

On stationary equivalent modons in an eastward flow

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Modons correspond to isolated dipole vortex solutions of the quasigeostrophic equations. They have been proposed as prototype models for some geophysical (and plasma) vortices. The classical modon solution on a β plane does not permit a Rossby wave field in the exterior or far-field region of the modon. However, it is qualitatively known that the gravest mode associated with a normal mode decomposition of a stationary modon in a continuously stratified fluid of finite depth necessarily contains a Rossby wave tail in the downstream region if the background flow is eastward. The same effect can be formally recreated in an equivalent-barotropic model of a stationary modon embedded in a constant eastward zonal flow. An analytical solution to this problem satisfying the correct upstream radiation condition is presented and its dynamical characteristics are discussed.

I. INTRODUCTION

Butchart *et al.*¹ (also see Haines and Marshall²), have qualitatively proved that the exterior-region horizontal streamfunction field associated with the gravest mode in a vertical mode decomposition of a stationary baroclinic modon embedded in an eastward flow of a continuously stratified fluid of finite depth necessarily contains Rossby waves. In fact, the Butchart *et al.*¹ result is rather general and applies other eddy solutions besides modons. The key dynamical assumptions are that the quasigeostrophic eddy is stationary and embedded in an eastward zonal flow that is not meridionally sheared on a β plane, and that the geostrophic pressure associated with the eddy decays to zero at infinity in both the upstream and downstream regions. The Butchart *et al.*¹ result has important implications in attempting to model large-scale anomalous atmospheric circulation patterns such as atmospheric blocking.²⁻⁵ The principal purpose of this paper is to present an explicit solution for a stationary modon in the equivalent-barotropic context, which contains a downstream Rossby wave tail and that satisfies the correct upstream radiation condition.

The equations of motion governing the horizontal structure of the streamfunction for the gravest mode associated with the normal mode decomposition introduced by Butchart *et al.*¹ are formally the same as those obtained from the equivalent-barotropic potential vorticity equation. In classical *steadily traveling* modon theory,^{6,7} the exterior Rossby wave field is eliminated by demanding that the propagation velocity of the modon *not* be an allowed Rossby wave phase velocity. In the context of a *stationary* isolated equivalent modon in an ambient zonal flow, the exterior Rossby wave field is eliminated if the ambient zonal flow is westward. If the ambient or background zonal flow is eastward, it necessarily follows that the exterior streamfunction field contains Rossby waves. The reason that these two physical situations are not isomorphic is because the equivalent-barotropic equation is not invariant under Galilean transformations. By an *isolated* solution we mean, following Flierl *et al.*,⁷ a solution that contains re-

gions with closed streamlines in the stationary context, or closed streaklines in the steadily traveling context, *and* for which the eddy component of the total streamfunction decays to zero at infinity. In such a configuration, the fluid associated with the eddy interior or closed streamline region is isolated from the surrounding exterior or open streamline fluid, in the sense that there is no exchange of fluid between the two regions.

The subtlety associated with determining a solution for a modon with a Rossby wave tail resides in constructing the external Rossby wave field in such a manner so as to satisfy the appropriate radiation condition upstream of the modon and the matching conditions on the modon boundary. The solution presented here will satisfy the correct upstream radiation condition. The pressure is continuous at the modon boundary. However, unlike the classical modon solution having no exterior wave tail, the azimuthal velocity of the wave-like solution presented here will *not* be continuous at the modon boundary. Consequently, the modon boundary in our solution will correspond to a vortex sheet with a zero potential vorticity jump.

Because of the mathematical similarity between the solution for a steadily traveling equivalent modon written with respect to the comoving reference frame, and a stationary modon embedded in a constant zonal flow, the solution presented here also solves the problem of determining the structure of a steadily traveling radiating equivalent modon when the translation velocity is an allowed Rossby wave phase velocity. The solution presented here will also be presumably useful in modon perturbation and modulation theory in the situation where the modon is undergoing nonadiabatic adjustment and a field of external Rossby waves will be an intrinsic aspect of the time-dependent behavior. Examples of this kind of perturbation might include north-south oscillations observed in eastward-traveling modons if the initial propagation direction is tilted slightly away from an exact east-west configuration,⁹ or the problem of determining the topographic steering of a modon¹⁰ over slowly varying topography.

II. PROBLEM FORMULATION AND SOLUTION

The nondimensional equivalent-barotropic potential vorticity equation can be written in the form⁸

$$(\Delta - 1)\psi_t + \psi_x + J(\psi, \Delta\psi) = 0, \quad (1)$$

where $\Delta \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$ and $J(A, B) \equiv A_x B_y - B_x A_y$ with (x, y) the eastward and northward coordinates, respectively, and t is the time. Subscripts with respect to (x, y, t) indicate the appropriate partial derivatives. The two-dimensional velocity field $\mathbf{v} = (u, v)$ is related to the geostrophic pressure $\psi(x, y, t)$ via $\mathbf{v} = \hat{e}_3 \times \nabla\psi \equiv (-\psi_y, \psi_x)$.

We seek a *steady* solution to (1) in the form

$$\psi = -Uy + \varphi(x, y), \quad (2)$$

where $\varphi(x, y)$ will be referred to as the eddy streamfunction and the $-Uy$ term corresponds to a constant eastward zonal flow if $U > 0$ (which is what we assume). Substitution of (2) into (1) leads to

$$J(\varphi - Uy, \Delta\varphi + y) = 0, \quad (3)$$

which can immediately be integrated to imply

$$\varphi - Uy = \mathcal{F}(\Delta\varphi + y), \quad (4)$$

where $\mathcal{F}(\ast)$ determines the relationship between the vorticity and streamfunction fields.

Following classical modon theory, we define the *exterior* region to be the region containing all those streamlines that extend to infinity [given by $r \equiv (x^2 + y^2)^{1/2} > a$], the *interior* region to be the region containing all those streamlines that do not extend to infinity (given by $r < a$), and the modon boundary will be given by $r = a$. The modon boundary must correspond to a streamline and we may, without loss of generality, demand

$$\varphi = Uy, \quad \text{on } r = a. \quad (5)$$

It is possible to interpret (5) as a necessary condition for the continuity of the potential vorticity at $r = a$.

It follows from (4) and the fact that $\varphi \rightarrow 0$ as $r \rightarrow \infty$, that

$$\mathcal{F}(\ast) = -U\ast, \quad \text{for } r > a, \quad (6a)$$

and we introduce the ansatz⁶

$$\mathcal{F}(\ast) = -k^{-2}\ast, \quad \text{for } r < a, \quad (6b)$$

where k is called the modon wave number. Substitution of (6) into (4) yields, respectively,

$$\Delta\varphi + U^{-1}\varphi = 0, \quad \text{for } r > a, \quad (7a)$$

$$\Delta\varphi + k^2\varphi = (Uk^2 - 1)y, \quad \text{for } r < a. \quad (7b)$$

It follows from (7a) that if $U < 0$, there is no Rossby wave field in $r > a$. If $U > 0$, it follows that necessarily a Rossby wave field exists in $r > a$. This latter situation is the case we are interested in here. The solution to (7) must be obtained subject to (5) and the "no upstream waves" condition¹¹⁻¹³

$$\lim_{r \rightarrow \infty} r^{1/2}\varphi(r, \theta) = 0, \quad \forall \theta \in (\pi/2, 3\pi/2), \quad (8)$$

where $\tan(\theta) = y/x$ since we assume $U > 0$.

The solution to (7b) subject to (5) is given by

$$\varphi = [(a/k^2)J_1(kr)/J_1(ka) + k^{-2}(Uk^2 - 1)r]\sin(\theta), \quad (9)$$

for $r < a$. The solution to (7a) subject to (5) and (8) can be constructed in the form¹¹

$$\varphi = \sum_{n=1}^{\infty} \alpha_n \frac{[Y_n(r/U^{1/2})\sin(n\theta) + h_n(r, \theta)]}{Y_n(a/U^{1/2})}, \quad (10a)$$

where

$$h_n(r, \theta) = \sum_{m=1}^{\infty} \beta_{n,m} J_m\left(\frac{r}{U^{1/2}}\right) \sin(m\theta). \quad (10b)$$

There are no cosine terms in the expansion for φ given by (10) because of the boundary condition (5). This will mean that the Rossby wave field will be antisymmetric about $y = 0$, as is the interior solution (9). This is precisely the symmetry observed in Haines and Marshall's² numerical simulations of modons with a downstream Rossby wave tail on a β plane.

Recalling that even (odd) $J_n(\ast)$ functions have the same asymptotic form as the odd (even) $Y_n(\ast)$ as $r \rightarrow \infty$ (see Abramowitz and Stegun¹⁴), the no-upstream waves condition (8) will imply that the coefficients $\beta_{n,m}$ must satisfy the constraints

$$\sin(2n\theta) = \sum_{m=0}^{\infty} (-1)^{m+n+1} \beta_{2n, 2m+1} \sin[(2m+1)\theta], \quad (11a)$$

$$\sin[(2n+1)\theta] = \sum_{m=0}^{\infty} (-1)^{m+n} \beta_{2n+1, 2m} \sin(2m\theta), \quad (11b)$$

for $n = 0, 1, 2, \dots$, in the sector $\pi/2 < \theta < 3\pi/2$. Since the sets $\{\sin[(2n+1)\theta]\}_{n=0}^{\infty}$ and $\{\sin(2n\theta)\}_{n=1}^{\infty}$ both form a complete set of antisymmetric basis functions in the interval $\pi/2 < \theta < 3\pi/2$, it follows from (11a) and (11b) that

$$\beta_{n,m} = \begin{cases} (4/\pi)n(m^2 - n^2)^{-1}, & (n \text{ even}, m \text{ odd}), \\ (4/\pi)m(m^2 - n^2)^{-1}, & (n \text{ odd}, m \text{ even}), \\ 0, & (n - m \text{ even}). \end{cases}$$

It remains to determine the α_n coefficients. We rewrite (10a) in the form

$$\varphi = \sum_{m=1}^{\infty} \left(\sum_{n=1}^{\infty} \alpha_n \Gamma_{n,m}(r) \right) \sin(m\theta), \quad (12a)$$

where

$$\Gamma_{n,m}(r) = \frac{[\delta_{n,m} Y_m(r/U^{1/2}) + \beta_{n,m} J_m(r/U^{1/2})]}{Y_n(a/U^{1/2})}, \quad (12b)$$

where $\delta_{n,m}$ is the Kronecker delta function between n and m . If we apply the boundary condition (5), it follows that

$$\sum_{n=1}^{\infty} \alpha_n \Gamma_{n,m}(a) = Ua\delta_{m,1}, \quad (13)$$

with $m = 1, 2, 3, \dots$. The coefficients in the exterior solution are thus completely determined. As it turns out, relatively

few α_n values need to be computed to be able to give very accurate results. If we recall the fact that $Y_n(a/U^{1/2}) \rightarrow -\infty$ and $J_n(a/U^{1/2}) \rightarrow 0$ as $n \rightarrow \infty$ (Abramowitz and Stegun¹⁴), then $\Gamma_{n,m}(a) \approx \delta_{n,m}$ for sufficiently large n or m . This property will imply relatively few terms of the matrix $[\Gamma_{n,m}]$ are needed for very accurate results. In practice, we found for $n, m \geq 12$ this property held. As a result, when it came to solving for the α_n coefficients from (13), very good results were obtained by approximating the infinite system with the leading 20×20 finite system of equations. (For all cases examined we found $|\alpha_n| < 10^{-7}$ for $n > 15$.) In particular, for $U=a=1$ we found $\alpha_1 \approx 0.97$, $\alpha_2 \approx 0.12$, $\alpha_3 \approx 7.3 \times 10^{-4}$, $\alpha_4 \approx 1.04 \times 10^{-3}$, $\alpha_5 \approx 2.23 \times 10^{-6}$, and so on.

In classical modon theory, the modon wave number is determined by demanding that the radial derivative of the total streamfunction is continuous at the modon boundary.⁶ In the solution presented here, it will *not* be possible to choose the modon wave number k , so that ψ_r is continuous at $r=a$ because there are an infinity of trigonometric terms $[\sin(n\theta)]_{n=1}^{\infty}$ in the exterior region, and only a single $\sin(\theta)$ term in the interior region. There are, consequently, two choices one can make; either determine k by making an additional closure assumption, or do nothing and leave the modon wave number as a free parameter in the solution. We have chosen to do the former by determining the modon wave number by demanding that the $\sin(\theta)$ component of ψ_r be continuous at $r=a$. It is important to emphasize that this choice is arbitrary. Because the solution as constructed satisfies $\psi=0$ on $r=a$, where ψ is the total streamfunction (2), it follows that the continuity of ψ ensures that the *physical* matching conditions of continuity of leading-order geostrophic pressure and normal mass flux required in inviscid fluid dynamics hold at the wave-like modon boundary. The continuity of the azimuthal velocity at the modon boundary, here determined by $\psi_r(a, \theta)$, is *not* a required condition in inviscid fluid dynamics. The implications of this will be more fully discussed in the next section.

Formally, our matching condition on the eddy streamfunction can be expressed as

$$\lim_{r \rightarrow a^-} \int_0^{2\pi} \sin(\theta) \varphi_r(r, \theta) d\theta = \lim_{r \rightarrow a^+} \int_0^{2\pi} \sin(\theta) \varphi_r(r, \theta) d\theta. \quad (14)$$

If (9) and (12a) are substituted into (14), it follows that the modon wave number k will be a nonzero solution of

$$\frac{J_2(ka)}{J_1(ka)} = \frac{k}{aU^{1/2}} \times \sum_{n=1}^{\infty} \alpha_n \frac{[\delta_{n,1} Y_2(a/U^{1/2}) + \beta_{n,1} J_2(a/U^{1/2})]}{Y_n(a/U^{1/2})}. \quad (15)$$

It is straightforward to numerically verify that there will exist a countable infinity of solutions $k=k(a, U)$ to (15)

with $k=0$ the smallest non-negative solution. The *ground-state* wave-like modon will correspond to the first nonzero k that solves (15). For example, if $a=U=1$ we find that the ground-state wave number is given by $k \approx 3.69$.

If there is no wave field in the exterior region, then (15) reduces exactly to the barotropic limit of the Larichev and Reznik⁶ classical modon dispersion relationship. This can be easily seen as follows. A stationary modon with no wave tail, in the present context, implies that we must choose $U < 0$. However, in this situation, the exterior region solution will be written in terms of exponentially decaying modified Bessel functions, which trivially satisfy the no-waves constraint (8). Consequently, there is no $h_n(r, \theta)$ term in (10a) and the $Y_n(r/U^{1/2})$ terms are replaced by $K_n(r/U^{1/2})$ terms. The boundary condition (5) implies that only $K_1(r/U^{1/2})$ contributes to the solution. Substitution of the resulting solution into (14), realizing the integration becomes superfluous, yields the classical barotropic modon dispersion relationship.

The solution just presented is similar in some respects to the stationary modon solution on a rotating sphere presented by Verkley.¹⁵ There are, however, important differences. On the sphere, in contrast to the infinite β plane, it is not necessary to satisfy an upstream radiation condition (for the stationary solution), since presumably energy associated with those spectral components of the downstream Rossby wave tail that propagate (or are advected) eastward would eventually travel entirely around the sphere, and thus would also appear in the upstream region. There is, therefore, no physical requirement to eliminate the upstream Rossby waves associated with the spherical modon solution presented by Verkley. However, as a result, the β -plane *approximations* to the spherical solutions presented in Appendix A of Verkley¹⁵ will not satisfy the correct upstream radiation condition (8) and therefore are not physically admissible solutions on the β plane.

III. DYNAMICAL CHARACTERISTICS

The solution obtained in Sec. II possesses the property that the leading-order geostrophic pressure and normal mass flux is continuous at the modon boundary $r=a$. This is illustrated in Fig. 1, where we show a cross section for $-2 \leq y \leq 2$ on $x=0$ of the *total* streamfunction $\psi \equiv -Uy + \varphi(x, y)$ for $a=U=1.0$ and $k \approx 3.69$. The modon boundary is located at $y=-1$ and $+1$, respectively. We remark that ψ is continuous but *not* differentiable at $r=a$, reflecting the fact that ψ_r is *not* continuous at $r=a$ (however, ψ_θ is continuous at $r=a$). The apparent near smoothness at the modon boundary in the total streamfunction seen in Fig. 1 is a simple numerical consequence of the fact that the radial derivatives associated with the higher harmonics make a relatively small contribution to the overall solution in comparison the $\sin(\theta)$ mode near the modon boundary for these parameter values.

The continuity of ψ and ψ_θ at $r=a$ has an interesting consequence for the regularity of the total potential vorticity $PV \equiv \Delta\psi - \psi + y$ at $r=a$. Clearly, since ψ_r is *not* continuous at $r=a$, it follows that the PV will *not* be continuous

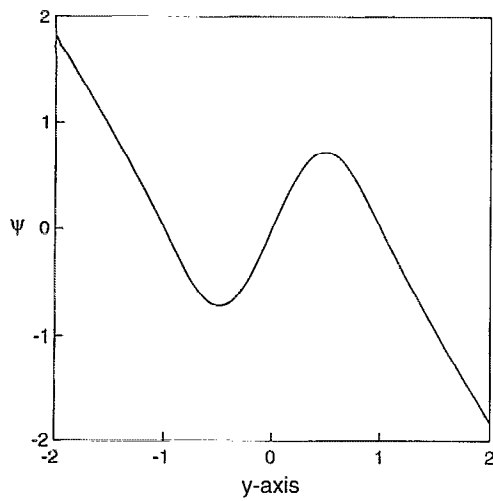


FIG. 1. A y cross section of the total streamfunction field $\psi \equiv -Uy + \varphi$ on $x=0$ for $-2 \leq y \leq 2$. The parameter values are $a=U=1.0$ and $k \approx 3.69$. The modon boundary is located at $y=-1$ and $+1$, respectively.

at $r=a$. However, the limit of the PV as $r \rightarrow a$ does exist. It follows from (6) that

$$\lim_{r \rightarrow a^+} (\Delta\psi - \psi + y) = \lim_{r \rightarrow a^+} -(1 + U^{-1})\psi = 0, \quad (16a)$$

$$\lim_{r \rightarrow a^-} (\Delta\psi - \psi + y) = \lim_{r \rightarrow a^-} -(1 + k^2)\psi = 0, \quad (16b)$$

since ψ is continuous at $r=a$ and $\psi(a, \theta) = 0$ on account of (5). Thus, the required limit exists. Because the limit exists, it follows that the jump in the potential vorticity across the modon boundary, defined by

$$[PV]_a \equiv \lim_{r \rightarrow a^+} PV - \lim_{r \rightarrow a^-} PV,$$

is identically zero, that is $[PV]_a = 0$. The modon boundary, therefore, corresponds to a vortex sheet with a zero potential vorticity jump. Figure 2 illustrates the potential vorticity field for $-2 \leq y \leq 2$ on $x=0$ (for the same parameter values as that used in Fig. 1). The modon boundary corresponds to the two cusps located at $y=-1$ and $+1$, respectively.

The solution for ψ as constructed above is therefore a proper solution of the potential vorticity equation. It is a classical solution everywhere except on the set of measure zero given by $r=a$. Note that as $r \rightarrow a$, our solution satisfies (3), since

$$\begin{aligned} \lim_{r \rightarrow a} J(\psi, \Delta\psi + y) &= a^{-1} [\lim_{r \rightarrow a} \psi_r (\Delta\psi + y)_\theta - \lim_{r \rightarrow a} \psi_\theta (\Delta\psi + y)_r] \\ &= 0, \end{aligned} \quad (17)$$

since $\lim_{r \rightarrow a^+} |\psi_r (\Delta\psi + y)_\theta| < \infty$ and $\lim_{r \rightarrow a} \psi_\theta = \lim_{r \rightarrow a} (\Delta\psi + y)_\theta = 0$.

In Figs. 3(a) and 3(b) we present “close-up” and “large-scale” contour plots of the total potential vorticity field $\Delta\psi - \psi + y$ for $-2 \leq x, y \leq 2$ and $-10 \leq x, y \leq 10$, respectively. The modon boundary corresponds to the closed cir-

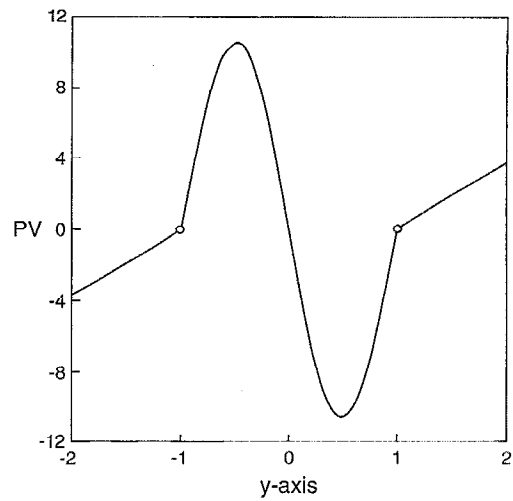


FIG. 2. A y cross section of the potential vorticity field $PV \equiv \Delta\psi - \psi + y$ on $x=0$ for $-2 \leq y \leq 2$. The parameter values are as in Fig. 1. The modon boundary, which is a vortex sheet in the solution presented here, corresponds to the two open circles located at $y=-1$ and $+1$, respectively.

cular contour (with value 0). The zero value on the contour associated with the modon boundary represents the (interior and exterior) limits of the potential vorticity at the modon radius $r=a$. The potential vorticity at $r=a$ does not formally exist, since the modon boundary corresponds to a vortex sheet.

The Rossby wave tail, which is not very pronounced in Fig. 3(a), is clearly seen in Fig. 3(b). Note how the wave tail is confined to the downstream region, in accordance with the radiation condition (8). Also, as pointed out earlier, note that the potential vorticity field is an odd function with respect to y . This is the pattern observed in the numerical experiments reported by Haines and Marshall.²

In Figs. 4(a) and 4(b) we present “close-up” and large-scale contour plots of the total streamfunction field, $\psi(x, y) \equiv -Uy + \varphi(x, y)$ for $-2 \leq x, y \leq 2$ and $-10 \leq x, y \leq 10$, respectively. Here, again, the modon boundary corresponds to the closed circular (with value 0). The values for a and U for both Figs. 3 and 4 are, as in Figs. 1 and 2, given by $a=U=1$ and $k \approx 3.96$. The ambient mean flow moves from left to right and the modon is stationary.

IV. DISCUSSION

We have presented a solution for a stationary equivalent modon on a β plane that is embedded in an eastward flow, which satisfies the correct upstream radiation condition. The solution has continuous geostrophic pressure and normal mass flux across the modon boundary. It is not possible to select the modon wave number, as in classical modon theory, to ensure that the azimuthal velocity field is continuous at the modon boundary. We have chosen to assign the modon wave number by demanding that the component of the azimuthal velocity field associated with the $\sin(\theta)$ mode is continuous at $r=a$. In the absence of an exterior wave field this matching condition reduces to the classical barotropic modon dispersion relationship. The so-

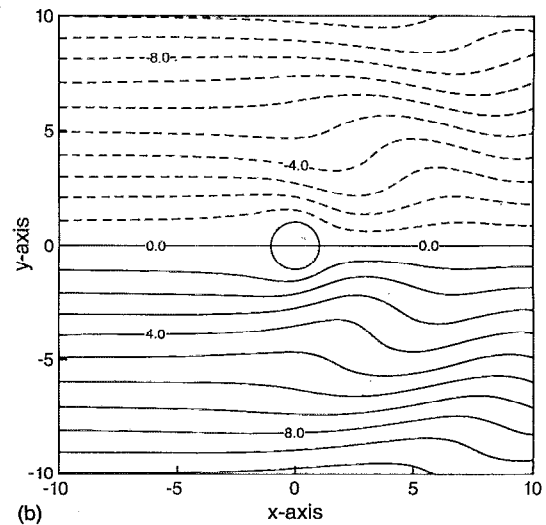
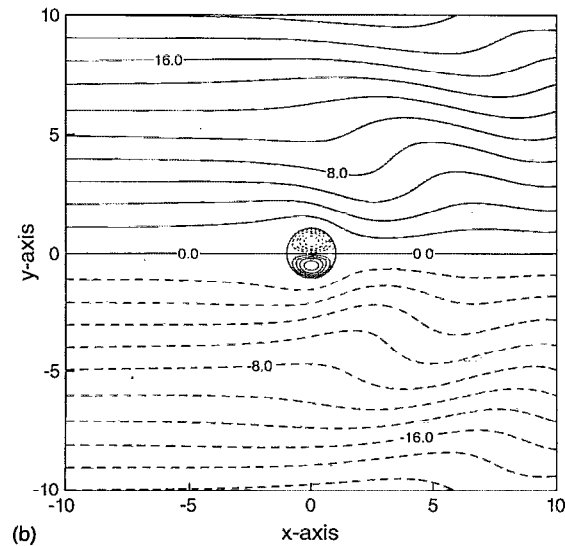
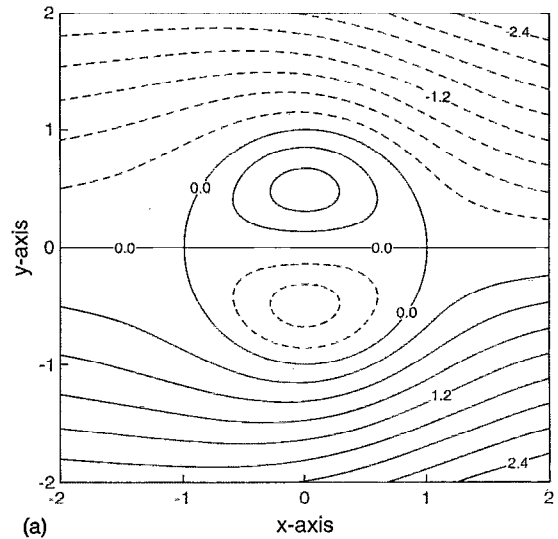
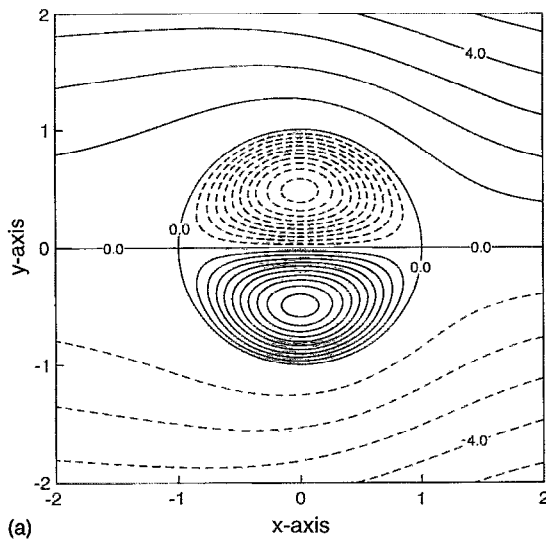


FIG. 3. (a) A "close-up" contour plot of the potential vorticity field for $-2 < x, y < 2$ with the same parameter values as in Fig. 1. The solid and dashed contours correspond to non-negative and negative isolines of potential vorticity, respectively. The contour increment is about ± 1.0 . The modon boundary corresponds to the circular zero-value contour. The reader is reminded that the zero value on the circular contour associated with the modon boundary corresponds to the limiting contour value and does not represent the actual value of the potential vorticity on the modon boundary, which does not exist since the modon boundary is a vortex sheet. The maximum and minimum values of potential vorticity within the modon interior are about $+10.5$ and -10.5 , respectively. (b) A "large-scale" contour plot of the potential vorticity field for $-10 < x, y < 10$ with the same parameter values as in Fig. 1. The solid and dashed contours are as in (a). The contour increment is about ± 2.0 .

FIG. 4. (a) A "close-up" contour plot of the total streamfunction field for $-2 < x, y < 2$ with the same parameter values as in Fig. 1. The solid and dashed lines correspond to non-negative and negative streamline values, respectively. The contour increment is about ± 0.3 . The modon boundary corresponds to the circular zero-value contour. The relative maximum and minimum values of the streamfunction within the modon interior are about $+0.72$ and -0.72 , respectively. (b) A "large-scale" contour plot of the total streamfunction field for $-10 < x, y < 10$ with the same parameter values as in Fig. 1. The solid and dashed contours are as in Fig. 4(a). The contour increment is about ± 1.0 .

lution obtained is very similar to those observed in numerical simulations of radiating modons on a β plane.

The fact that we cannot choose the modon wave number is such a way as to ensure that the azimuthal velocity field is continuous at the modon boundary implies that the modon boundary in our solution is a vortex sheet. The implications of this property on the stability of the present solution needs to be further examined in order to better determine the physical importance of this solution as a

model for a stationary dipole with a Rossby wave tail on a β plane. Preliminary numerical time integrations of (1) using the stationary solution presented here as an initial condition suggests that the qualitative features of the solution remain coherent for several eddy circulation time scales. However, it needs to be emphasized again that these observations are preliminary and further study is required.

Another issue we have not examined here is the effect of the wave drag associated with the wave tail. Because there will be a nonzero downstream momentum flux associated with the wave tail, there must be an energy source for the waves. Consequently, there will be a decay in the

strength of the dipole over time. The stationary ansatz introduced here will make sense only if the decay time scale is long in comparison with the eddy circulation time scale. A detailed calculation of this decay is required to verify that our ansatz has physical merit.

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- ¹N. Butchart, K. Haines, and J. C. Marshall, "A theoretical and diagnostic study of solitary waves and atmospheric blocking," *J. Atmos. Soc.* **46**, 2063 (1989).
- ²K. Haines and J. Marshall, "Eddy-forced coherent structures as a prototype of atmospheric," *Q. J. R. Meteorol. Soc.* **113**, 681 (1987).
- ³J. C. McWilliams, "An application of equivalent modons to atmospheric blocking," *Dyn. Atmos. Oceans* **5**, 43 (1980).
- ⁴K. Haines, "Baroclinic modons a prototypes for atmospheric blocking," *J. Atmos. Sci.* **46**, 3202 (1989).

- ⁵N. Ek and G. E. Swaters, "Geostrophic scatter diagrams and the application of quasi-geostrophic free-mode theory to a northeast Pacific blocking episode," *J. Atmos. Sci.* (in press).
- ⁶V. P. Larichev and G. M. Reznik, "Two-dimensional Rossby soliton: An exact solution," *Rep. USSR Acad. Sci.* **231**, 1077 (1976).
- ⁷G. R. Flierl, V. D. Larichev, J. C. McWilliams, and G. M. Reznik, "The dynamics of baroclinic and barotropic solitary eddies," *Dyn. Atmos. Oceans* **5**, 1 (1980).
- ⁸J. Pedlosky, *Geophysical Fluid Dynamics* (Springer-Verlag, New York, 1987).
- ⁹M. Makino, T. Kamimura, and T. Taniuti, "Dynamics of two-dimensional solitary vortices in a low- β plasma with convective motion," *J. Phys. Soc. Jpn.* **50**, 980 (1981).
- ¹⁰G. F. Carnevale, G. K. Vallis, R. Purini, and M. Briscolini, "Propagation of barotropic modons over topography," *Geophys. Astrophys. Fluid Dyn.* **41**, 45 (1988).
- ¹¹J. W. Miles, "Lee waves in a stratified flow. Part 2. Semi-circular obstacle," *J. Fluid Mech.* **33**, 803 (1968).
- ¹²G. R. Flierl, "Rossby wave radiation from a strongly nonlinear warm eddy," *J. Phys. Oceanogr.* **14**, 47 (1984).
- ¹³G. E. Swaters and G. R. Flierl, "Dynamics of ventilated coherent cold eddies on a sloping bottom," *J. Fluid Mech.* **223**, 565 (1991).
- ¹⁴M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965).
- ¹⁵W. T. M. Verkley, "Stationary barotropic modons in westerly background flows," *J. Atmos. Sci.* **44**, 2383 (1987).

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