

On the meridional flow of source-driven abyssal currents along a continental slope

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Abstract

A hybrid 3-layer quasi-geostrophic/planetary geostrophic (QG/PG) model is introduced to examine the baroclinic evolution and meridional flow of abyssal currents in a wind-driven, stratified and differentially-rotating basin with variable bottom topography. The model resolves mesoscale processes and allows for the formation of a wind driven surface intensified ocean circulation, with a poleward western boundary current, and a source driven equatorward flowing deep western boundary undercurrent. Baroclinic and barotropic instability within the surface intensified western boundary current, and baroclinic instability between the abyssal current and the overlying wind driven circulation, is resolved. The model allows for finite amplitude variations in the height field of the abyssal current so that groundings in the thickness or isopycnal field associated with the undercurrent are resolved. Downwelling and upwelling between the abyssal water mass and the overlying ocean conserves vertical mass exchange. A southern boundary upwelling scheme is introduced, within the context of a closed basin with no-slip boundary conditions, to balance the northern source of abyssal water thereby allowing the meridional transport of abyssal water to evolve toward a steady state.

1 Introduction

On the large scale, ocean currents can be characterized into two broad groups. The first are the wind driven currents that are most intense near the surface of the ocean. Their principal role is to transport warm equatorial waters toward the polar regions. The second group of currents are those that are driven by density contrasts with the surrounding waters. Among these are the deep, or abyssal, currents flowing along or near the bottom of the oceans. Their principal role is to transport cold, dense waters produced in the polar regions back toward the equator. Taken together, the warm surface currents and the cold abyssal currents are the means by which the ocean horizontally mixes and convectively overturns, distributing incoming solar heating meridionally and deep into the oceans. The importance of this process in understanding climate dynamics is obvious.

Stommel and Arons [1] showed that the Sverdrup vorticity balance predicts the equatorward flow of a source driven abyssal water mass. Thus, in the immediate vicinity of the region of deep water production in high latitudes, there is an intrinsic tendency for preferential equatorward abyssal flow. Away from the source region, much of the abyssal circulation is strongly characterized by the isopycnal field being grounded against sloping topography (e.g., the deep western boundary undercurrent in the North Atlantic, Richardson [2]) and the flow being in geostrophic balance. Indeed, as shown by Nof [3], a fully grounded (i.e., compactly supported) abyssal water mass, in the fully nonlinear but reduced gravity dynamical limit, moves nondispersively and steadily in the along slope direction (in a right (left) handed sense in the northern (southern) hemisphere), regardless of the height or vorticity field within the abyssal water mass.

These two results provide a compelling scenario for the initiation and maintenance of grounded abyssal flow. That is, in high latitude source regions where deep water is produced (often over sloping topography), the Sverdrup vorticity balance initiates equatorward flow. Once produced, this abyssal flow can become grounded and geostrophically adjusted, maintaining a Nof balance which permits sustained basin scale meridional quasi-steady and coherent propagation regardless of the spatial structure of the water mass. Of course, this picture leaves out many important dynamical processes such as diabatic effects, baroclinicity, instability and mixing. In addition, such a scenario cannot explain cross-equatorial abyssal currents where the underlying assumptions of geostrophically balanced grounded flow must necessarily breakdown.

The principal purpose of the present contribution is to briefly describe a

model for the sub-inertial evolution and meridional flow of source driven grounded abyssal currents over sloping topography and their baroclinic interaction with the overlying wind driven circulation. Our discussion here, due to space considerations, is purely theoretical and focusses on model development. During the oral presentation we shall describe results from some numerical simulations.

2 Model equations

Succinctly summarized, the model is an amalgamation, with the inclusion of variable topography and mass conserving up and downwelling, of the two layer QG model used by Holland [4] to investigate the baroclinic evolution of the wind driven circulation and the QG/PG abyssal current model of Swaters [5] used to investigate the baroclinic instability of grounded geostrophic flow. A continuously stratified version of the present model is described by Poulin and Swaters [6] and Reszka *et al.* [7].

Assuming a Boussinesq, rigid-lid approximation with wind stress, horizontal and bottom friction, topography, and mass conserving up or downwelling, the nondimensional model can be written in the form

$$[\partial_t + J(\phi_1, \cdot)] [\Delta\phi_1 - F_1(\phi_1 - \phi_2) + \beta y] = \Upsilon \nabla \times \boldsymbol{\tau} + \frac{F_1}{F_1 + F_2} Q + \frac{1}{R_e} \Delta^2 \phi_1, \quad (1)$$

$$[\partial_t + J(\phi_2, \cdot)] [\Delta\phi_2 - F_2(\phi_2 - \phi_1) + h + h_B + \beta y] = -r_2 \Delta\phi_2 + \frac{F_1}{F_1 + F_2} Q + \frac{1}{R_e} \Delta^2 \phi_2, \quad (2)$$

$$h_t + \frac{1}{1 + s\beta y} J(\phi_2 + h_B, h) - \frac{s\beta h}{(1 + s\beta y)^2} (\phi_2 + h_B + h)_x = Q + r_3 \Delta(\phi_2 + h_B + h), \quad (3)$$

with the auxiliary diagnostic relations

$$\left. \begin{aligned} \mathbf{u}_{1,2} &= \mathbf{e}_3 \times \nabla \phi_{1,2}, \quad \mathbf{u}_3 = \frac{\mathbf{e}_3 \times \nabla (\phi_2 + h_B + h)}{1 + s\beta y}, \\ p &= \phi_2 + h_B + h, \quad \eta = \phi_2 - \phi_1, \end{aligned} \right\} \quad (4)$$

with $J(A, B) = A_x B_y - A_y B_x$, and where the 1, 2 or 3 subscript on a *physical variable* refers to the upper, middle and abyssal layer, respectively, alphabetical subscripts (unless otherwise noted) indicate partial differentiation, $\mathbf{u}_{1,2,3} = (u_{1,2,3}, v_{1,2,3})$, $\nabla = (\partial_x, \partial_y)$, $\Delta = \nabla \cdot \nabla$, h_B is the height of

the bottom topography, h is the height of the abyssal layer relative to h_B , η is the deflection (measured positively upward) of the interface between the upper two QG layers from its equilibrium position, $\boldsymbol{\tau}$ is the wind stress and Q is the down or upwelling term, respectively. The dynamic pressures (i.e., the total pressure minus the hydrostatic pressure) in the upper two layers is given by $\phi_{1,2}$, and in the abyssal layer by p , respectively. Equations (4c,d) express the continuity of total pressure across the deforming interfaces between the middle and abyssal layers and the upper and middle layers, respectively.

The dynamical parameters in the model are defined by

$$\left. \begin{aligned} s &= \frac{s^* L}{H_2}, \quad \beta = \frac{\beta^* L^2}{U_*}, \quad R_e = \frac{U_* L}{A_H}, \quad \Upsilon = \frac{\Upsilon^* L}{\rho_* H_1 U_*^2}, \\ F_1 &= \frac{g' H_2}{\tilde{g} H_1}, \quad F_2 = \frac{g'}{\tilde{g}}, \quad r_{2,3} = \frac{r_{2,3}^*}{s H_2}, \end{aligned} \right\} \quad (5)$$

where $H_{1,2}$ are the constant reference layer thicknesses in the upper two layers, ρ_* is the reference Boussinesq density and $g' = (\rho_3 - \rho_2)g/\rho_*$ and $\tilde{g} = (\rho_2 - \rho_1)g/\rho_*$ where $\rho_{1,2,3}$ correspond to the constant density in each individual layer with $0 < \rho_1 < \rho_2 < \rho_3$, $L = \sqrt{g' H_2}/f_0$ (the internal deformation radius for the middle layer based on the density difference with the abyssal layer), $U_* = s^* g'/f_0$ (the Nof velocity), $s^* \approx O(\nabla^* h_B^*)$ (a representative value for the topographic slope), f_0 is the reference Coriolis parameter, β^* is the northward gradient of the meridionally varying Coriolis parameter and Υ^* is a typical value for the wind stress, respectively. In addition, A_H is the upper layers' horizontal eddy coefficient and $r_{2,3}^*$ are “bottom friction coefficients” for the middle and abyssal layers, respectively. Ekman boundary layer theory (Pedlosky [8]) suggests that $r_{2,3}^* = H_{2,3} \sqrt{E_{2,3}^V}$, where $H_{2,3}$ is a vertical length scale and $E_{2,3}^V$ is a vertical Ekman number for, respectively, the middle and abyssal layers. Accordingly, $r_{2,3}^*$ are the scale vertical thicknesses of the Ekman bottom boundary layer in the middle and abyssal layers, respectively.

The wind stress is explicitly distributed only over the upper layer and the upper layer does not have surface Ekman friction. The upper two layers have horizontal friction, but this has been neglected in the abyssal layer. The abyssal layer does, however, include bottom friction. It can be shown that, in the unforced, inviscid limit, the horizontal divergence of the barotropic mass flux is zero. That is, whatever mass is accumulated into or lost from, say, the abyssal layer, is assumed to have been instantaneously gained from or lost into the upper two layers (where the distribution over the upper two

layers is in proportion to the individual upper layer volume fractions). The net barotropic QG mass transport is forced only by wind stress and bottom friction. The unforced, inviscid dynamics of the model is, therefore, purely baroclinic. Though in geostrophic balance, the leading order abyssal layer equations are not geostrophically degenerate and allow for finite amplitude dynamic deflections in the abyssal layer height.

The model (1), (2) and (3) can be formally derived in a small Rossby number limit (i.e., $s \rightarrow 0$) of the 3-layer shallow water equations using the scaling arguments of Swaters [5,9], Poulin and Swaters [6] and Reszka *et al.* [7]. Briefly, the abyssal layer equations are scaled assuming that the dynamics is principally governed by a geostrophic balance between the down slope gravitational acceleration of the abyssal water mass and the Coriolis term (which implies that the velocity field scales with the Nof [3] velocity). The upper layers are scaled assuming that the baroclinic stretching associated with deformations associated with the interface between the abyssal and middle layers is the same order of magnitude as the velocity field in the upper layers. This will imply that the appropriate length scale is the internal deformation radius associated with the middle layer and that time will be scale advectively. All other variables are scaled assuming an underlying geostrophic balance to leading order.

Although there is considerable geographical variability, typical basin scale values of the physical parameters are about

$$\left. \begin{aligned} \tilde{g} &\simeq 9.5 \times 10^{-3} \text{ m/s}^2, \quad g' \simeq 4.8 \times 10^{-4} \text{ m/s}^2, \quad s^* \simeq 5.6 \times 10^{-3}, \\ \Upsilon^* &\simeq 10^{-1} \text{ N/m}^2, \quad H_1 \simeq 1 \text{ km}, \quad A_H \simeq 1.6 \times 10^4 \text{ m}^2/\text{s}, \\ H_2 &\simeq 4 \text{ km}, \quad f_0 \simeq 9.35 \times 10^{-5} \text{ s}^{-1}, \quad \beta^* \simeq 1.6 \times 10^{-11} \text{ (ms)}^{-1}, \end{aligned} \right\} \quad (6)$$

which implies that

$$\left. \begin{aligned} L &\simeq 15 \text{ km}, \quad U_* \simeq 3 \text{ cm/s}, \quad T \equiv \frac{L f_0}{s^* g'} \simeq 6 \text{ days}, \\ (H_3, h_B^*) &\equiv s^* L \simeq 84 \text{ m}, \quad Q_* \equiv s^{*2} g' / f_0 \simeq 1.6 \times 10^{-2} \text{ cm/s}, \end{aligned} \right\} \quad (7)$$

where T , Q_* , H_3 , h_B^* are the formal scalings for the time, upwelling, abyssal layer thickness and topographic height, respectively, used in the derivation of (1), (2) and (3). These scalings imply that the nondimensional parameters are about

$$\left. \begin{aligned} s &\simeq 0.02, \quad F_1 \simeq 0.2, \quad F_2 \simeq 0.05, \quad r_2 \simeq 0.08, \\ \beta &\simeq 0.12, \quad R_e \simeq 1/35, \quad \Upsilon \simeq 1.62, \quad r_3 \simeq 0.001. \end{aligned} \right\} \quad (8)$$

The above gives order of magnitude estimates for the forcing and dissipation parameters. In the numerical simulations, Υ , Q and R_e were chosen so that the wind driven northward barotropic QG mass transport and zonal width of the numerically computed western boundary current (i.e., the “Gulf Stream”), and the source driven equatorward abyssal transport is consistent with observations.

The inclusion of the $O(s\beta)$ terms in (3) is, formally, an *ad hoc* approximation. Although the internal deformation associated with the middle layer is the length scale, which is the appropriate scaling for the $O(1)$ baroclinic coupling of interest here, the numerical simulations will be done in an $40^\circ \times 40^\circ$, or equivalently, an approximately $3660 \text{ km} \times 3660 \text{ km}$ (or, *nondimensionally*, a 244×244) basin. Over a meridional range this large, the $O(s\beta)$ terms in (3) make a not insignificant contribution and thus they will be retained them even if, formally at least, the scaling suggests they can be ignored to leading order with respect to s . It is possible that a complete multiple scale asymptotic theory, similar to that described by Pedlosky [10] for geostrophic flow, could be developed to make this *ad hoc* approximation rational, i.e., uniformly valid with respect to y .

Additionally, the inclusion of the $O(s\beta)$ terms allows that, in a steady, reduced gravity and flat bottom approximation, (3) reduces to the (vertically integrated) Sverdrup vorticity equation with a source term, which forms the basis of the Stommel and Arons [1] theory for the equatorward flow of abyssal water. Thus, the classical Stommel-Arons theory is retained in a sub-approximation of the model.

The equations (1), (2) and (3) are, although not completely unexpected, a new model in this context, individual sub-approximations are very well known in the literature either explicitly or in closely related form. The upper layer equations (1) and (2) are simply, of course, the Phillips two-layer model, with topography, friction, diabatic forcing, and wind stress present, which has been extensively used, and is a useful pedagogical paradigm, to examine various aspects of baroclinic instability on a β -plane (see, e.g., Pedlosky [8]) as well as the spin up of stratified basin (Holland [4]).

The abyssal equation (3) is simply the planetary geostrophic equation for a single layer, with baroclinic coupling, topography, friction and a mass source term, on a β -plane (Pedlosky [10]). The planetary geostrophic approximation has been used widely to investigate aspects of abyssal and other long length scale flow (e.g., Stommel and Arons [1], Anderson and Killworth [11], Dewar [12], Kawase [13], Kawase and Straub [14], Rhines [15], Speer *et al.* [16], among many others) and the thermohaline circulation (e.g., Edwards *et al.* [17], Samelson and Vallis [18] and Samelson [19], among many others).

In particular, (3) can describe an abyssal current which possesses distinct incroppings or groundings (Speer and McCartney [20]) in its thickness or height, that is, $h = 0$ along a curve $\psi(x, y, t) = 0$. This property is an important characteristic of the cross slope structure in deep western boundary undercurrents.

It is known that, in the purely inertial limit, the planetary geostrophic approximation exhibits an ultra violet catastrophe in the linear instability problem (i.e., the most unstable occurs for an infinite wavenumber; see de Verdiere [21]). While the inclusion of Rayleigh damping can remove the ultra violet catastrophe (Samelson and Vallis [18]), Swaters [5] has shown that an inviscid baroclinic model, that couples an abyssal planetary geostrophic layer to an overlying quasigeostrophic layer (with its implicit finite deformation radius), also ensures that the most unstable mode occurs at a finite (mesoscale) wavenumber. This property continues, of course, to hold if the overlying quasigeostrophic fluid is multilayered (as occurs here) or is continuously stratified (Poulin and Swaters [6] and Reszka *et al.* [7]).

3 Theoretical properties

There are a number of sub-dynamical, or distinguished, limits in (3) which are well known such as the *Nof balance* describing grounded abyssal flow (Nof [3]), the *Stommel-Arons Sverdrup vorticity balance* describing the equatorward flow of a source driven abyssal water mass (Stommel and Arons [1]), the *planetary shock wave balance* (Anderson and Killworth [11] and Dewar [12]) describing the amplitude- β induced isopycnal steepening associated with the frontal geostrophic dynamical regime (Karsten and Swaters [22,23]) and, finally, the coupling between the abyssal layer and the overlying ocean which can lead to baroclinic instability and mixing (Swaters [5,9]). Each of these processes plays an important role in determining aspects of the spatial and temporal structure of the abyssal currents. It is useful to briefly review these approximations.

The area integrated energy for the model is given by

$$\mathcal{E} = \iint_{\Omega} \frac{F_2 \nabla \phi_1 \cdot \nabla \phi_1 + F_1 \left[\nabla \phi_2 \cdot \nabla \phi_2 + (h + h_B)^2 - h_B^2 \right]}{2(F_1 + F_2)} dx dy, \quad (9)$$

where Ω is the fixed spatial horizontal domain. In the absence of wind forcing, interlayer mass exchange and dissipation (i.e., $\Upsilon = Q = r_{2,3} = 0$, $R_e \rightarrow \infty$), it follows that $d\mathcal{E}/dt = 0$. In the unforced situation, (1), (2) and (3) is an infinite dimensional Hamiltonian dynamical system (Swaters

[24,25]), in which \mathcal{E} will be the Hamiltonian functional, and it is possible to establish a set of variational principles and for steady and steadily travelling solutions for the model and use these to examine general stability criterion.

In the inviscid and dynamically uncoupled, or reduced gravity, limit, the abyssal equation (3) reduces to

$$h_t + \frac{h'_B h_y}{1 + \tilde{\beta}y} - \frac{\tilde{\beta}h (h'_B + h_x)}{(1 + \tilde{\beta}y)^2} = Q, \quad (10)$$

where it has been assumed, for convenience, that $\tilde{\beta} = s\beta$, $h_B = h_B(x)$ and $h'_B = dh_B(x)/dx$, i.e., the topography varies only zonally. Although it is straightforward to explicitly solve (10) using the method of characteristics, the highly implicit form of the solution suggests that it is more valuable to highlight various important sub-approximations and to describe these in terms of the existing literature.

The first approximation of interest is the “Stommel-Arons balance” (Stommel and Arons [1], see, also, Pedlosky [26]) for abyssal flow with a source, in which topography and time dependence is neglected in (10), given by

$$vh = -\frac{(1 + \tilde{\beta}y)Q}{\tilde{\beta}} \iff \beta^* v^* = -\frac{fQ^*}{h^*}, \quad (11)$$

where $v = h_x / (1 + \tilde{\beta}y)$. Thus, for a source $Q > 0$, there is an equatorward (in either hemisphere) abyssal mass transport induced by the β -effect.

The next approximation of interest is the “Nof balance” (Nof [3]) for abyssal flow with a source, in which the β -effect is neglected in (10), given by

$$h_t + h'_B(x) h_y = Q(x, y). \quad (12)$$

In the homogeneous and constant slope limit, (12) should be considered only a parsimonious reflection of the full Nof [3] result which showed that, for the fully nonlinear reduced gravity shallow water equations, any isolated (i.e., compactly supported or, equivalently, fully grounded) steadily travelling abyssal water mass, moves with the *Nof velocity*, $\mathbf{c}_{NOF} = (0, h'_B)$ (it is here assumed that h'_B is constant), irrespective of the particular height and vorticity distribution within the abyssal water mass.

That is, importantly and perhaps even surprisingly, even in the fully nonlinear regime (but dynamically uncoupled from the overlying ocean), Nof [3] showed that a grounded abyssal water mass travels nondispersively in

the along slope direction. The Nof [3] balance provides a dynamical process which can sustain the coherent equatorward movement of *grounded* deep western boundary abyssal flows even when far removed from a source region where, presumably, the Stommel-Arons balance initiates equatorward movement.

In the large time limit, after the transients have propagated away, the Nof balance will be approximately steady in the near source region, and governed by

$$uh'_B = -\frac{Q}{(1 + \tilde{\beta}y)} \iff u^* = -\frac{fQ^*}{f_0(h'_B)^*}, \quad (13)$$

where $u = -h_y / (1 + \tilde{\beta}y)$. Thus, for a source $Q > 0$, there is an eastward (in either hemisphere) abyssal mass transport induced sloping when $h'_B(x) < 0$. Indeed, this is really a Stommel-Arons-like Sverdrup vorticity balance for source driven abyssal flow in which sloping topography replaces the planetary vorticity gradient.

A third approximation of interest is the “planetary shock wave balance” for abyssal flow with a source (Anderson and Killworth [11] and Dewar [12]), in which topography is neglected in (10), given by

$$h_t - \frac{\tilde{\beta}h h_x}{(1 + \tilde{\beta}y)^2} = Q(x, y). \quad (14)$$

The steady state limit of the planetary shock wave balance is simply the Stommel-Arons balance. The solution to (14) corresponds to westward propagating (in either hemisphere) abyssal anomalies in which the speed of propagation is proportional to h itself. Thus, as time proceeds, the height profile will tend to steepen on the western side and it is possible for a shock to form, that is, it is possible that $|v| = |h_x| \rightarrow \infty$ in finite time at a discrete location(s).

Of particular interest is the baroclinic evolution of an initially steady deep western boundary current. To this end it is useful to examine the structure of the solutions to (10) in the time independent limit. This balance can be thought of as a Stommel-Arons approximation with variable bottom topography. Neglecting h_t in (10) leads to the quasi-linear hyperbolic equation

$$\frac{(1 + \tilde{\beta}y)}{\tilde{\beta}} h_y - \frac{h}{h'_B} h_x = h + \frac{(1 + \tilde{\beta}y)^2 Q(x, y)}{\tilde{\beta}h'_B}. \quad (15)$$

The solution to (15) corresponds to a steady equatorward source driven abyssal flow in which the stream lines are oriented, in the northern hemisphere, in the southwest to northeast direction.

Finally, we remark that the baroclinic instability characteristics that the model predicts for grounded abyssal flow have been described in Swaters [5,9,24], Poulin and Swaters [6], Reszka *et al.* [7] and Mooney and Swaters [27].

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