_{uu}d\xi_u + b_{uw}d\xi_w$, $dx = (U(z) + u')dt$ for which case the b's are independent of the mean shear. Predicted mean concentrations were as good as those given here, but the predicted alongstream fluxes were less satisfactory.

released at the origin with a random velocity from the Eulerian pdf, the initial value of the joint pdf is

$$p = \delta(x,0)\delta(z,0)\frac{1}{2\pi\sigma}exp[-\frac{(u-U_o)^2\sigma_w^2 + w^2\sigma_u^2 + 2(u-U_o)wu_*^2}{2\sigma^2}]$$

the solution is:

$$p = \frac{1}{(\sqrt{2\pi})^4\sqrt{detM}}exp[-\frac{1}{2}(y_i - m_i)m_{ij}^{-1}(y_j - m_j)].$$

The y_i denote coordinates in phase space (x, u, z, w); the m_i expected values for the particle coordinates at time t ($m_z = m_w = 0, m_x = U_o t, m_u = U_o$); and m_{ij} and m_{ij}^{-1} are the elements of the covariance matrix M (given in the Appendix) and its inverse.

An alternative expression of the solution is:

$$p = \frac{1}{(\sqrt{2\pi})^4\sqrt{m_{xx}m_{zz} - m_{xz}^2}\sqrt{2\Psi}\sqrt{2X}}\cdot$$

$$\cdot exp[-\frac{m_{xx}z^2 + m_{zz}(x - U_o t)^2 - 2m_{xz}z(x - U_o t)}{2(m_{xx}m_{zz} - m_{xz}^2)}]exp[-\frac{(u+\Lambda)^2}{4\Psi} - \frac{(w+\Omega)^2}{4X}]$$

where the velocity moments are:

$$\Lambda(x,z,t) = \frac{(x - U_o t)(m_{xz}m_{uz} - m_{zz}m_{ux}) + z(m_{xz}m_{ux} - m_{xx}m_{uz})}{m_{xx}m_{zz} - m_{xz}^2} - U_o$$

$$\Psi(t) = \frac{m_{xx}m_{uu}(m_{xx}m_{zz} - m_{xz}^2) - m_{ux}^2(m_{xx}m_{zz} - m_{xz}^2) - (m_{xz}m_{ux} - m_{xx}m_{uz})^2}{2m_{xx}(m_{xx}m_{zz} - m_{xz}^2)}$$

$$\Omega(x,z,u,t) = \frac{(x - U_o t)(m_{xz}m_{wz} - m_{zz}m_{wx}) + z(m_{xz}m_{wx} - m_{xx}m_{wz})}{m_{xx}m_{zz} - m_{xz}^2} + \nu(u + \Lambda)$$

$$X(t) = [\frac{m_{xx}m_{ww} - m_{wx}^2}{2m_{xx}} - \frac{(m_{xz}m_{wx} - m_{xx}m_{wz})^2}{2m_{xx}(m_{xx}m_{zz} - m_{xz}^2)}] -$$

$$-\nu[\frac{(m_{xz}m_{wx} - m_{xx}m_{wz})(m_{xz}m_{ux} - m_{xx}m_{uz}) + (m_{xx}m_{zz} - m_{xz}^2)(m_{ux}m_{wx} - m_{xx}m}{m_{xx}(m_{xx}m_{zz} - m_{xz}^2)}$$

where:

$$\nu(t) =$$

$$= \frac{(m_{xz}m_{wx} - m_{xx}m_{wz})(m_{xz}m_{ux} - m_{xx}m_{uz}) + (m_{ux}m_{wx} - m_{xx}m_{uw})(m_{xx}m_{zz} - m_x^2}{(m_{xx}m_{zz} - m_{xz}^2)(m_{xx}m_{uu} - m_{ux}^2) - (m_{xz}m_{ux} - m_{xx}m_{uz})^2}$$

Mean concentration is obtained from the joint position-velocity pdf by integrating out the velocity dependence:

$$C(x,z,t) = \frac{1}{2\pi\sqrt{m_{xx}m_{zz} - m_{xz}^2}} e^{-\frac{m_{xx}z^2 + m_{zz}(x-U_o t)^2 - 2m_{xz}z(x-U_o t)}{2(m_{xx}m_{zz}-m_{xz}^2)}} .$$

By calculating $\int C dx$, one correctly recovers the Gaussian distribution in z with variance $m_{zz}(t)$. Similarly, $\int C dz$ yields a Gaussian for the distribution about the mass-weighted mean streamwise position

$$< x >= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x C(x,z,t) dx dz = U_o t$$

with variance $m_{xx}(t)$. In the eventuality that α=0, $u_* = 0$, (no mean shear and no covariance), $m_{xz}(t) = 0$ and $C(x,z,t)$ reduces to a product of independent Gaussian distributions about the drifting point $z = 0$, $x = U_o t$. This is a Taylor solution with independent diffusion in z and x. Should $\sigma_u \to 0$, the Gaussian in x reduces to $\delta\ (x - U_o t)$, and Taylor's solution (strictly speaking, the Gaussian distribution with Taylor's solution for the variance) is again recovered.

For large t, the dominant term in the streamwise position-variance $m_{xx}(t)$ about the centre of mass is $(2/3)\alpha^2 U_o^2 \sigma_w^2 \tau t^3$ (see Appendix). Streamwise spread is dominated not by alongwind "diffusion" (which would involve the streamwise velocity variance σ_u^2) but by the joint action of vertical turbulent convection and differential advection in the mean shear. The earliest demonstrations that the alongwind variance of a puff released into an unbounded, linearly-sheared atmosphere increases with t^3 were given by Saffman (1962) and Smith (1965).

In the next section solutions for a continuous source are obtained by integrating in time the instantaneous-source solution for unit release at all previous release times. Similarly, fluxes for a steady source are built from the instantaneous flux densities, eg. the instantaneous along-stream turbulent flux density is

$$F_x = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [u - U(z)] p(x,z,u,w,t) dw du.$$

3. Comparison with Observations

Measurements of the heat plume from a continuous line source in uniformly-sheared, approximately homogeneous turbulence have been reported by Tavoularis and Corr- sin (1981; TC), Stapountzis and Britter (1989; SB), Karnik and Tavoularis (1989); KT), and Chung and Kyong (1989); CK). But the data do not provide a useful test of the present solution with respect to Durbin's, due to factors such as (i) flow disturbance by the source wire (CK); (ii) unreported source strength (TC); (iii) use of scaling that cannot be "undone" and forces the model and observed concentrations to overlap (KT); (iv) horizontal inhomogeneity; (v) very small turbulence intensity σ_u/U; and (vi) incompletely reported flow statistics. All solutions we have examined (ours, Durbin's, or our implementation of the numerical model of SB) predict displacement of the plume centreline (location of maximum temperature) in the direction of lower mean velocity. The opposite displacement was observed by SB and KT near the source (buoyancy was a possible factor). If the observed asymmetry is not due to buoyancy, it is also not due to any mechanism encapsulated in our model or that of SB, and the simulation reported by SB is puzzling.

Flow properties for simulating the KT experiments, derived from KT and companion papers, were: $\sigma_u = 0.06$, $\sigma_w = 0.04$, $U_o = 1$, $\alpha = 0.016$, $u_* = 0.021$, $\tau = 13$, where numbers are dimensionless on the mean centreline velocity U_o (varying from run to run) and the constant "mesh length" M=0.0254m. The small turbulence intensity implies little advantage of our model relative to Durbin's. Figure 1 compares our solutions with the KT data, at scaled downstream distances (measured from the source) of x/M=0.25, 10, 80. KT gave their data in the form of a dimensionless temperature perturbation χ formed by scaling the observed perturbation temperature on the maximum observed perturbation temperature for that cross-section of the flow. The cross-stream position was scaled on the local half-width of the plume ("x_2/w"). Unfortunately that presentation discarded much information (centreline temperature, plume width), leaving a shape largely constrained by the scaling. Differences between the different models are invisible. However at x/M=80 there is a fourfold difference between the present and the Taylor (cross-stream dispersion only) solution for (unscaled) centreline perturbation temperature.

Legg et al. (1988; LRC) performed dispersion experiments in the vertically-inhomogeneous flow about an artificial crop in a boundary-layer wind tunnel. Flow statistics were well-specified and adequately uniform alongstream, and all information needed to predict actual temperature rise in the tunnel can be gleaned from LRC and related work cited below. The features of the LRC experiment useful to us are the large turbulence intensity (σ_u/U at the source was 0.67) and shear stress, which might allow discrimination between models. The objection that homogeneous flow models should not be compared with data from inhomogeneous flows can be subdued by comparing model and observation near the source (where the plume has sampled a narrow range on the inhomogeneous axis). But a short flight does not exclude the influence of other factors not accounted for in the present model, eg. velocity skewness.

Details of the experiments are found in Legg et al. (1986), Raupach et al. (1986), and Raupach et al. (1987). A heating wire was stretched across the flow at height z_s=51 mm, within a canopy of height h_c=60 mm. At the source height U=2.6 ms^{-1}, $< u'w' >= -0.8\ m^2s^{-2}$, $\sigma_u = 1.7\ ms^{-1}$, and $\sigma_w = 1.3\ ms^{-1}$. We formed dimensionless variables using a friction velocity $u_{*s} = 0.9\ ms^{-1}$ based on the shear stress at the source height, and on LRC's estimate for the

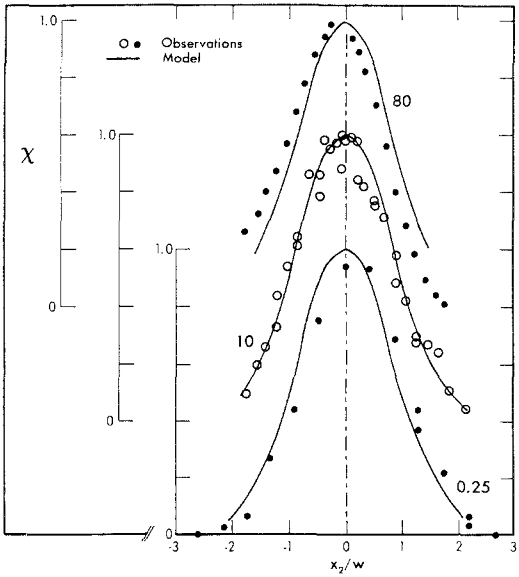

Fig. 1. Comparison of solution with the observations of Karnik and Tavoularis (1989) of the normalised mean temperature rise downstream of a heater-wire in uniformly-sheared homogeneous turbulence. x_2/w is the cross-stream coordinate scaled on the plume half-width. Comparisons at distances from the source (scaled on mesh length) x/M=0.25, 10, 80.

Lagrangian timescale within the crop (derived from the dispersion data), namely $\tau = 0.3h_c/u_{*c}$=0.018 s, where $u_{*c} = 1.0ms^{-1}$ is the friction velocity based on the shear stress measured immediately above the crop. The dimensionless variables are: $U_o = 2.8$, $\alpha = 0.44$, $\sigma_u = 1.9$, $\sigma_w = 1.4$, $u_* = 1$, $\tau = 1$. We examine LRC's results at the closest point of observation downwind from the source, in their terminology x_L=0.023m, or in our dimensionless notation x=1.44. Advection time from the source to this station is 0.5 (i.e. half a timescale). Even at this short fetch, the plume covers roughly a height range of 20 to 80 mm. Inspection of the velocity statistics given by the experimenters shows that plume depth is too great to justify a claim of homogeneity. Down at 20 mm the plume has encountered a mean velocity larger (by about 50%) than given by our linear profile, and much attenuated turbulent velocities.

It is necessary to multiply our dimensionless concentration and turbulent alongwind flux by $U_o z_s/(\tau u_{*s}^2)$=9.0 to obtain the normalised mean temperature θ/θ_* and flux $< u'\theta' > /u_*\theta_*$ presented by LRC (their Figure 16a). Figure 2a compares our solution, Durbin's, and Taylor's (1-dimensional) against the data. Though it accounts for shear, Durbin's solution performs little better than Taylor's. Its overprediction at low heights may be because, omitting u', it is more sensitive to underestimation of true windspeed by our shear profile.

Figure 2b shows improvements (relative to the 1-dimensional Taylor solution) as first streamwise diffusion is accounted for (using the present solution with

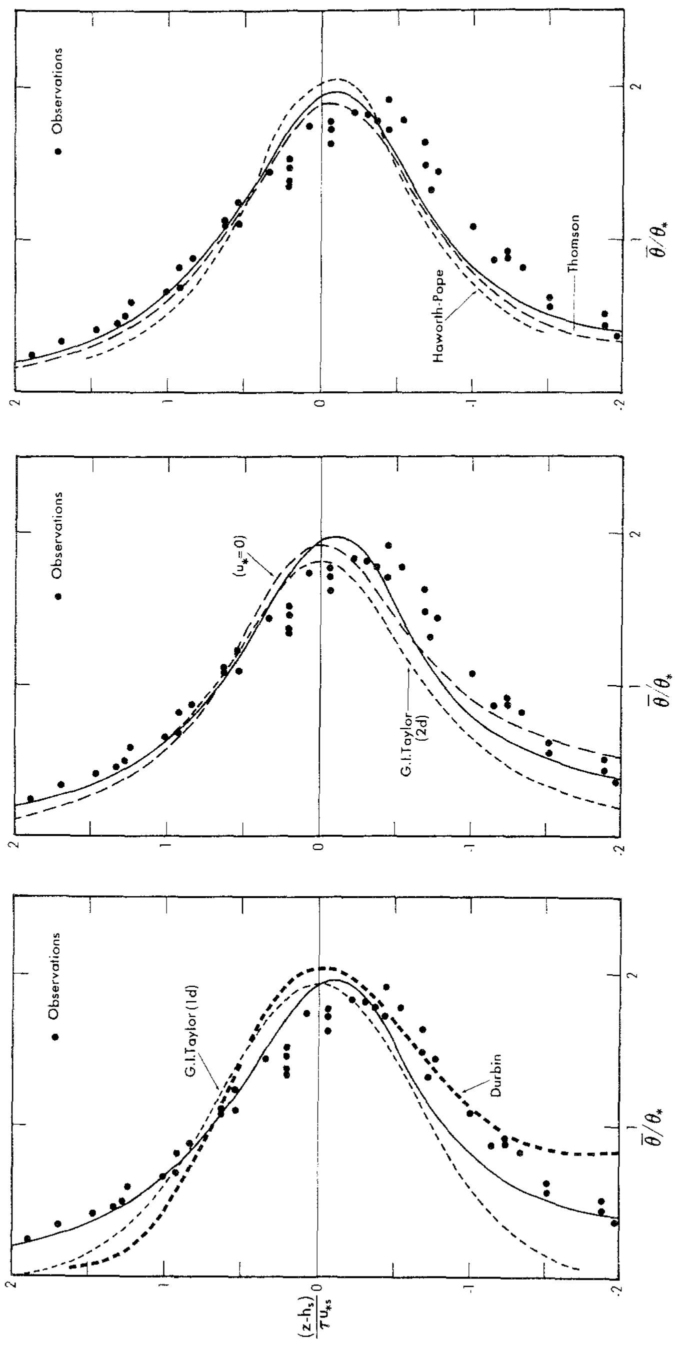

Fig. 2. Comparison of models with observations of Legg et al. (1986) of dispersion from a line source placed in strong velocity shear near the top of a wind-tunnel crop. Height axis spans z=20 mm to z=82 mm, and $(h_C - h_S)/(\tau u_{*s})$=0.58. Solid lines: present solution with shear and covariance.

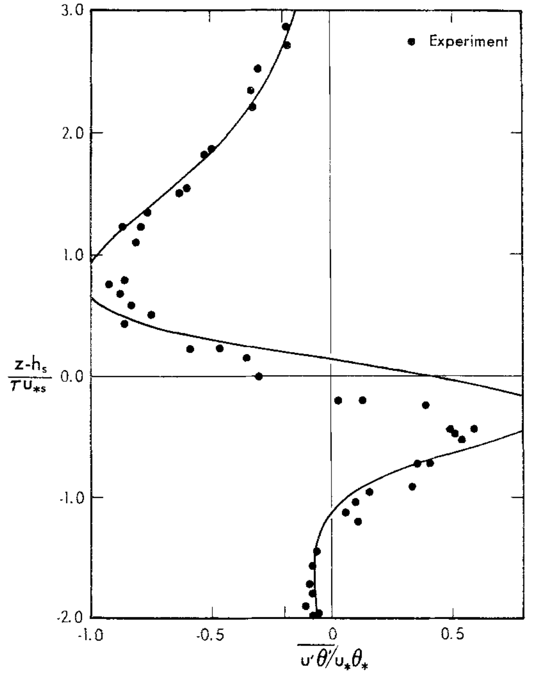

Fig. 3. Comparison of the present solution for the turbulent streamwise heat flux with the observations of Legg et al. (1986).

$\alpha = u_* = 0$; equivalently Taylor's solution both for vertical diffusion and for streamwise diffusion about the advected centre of mass), then wind shear, and finally velocity covariance are introduced. Comparing with the Durbin solution we see that it is better (in this case) to account for shear and alongwind turbulence than shear alone. In Figure 2c the LRC data are compared with our model and with numerical simulations using the random flight models of Haworth and Pope (1986) and Thomson (1987; multi-dimensional Gaussian model). Homogeneous flow properties were used in the random flight simulations. Collectively Figures 2a-2c indicate that for this experiment it is advantageous to account for streamwise dispersion, mean shear and velocity covariance, and that the present solution is as good as alternative models permitting these complications. In fact comparing with LRC's Figure 16a, our solution is better than the prediction of LRC's numerical, height-dependent LS model.

Figure 3 compares LRC's observations of the turbulent streamwise heat flux density at x_L=0.023 m with our solution. The mean streamwise flux densities UC are an order of magnitude larger than the turbulent, and the height-integrated lateral flux at this position is (according to our solution) composed of 116% due to advection by the mean flow and -16% due to the turbulent component. Our solution overestimates the height at which the turbulent flux changes sign, as did the numerical, height-dependent LS model of LRC.

4. Conclusion

Although the Langevin model we assumed is open to criticism, the solution it implies matches the observed near-field of a source in high-intensity turbulence better than simpler solutions (Taylor, 1921; Durbin, 1983) and as well as any numerical Lagrangian stochastic model yet reported. Models (like Raupach, 1989) which for simplicity use an assumption of local homogeneity, could in principle be improved by adopting this extended Gaussian solution.

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Appendix

Determination of the moments $m_{xz}(t) =< xz > - < x >< z >$ (etc) is laborious but straightforward. We give the results for the case $\tau_u = \tau_w = \tau$. In none of the experiments that we considered was there any benefit from distinguishing the timescales.

$$m_{zz} = 2\sigma_w^2 \tau (t - \tau(1 - e^{-t/\tau}))$$

$$m_{xx} = 2\sigma_u^2 \tau (t - \tau(1 - e^{-t/\tau})) +$$

$$+4\alpha U_o u_*^2 \tau^2 t - 2(\alpha U_o u_*^2 \tau + \alpha^2 U_o^2 \sigma_w^2 \tau^2) t^2 + (2/3)\alpha^2 U_o^2 \sigma_w^2 \tau t^3 +$$

$$+(4\alpha U_o u_*^2 \tau^3 - 8\alpha^2 U_o^2 \sigma_w^2 \tau^4) e^{-t/\tau} - 8\alpha^2 U_o^2 \sigma_w^2 \tau^3 t e^{-t/\tau}$$

$$-2\alpha^2 U_o^2 \sigma_w^2 \tau^2 t^2 e^{-t/\tau} + 8\alpha^2 U_o^2 \sigma_w^2 \tau^4 - 4\alpha U_o u_*^2 \tau^3$$

$$m_{ww} = \sigma_w^2, \quad m_{uu} = \sigma_u^2, \quad m_{wz} = \sigma_w^2 \tau (1 - e^{-t/\tau}),$$

$$m_{ux} = \sigma_u^2 \tau (1 - e^{-t/\tau}) - 2\alpha U_o \tau (u_*^2 + \alpha U_o \tau \sigma_w^2) t + \alpha^2 U_o^2 \sigma_w^2 \tau t^2 +$$

$$+2\alpha^2 U_o^2 \sigma_w^2 \tau^2 t e^{-t/\tau} + \alpha^2 U_o^2 \sigma_w^2 \tau t^2 e^{-t/\tau} + 2\alpha U_o u_*^2 \tau^2 (1 - e^{-t/\tau})$$

$$m_{wx} = (\alpha U_o \sigma_w^2 \tau^2 - u_*^2 \tau)(1 - e^{-t/\tau}) - \alpha U_o \sigma_w^2 \tau t e^{-t/\tau}$$

$$m_{uz} = -(3\alpha U_o \sigma_w^2 \tau^2 + u_*^2 \tau)(1 - e^{-t/\tau}) + \alpha U_o \sigma_w^2 \tau t e^{-t/\tau} + 2\alpha U_o \sigma_w^2 \tau t$$

$$m_{xz} = -2(u_*^2 \tau + \alpha U_o \sigma_w^2 \tau^2)(t - \tau(1 - e^{-t/\tau})) + \alpha U_o \sigma_w^2 \tau t^2$$

$$m_{uw} = -u_*^2 + \alpha U_o \sigma_w^2 \tau (1 - e^{-t/\tau}).$$

On first sight it may confuse the reader that m_{uw} does not tend to -u_x^2 for large t. The explanation is that

$$m_{uw} =< uw > - < u >< w >=< uw >$$

since $< w >= 0$. Therefore

$$m_{uw} =< (u - U)w > + < Uw >=< u'w > + < U_o(1 + \alpha z)w >=$$

$$=< u'w' > + \alpha U_o m_{wz},$$

i.e. the term $< Uw >$ contributing to m_{uw} does not vanish at large t.

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