THE SOURCE AREA INFLUENCING A MEASUREMENT IN THE PLANETARY BOUNDARY LAYER: THE "FOOTPRINT" AND THE "DISTRIBUTION OF CONTACT DISTANCE"

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(Received in final form 25 September, 1990)

Abstract. This paper considers the ground area which affects the properties of fluid parcels observed at a given spot in the Planetary Boundary Layer (PBL). We examine two source-area functions: the "footprint," giving the source area for a measurement of vertical flux; and the distribution of "contact distance", the distance since a particle observed aloft last made contact with the surface. We explain why the distribution of contact distance extends vastly farther upwind than the footprint, and suggest for the extent of the footprint the inequalities:

$$U\frac{h}{\sigma_w(h)} < x < U\begin{cases}\tau_L(h), & h \text{ above surface-layer}\\ h/\dot{z}, & \text{otherwise}\end{cases}$$

$$|y| < \sigma_v(h)\frac{x}{U}$$

where U is the mean streamwise (x) velocity, h is the observation height, τ_L is the Lagrangian timescale, σ_v and σ_w are the standard deviations of the cross-stream horizontal (y) and vertical (z) velocity fluctuations, and $\dot{z}$ is the Lagrangian Similarity prediction for the rate of rise of the centre of gravity of a puff released at ground.

Simple analytical solutions for the contact-time and the footprint are derived, by treating the PBL as consisting of two sub-layers. The contact-time solutions agree very well with the predictions of a Lagrangian stochastic model, which we adopt in the absence of measurements as our best estimate of reality, but the footprint solution offers no improvement over the above inequality.

1. Introduction

The concept of an upwind "source area", which exerts a dominating influnce on the properties sampled in an elevated measurement, was introduced by Pasquill (1972); and the point was made that "measurements made at an elevated point may be accepted as representative of underlying patchy terrain only if the patchiness is predominantly on a scale small compared to the effective source area". The ground is normally patchy on scales from the minute to the continental: so we must place our instruments high to average the evaporation from a cornfield which is patchy with respect to soil moisture, plant spacing, etc. – but not so high as to be confounded by the contribution from a lake, or whatever, upstream.

Aircraft measurements of the "surface" fluxes (of diverse properties including water vapour, heat, ozone) are, according to Schuepp *et al.* (1990), likely to become more common. Knowledge of the source area is obviously important. Accordingly, Schuepp *et al.* (1990) identify "a pressing need for manageable analytical solutions capable of giving order-of-magnitude predictions of upwind

Boundary-Layer Meteorology **55**: 25–46, 1991.

areas most likely to affect a point measurement at a given height", solutions which should "specifically include the effect of stability". This then – provision of analytic solutions for the source area – is our concern herein.

Schuepp *et al.* (1990) have reviewed efforts to quantify source area, so we shall be schematic in our survey. Pasquill used a Gaussian Plume dispersion model to quantify broadly the dimensions of the source area with respect to its influence on concentration, for three classes of atmospheric stability and for smooth or rough terrain. Pasquill's approach has been extended by Gash (1986) and Schmid and Oke (1990), the latter using a K-theory model which permits specification of the surface-layer scales ($u*$, L_{MO}). Leclerc and Thurtell (1990) and Schuepp *et al.* (1990; hereafter SLMD) have investigated the source area for a flux measurement using a two-dimensional Lagrangian stochastic dispersion model, in which an underlying canopy is parameterised by effective roughness and displacement lengths. These authors coined the term "footprint" for the source area affecting a vertical flux.

We begin in Section 2 with a discussion of source area functions, to stress why differently-defined functions may have very different range. Using Lagrangian terminology, we define the footprint, and discuss its relationship to the probability density $\Phi_T(t \mid h)$ for the time t which has elapsed since a particle, spotted in the PBL at height h, last touched the ground (by symmetry this equals the distribution of the time until next contact for a particle let loose at h – at least provided we are many length scales from ground). In the language of stochastic processes, $\Phi_T(t \mid h)$ is called the "distribution of first passage time".

In Sections 3 and 4 we derive analytical solutions for the source area functions. These employ one or two layers within which dispersion is parameterised by the gradient-diffusion model (K-theory). This is a fundamentally flawed model, but for present purposes we hoped (with only partial gratification) that it would suffice; we are interested in the source-ground pathway, which extends over many turbulence timescales: therefore travel times are expected to be long compared to the timescale at either end of the path, and K-theory is expected to perform adequately. The advantage of the choice of K-theory is that the results have a degree of universality: the eddy diffusivities may be chosen at will, with the guidance, for example, of Monin-Obukhov and mixed-layer similarity.

2. Definition of Source Area Functions

2.1. Footprint

Figure 1 lays out the experimental situation envisaged. We have an observation point at $(x, z) = (0, h)$ where we may measure a vertical flux, measure a mean concentration, or simply spot a parcel. Each element dx of the surface upstream $(x < 0)$, which is in general covered by a crop of height h_c, can be considered to be a source of marked fluid elements which may or may not contribute to what

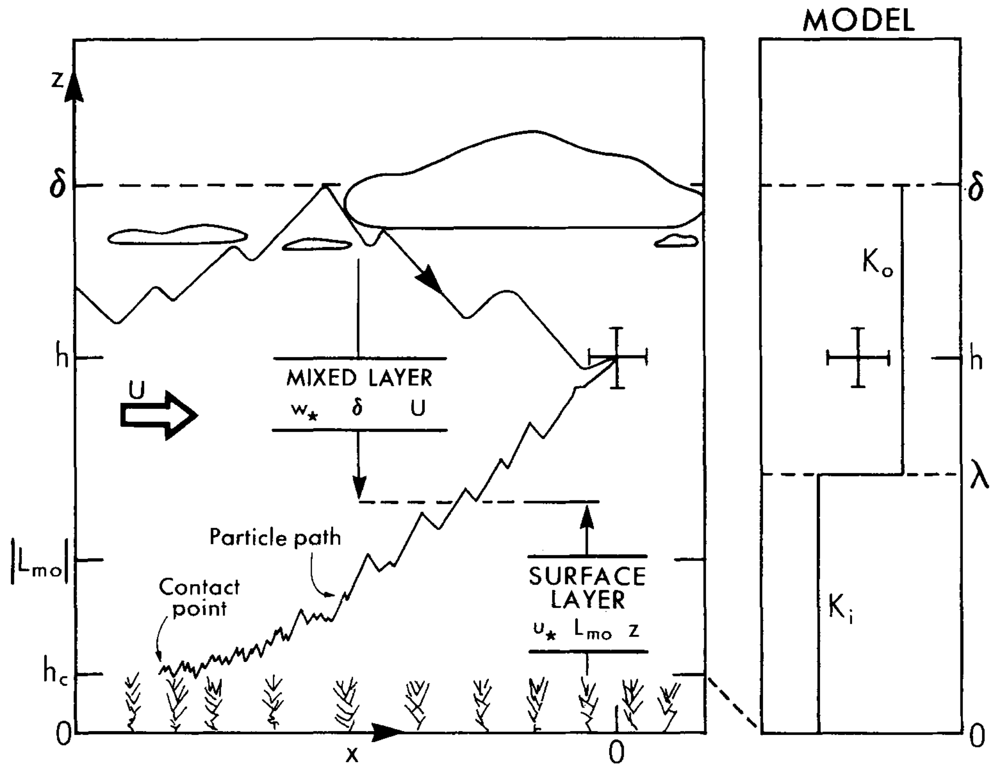

Fig. 1. Showing the PBL in its convective state ($\delta \gg |L_{MO}|$), subdivided (for simplicity) into a surface layer (Monin-Obukhov scaling) and a mixed layer (mixed-layer scaling). At $(x, z) = (0, h)$, we have an observation point where we may measure a mean vertical flux or concentration (each of which is influenced by some upwind range of the surface which we would like to be able to calculate) or simply tag a parcel and ask: "How long since it last touched (until it next will touch) the surface $z = h_c$?"
Also indicated is a two-layer model of the PBL where at height $z = \lambda$ the eddy diffusivity jumps from its inner to its outer value. The model is capable of (crudely) representing the PBL, whatever its state, by appropriate selection of λ, K_i, and K_o.

we see at $(0, h)$. As in earlier approaches to the source area problem, we assume that the flow is horizontally uniform.

Let $F(0, h)$ be the measured vertical flux density (of whatever constituent concerns us) at $(0, h)$. Then if $Q_0(x)$ is the source strength for this constituent, the footprint $\Psi_x(x \mid h)$ [m^{-1}] is defined by:

$$F(0, h) = \int_{x=-\infty}^{\infty} Q_0(x')\, \Psi_x(x' \mid h)\, \mathrm{d}x' \tag{1}$$

and is simply the vertical flux at $(0, h)$ due to a continuous line source of unit strength at x. It can also be seen that:

$$\int_{x'=-\infty}^{\infty} \Psi_x(x' \mid h)\, \mathrm{d}x' = 1 \tag{2}$$

and that if Q_0 is independent of x (uniform source extending to infinity), $F(0, h) = Q_0$.

Denoting the source height by z_c, the footprint may be expressed (Van Dop *et al.*, 1985):

$$\Psi_x(x \mid h) = \int_{t'=-\infty}^{t} \int_{w=-\infty}^{\infty} w\, p(0, h, w, t \mid x, z_c, t')\, \mathrm{d}w\, \mathrm{d}t' \tag{3}$$

where $p(x, z, w, t \mid x_0, z_0, t_0)\, \mathrm{d}x\, \mathrm{d}z\, \mathrm{d}w$ is the probability that a fluid element released at (x_0, z_0, t_0) will at time t lie in "volume" $\mathrm{d}x\ \mathrm{d}z$ centred on (x, z) with vertical velocity between $w - \mathrm{d}w/2$ and $w + \mathrm{d}w/2$. Note that the dimensionality of the other pdf's which will appear in the following is implied by the variables appearing.

Throughout the present work, we make the approximation that streamwise motion occurs at a constant velocity U, thus neglecting mean wind shear and the effect of the streamwise velocity fluctuation u' (whose standard deviation σ_u is in any case small compared to the mean streamwise velocity except very near the surface). Under this approximation, the observation point is at successive times t under the influence of elementary filaments of air (advecting without horizontal distortion) into which the additive was instantaneously released at the ground at location $x = -Ut$ and time $t = 0$. Then:

$$p(0, h, w, t \mid x, z_c, t') = p(h, w, t \mid z_c, t')\, \delta(x, -U(t - t')) \tag{4}$$

and the footprint simplifies to:

$$\Psi_x(x \mid h) = \frac{1}{U} \int_{w=-\infty}^{\infty} w\, p\left(h, w, \frac{-x}{U} \mid z_c, 0\right) \mathrm{d}w, \tag{5}$$

which, within the factor U, is simply the ensemble mean vertical flux $\Psi_T(t \mid h)$ at h at time $t = -x/U$ due to an instantaneous unit release at $t = 0$, $z = z_c$.

Now we may rewrite the footprint (van Dop *et al.*, 1985) as

$$\Psi_x(x \mid h) = \frac{1}{U} \langle w \rangle\, C/U \tag{6}$$

where $C = p(h, -x/U \mid z_c, 0)$ is the mean concentration and $\langle w \rangle$ is the mean vertical velocity of particles which, having been released at $z = z_c$ at time $t = 0$, are at $z = h$ at time $t = -x/U$. If the conditional mean velocity $\langle w \rangle$ vanishes, so must the footprint for the corresponding source location. We therefore make the following general hypothesis:

> "Only those elementary line sources contribute to the vertical flux at $(0, h)$ for which it is simultaneously the case that (i) particles released therefrom are able to attain the required altitude h before being blown past the observation point; and (ii) when the required displacement x is achieved, there remains a mean vertical velocity for particles happening to be at the observation point."

Part (i) of our hypothesis is obviously valid; we have demonstrated part (ii) for the special case ($x = Ut$), and it is equally true when we remove that limitation. On the basis of this hypothesis, we suggest for the alongwind extent of the footprint the general limits:

$$Ut_{\uparrow} < x < Ut_{\sim} \tag{7a}$$

where the lower limit ensures adequate "time to climb" ($t_{\uparrow}$) to the observation height, the outer that particles from sources within the footprint arrive with a preferentially-upward velocity ($t_{\sim}$ representing the time-of-flight necessary to ensure small mean vertical velocity).

A rough estimate of $t_{\uparrow}$ is $h/\sigma_w(h)$. As for $t_{\sim}$, if h is far from the ground, and many timescales $\tau_L(h)$ elapse before arrival at h from a given source, that source is not likely to contribute to the vertical flux ($t_{\sim} \approx \tau_L(h)$). But if h is close to ground, we may estimate $t_{\sim} \approx h/\dot{z}$, where $\dot{z} = k_v u_* \phi$ is the prediction of Lagrangian Similarity theory for the rate of rise of the centre of gravity of a puff released at ground ($k_v = 0.4$, von Karman's constant; u_*, the friction velocity; ϕ, a correction factor for atmospheric stratification, unity in neutral stratification). The latter estimate assumes upward and downward motion across h are about equally likely for sources whose "puffs" will, on achieving the downwind displacement x, have a centre of gravity at or above h.

With these estimates, the domain of the footprint is:

$$U\frac{h}{\sigma_w(h)} < x < U\begin{cases}\tau_L(h), & h \text{ above surface-layer} \\ h/\dot{z}, & \text{otherwise}\end{cases}$$

$$|y| < \sigma_v(h)\,\frac{x}{U} \tag{7b}$$

Within these limits, the footprint will be skewed towards the "nearby" end, because of the importance of the large vertical velocity associated with the nearby contributions. (It was noted by a reviewer that, since if the observation point is within the convective boundary layer $\tau_L(h) \sim \delta/w*$ (where $w*$ is the convective velocity scale defined in Section 6.1), the upper limit amounts to saying that the vertical flux at h due to a ground source decays on the convective timescale.)

2.2. Contact Time

Although our solutions do not constrain us to do so, we shall here regard "contact" with the ground as passage across the plane $z_c = h_c$, where h_c is crop height. Since lateral motion below this level is very slow, it is pointless to wrestle with the conundrum of what is true contact: (grazing a leaf boundary layer? Entering a stomate?)

Let $p_*(z, w, t \mid h, 0)$ be the joint position-velocity probability density function for a marked fluid element which is released at $t = 0$, $z = h < \delta$. The "$*$" signifies that the particle is reflected if it arrives at $z = \delta$, and more importantly, *is absorbed if it arrives at the "contact height"* z_c. The "marginal" pdf $p_*(z, t \mid h, 0)$ is simply the ensemble mean concentration of marked fluid elements, and satisfies:

$$\int_{z=z_c}^{\infty} p_*(z, t \mid h, 0)\, \mathrm{d}z \le 1 \tag{8}$$

Now let $P[T < t]$ denote the cumulative probability that the time T till contact with the ground is less than t. Then the probability density for the contact time is:

$$\begin{aligned} \Phi_T(t \mid h) &= \frac{\partial P[T < t]}{\partial t} = \frac{\partial}{\partial t}\left[1 - \int_{z_c}^{\delta} p_*(z, t \mid h, 0)\, \mathrm{d}z\right] \\ &= -\int_{w=-\infty}^{\infty} w\, p_*(z_c, w, t \mid h, 0)\, \mathrm{d}w \end{aligned} \tag{9}$$

and so is given simply as (minus) the flux at the contact height. If we assume that the ensemble mean vertical flux is given by $-K\partial p_*/\partial z$ (where here and elsewhere if the variables are not indicated, p_* is to be interpreted as the concentration) the distribution of contact time reduces to

$$\Phi_T(t \mid h) = \left(K \frac{\partial p_*}{\partial z}\right)_{z=z_c} \tag{10}$$

and $p*$ evolves in time according to the diffusion equation:

$$\frac{\partial p_*}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial p_*}{\partial z}\right) \tag{11}$$

subject to initial and boundary conditions:

$$p_*(z, 0 \mid h, 0) = \delta(z, h), \quad p_*(z_c, t \mid h, 0) = 0, \quad \left(\frac{\partial p_*}{\partial z}\right)_\delta = 0. \tag{12}$$

Hence the distribution of contact time can be obtained by solving the diffusion equation to determine the ensemble mean flux of the marked fluid elements across

z_c under the condition of absorption at z_c. We give several such solutions in section 3. The distribution of contact distance, supposing $x = Ut$ to be valid, is:

$$\Phi_x(x \mid h) = \frac{1}{U}\Phi_T(t \mid h). \tag{13}$$

2.3. Relation between footprint and contact distance

When our solutions (below) for the contact distance were first obtained, we were concerned that they spanned a far greater upstream range than do published footprints. This however can be understood as follows.

Those particles originating from very far upstream, and constituting the tail of the distribution of contact distance, arrive at the observation point (perhaps after one or more reflections off $z = \delta$, but in any case) with, on average, no mean vertical velocity: hence they make little contribution to the vertical flux. On the other hand, the very few particles arriving at $(0, h)$ from close upstream, having managed to attain the height h in so short a time (distance), have large upward velocity and thus make large contributions to the flux.

The formal connection between these two very different source functions is:

$$p_*(z_c, w, t \mid h, 0) = p(z_c, w, t \mid h, 0) - \\ - \int_{t'=0}^{t} \Phi_T(t' \mid h)\, p(z_c, w, (t - t') \mid z_c, 0)\, \mathrm{d}t' \tag{14}$$

or, in terms of the Laplace transforms of the functions (denoted by overbars):

$$\overline{p_*}(z_*, w, s \mid h, 0) = \bar{p}(z_c, w, s \mid h, 0) - \\ - \overline{\Phi_T}(s \mid h)\, \bar{p}(z_c, w, s \mid z_c, 0) \tag{15}$$

where s is the complex frequency. Probably the one source function can be calculated given the other. We have not explored this possibility.

2.4. The influence function for concentration

By analogy to the footprint, we may define the influence function $\Sigma_x(x \mid h)$ for concentration by:

$$C(0, h) = \int_{x'=-\infty}^{\infty} Q_0(x')\, \Sigma_x(x' \mid h)\, \mathrm{d}x'. \tag{16}$$

It can be shown that under the constant advection restriction $x = Ut$:

$$\Sigma_x(x \mid h) = \int_{w=-\infty}^{\infty} p\left(h, w, \frac{-x}{U} \mid z_c, 0\right) \mathrm{d}w. \tag{17}$$

For a flow bounded at $z = \delta$, the influence function for concentration does not vanish for large x. It is given by the mean concentration at h due to a unit release at ground time $t = x/U$ ago, and must tend to the value $1/\delta$.

3. Analytical Solutions for the Distribution of Passage Time

For completeness we shall give simple, single-homogeneous-layer solutions, before progressing in complexity to secure a better agreement with the inhomogeneous atmospheric boundary layer.

3.1. A single, infinitely-deep layer

Let $z_c = 0$. The solution to the diffusion Equation (11) subject to the chosen initial and boundary-conditions is:

$$p_*(z, t \mid h, 0) = \frac{1}{2\sqrt{\pi K t}} \left[\exp\left(-\frac{(z-h)^2}{4Kt}\right) - \exp\left(-\frac{(z+h)^2}{4Kt}\right)\right]. \tag{18}$$

This solution is composed simply of the difference between the "source solutions" (Carslaw and Jaeger, 1959) for the physical source and its image at $z = -h$, i.e., the solution is just the Green's function for the diffusion equation in a one-dimensional half-space with the source at $z = h$ and formed so as to ensure no concentration at $z = 0$. From Equation (18) it follows that:

$$\Phi_T(t \mid h) = \frac{h}{2t\sqrt{\pi K t}} \exp\left(-\frac{h^2}{4Kt}\right). \tag{19}$$

The integral

$$\bar{t} = \int_{t'=0}^{\infty} t' \, \Phi_T(t' \mid h) \, \mathrm{d}t' \tag{20}$$

does not exist. Mean passage times calculated from a distribution pursued out to finite time will be in error. In this case, the cumulative probability is:

$$P[T \leq t] = \operatorname{erfc}\left(\frac{h}{2\sqrt{Kt}}\right). \tag{21}$$

3.2. A single layer of finite depth

When the upper boundary δ is not located at infinity, the required solution is composed of an infinite series of source solutions, chosen to ensure vanishing concentration at $z = 0$ and vanishing flux at $z = \delta$. The distribution of the contact time is found to be:

$$\Phi_T(t\,|h) = \frac{1}{4t\sqrt{\pi K t}} \left\langle 2h \exp\left(-\frac{h^2}{4Kt}\right) + 2 \sum_{k=1}^{\infty} (-1)^k \right. \tag{22}$$

$$\left. \left[(2k\delta + h) \exp\left(-\frac{(2k\delta + h)^2}{4Kt}\right) - (2k\delta - h) \exp\left(-\frac{(2k\delta - h)^2}{4Kt}\right)\right] \right\rangle$$

and the cumulative probability is:

$$P[T \leq t] = \operatorname{erfc}\left(\frac{h}{2\sqrt{Kt}}\right) + $$

$$+ \sum_{k=1}^{\infty} (-1)^k \left[\operatorname{erfc}\left(\frac{2k\delta + h}{2\sqrt{Kt}}\right) - \operatorname{erfc}\left(\frac{2k\delta - h}{2\sqrt{Kt}}\right)\right]. \tag{23}$$

3.3. Two-layer solution, both layers homogeneous

Let the inner layer (whose properties are denoted with a subscript "i") span the range $0 \leq z \leq \lambda$ (it makes no difference to the solution whether the inner layer is based at 0 or at z_c – the important thing will be seen to be the ratio of its depth to its diffusivity). The outer layer (properties denoted by subscript "o") spans the range $\lambda \leq z \leq \delta$. We solve the diffusion equation in each domain for $p_i(z, t)$ and $p_o(z, t)$, matching the flux and concentration at the interface $z = \lambda$. Details are given in Appendix 1a.

The result for the Laplace transform of the distribution of contact time is:

$$\overline{\Phi_T}(s \mid h) = \frac{\cos[\beta(\delta - h)]}{\cos[k\beta\lambda] \cos[\beta(\delta - \lambda)] - k \sin[k\beta\lambda] \sin[\beta(\delta - \lambda)]} \tag{24}$$

where:

$$k = \sqrt{K_o/K_i}, \quad \beta = iq_o = i\sqrt{s/K_o}. \tag{25}$$

Our method of inversion to obtain $\Phi_T(t \mid h)$ is given in Appendix 2. This has been performed numerically, since no simple closed form can be given.

3.4. Two layer solution, surface layer inhomogeneous

As the marked particle approaches the surface, it encounters turbulence of shorter timescale and thus makes more "velocity choices" in traversing a given vertical distance. Therefore the contact time is highly affected by the most resistant part of the diffusion pathway. We anticipated obtaining a superior estimate of the distribution of contact time by allowing the surface layer to be inhomogeneous.

We write $K_i = v(z - d)$, where the velocity scale v and the offset length d (which is not necessarily equal to the normal micrometeorological displacement length) may be chosen at will; it is not necessary that $K_i(\lambda) = K_o$, i.e., a discontinuity in the eddy diffusivity at the interface between the two layers is permitted.

Details of the derivation are given in Appendix (1b). The Laplace transform of the distribution of contact time is found to be:

$$\overline{\Phi_T}(s \mid h) = \frac{\dfrac{v}{\pi K_o \beta} \cos[\beta(\delta - h)]}{\sin[\beta(\delta - \lambda)]\, Q_0(\beta) + \sqrt{\dfrac{v(\lambda - d)}{K_o}} \cos[\beta(\delta - \lambda)]\, Q_1(\beta)} \tag{26}$$

where, as earlier, $\beta = i\sqrt{(s/K_o)}$. The Q's are functions of β, given by:

$$Q_0(\beta) = J_0[l_\lambda \beta] N_0[l_c \beta] - N_0[l_\lambda \beta]] J_0[l_c \beta] \tag{27a}$$

$$Q_1(\beta) = J_1[l_\lambda \beta] N_0[l_c \beta] - N_1[l_\lambda \beta]] J_0[l_c \beta] \tag{27b}$$

where J and N are Bessel functions of the first and second kind, and:

$$l_\lambda = 2\sqrt{\frac{K_o(\lambda - d)}{v}}, \qquad l_c = 2\sqrt{\frac{K_o(z_c - d)}{v}}. \tag{28}$$

4. Analytical Solution for the Footprint

Our solution for the footprint again splits the PBL into outer and inner layers, both homogeneous. We follow much the same procedure as in Appendix (1a). The inner layer is again defined to span $0 \leq z \leq \lambda$, and we write the inner-layer concentration as the sum of a source solution:

$$u_i(z, t) = \frac{1}{\sqrt{\pi K_i t}} \exp\left(\frac{-z^2}{4K_i t}\right) \tag{29}$$

chosen to ensure unit mass above $z = 0$ and zero vertical flux at $z = 0$) and a solution w_i vanishing at $t = 0$ and ensuring satisfaction of boundary and matching conditions. The solution for the Laplace transform of the vertical flux at h due to a source at $z = 0$ is:

$$\overline{\Psi_T}(s \mid h) = \frac{\sin[\beta(\delta - h)]}{\cos\left[\dfrac{\beta\lambda}{k}\right] \sin[\beta(\delta - \lambda)] + k \sin\left[\dfrac{\beta\lambda}{k}\right] \cos[\beta(\delta - \lambda)]} \tag{30}$$

where again $\beta = i\sqrt{(s/K_o)}$, and (note the difference from earlier) $k = \sqrt{(K_i/K_o)}$.

5. Lagrangian Stochastic Model

No experimental data exist with which to test the preceding theory. Nor does a direct test ever seem likely to be possible, because, in the case of the distribution of contact time, one would need an absolutely non-buoyant tracer. We decided therefore to compare the K-theory results with the prediction of a Lagrangian stochastic (LS) model.

We have used a model which has been proven to be appropriate for a turbulent medium in which the pdf for the Eulerian vertical velocity is Gaussian. The velocity pdf in the PBL is skewed. Luhar and Britter (1989) and Weil (1990) present (similar) Lagrangian stochastic models conforming to the selection criteria laid out by Thomson (1987), but in both cases the surface layer is neglected and the lower boundary condition is a vanishing turbulent velocity scale σ_w at $z = 0$. For our problems, the critical region is the surface layer: the distribution of contact time is strongly dependent on surface roughness. Therefore we need a good model of the surface layer. A general formulation of the surface-layer velocity pdf and subsequent development of an appropriate LS model allowing for velocity skewness, while certainly warranted, is outside the scope of this paper.

We are concerned only with vertical motion. Our LS model is that given originally (in the form of a Markov chain for the dimensionless velocity $w/\sigma_w(z)$) by Wilson *et al.* (1983: the model WTK″) and proven satisfactory (for the case of Gaussian inhomogeneous turbulence) by Thomson (1984, 1987), namely:

$$\mathrm{d}w = -\frac{\mathrm{d}t}{\tau_L} w + \frac{1}{2}\frac{\partial \sigma_w^2}{\partial z}\left[1 + \frac{w^2}{\sigma_w^2}\right]\mathrm{d}t + \sqrt{\frac{2\sigma_w^2}{\tau_L}}\,\mathrm{d}\xi \tag{31a}$$

$$\mathrm{d}z = w\,\mathrm{d}t \tag{31b}$$

$$\mathrm{d}t = \mu\tau_L(z) \tag{31c}$$

where $\mathrm{d}\xi$ is chosen from a Gaussian distribution with mean zero and variance $\mathrm{d}t$, and $\mu = 0.1 \ll 1.0$. The model is fully determined by specifying the scales $\sigma_w(z)$ and $\tau_L(z)$, and we given our choice in Section (6). Please note that we have treated the advection velocity U (required to obtain the footprint) as a parameter which can be specified independently of the surface roughness or canopy height.

6. Results

Our analytical solutions apply most naturally to the Convective Planetary Boundary Layer (CBL), which in essence consists of an outer, well-stirred layer (the "mixed layer") and a surface layer. There is nothing in principle to prevent application of our solution to the neutral and stable cases. The inner layer in those cases will be as easy to parameterise as for the convective case, because the surface-layer profiles are well established; greater difficulty might surround choice of an outer-layer K. In any case, we demonstrate our solutions here for the unstable case only.

6.1. Velocity statistics and eddy diffusivities in the outer CBL

It has been found that the turbulence in the outer part of the CBL, or "mixed-layer," obeys "mixed-layer scaling". The mixed-layer velocity and length scales are:

$$w_* = \left(\delta \frac{g}{T_o} \frac{Q_H}{\rho C_p}\right)^{1/3}; \qquad \delta. \tag{32}$$

Hicks (1985) suggests for the standard deviation of vertical velocity in the mixed-layer, the formula

$$\sigma_{wo} = (1.2\, u_*^2 + 0.35\, w_*^2)^{1/2} \qquad 0.1 < z/\delta \leq 0.9 \tag{33}$$

which we have used for the outer layer right up to $z = \delta$.

Luhar and Britter recommend for the Lagrangian timescale in the CBL the relationship:

$$\tau_L = \left[1.5 - 1.2\left(\frac{z}{\delta}\right)^{1/3}\right] \frac{\sigma_w^2 \delta}{w_*^3} \tag{34}$$

but we have here used the simpler recommendation of Sawford and Guest (1987), namely:

$$\tau_{Lo} = \frac{\delta}{w_*} \alpha_{SG} \frac{\sigma_w^2}{w_*^2}. \tag{35}$$

We chose $\alpha_{SG} = 2.5$. We found only very small alterations in the contact time if the more complex formula is used.

The outer-layer diffusivity is $K_o = \sigma_w^2 \tau_L$.

6.2. Velocity Statistics and Eddy Diffusivities in the Surface Layer

In keeping with the analysis of flux-gradient relationships given by Dyer and Bradley (1982), we use for the profile of eddy diffusivity in the inner layer:

$$K(z) = \max\left[\frac{u_* h_c}{2}, 0.4 u_* (z - d_*) \sqrt{1.0 - 14.0 \frac{z - d_*}{L_{MO}}}\right]. \tag{36}$$

Here we have modified their expression for the eddy diffusivity for water vapour, which we generalise to any mass constituent, for application well above the roughness elements by the standard though artificial device of allowing a displacement length $d_* = (2/3)h_c$. We have estimated K at the crop height by $K(h_c) = 0.5 u_* h_c$ in accordance with the findings of Legg *et al.* (1986) for a wind-tunnel canopy. This formulation remains satisfactory if the crop height becomes very small.

For the unstable surface layer, Panofsky *et al.* (1977) recommend

$$\sigma_w = u_*\left(1.6 + 2.9\left(\frac{z - d_*}{-L_{MO}}\right)^{2/3}\right)^{1/2} \tag{37}$$

which they show to provide a reasonable fit to observations as high as $z/|L_{MO}| = 7$. For the timescale in the inner layer, we use:

$$\tau_L(z) = \frac{K(z)}{\sigma_W^2} \tag{38}$$

where K and σ_W are given by Equations (36, 37).

6.3. Matching the Inner and Outer Layers

In applying our analytical solutions, we identify the inner layer with the well-understood atmospheric surface layer (ASL), writing $\lambda = n|L_{MO}|$. It is desirable that the solutions be independent of n, and this proves to be the case for the contact-time solutions, at least over the range $1 \leq n \leq 4$. We usually use $n = 2$, because for $z < \sim 2|L_{MO}|$, surface-layer statistics are well described by Monin-Obukhov scaling.

In order to avoid discontinuities in σ_w and τ_L in the Lagrangian stochastic simulations, we used (at given z) the *minimum* of the inner- and outer-layer values. In the case of σ_w, the surface-layer formulation (which as noted above has been shown to fit observations out to $7|L_{MO}|$) matches the outer-layer expression at a height of order $10|L_{MO}|$. There is no reason to believe that this convenient interpolation is unreasonable. In the case of τ_L, matching occurs even higher. But since the Lagrangian stochastic solutions for the contact time are negligibly altered by adopting the alternative prescription of Luhar and Britter (1989), with which the inner-layer τ_L matches at about $10|L_{MO}|$, we have been content to use the simpler formula.

6.4. Aerodynamic Resistance of the Inner Layer

Where an inner layer is recognised in the following results, it spans the range $h_c \leq z \leq \lambda$ of the real world, but $0 \leq z \leq \lambda$ in the "model" world. Then if $K(z)$ is the *actual* eddy diffusivity, the *actual* aerodynamic resistance to motion across the inner layer is defined by:

$$r_{\lambda - h_c} = \int_{z=h_c}^{\lambda} \frac{\mathrm{d}z}{K(z)}. \tag{39}$$

If, in the case of a homogeneous inner layer, we wish the inner diffusivity K_i to be such that the total aerodynamic resistance of the layer is correct, we need only choose K_i to satisfy

$$\frac{\lambda}{K_i} = r_{\lambda - h_c} \tag{40}$$

with $r_{\lambda - h_c}$ specified by Equation (39).

The idea of pinning down the inner-layer properties by enforcing correct resistance has been found to be almost good enough. But in fact it is better to follow what we initially did by mistake, and write:

$$\frac{\lambda - h_c}{K_i} = r_{\lambda - h_c}. \tag{40'}$$

This is the choice used in the results to follow. Should one be interested in the time until penetration of a stomatal cavity, for example, one could add to the total inner-layer resistance the appropriate aerodynamic, leaf boundary-layer, and stomatal resistances.

6.5. Solutions for the Contact Time

Figure (2) compares the Lagrangian stochastic and analytical solutions for the case of a particle marked at $h = 100$ m in a CBL characterised by $\delta = 2$ km, $Q_H = 200$ W m^{-2}, $u_* = 0.35$ m s^{-1} above crops of height $h_c = (0.1, 0.5, 5.0, 20)$ m. Under these conditions, $w_* = 2.25$ m s^{-1}, $\sigma_{wo} = 1.37$ m s^{-1}, $\tau_{Lo} = 826$ s, and the mixed-layer diffusivity is $K_o = 1550$ m^2 s^{-1}.

Fastest passage time is of course predicted by the 1-layer model using the mixed-layer (outer) diffusivity, and only the tail end of that solution can be seen in Figure 2. We have not bothered to display the two-layer solutions with K_i set to $K(\lambda)$, $K(h_c)$ which (underestimate, overestimate) the contact time.

With the earlier-given choice (Equation 40′) for inner-layer resistance, the two-layer analytical solutions agree well with the Lagrangian stochastic model, there being notable discrepancy only for the "smoothest" case ($h_c = 0.1$). For reasonable values of n (the ratio of inner layer depth to the magnitude of the Monin-Obukhov length), the solutions do not depend on n. The more complex solution, with the inhomogeneous inner layer, has been applied with the parameters ν, d chosen to ensure the correct diffusivity at canopy height ($0.5 u_* h_c$), and an effective resistance for the inner-layer given, as for the case when both layers are homogeneous, by equation (40′). For this problem, the more complex approach of allowing an inhomogeneous inner layer yields so little advantage over the simpler treatment as to be not worth the extra trouble of needing access to subroutines for the Bessel functions.

As for the meteorological implications, a very marked dependency of the contact time upon surface roughness can be seen. And even in the "forest" case, $h_c = 20$ m, 20% of all particles spotted at $h = 100$ m last "contacted" the surface more than 5 hours prior to being sighted.

Figure 3 examines the sensitivity of the contact time to observation height, h. Large changes in h have little impact on contact time. The explanation is that motion in the outer layer is very rapid, and most of the resistance to surface contact is offered near the top of the crop.

An anonymous reviewer gave the following order-of-magnitude argument, which exploits this insensitivity to the details of mixing in the outer layer and leads to a reasonable prediction of the contact time. Assume that the outer layer is well mixed, with concentration $C_o(t)$, and that the concentration gradient in the inner layer is:

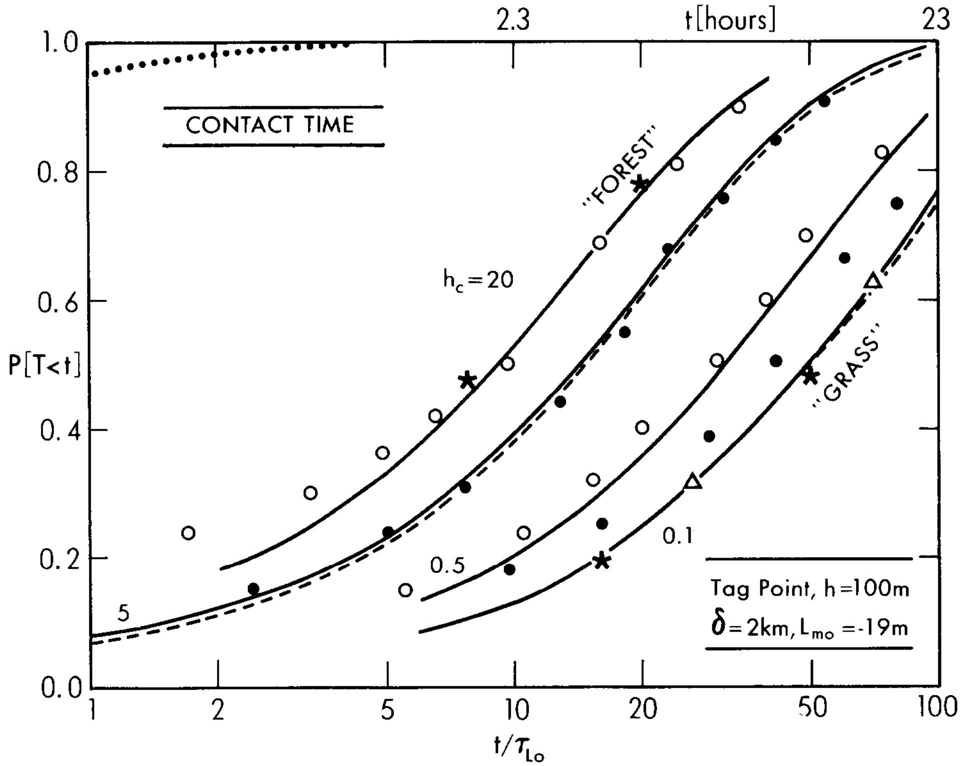

Fig. 2. Lagrangian stochastic and analytical solutions for the cumulative distribution of contact time

$$P[T<t] = \int_0^t \Phi_T(t' \mid h)\, dt'$$

versus t/τ_{Lo} (time scaled on the mixed-layer timescale) for a marking height $h = 100$ m. CBL and surface layer characterised by: $\delta = 2$ km, $w_* = 2.25$ m s^{-1}, $\sigma_{wo} = 1.38$ m s^{-1}, $\tau_{Lo} = 843$ s, $K_o = 1615$ m^2 s^{-1}, $u_* = 0.35$ m s^{-1}, $L_{MO} = -19$ m, $Q_H = 200$ W m^{-2}. For the case with $h_c = 5$ m, the "real world" resistance between the crop and the height $\lambda = 2|L_{MO}| = 37.7$ m is 7.68 s m^{-1}; thus the analytical solution with $n = 2$ uses, in accordance with Equation (40′), $K_i = 4.26$ m^2 s^{-1} and $k = \sqrt{(K_o/K_i)} = 19.5$. ○ ●, Lagrangian stochastic model. ————, 2-layers, both homogeneous. – – – – – – –, 2-layers, inner inhomogeneous. ········, single-layer (Equation (23)).

$$\frac{\partial C_i}{\partial z} = \frac{C_o(t)}{\lambda - h_c}.$$

The total mass remaining is approximately $C_o(t)\delta$, so conservation of mass is expressed (approximately) by

$$\frac{\partial C_o \delta}{\partial t} = -K_i \frac{C_o}{\lambda - h_c}$$

and a "mass-loss timescale" is therefore $(\lambda - h_c)\ \delta/K_i$. One expects that 63% of contact times will be smaller than this timescale. This supports the idea that the

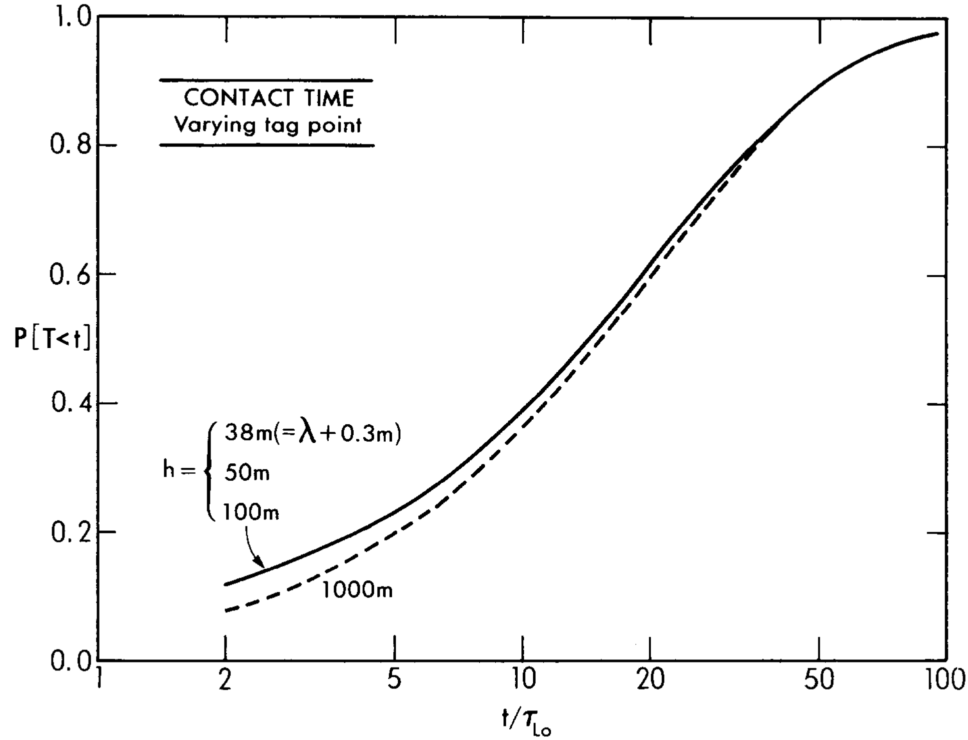

Fig. 3. Solutions showing the sensitivity of the cumulative contact time, $P[T < t]$, to the height h at which the parcel is tagged. All solutions were obtained with the two-layer model (both layers homogeneous) and $n = 2$. The CBL (and surface layer) are characterised by: $\delta = 2$ km, $w_* = 2.25$ m s^{-1}, $\sigma_{wo} = 1.38$ m s^{-1}, $\tau_{Lo} = 843$ s, $K_o = 1615$ m^2 s^{-1}, $u_* = 0.35$ m s^{-1}, $L_{MO} = -19$ m, $Q_H = 200$ W m^{-2}, $h_c = 5$ m. The "real world" resistance between the crop height and the height $\lambda = 2|L_{MO}| = 37.7$ m is 7.68 s m^{-1}; thus the analytical solutions use, in accordance with Equation (40′), $K_i = 4.26$ m^2 s^{-1} and $k = \sqrt{(K_o/K_i)} = 19.5$.

details of mixing in the outer layer are not crucial to a prediction of the contact time. The reviewer also argues that since in the case of the footprint the surface is a reflector, the timescale for decay of the flux (at h due to a source at h_c) is much shorter, simply δ/w_*.

6.6. Solution for the footprint

We examine the footprint when the atmospheric condition is as defined in Section 6.4, and we assume an advection velocity of $U = 3$ m/s. Our intuitive inequality for the range of the footprint gives 0.2 km $\leq x \leq$ 2.5 km for this case, which agrees reasonably well with the Lagrangian solution.

The analytical solution for the footprint (see Figure 4) is disappointing. There is an unacceptable dependence upon the choice of inner-layer depth; and no choice of n gives satisfactory agreement with the Lagrangian solution for more than one choice of source height (which is set equal to crop height). The poor solution for the footprint is a mystery, given the successful prediction of contact time; perhaps

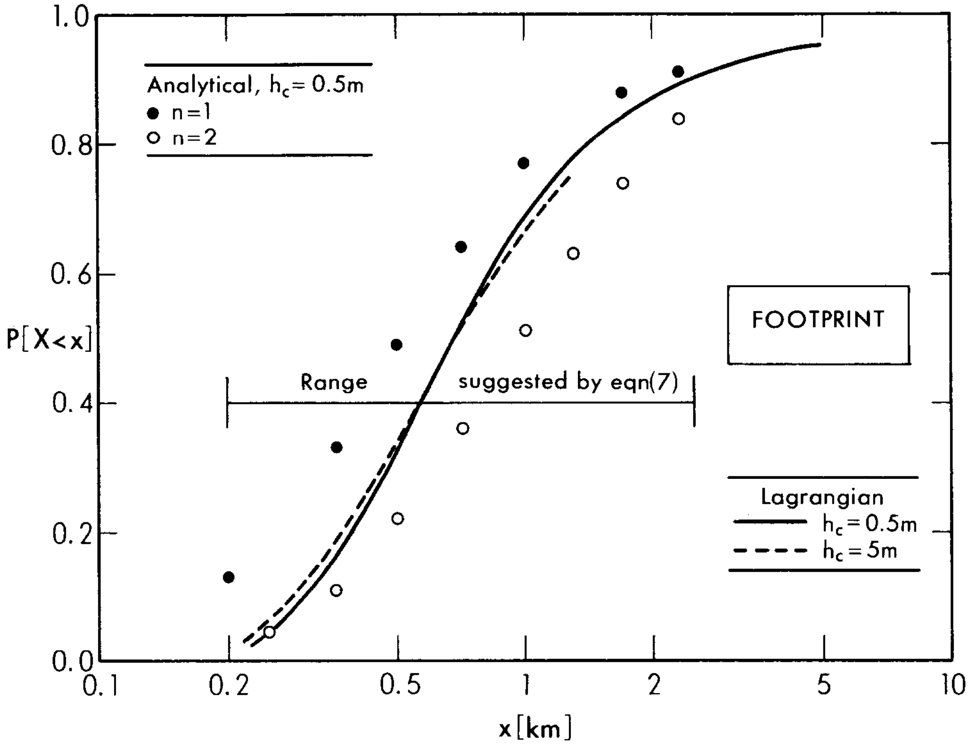

Fig. 4. Lagrangian stochastic and analytical solutions for the cumulative footprint

$$P[X < x] = \int_0^x \Psi_x(x' \mid H)\, dx'$$

versus x for an observation height $h = 100$ m. Analytical solutions assume canopy height (=source height) $h_c = 5$ m, $\delta = 2$ km, $w_* = 2.25$ m s^{-1}, $\sigma_{wo} = 1.38$ m s^{-1}, $\tau_{Lo} = 843$ s, $K_o = 1616$ m^2 s^{-1}, $u_* = 0.35$ m s^{-1}, $L_{MO} = -19$ m, $Q_H = 200$ W m^{-2}.

the explanation is that much longer travel times are involved in the contact time – but in the case of the footprint, the source is at the ground (or h_c at least) and so the travel time for most particles does remain large with respect to the Lagrangian timescale at the source. The reviewer is of the opinion that the difficulty with the analytical solution is that in the case of the footprint, the details of mixing in the outer layer *are* important.

For a tenfold reduction in crop height, our Lagrangian stochastic solution is little changed. At first sight this seems to contradict the contention of Leclerc and Thurtell (1990) that the footprint is highly sensitive to surface roughness. The explanation is simply that we have treated the lateral velocity U as an independent parameter, whereas Leclerc and Thurtell, in their two-dimensional Lagrangian stochastic simulations of the footprint, included a height-dependent advection velocity with standard dependence upon roughness length.

7. Conclusion

We have defined several "source area functions" in Lagrangian terminology and explored how these differ in the range of the upwind surface they cover. Simple two-layer models have been shown to give satisfactory estimates of the distribution of the time since last (or until next) surface contact for a parcel spotted aloft – at least if the Lagrangian stochastic model which we have used is adequate. These contact times are amazingly long. By way of example, for a parcel tagged at $h = 100\,\mathrm{m}$, in a CBL characterised by depth $\delta = 2\,\mathrm{km}$, surface friction velocity $u_* = 0.35\,\mathrm{m\,s^{-1}}$, and $L_{MO} = -19\,\mathrm{m}$, we find a 20% probability that the most recent excursion below crop height $h_c = 20\,\mathrm{m}$ occurred more than 5 hours earlier. For the same conditions over a shorter crop, $h_c = 0.5\,\mathrm{m}$, this increases to 20 hours. Such statistics bear some relevance to problems involving diffusion of an atmospheric species which is subject to rapid decay in activity or toxicity: for example, numerous pests and diseases affecting plants and animals (foot-and-mouth disease, etc.) can be spread by the wind, but are subject to decay through dehydration.

Our analytical solution for the footprint is not very good; but we have suggested an intuitive inequality (which could be refined) for the footprint which might perhaps serve as the "manageable analytical expression capable of giving order-of-magnitude predictions of upwind areas most likely to affect a point measurement at a given height" which was identified as needed by Schuepp *et al.* (1990).

The effect of velocity skewness has been neglected here, both in the analytical solutions and in the Lagrangian stochastic model. The footprint and the distribution of contact time should be re-examined with proper allowance for skewness. That task awaits the development of a soundly-based Lagrangian stochastic model.

Appendix 1. Details of Two-Layer Solutions for Passage Time

(a) Both Layers Homogeneous:

In the outer layer the solution can be decomposed as

$$p_o(z, t) = u_o(z, t) + w_o(z, t) \tag{A1–1}$$

where u_o is the solution for a unit source at $z = h$, $t = 0$:

$$u_o(z, t) = \frac{1}{2\sqrt{\pi K_o t}} \exp\left[-\frac{(z-h)^2}{4K_o t} \right]. \tag{A1–2}$$

Then w_o must satisfy the diffusion equation in the outer layer, must vanish at $t = 0$, and must be such that the boundary and matching conditions on $u_o + w_o$ at $z = \lambda$ and $z = \delta$ are satisfied. In the inner layer, no source solution is needed, so we write $p_i(z, t) = w_i(z, t)$ with appropriate boundary conditions.

Taking the Laplace transform of the diffusion equations with respect to time, we obtain:

$$\frac{\partial^2 \overline{w_o}}{\partial z^2} - q_o^2 \overline{w_o} = 0 \tag{A1-3}$$

$$\frac{\partial^2 \overline{w_i}}{\partial z^2} - q_i^2 \overline{w_i} = 0 \tag{A1-4}$$

where

$$q_o^2 = \frac{s}{K_o} \qquad q_i^2 = \frac{s}{K_i}. \tag{A1-5}$$

The Laplace transform of the source solution is:

$$\overline{u_o}(z, s) = \frac{1}{2\sqrt{sK_o}} \exp\left(-\sqrt{\frac{s}{K_o}} |z - h| \right) \tag{A1-6}$$

and the boundary and matching conditions to be applied are:

$$\overline{w_i}(0, s) = 0 \tag{A1-7}$$

$$\overline{w_i}(\lambda, s) = \overline{u_o}(\lambda, s) + \overline{w_o}(\lambda, s) \tag{A1-8}$$

$$K_i \left(\frac{\partial \overline{w_i}}{\partial z} \right)_{z=\lambda} = K_o \left(\frac{\partial \overline{u_o}}{\partial z} \right)_{z=\lambda} + K_0 \left(\frac{\partial \overline{w_o}}{\partial z} \right)_{z=\lambda} \tag{A1-9}$$

$$\left(\frac{\partial (\overline{u_o} + \overline{w_o})}{\partial z} \right)_{z=\delta} = 0. \tag{A1-10}$$

A good starting point is:

$$\overline{w_i}(z, s) = A \sinh[q_i z] \tag{A1-11}$$

$$\overline{w_o}(z, s) = C \cosh[q_o(z - \delta)] - \frac{B}{q_o} \sinh[q_o(z - \delta)]. \tag{A1-12}$$

The concentration boundary condition at $z = 0$ is then automatically satisfied, and provided we choose

$$B = \left(\frac{\partial \overline{u_o}}{\partial z} \right)_\delta = -\frac{q_o}{2\sqrt{sK_o}} \exp(-q_o(\delta - h)), \tag{A1-13}$$

so also is the flux condition at $z = \delta$. A and B are determined by the matching conditions.

(b) Inner Layer Inhomogeneous:

The inner-layer solution now satisfies:

$$\frac{1}{(z-d)}\frac{\partial}{\partial(z-d)}\left[(z-d)\frac{\partial\overline{w_i}}{\partial(z-d)}\right]-\frac{s}{\nu(z-d)}\overline{w_i}=0. \tag{A1–14}$$

The solution is of the form (Gradshteyn and Ryzhik, 1980):

$$\overline{w_i}(z,s)=aJ_0[\gamma\sqrt{z-d}]+bN_0[\gamma\sqrt{z-d}] \tag{A1–15}$$

where J_0 and N_0 are Bessel functions of the first and second kind and

$$\gamma^2=-4\frac{s}{\nu}. \tag{A1–16}$$

Since a, b are arbitrary, we choose $\gamma = +2i\sqrt{(s/\nu)}$.

Applying the boundary and matching conditions at $z = z_c$, $z = \lambda$, and $z = \delta$, we obtain, after some tedious algebra, Equation (26).

Appendix 2. Inversion of Laplace Transforms

Our method of inverting the Laplace transforms is standard, and is well covered by Churchill *et al.* (1974). Assume that the function to be inverted is $f(s)$ and that it may be expressed as:

$$f(s)=\frac{q(s)}{p(s)}. \tag{A2–1}$$

In our problems, it is convenient to regard p and q as functions of β rather than of s; the connection is straightforward because we have in all (our) cases $s = -K_o\beta^2$.

We begin by seeking the positive roots of $p(\beta) = 0$, which we label β_{*j}; these we have obtained by brute force: $p(\beta)$ has been evaluated as β steps upwards from zero in small increments (typically $\Delta\beta = 2\pi/(10^4\ (\delta - h_c))$ [radians m^{-1}]).

By virtue of the Residue Theorem, the desired solution is

$$f(t)=\sum_j R_j \tag{A2–2}$$

where the R_j are the residues of $e^{st} f(s)$ at the roots of $p(s)$. Then provided the poles are simple (which is the case in our problems), we may write:

$$f(t)=\sum_j \frac{\exp(s_{*j}t)q(s_{*j})}{\left(\frac{\partial p}{\partial\beta}\right)_{\beta_{*j}}\left(\frac{\partial\beta}{\partial s}\right)_{\beta_{*j}}} \tag{A2–3}$$

where, as mentioned above, $s_{*j} = -K_o\beta_{*j}^2$. To give a specific example, the residues for the case of the contact-time solution involving two homogeneous layers are:

$$R_j=\frac{2K_o\beta_{*j}\exp[-K_o\beta_{*j}^2t]\cos[\beta_{*j}(\delta-h)]}{k\delta\sin[k\beta_{*j}\lambda]\cos[\beta_{*j}(\delta-\lambda)]+(\delta-\lambda+k^2\lambda)\cos[k\beta_{*j}\lambda]\sin[\beta_{*j}(\delta-\lambda)]} \tag{A2–4}$$

Because of the weighting $\exp(-K_o t\beta_{*j}^2)$, except for very small t, only the first several roots make a significant contribution.

Acknowledgements

Both authors gratefully acknowledge the support of the Natural Sciences and Engineering Research Council of Canada (NSERC) and the Atmospheric Environment Service (AES) of Environment Canada.

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