Ekman destabilization of inertially stable baroclinic abyssal flow on a sloping bottom

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Baroclinic abyssal currents on a sloping bottom, which are nonlinearly stable in the sense of Liapunov in the absence of dissipation, are shown to be destabilized by the presence of a bottom Ekman boundary layer for any positive value of the Ekman number. When the abyssal flow is baroclinically unstable, the dissipation acts to reduce the inviscid growth rates except near the marginal stability boundary where it acts to increase the inviscid growth rates. It is shown that when the abyssal flow is baroclinically stable, the Ekman destabilization corresponds to the kinematic wave phase velocity lying outside the range of the inertial topographic Rossby phase velocities. The transition mechanism described here might provide a dynamical bridge between the nonrotational roll-wave instability that can occur in supercritical abyssal overflows and frictionally induced destabilization in subinertial geostrophically balanced baroclinic abyssal currents. In addition, the theory presented here suggests a dissipation-induced destabilization mechanism for coastal downwelling fronts whose cross-slope potential vorticity gradient does not satisfy the necessary condition for baroclinic instability. © *2009 American Institute of Physics*.
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I. INTRODUCTION

The flow of dense water over deep sills is a source point for the formation of grounded abyssal currents, which form an important component of the thermohaline ocean circulation and hence climate dynamics. The instability of these flows plays a fundamental role in planetary scale thermal transport. The parametrization of the mixing properties of these flows remains a challenge in oceanic general circulation and climate modeling.

Experiments[1–4] and numerical simulations based on the primitive equations[5–9] have shown that these flows progress through a sequence of dynamical regimes described as laminar, wavy, and eddy forming. The vortices seen in the laboratory and in the numerical simulations possess features very similar to those associated with, for example, the mesoscale eddies observed in the Denmark Strait overflow[10] and are produced by subinertial baroclinic instability and vortex stretching.[11–15] The onset of the wavy regime[4] is consistent with a baroclinic roll-wave instability[16–18] in which superinertial internal gravity wave processes cannot be neglected. The above experiments and numerical simulations clearly suggest the importance of an Ekman boundary layer in the flow evolution. However, the interaction between Ekman boundary layer processes and the destabilization of baroclinic grounded abyssal flow has yet to be described.

The principal purpose of this contribution is to briefly describe a previously unexplored mode of transition for baroclinic abyssal currents on a sloping bottom. We describe the instability that explicitly arises from the presence of an Ekman bottom boundary layer for grounded baroclinic abyssal flows *that are otherwise inertially linearly and nonlinearly stable*. (By inertial stability we mean that in the absence of dissipation, the flow is stable.) Of course, in the parameter regime where the flow is baroclinically unstable, the presence of the Ekman boundary layer will reduce the inviscid growth rates. Given the physical relevance of the Ekman layer processes in laboratory experiments and observed oceanographic overflows, the instability described here could be phenomenologically significant.

The Ekman destabilization described here is *not* a viscous modification of an existing *inertial* baroclinic instability. Nor is this transition a Tollmien–Schlichting instability, which explicitly requires a normal shear in the tangential velocity within a Blasius boundary layer.[19] It will be shown that in the case where the flow is baroclinically nonlinearly stable (in the inviscid limit), the stability conditions associated with the presence of an Ekman layer are that the flow is dissipatively stable *if and only if* the kinematic wave velocity lies within the range of the inertial inviscid phase velocities. This is, qualitatively, precisely the same stability condition as that associated with roll-wave formation in the down slope flow of a nonrotating fluid with quadratic bottom friction,[20] although the inertial modes in that situation correspond to internal gravity waves and not topographic Rossby waves as they do here.

The role of Ekman dissipation as a potential source of destabilization in quasigeostrophic baroclinic flows was originally identified by Romea.[21] Further work by, among others, Klein and Pedlosky[22] greatly clarified the rather subtle dynamical implications of various dissipation parametrizations in baroclinic instability. Krechetnikov and Marsden[23,24] recently described the mathematical foundations for this counterintuitive dissipative destabilization within the framework of general finite and infinite-dimensional dynamical systems. The potential dynamical im-

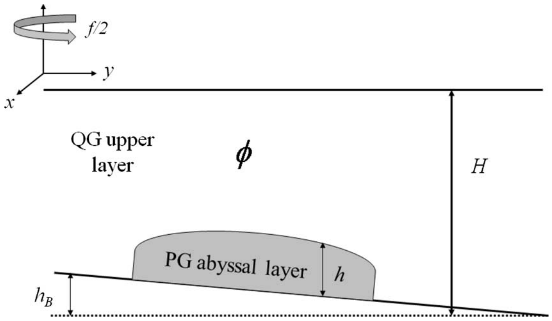

FIG. 1. Model geometry used in this paper.

plications of dissipative destabilization could be quite important for the long time evolution of grounded baroclinic abyssal flow in the ocean over basin length scales and may play a role in their low frequency variability in climate-related processes in global numerical simulations. Our goal here is to show how this instability mechanism arises in the subinertial transition problem for geostrophic baroclinic abyssal currents on a sloping bottom. In addition, the theory presented here suggest a dissipation-induced destabilization mechanism for coastal upwelling fronts whose cross-slope potential vorticity gradient does not satisfy the necessary condition for baroclinic instability.

II. MODEL EQUATIONS

The governing equations is a two-layer hybrid planetary-geostrophic quasigeostrophic model[11] that describes the subinertial baroclinic dynamics of grounded abyssal flow over variable topography on an f-plane including a bottom Ekman boundary layer, which in nondimensional form can be written as

$$[\partial_t + J(\phi,\cdot)][\Delta\phi + h + h_B] = 0, \tag{1}$$

$$h_t + J(\phi + h_B, h) = r\Delta(\phi + h + h_B) + Q, \tag{2}$$

with the auxiliary diagnostic relations

$$\mathbf{u}_{\text{upper}} = \mathbf{e}_3 \times \nabla\phi, \quad \mathbf{u}_{\text{abyssal}} = \mathbf{e}_3 \times \nabla(\phi + h + h_B),$$
$$p = \phi + h + h_B, \tag{3}$$

with $J(A,B)=A_xB_y-A_yB_x$, alphabetical subscripts (unless otherwise noted) indicate partial differentiation, $\nabla=(\partial_x,\partial_y)$, $\Delta=\nabla^2$, $h_B(x,y)$ is the height of the topography, $h(x,y,t)\geq 0$ is the height of the abyssal layer relative to h_B, and $Q(x,y)$ is a forcing term that is potentially necessary to maintain a steady abyssal flow in the presence of friction. The geostrophic pressure in the upper layer is given by $\phi(x,y,t)$, and in the abyssal layer by $p(x,y,t)$, respectively. The model geometry is shown in Fig. 1.

In addition, $r \equiv r_*/h_*$, where r_* and h_* are the bottom friction coefficient and scale thickness for the abyssal layer, respectively. The Ekman boundary layer theory[25] suggests that $r_*=h_*\sqrt{E_V}$, where E_V is the vertical Ekman number for the abyssal layer. Accordingly, r_* is the scale vertical thickness of the Ekman bottom boundary layer in the abyssal layer. Typical dimensional values appropriate for grounded overflows are of the order $H\simeq 1-4$ km, $h_*\simeq 100-500$ m, and $r_*\simeq 5-10$ m with the velocity, length, and time scale associated with the above equations on the order of about $U\simeq 3$ cm/s, $L_D\simeq 15$ km (L_D is the internal deformation radius based on the total water column depth H), and $T\simeq 6$ days, respectively.[13,14,18] Multilayered, continuously stratified, and β-plane versions of Eqs. (1) and (2) have been also derived and analyzed.[14,26,27]

The model equations (1) and (2) correspond to an intermediate length scale dynamical limit in the sense of Charney and Flierl,[28] in which the dominant nonlinearity arises due to isopycnal steepening and not momentum advection. This dynamical property occurs since the dynamic deflections of the abyssal layer height are on the same order of magnitude as the scale thickness for the abyssal layer itself (see Fig. 1). In particular, it is important to note that Eqs. (1) and (2) allow for dynamically evolving groundings or incroppings in the height field (locations where h intersects the bottom) associated with the abyssal layer, as shown in Fig. 1. The model assumes that the dynamics of the overlying water column is quasigeostrophic (because the overlying water column is deeper than the abyssal layer thickness) and driven by a balance between relative vorticity, vortex-tube stretching, and the background topographic vorticity gradient. However, the abyssal current, while in geostrophic balance, is *not* quasigeostrophic because of the order-one variations in the abyssal current height. That is, the abyssal layer is not geostrophically degenerate. This balance represents a middle dynamical regime between full two-layer shallow-water theory and the kinematic assumptions of quasigeostrophic theory[28] and is associated with a length scale that is longer than the internal deformation radius based on the mean abyssal current thickness but less than a basin length scale where planetary spherical effects cannot be neglected.[28]

The model equations (1) and (2) can be derived as a systematic asymptotic reduction in the full two-layer shallow-water equations for a rotating fluid on a sloping bottom.[11,14,25] Implicit in the derivation of Eq. (2) is the assumption that the Ekman layer corresponds to a thin boundary layer between the geostrophically balanced abyssal layer and the ocean bottom. The above parameter estimates suggest a scale thickness for the Ekman boundary layer that is about 5%–10% of the scale thickness of the abyssal layer. Within this dynamical paradigm, the leading order dynamical effect of the *Ekman layer* on the interior dynamics of the geostrophic abyssal layer is through vortex stretching/compression induced by vertical motion at the top of the Ekman layer that is created as a consequence of mass conservation and cross-geostrophic streamline flow within the Ekman layer required to satisfy the no-slip boundary condition along the gently sloping bottom [see Eq. (4.5.37) in Ref. 25]. The asymptotic matching condition on the vertical velocity is that the leading order vertical velocity in the abyssal layer as it exits the geostrophic abyssal layer and enters the Ekman boundary layer must asymptotically match with the leading order vertical velocity within the Ekman layer as it exits the Ekman boundary layer and enters the geostrophic abyssal layer [see Eq. (4.5.38) in Ref. 25]. The leading order vertical velocity within an Ekman layer on a gently sloping bottom as it exits the Ekman boundary layer and enters a geostrophic flow is the *sum* of the Ekman-induced vertical velocity, which is proportional to the relative vorticity of the interior geostrophic flow, which in the situation described here, is given by

$$\zeta_{\text{abyssal}} \equiv \mathbf{e}_3 \cdot \nabla \times \mathbf{u}_{\text{abyssal}} = \Delta(\phi + h + h_B),$$

and the inviscid vertical velocity associated with the kinematic boundary condition on the sloping bottom [see Eq. (4.9.36) in Ref. 25]. Thus, when the $3D$ continuity equation for the abyssal layer is vertically integrated over the height of the abyssal layer, as must be done in the derivation of Eq. (2) (see Refs. 11, 14, and 25), the (nondimensional) vertical velocity at the base of the abyssal layer will be determined by

$$w_{\text{abyssal}}|_{z=h+h_B} = \mathbf{u}_{\text{abyssal}} \cdot \nabla h_B + r\zeta_{\text{abyssal}},$$

and Eq. (2) follows.

The *inviscid* instability mechanism modeled by Eqs. (1) and (2) is the release of the available gravitational potential energy associated with the down slope motion of the cross-slope position of the center of mass associated with a pool of dense water sitting *directly* on a sloping bottom surrounded by relatively lighter water (see Fig. 1). For an unstable abyssal current which has a transverse thickness profile shaped like a coupled front (like that shown in Fig. 1), the instabilities take the form of along-slope traveling waves that preferentially amplify on and over the downslope incropping (the location where the abyssal layer height intersects the bottom) which subsequently develop into downslope propagating plumes, which can roll up into along-slope propagating abyssal dome eddies with a cyclonic surface signature.[11,12]

The amplifying perturbations on the upslope and downslope incroppings are asymmetric in contrast to the sinuous or varicose symmetry that would be predicted by an ageostrophic or semigeostrophic barotropic instability theory.[29,30] This asymmetry is a unique signature of the baroclinic destabilization of these currents. From a modal point of view, the instability may be thought of as the coalescence of two topographic vorticity waves[31] that have been excited in the upper layer. Numerical simulations based on the full primitive equations and laboratory experiments have concluded that the subinertial instabilities observed in oceanographic overflows on a sloping bottom arise due to the baroclinic instability mechanism modeled by Eqs. (1) and (2).[2–9,32]

When $r=Q=0$, Eqs. (1) and (2) are a 2×2 infinite-dimensional noncanonical Hamiltonian dynamical system[33]

$$\mathbf{q}_t = \mathcal{J}\frac{\delta H}{\delta \mathbf{q}},$$

where the Hamiltonian H is given by

$$H(\mathbf{q}) \equiv \frac{1}{2}\int\int_{\Omega} |\nabla\phi|^2 + (h+h_B)^2 - h_B^2 dxdy > 0, \tag{4}$$

and

$$\mathcal{J} = \begin{bmatrix} -J(q_1+h_B,\cdot) & 0 \\ 0 & J(q_2,\cdot) \end{bmatrix},$$

with $\mathbf{q}=(q_1,q_2)^{\mathsf{T}}=(\Delta\phi+h,h)^{\mathsf{T}}$ where, for the problem considered here, Ω is the periodic channel given by

$$\Omega = \{(x,y)| -x_0 < x < x_0, \quad 0 < y < L\}, \tag{5}$$

and where

$$\frac{\delta H}{\delta \mathbf{q}} \equiv \left(\frac{\delta H}{\delta q_1}, \frac{\delta H}{\delta q_2}\right)^{\mathsf{T}},$$

with $\delta H/\delta q_{1,2}$ the variational derivatives of H with respect to $q_{1,2}$, respectively. The Poisson bracket associated with the Hamiltonian structure is given by

$$[F,G] = -\int\int_{\Omega}\left\{\frac{\delta F}{\delta q_1}J\left(q_1+h_B,\frac{\delta G}{\delta q_1}\right) + \frac{\delta F}{\delta q_2}J\left(q_2,\frac{\delta G}{\delta q_2}\right)\right\}dxdy,$$

where F and G are arbitrary functionals (some differentiability and appropriate boundary conditions are required[31]) with respect to $\mathbf{q}$.

Even though any potential incropping or grounding is a free boundary, it can be shown[33] that there exists a degeneracy between the governing equations (1) and (2) and the free-boundary conditions (due to the underlying geostrophic balance that implies the incropping must correspond to a streamline) that ensure that the appropriate kinematic and dynamic boundary conditions are automatically satisfied. That is, if the grounding (i.e., where $h=0$) is given by, say, $\psi(x,y,t)=0$, then the dynamic and kinematic conditions, given by, respectively,

$$h = 0 \quad \text{on} \quad \psi(x,y,t) = 0,$$

$$\psi_t + J(\phi + h + h_B, \psi) = 0 \quad \text{on} \quad \psi(x,y,t) = 0,$$

imply that

$$h_t + J(\phi + h_B, h) = 0 \quad \text{on} \quad \psi(x,y,t) = 0,$$

i.e., Eq. (2) is necessarily satisfied. Conversely, Eq. (2) together with $h=0$ on $\psi(x,y,t)=0$ implies that the kinematic condition holds on the incropping. This point is very important since it means that many of the technicalities associated with the Hamiltonian formulation of true free-boundary problems are avoided here.[34]

Irrespective of whether $r=0$ or not, the domain boundary conditions, in general, are that h and ϕ (and all their derivatives) are periodic along $x=\pm x_0$ and, for the upper layer,

$$\phi_x = \frac{d}{dt}\int_{-x_0}^{x_0} \phi_y = 0 \quad \text{on} \quad y = 0 \ \text{and} \ L. \tag{6}$$

However, in order to focus on baroclinic processes it will be assumed that $\phi|_{y=0,L}=0$ so that there will be no net barotropic mass flux.

III. GENERAL STABILITY CONSIDERATIONS

The steady baroclinic abyssal flow examined here is a topographically steered parallel current where Q balances the dissipation and the topography varies in the cross-channel direction, i.e.,

$$\phi = \phi_0 = 0, \quad h = h_0(y), \quad h_B = h_B(y),$$

and

$$Q = r(h_0 + h_B)_{yy}, \tag{7}$$

from which it trivially follows that

$$J(h_B, h_0) = 0. \tag{8}$$

These are the flows that in the inviscid ($r=0$) limit correspond to unforced ($Q=0$) steady *abyssal* solutions of Eqs. (1) and (2) and are the configuration shown in Fig. 1. The assumption $\phi_0=0$ has been made in order to filter out any possible barotropic instability in the upper layer and thus to explicitly focus attention on the Ekman destabilization of grounded abyssal flow within the context of the baroclinic instability theory. The analytical general inviscid (linear and nonlinear) instability theory and related numerical simulations with $Q=0$, $h_0=h_0(x,y)$, and $h_B=h_B(x,y)$ has been described previously.[11–14,26–28]

The fact that $J(h_B,h_0)=0$ implies that

$$h_B = \Phi(h_0), \tag{9}$$

for some function Φ [this is true irrespective of whether or not Eq. (7) holds]. For the parallel shear flow (7)

$$\Phi(\xi) = h_B[h_0^{-1}(\xi)], \tag{10}$$

where h_0^{-1} is the inverse function associated with h_0. As an example, for the constant velocity abyssal flow over linearly sloping topography given by

$$h_0 = h_{\max} - \gamma y \quad \text{and} \quad h_B = -y, \tag{11}$$

where $h_{\max}$ and γ are constants satisfying $h_0 \geq 0$ for $y \in (0,L)$ (the abyssal thickness can never be negative; see Fig. 1), it follows that

$$\Phi(\xi) = (\xi - h_{\max})/\gamma. \tag{12}$$

It is necessary to establish conditions for the linear and nonlinear stability of solutions of form (9) when $Q=r=0$ in order to unambiguously show that the presence of an Ekman boundary layer can destabilize these flows even when the flow is baroclinically stable. When $Q=r=0$, flow (9) (where it is understood that $\phi_0 \equiv 0$) satisfies the first-order necessary conditions for an extremal to the conserved functional (for the full nonlinear dynamics when $Q=r=0$) (Ref. 33)

$$\begin{aligned}\mathcal{H}(\mathbf{q}) &\equiv H - \int\int_\Omega \left\{\frac{1}{2}h^2 + \int_0^h \Phi(\xi)d\xi\right\}dxdy \\ &= \int\int_\Omega \left\{\frac{1}{2}|\nabla\phi|^2 + hh_B - \int_0^h \Phi(\xi)d\xi\right\}dxdy,\end{aligned} \tag{13}$$

i.e.,

$$\delta\mathcal{H}|_{\phi=\phi_0,h=h_0} = 0.$$

The functional $\mathcal{H}$ is the Hamiltonian constrained by an appropriately chosen *Casimir*.[33,35]

The second variation of $\mathcal{H}$ evaluated at $\phi=\phi_0=0$ and $h=h_0$ is given by

$$\delta^2\mathcal{H}|_{\phi=\phi_0,h=h_0} = \int\int_\Omega |\nabla\delta\phi|^2 - \frac{\partial_y h_B}{\partial_y h_0}(\delta h)^2 dxdy, \tag{14}$$

where Eq. (9) has been used. Since $\delta^2\mathcal{H}|_{\phi=\phi_0,h=h_0}$ is an invariant of the associated linear stability problem (when $Q=r=0$), if there exist constants α and β for which

$$-\infty < \alpha \leq \frac{\partial_y h_B}{\partial_y h_0} \leq \beta < 0 \ \text{ for all } \ (x,y) \in \Omega, \tag{15}$$

then

$$\delta^2\mathcal{H}|_{\phi=\phi_0,h=h_0} > 0,$$

for all perturbations $(\delta\phi, \delta h)$ and the steady solution (ϕ_0, h_0) is linearly stable in the sense of Liapunov with respect to the disturbance norm

$$\|(\delta\phi,\delta h)\|^2 \equiv \int\int_\Omega |\nabla\delta\phi|^2 + (\delta h)^2 dxdy. \tag{16}$$

Condition (15) is the analog of Fjørtoft's stability theorem for this problem.[11,20,33,35]

Conditions for the *nonlinear* stability in the sense of Liapunov are obtained from examining the conserved functional (for the full nonlinear dynamics when $Q=r=0$) (Refs. 33 and 35)

$$\begin{aligned}\mathcal{L} &\equiv \mathcal{H}(\mathbf{q}_0 + \delta\mathbf{q}) - \mathcal{H}(\mathbf{q}_0) - \delta\mathcal{H}|_{\phi=\phi_0,h=h_0} \\ &= \int_\Omega \left\{\frac{1}{2}|\nabla\delta\phi|^2 - \int_{h_0}^{h_0+\delta h} \Phi(\xi)d\xi + \Phi(h_0)\delta h\right\}dxdy,\end{aligned} \tag{17}$$

where $q_{10} \equiv \Delta\phi_0 + h_0$ and $q_{20} \equiv h_0$. If there exist constants α and β for which

$$-\infty < \alpha \leq \Phi'(\xi) \leq \beta < 0 \ \text{ for all } \ \xi \in \mathbb{R}, \tag{18}$$

where $\Phi'(\xi)=d\Phi/d\xi$, then the steady solution (ϕ_0,h_0) is nonlinearly stable in the sense of Liapunov with respect to the disturbance norm (16).[33] The stability condition (18) is the analog of Arnol'd's *first* nonlinear stability theorem for this problem.[33,35,36]

For the constant abyssal velocity profile on the linearly sloping topography given in Eq. (11), $\Phi'(\xi)=1/\gamma$ so that the flow is, in fact, inertially *nonlinearly stable* if $\gamma<0$. We are now in a position to unambiguously show that for the abyssal flow given by Eq. (11) with $\gamma<0$, if $r>0$ (no matter how small) the presence of an Ekman boundary layer leads to destabilization.

IV. EKMAN DESTABILIZATION

For the baroclinic constant velocity abyssal flow profile on the linearly sloping topography given by Eq. (11), it follows that $Q\equiv 0$ so that Eqs. (1) and (2) reduce to simply

$$\Delta\phi_t-(\phi+h)_x+J(\phi,\Delta\phi)=-r\Delta(\phi+h), \quad (19)$$

$$(\partial_t+\partial_x)h+J(\phi,h)=r\Delta(\phi+h). \quad (20)$$

Even if $Q=0$ there is still an Ekman flux at the base of the layer and fluid must be supplied at the up slope side and removed at the down slope side.

An indication that Ekman dissipation has the potential to destabilize is seen by computing the time evolution of the area-integrated energy H, i.e.,

$$\frac{dH}{dt}=r\int\int_{\Omega}(h_0+h_B)\Delta(\delta\phi+\delta h)-|\nabla(\delta\phi+\delta h)|^2dxdy, \quad (21)$$

where $\phi=\phi_0+\delta\phi$ and $h=h_0+\delta h$ have been substituted into the right hand side (RHS) of Eq. (21). Clearly, the RHS of Eq. (21) is not necessarily negative definite. Heuristically, for sufficiently small perturbations the first term in the integral on the RHS in Eq. (21) could be positive and larger in absolute value than the second term in the integral leading to initial growth in the perturbations (note that $r\geq 0$). However, Eq. (21) suggests that the growth in the amplitude of an initial unstable perturbation will be arrested by frictional effects once the nonlinear term on the RHS of Eq. (21) begins to dominate.

The linear stability equations are obtained by substituting

$$\phi=\tilde{\phi}(x,y,t) \quad \text{and} \quad h=h_0(y)+\tilde{h}(x,y,t)$$

into Eqs. (19) and (20) and neglecting all the nonlinear terms with respect to $\tilde{\phi}$ and $\tilde{h}$, yielding,

$$\Delta\phi_t-(\phi+h)_x=-r\Delta(\phi+h), \quad (22)$$

$$(\partial_t+\partial_x)h+h_0'\phi_x=r\Delta(\phi+h), \quad (23)$$

where $h_0'=dh_0/dy$ and the tildes have been dropped. It is remarked that Eqs. (22) and (23) are in fact the linear stability equations for *all flows of the form* $h_0=h_0(y)$, where $Q\equiv -rh_0''$ and $h_B=-y$.

The area-averaged perturbation energy equation for the upper layer is obtained by multiplying Eq. (22) by ϕ and integrating with respect to $(x,y)\in\Omega$, yielding

$$\begin{aligned}\frac{d}{dt}\int\int_{\Omega}\nabla\phi\cdot\nabla\phi dxdy &=-2\int\int_{\Omega}h_x\phi dxdy-2r\int\int_{\Omega}\nabla(\phi+h)\cdot\nabla\phi dxdy\\ &=2\int\int_{\Omega}hv_{\text{upper}}dxdy-2r\int\int_{\Omega}\nabla(\phi+h)\cdot\nabla\phi dxdy\\ &=2\int\int_{\Omega}hv_{\text{upper}}dxdy-2r\int\int_{\Omega}\left[\left|\nabla\left(\phi+\frac{1}{2}h\right)\right|^2-\frac{1}{4}|\nabla h|^2\right]dxdy,\end{aligned}$$

where a number of integration by parts have occurred, the boundary conditions along $y=0$ and L have been used, the periodicity conditions along $x=\pm x_0$ have been used, and the geostrophic relation $v_{\text{upper}}=\phi_x$ has been used. From the above balance we see that if $r=0$, it is necessary that, *on average*, h must be positively correlated with v_{upper} for baroclinic instability to occur. Since a positive h is a positive abyssal layer height anomaly and since the abyssal layer is colder or denser than the fluid in the overlying water column, it follows that baroclinic instability only occurs if, on average, cold abyssal anomalies are advected up the topographic slope or background vorticity gradient, i.e., classical baroclinic instability.[25] However, if $r\neq 0$, h need not be positively correlated with v_{upper} for instability to occur (note that second term in the above balance is not definite in sign). Thus, again, we conclude that when $r\neq 0$, the conditions for baroclinic instability need not occur and it is *not* possible to conclude that the dissipation-induced destabilization is a variant of conventional baroclinic instability simply modified by bottom friction.

The analog of Eq. (21) for the linearized dynamics is

$$\frac{d}{dt}\int\int_{\Omega}|\nabla\phi|^2+\frac{1}{h_0'}(\delta h)^2dxdy=-2r\int\int_{\Omega}\nabla\left(\phi+\frac{1}{h_0'}h\right)\cdot\nabla(\phi+h)dxdy. \quad (24)$$

The integral in the left hand side (LHS) of Eq. (24) is $\delta^2\mathcal{H}|_{\phi=\phi_0,h=h_0}$ given by Eq. (14) with $h_B=-y$. Again, in general, the RHS in Eq. (24) is not necessarily negative definite so that the Ekman layer could destabilize the flow even if $h_0'>0$ [the linear stability condition (15) since $h_B'=dh_B/dy=-1$].

In the case where h_0' is a constant the RHS in Eq. (24) can be diagonalized to yield

$$\frac{d}{dt}\int\int_{\Omega}|\nabla\phi|^2+\frac{1}{h_0'}(\delta h)^2dxdy=-2r\int\int_{\Omega}\left[\left|\nabla\left[\phi+\left(\frac{1+h_0'}{2h_0'}\right)h\right]\right|^2-\left(\frac{1-h_0'}{2h_0'}\right)^2|\nabla h|^2\right]dxdy.$$

The RHS of this expression is negative definite, in general, only if $h_0'=1$, which corresponds to the situation where the interface between the abyssal current and the overlying water column is exactly a geopotential (i.e., $h_B+h_0=\text{constant}$) so that the mean abyssal current velocity is identically zero (see Fig. 1) since $\phi_0=0$.

We now turn to the explicit determination of the conditions for Ekman destabilization. When $h_0'=-\gamma$, the normal-mode solution to Eqs. (22) and (23), subject to the perturbation boundary conditions $\phi=h=0$ on $y=0$ and L, can be written in the form

$$(\phi,h)=(A,B)\sin(n\pi y/L)\exp(ikx+\sigma t)+\text{c.c.}, \quad (25)$$

where the complex-valued growth rate σ satisfies the dispersion relation

$$\sigma = \sigma_{\pm} \equiv \frac{-(1+\mu^2)(r\mu^2+ik) \pm \sqrt{(1+\mu^2)^2(r\mu^2+ik)^2 - 4ik\mu^2(1+\gamma)(r\mu^2+ik)}}{2\mu^2}, \tag{26}$$

with

$$B = [\mu^2 + ik(1+\gamma)/\sigma]A,$$

where $k = m\pi/x_0$ (with m a nonnegative integer) is the along-channel wavenumber, $\mu^2 \equiv k^2 + (n\pi/L)^2$ (with $n \in \mathbb{Z}^+$), c.c. means the complex conjugate of the preceding term and where the branch cut is taken along the negative real axis for the square root.

If $r=0$, Eq. (26) is identical, of course, to the inviscid dispersion relation.[31] Moreover, when $r=0$, it follows from Eq. (26) that the marginal stability curve is given by

$$\gamma = \gamma_c(\mu) \equiv (\mu^2-1)^2/(4\mu^2) \geq 0. \tag{27}$$

The mode with wavenumber modulus μ becomes unstable when $\gamma > \gamma_c(\mu)$ and is stable when $\gamma \leq \gamma_c(\mu)$. The point of marginal stability is the minimum on the curve $\gamma = \gamma_c(\mu)$ located at $(\mu,\gamma)=(1,0)$ (see Fig. 2 in Ref. 31).

When $r>0$, stability occurs if and only if $\text{Re}(\sigma_+) \leq 0$, i.e.,

$$\text{Re}[\sqrt{(1+\mu^2)^2(r\mu^2+ik)^2 - 4ik\mu^2(1+\gamma)(r\mu^2+ik)}] \leq r\mu^2(1+\mu^2). \tag{28}$$

We now show that Eq. (28) holds if and only if

$$\gamma = -1 \Leftrightarrow h_0' = 1, \tag{29}$$

i.e., the current velocity in the abyssal is zero so that there is no mean flow whatsoever. Thus, even if the flow is baroclinically nonlinearly stable in the sense of Liapunov (i.e., $\gamma<0$ with the no flow situation exempted), the presence of an Ekman boundary layer will necessarily destabilize the flow. The Ekman layer is not modifying a pre-existing baroclinic instability, its presence *causes* the instability.

Introducing the Euler decomposition

$$\begin{aligned}\alpha \exp(i\beta) &\equiv (1+\mu^2)^2(r\mu^2+ik)^2 - 4ik\mu^2(1+\gamma)(r\mu^2+ik) \\ &= (r^2\mu^4 - k^2)(1+\mu^2)^2 + 4(\mu k)^2(1+\gamma) \\ &\quad + 2irk\mu^2[(1+\mu^2)^2 - 2\mu^2(1+\gamma)],\end{aligned}$$

which serves to define the real parameters $\alpha>0$ and $-\pi < \beta \leq \pi$, allows Eq. (28) to be written in the form

$$\sqrt{\alpha}\cos(\beta/2) \leq r\mu^2(1+\mu^2) \Rightarrow \alpha[1+\cos(\beta)] \leq 2r^2\mu^4(1+\mu^2)^2, \tag{30}$$

on account of the location of the branch cut. If α and β are substituted into Eq. (30), it follows that

$$\{[(r^2\mu^4-k^2)(1+\mu^2)^2 + 4(\mu k)^2(1+\gamma)]^2 + 4(rk\mu^2)^2[(1+\mu^2)^2 - 2\mu^2(1+\gamma)]^2\}^{1/2} \leq (r^2\mu^4+k^2)(1+\mu^2)^2 - 4(\mu k)^2(1+\gamma). \tag{31}$$

Note that for $r=0$, Eq. (31) reduces to

$$|(1+\mu^2)^2 - 4\mu^2(1+\gamma)| \leq (1+\mu^2)^2 - 4\mu^2(1+\gamma), \tag{32}$$

for $k \neq 0$ (and is trivially satisfied if $k=0$). Inequality (32) can only be satisfied if the RHS is non-negative which implies the stability condition $\gamma \leq \gamma_c(\mu)$, i.e., the inviscid stability condition associated with Eq. (27). It is straight forward to verify that if $k=0$, then Eq. (31) trivially satisfied for all γ and r. The case $\mu=0$ is not physically relevant since it corresponds to only the trivial solution for the normal modes.

The stability condition associated with $r>0$ is obtained as follows. It is noted that since the LHS of Eq. (31) is non-negative, the RHS must be non-negative, which implies

$$\gamma \leq -1 + (r^2\mu^4 + k^2)(1+\mu^2)^2/(4\mu^2k^2). \tag{33}$$

This constraint is *necessary* for stability but not sufficient. If Eq. (33) does not hold then Eq. (31) can never hold and the flow *must be unstable*.

If Eq. (33) holds, then both sides of Eq. (33) can be squared and it follows, after a little algebra, that for $\mu k r \neq 0$, stability occurs only if

$$(1+\gamma)^2 \leq 0 \Rightarrow \gamma = -1. \tag{34}$$

Clearly, if γ satisfies Eq. (34) it will satisfy Eq. (33). Thus, Eq. (34) is the *necessary and sufficient* stability condition on γ assuming $\mu k \neq 0$ and $r>0$.

The stability condition (34) implies that when $r>0$ the only stable abyssal flow is one for which there is an abyssal current with zero velocity (since the case where $\gamma=-1$ corresponds to the situation where the interface between the abyssal current and the overlying water column is a geopotential; see Fig. 1) and all other flows are necessarily unstable. If $\gamma>0$ the flow is linearly baroclinically unstable in the inviscid $r=0$ limit, but if $\gamma \in (-\infty,-1) \cup (-1,0]$ the (nonlinearly inertially stable) flow is explicitly destabilized by the Ekman boundary layer for any $r>0$.

Physically, coastal downwelling fronts correspond to flows for which $\gamma<0 \Leftrightarrow h_0'>0$ (see Fig. 2 in Ref. 11). Thus, we have shown that while downwelling fronts may be baroclinically stable (the cross-slope potential vorticity gradient need not satisfy the necessary conditions for instability), the front can be destabilized due to the presence of an Ekman boundary layer.

The stability condition (34) can be physically interpreted as the requirement that the kinematic wave phase velocity (this concept is described more completely below) lies within the range of the inviscid Rossby wave phase velocities. This is, qualitatively, precisely the same stability condition as that associated with roll-wave formation in the down slope flow of a nonrotating fluid with quadratic bottom friction (see Secs. 3.1 and 3.2 in Ref. 37 or Sec. 2.3.4 in Ref. 20), although the inertial modes in the roll-wave problem correspond to internal gravity waves and not topographic Rossby waves as they do here.

To show this, it is convenient to rewrite Eqs. (22) and (23) in the form

$$(\mu^2\partial_t+\partial_x+r\mu^2)\phi=-(\partial_x+r\mu^2)h,$$

$$(\partial_t+\partial_x+r\mu^2)h=(\gamma\partial_x-r\mu^2)\phi,$$

where we have assumed $h_0'=-\gamma$ and a normal-mode assumption for the perturbations in order, for convenience in the argument, to write $\Delta=-\mu^2<0$ where μ is the modulus of the total wavenumber vector. These two equations can be combined, after a little algebra, to yield

$$(\partial_t+c_+\partial_x)(\partial_t+c_-\partial_x)\phi=-r(1+\mu^2)(\partial_t+c_0\partial_x)\phi, \qquad (35)$$

where c_+ and c_- are the (inviscid) barotropic and baroclinic topographic Rossby wave velocities,[31] given by

$$c_+\equiv\frac{1+\mu^2+\sqrt{(1+\mu^2)^2-4\mu^2(1+\gamma)}}{2\mu^2}, \qquad (36)$$

$$c_-\equiv\frac{1+\mu^2-\sqrt{(1+\mu^2)^2-4\mu^2(1+\gamma)}}{2\mu^2}, \qquad (37)$$

and c_0 is the kinematic wave phase velocity,[20,37,38] given by

$$c_0\equiv\frac{1+\gamma}{1+\mu^2}. \qquad (38)$$

The phase velocities $c_\pm$ are the solutions $\sigma_\pm$ in Eq. (26) written in the form $\sigma_\pm=-ic_\pm k$, where $r=0$. In the argument presented here we want to explicitly focus on the case where the inviscid barotropic and baroclinic topographic Rossby wave velocities, $c_\pm$, are real-valued, i.e., where the modes are inertially baroclinically stable so that the dissipation is the only possible source of the destabilization. This only occurs for all μ when, based on Eq. (27), $\gamma\leq 0$. In the case where $\gamma>0$ the flow is baroclinically unstable and the dissipation acts to modify (i.e., reduce the growth rates—this is shown below) the existing inertial instability. Henceforth for the argument presented here, it will be assumed that $\gamma\leq 0$.

Lighthill and Whitham[38] introduced the concept of the kinematic wave as a traveling wave solution to a first-order (1+1 dimensional) conservation law in which a functional relationship exists between the density and flux in the conservation law. Kinematic waves are not usually classical or dynamic waves that are typically described by Newton's second law of motion in which the acceleration associated with particle displacement occurs against a background restoring force (e.g., gravity waves). Equations describing dynamic waves are typically higher order in space and time. The kinematic wave concept has been useful in understanding the dynamics and stability of, for example, roll-wave formation, continuous models for traffic flow, and spatially variable chemical reactions.[37,38]

In the context of the present problem, the kinematic wave part of the partial differential operator in Eq. (35) is the first-order wave operator on the RHS. Within this paradigm, the second-order partial differential operator on the LHS of Eq. (35) is the dynamic wave part that describes the along slope propagating vorticity waves that arise as a consequence of the cross-slope displacement of patches of anomalous vorticity against the background topographic vorticity gradient. In the absence of dissipation the RHS of Eq. (35) is zero and what remains are only the dynamic waves.

The low frequency/wave number approximation in Eq. (35) would be to neglect the LHS and retain only the kinematic wave part of the partial differential operator. Thus, roughly speaking, in the context of Eq. (35) and from the perspective of the solution to the pure initial-value problem, the low frequency/wavenumber part of the solution would be governed by the RHS and everything else by the LHS. However, as written, Eq. (35) corresponds to a hyperbolic partial differential equation in which the characteristics are *exclusively determined* by the second-order dynamic wave terms on the LHS. From this perspective, for the problem to be well posed, the characteristics associated with the first-order kinematic wave operator (governing the low frequency/wavenumber part of the solution) must be consistent with the characteristics associated with the second-order dynamic wave part of the operator (see the discussion of wave hierarchies in Chap. 10 in Ref. 37), which *alone* determines the signal propagation properties for the overall dynamical system. That is, the phase velocity associated with the dissipative kinematic wave operator must lie in the interval spanned by the phase velocities associated with the dynamic topographic Rossby wave operator.

Thus, the stability condition associated with Eq. (35) in the baroclinically stable $\gamma\leq 0$ case is given by

$$c_-\leq c_0\leq c_+. \qquad (39)$$

It is now shown that Eq. (39) is equivalent to Eq. (34). Equation (39) can be rewritten in the form

$$-\sqrt{(1+\mu^2)^2-4\mu^2(1+\gamma)}\leq\frac{2\mu^2(1+\gamma)-(1+\mu^2)^2}{1+\mu^2}\leq\sqrt{(1+\mu^2)^2-4\mu^2(1+\gamma)}. \qquad (40)$$

Since the flow is inertially stable, i.e., the $c_\pm$ are real since $\gamma\leq 0$, it follows that

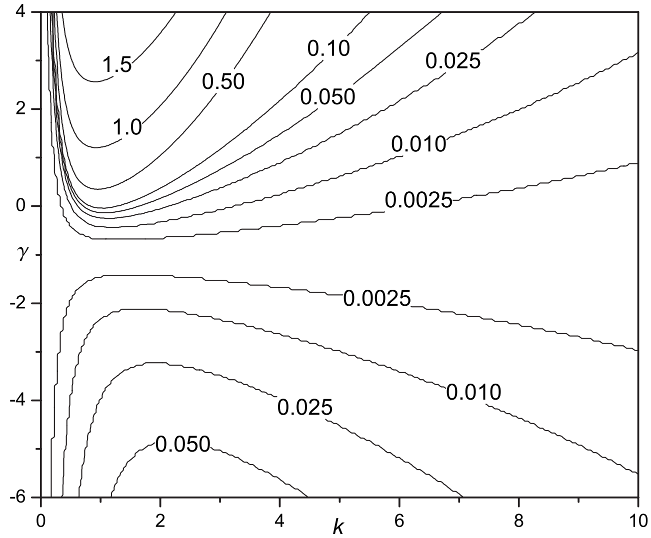

FIG. 2. Contour plot of $\mathrm{Re}[\sigma_+]$ in the (k,γ) plane with $r=0.1$ for selected contours.

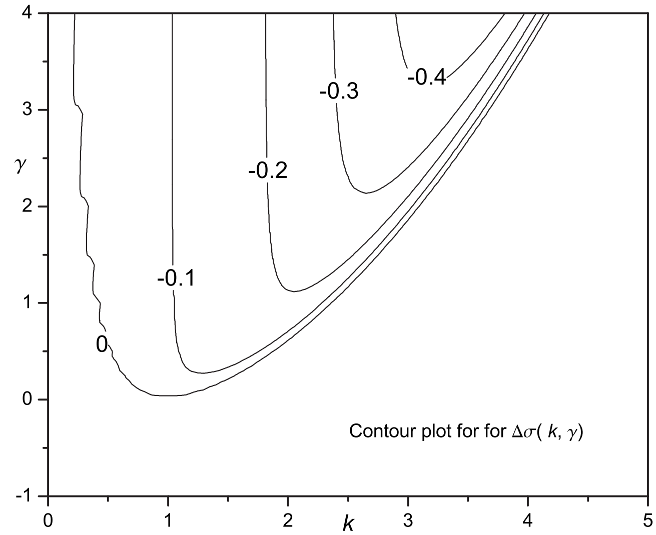

FIG. 3. Contour plot of $\Delta\sigma$ in the (k,γ) plane with $r=0.1$ for selected nonpositive contours. The negative contours are in the parameter region where the unstable modes with dissipation have a smaller growth rate than in the unstable modes in the inviscid theory. In the region outside the region with the negative contours (i.e., where $\Delta\sigma>0$), the modes are unstable due to the Ekman destabilization (except along $\gamma=-1$) and their growth rates are larger than the corresponding inviscid (mostly stable) modes (there is a very narrow region immediately adjacent to the 0-contour where there are some unstable inviscid modes; see Fig. 4).

$$(1+\mu^2)^2-4\mu^2(1+\gamma)\geq 0,$$

but this implies that

$$2\mu^2(1+\gamma)-(1+\mu^2)^2\leq -(1+\mu^2)^2/2<0,$$

so that the RHS inequality in Eq. (40) is trivially satisfied. Thus all that remains is the LHS inequality in Eq. (40), which can be written in the form

$$0\leq\frac{(1+\mu^2)^2-2\mu^2(1+\gamma)}{1+\mu^2}\leq\sqrt{(1+\mu^2)^2-4\mu^2(1+\gamma)},$$

which, if both sides are squared, results in

$$(1+\gamma)^2\leq 0,$$

which is exactly Eq. (34). Thus, we have shown that the stability condition (34) associated with the Ekman destabilization of inertially baroclinically abyssal stable flow is the requirement that the kinematic wave phase velocity lies in the range of the inertial topographic Rossby wave phase velocities.

Figure 2 is contour plot, for selected contours, of $\mathrm{Re}[\sigma_+]$ (the more unstable of the two $\sigma_\pm$ roots) in the (k,γ)-plane for $r=0.1$ (so that the Ekman boundary layer occupies about 10% of the total height of the abyssal current) and $l=0.1$ (so that the channel is several times wider than, as it turns out, the typical along-channel wavelength of the most unstable mode). In accordance with Eqs. (26) and (34), the only location where $\mathrm{Re}[\sigma_+]=0$ in Fig. 2 is along the γ-axis (where $k=0$) and along the line $\gamma=-1$. Everywhere else in the (k,γ)-plane $\mathrm{Re}[\sigma_+]>0$. In the inviscid ($r=0$) limit, all modes in the $\gamma<0$ region are stable (the flow is inertially nonlinearly stable when $\gamma<0$). The positive growth rate seen in Fig. 2 in the $\gamma<0$ region is the explicit consequence of Ekman destabilization.

Figure 2 shows that the wavenumber of the most unstable mode slightly decreases as γ increases. A typical value for the wavenumber of the most unstable mode is about $k\simeq 2$ corresponding to a dimensional wavelength of about 47 km (a little over three internal deformation radii). Whereas in the inviscid stability theory,[29] there is a nonzero low and finite high wavenumber cutoff for the baroclinic instability in the region $\gamma>0$, there are no such cutoffs when $r>0$. However, it follows from Eq. (26) that $\lim_{k\to\infty}\mathrm{Re}[\sigma_+]\to 0$ and, clearly, that $\mathrm{Re}[\sigma_+]|_{k=0}=0$ for all γ. Figure 2 also shows that the growth rates in the (baroclinically unstable) region $\gamma>0$ are about 40 times larger than those in the (baroclinically stable) region $\gamma<0$. Nevertheless, over the basin-scale (not to mention global) distances abyssal currents flow in the ocean spanning a Lagrangian time scale of months, the e-folding time scales associated with the Ekman destabilization is sufficiently rapid to be oceanographically relevant in the inertially baroclinically stable parameter regime.

There is a subtle transition in $\mathrm{Re}[\sigma_+]$ when $r>0$ as compared to when $r=0$ as γ increases past the point of inviscid marginal stability $\gamma_c[\mu(k)]$. In order to describe this property, the quantity $\Delta\sigma$ is introduced where

$$\Delta\sigma\equiv\mathrm{Re}[\sigma_+]-\mathrm{Re}[\sigma_+]|_{r=0}.$$

Where $\Delta\sigma<0$ (>0 or is equal to 0) the presence of an Ekman layer leads to a decrease (increase or no change) in the $r>0$ growth rate from its inviscid value.

Figure 3 is a contour plot of $\Delta\sigma$ in the (k,γ)-plane for $r=0.1$ for selected nonpositive contours (positive contours are not shown in Fig. 3 only because of crowding of the positive contours near the 0-contour). Outside the region with negative contours, i.e., for values of γ less than those associated with the 0-contour, $\Delta\sigma>0$ (except along $\gamma=-1$ and $k=0$ where $\Delta\sigma=0$ along these lines).

The 0-contour in Fig. 3 is the curve, denoted as $\gamma=\gamma_r(k)$, determined by

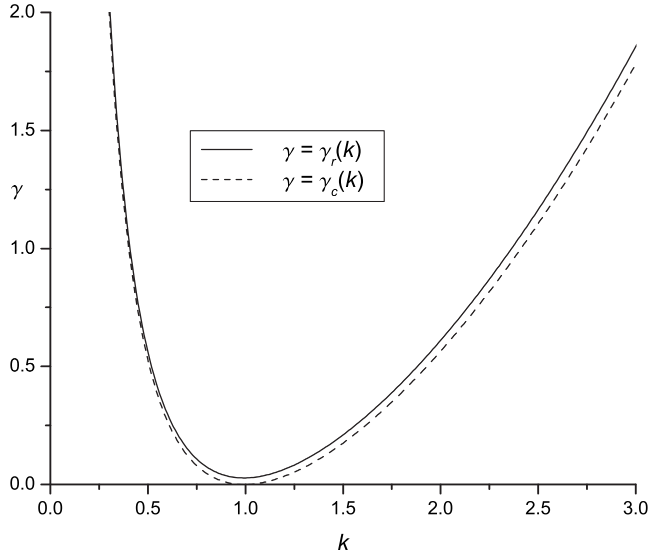

FIG. 4. Graphs of $\gamma=\gamma_c(k)$ and $\gamma=\gamma_r(k)$ vs k. The graph $\gamma=\gamma_c(k)$ is the marginally stability boundary separating the inertially ($r=0$) baroclinically stable ($\gamma\leq\gamma_c$) and unstable ($\gamma>\gamma_c$) regions, as given by Eq. (27). The graph $\gamma=\gamma_r(k)$ (which corresponds to the 0-contour in Fig. 3) is the boundary in the dissipative ($r>0$) theory separating the regions $\gamma<\gamma_r$ and $\gamma>\gamma_r$, where the unstable modes have a larger or smaller growth rate than in the inviscid theory, respectively.

$$\mathrm{Re}[\sigma_+] = \mathrm{Re}[\sigma_+]|_{r=0} \Rightarrow \gamma = \gamma_r(k), \tag{41}$$

is the boundary separating the regions where the growth rates associated with $r>0$ are greater or less than the corresponding inviscid values. It can be shown, manipulating Eq. (41), that γ_r satisfies a quartic for which it is possible to write an explicit solution. Figure 4 is a graph of the two curves $\gamma=\gamma_r$ and γ_c versus k for $r=0.1$. The property that $\gamma_r>\gamma_c$ for $r>0$ implies that in the narrow baroclinically unstable parameter region $\gamma_c<\gamma<\gamma_r$, the presence of the Ekman layer actually leads to an *increase* in the growth rate of the unstable mode compared to the inviscid value.

V. CONCLUSIONS

The role of an Ekman boundary layer in the transition to instability of grounded baroclinic abyssal currents on a sloping bottom has been examined. It was shown that such currents, when they are nonlinearly stable in the sense of Liapunov in the absence of dissipation, will be destabilized by the presence of a bottom Ekman boundary layer for any positive value of the Ekman number. When the abyssal flow is baroclinically unstable, the dissipation acts to reduce the inviscid growth rates except near the marginal stability boundary where the Ekman boundary layer increases the inviscid growth rates. The dissipative stability condition associated with baroclinically stable abyssal flow has been shown to correspond to the requirement that the kinematic wave phase velocity lies in the range of the inertial topographic Rossby wave phase velocities (the kinematic wave cannot travel faster than the fastest topographic Rossby wave or slower than the slowest topographic Rossby wave). The transition mechanism described here might provide a dynamical bridge between the nonrotational roll-wave instability that can occur in supercritical abyssal overflows and frictionally induced destabilization in subinertial geostrophically balanced baroclinic abyssal currents. In addition, the theory presented here suggests a mechanism for the dissipation-induced destabilization of coastal downwelling fronts whose cross-slope gradient in the mean-flow potential vorticity does not satisfy the necessary condition for baroclinic instability.

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[1] J. A. Whitehead, M. E. Stern, G. R. Flierl, and B. Klinger, "Experimental observations of a baroclinic eddy on a sloping bottom," J. Geophys. Res. **95**, 9585, DOI: 10.1029/JC095iC06p09585 (1990).
[2] G. F. Lane-Serff and P. G. Baines, "Eddy formation by dense flows on slopes in a rotating fluid," J. Fluid Mech. **363**, 229 (1998).
[3] D. Etling, F. Gelhardt, U. Schrader, F. Brennecke, G. Kühn, G. Chabert d'Hieres, and H. Didelle, "Experiments with density currents on a sloping bottom in a rotating fluid," Dyn. Atmos. Oceans **31**, 139 (2000).
[4] C. Cenedese, J. A. Whitehead, T. A. Ascarelli, and M. Ohiwa, "A dense current flowing down a sloping bottom in a rotating fluid," J. Phys. Oceanogr. **34**, 188 (2004).
[5] L. Jiang and R. W. Garwood, "Three-dimensional simulations of overflows on continental slopes," J. Phys. Oceanogr. **26**, 1214 (1996).
[6] J. H. Jungclaus, J. Hauser, and R. H. Käse, "Cyclogenesis in the Denmark Strait overflow," J. Phys. Oceanogr. **31**, 3214 (2001).
[7] W. Krauss and R. H. Käse, "Eddy formation in the Denmark Strait overflow," J. Geophys. Res. **103**, 15525, DOI: 10.1029/98JC00785 (1998).
[8] A. V. Wilchinsky and D. L. Feltham, "Generation of a Buoyancy-driven coastal current by an Antarctic polyna," J. Phys. Oceanogr. **38**, 1011 (2008).
[9] S. Kida, J. Yang, and J. F. Price, "Marginal sea overflows and the upper ocean interaction," J. Phys. Oceanogr. **39**, 387 (2009).
[10] J. G. Bruce, "Eddies southwest of Denmark Strait," Deep-Sea Res., Part I **42**, 13 (1995).
[11] G. E. Swaters, "On the baroclinic instability of cold-core coupled density fronts on a sloping continental shelf," J. Fluid Mech. **224**, 361 (1991).
[12] G. E. Swaters, "Numerical simulations of the baroclinic dynamics of density-driven coupled fronts and eddies on a sloping bottom," J. Geophys. Res. **103**, 2945, DOI: 10.1029/97JC02441 (1998).
[13] M. K. Reszka, G. E. Swaters, and B. R. Sutherland, "Instability of abyssal currents in a continuously stratified ocean with bottom topography," J. Phys. Oceanogr. **32**, 3528 (2002).
[14] G. E. Swaters, "The meridional flow of source-driven abyssal currents in a stratified basin with topography. Part I. Model development and dynamical characteristics," J. Phys. Oceanogr. **36**, 335 (2006).
[15] G. E. Swaters, "The meridional flow of source-driven abyssal currents in a stratified basin with topography. Part II. Numerical simulation," J. Phys. Oceanogr. **36**, 356 (2006).
[16] G. E. Swaters, "Baroclinic characteristics of frictionally destabilized abyssal overflows," J. Fluid Mech. **489**, 349 (2003).
[17] G. E. Swaters, "On the frictional destabilization of abyssal overflows dynamically coupled to internal gravity waves," Geophys. Astrophys. Fluid Dyn. **100**, 1 (2006).
[18] G. E. Swaters, "Mixed bottom-friction-Kelvin-Helmholtz destabilization of source-driven abyssal overflows in the ocean," J. Fluid Mech. **626**, 33 (2009).
[19] P. G. Baines, S. J. Majumdar, and H. Mitsudera, "The mechanics of the Tollmien-Schlichting wave," J. Fluid Mech. **312**, 107 (1996).
[20] P. G. Baines, *Topographic Effects in Stratified Flows* (Cambridge University Press, Cambridge, 1995).

[21] R. D. Romea, "The effects of friction and β on finite-amplitude baroclinic waves," J. Atmos. Sci. **34**, 1689 (1977).
[22] P. Klein and J. Pedlosky, "The role of dissipation mechanisms in the nonlinear dynamics of unstable baroclinic waves," J. Atmos. Sci. **49**, 29 (1992).
[23] R. Krechetnikov and J. E. Marsden, "Dissipation-induced instabilities in finite dimensions," Rev. Mod. Phys. **79**, 519 (2007).
[24] R. Krechetnikov and J. E. Marsden, "Dissipation-induced instability phenomena in infinite dimensional systems," Arch. Ration. Mech. Anal. (in press).
[25] J. Pedlosky, *Geophysical Fluid Dynamics* (Springer-Verlag, New York, 1987).
[26] F. J. Poulin and G. E. Swaters, "Sub-inertial dynamics of density-driven flows in a continuously stratified fluid on a sloping bottom. I. Model derivation and stability characteristics," Proc. R. Soc. London, Ser. A **455**, 2281 (1999).
[27] F. J. Poulin and G. E. Swaters, "Sub-inertial dynamics of density-driven flows in a continuously stratified fluid on a sloping bottom. II. Isolated eddies and radiating cold domes," Proc. R. Soc. London, Ser. A **455**, 2305 (1999).
[28] J. G. Charney and G. R. Flierl, "Oceanic analogues of large-scale atmospheric motion," in *Evolution of Physical Oceanography: Scientific Surveys in Honor of Henry Stommel*, edited by B. A. Warren and C. Wunsch (MIT Press, Cambridge, 1981), p. 504.
[29] R. W. Griffiths, P. D. Killworth, and M. E. Stern, "Ageostrophic instability of ocean currents," J. Fluid Mech. **117**, 343 (1982).
[30] N. Paldor and P. D. Killworth, "Instabilities of a two-layer coupled front," Deep-Sea Res., Part A **34**, 1525 (1987).
[31] C. J. Mooney and G. E. Swaters, "Finite amplitude baroclinic instability of a mesoscale gravity current in a channel," Geophys. Astrophys. Fluid Dyn. **82**, 173 (1996).
[32] P. F. Choboter and G. E. Swaters, "On the baroclinic instability of axisymmetric rotating gravity currents with bottom slope," J. Fluid Mech. **408**, 149 (2000).
[33] G. E. Swaters, "Nonlinear stability of intermediate baroclinic flow on a sloping bottom," Proc. R. Soc. London, Ser. A **442**, 249 (1993).
[34] D. Lewis, J. E. Marsden, R. Montgomery, and T. Ratiu, "The Hamiltonian structure for dynamic free boundary problems," Physica D **18**, 391 (1986).
[35] D. D. Holm, J. E. Marsden, T. Ratiu, and A. Weinstein, "Nonlinear stability of fluid and plasma equilibria," Phys. Rep. **123**, 1 (1985).
[36] V. I. Arnol'd, "On an a priori estimate in the theory of hydrodynamic stability," Am. Math. Soc. Transl. **79**, 267 (1969).
[37] G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1973).
[38] M. J. Lighthill and G. B. Whitham, "On kinematic waves: I. Flood movement in long rivers; II. Theory of traffic flow on long crowded roads," Proc. R. Soc. London, Ser. A **229**, 281 (1955).